



A semi-classical estimate for the q -parameter and decay time with Tsallis entropy of black holes in quantum geometry

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Received: 20 September 2020 / Accepted: 31 October 2020 / Published online: 18 November 2020
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Abstract In this letter, using the non-extensive entropy of Tsallis, we study some properties of the Schwarzschild black holes (BHs), based on the loop quantum gravity (LQG), some novel characteristics and results of the Schwarzschild BH can be obtained in Mejrhit and Ennadifi (Phys Lett B 794:45–49, 2019). Here we find that these findings are strikingly identical to ones obtained by Hawking and Page in anti-de Sitter space within the original of the Boltzmann entropy formula. By using the semi-classical estimate analysis on the energy at this minimum M_{min} , an approximate relationship between the q and γ parameters of BHs can be found, ($q \approx \frac{\sqrt{3}\gamma}{\pi \ln 2} + 1$), which is remarkable approaching to q -parameters of cosmic ray spectra and quarks coalescing to hadrons in high energy.

1 Introduction

Loop quantum gravity presents the spectrum of kinematic geometry operators such as the area operator and the volume operator [2]. It is a canonical quantification of general relativity within the framework of this formalism which leads to interesting applications of spin networks as the Hilbert space of the canonically quantized metric, the most fruitful implications that may arise from the application of LQG to BHs is to obtain a plausible explanation of BH thermodynamics based on non-extensive statistical mechanics taking into account strong gravitational couplings [3–8]. A comparative study of the results obtained with those based on Bekenstein–Hawking entropy shows the deep links between the entropy of BHs and their horizon surface, which clearly illustrates the underlying unity of the proposed cosmological models.

In a similar way to that based on the Sharma–Mittal entropy formalism [9], which can be considered as a combination of Tsallis and Rényi entropy with the horizon sur-

face of the BH is not quantified, we have achieved important results on the thermodynamics of the BH in relation to its stability using Tsallis non-extensive statistical mechanics in the framework of quantum gravity theory.

The LQG provides a guess of the microstates of a BH with the classical surface (A), for the calculation of the entropy for BHs. The principal critique of this approach was the need to take a particular value of a free dimensionless parameter which is called the Barbero–Immirzi (BI) parameter (γ) [10], to obtain the Bekenstein–Hawking entropy Horizon [11].

In LQG the event horizon of BH is described by a 2D–sphere which is a topological defects called punctures, where every edge of the global quantum geometry is represented with a spin carried by one puncture. In this work we will use Bekenstein–Hawking formula of the non-extensive Tsallis entropy [12, 13] to develop the work done before, all motivations for the use of non-extensive statistic in LQG are given in [1, 14]. This work is a complementary to the previous article [1], where we used the ideas discussed in [14], to study some thermodynamic properties of the Schwarzschild–BH [15–17] meeting the non-extensive Tsallis entropy in the framework of the LQG [3] instead of the Bekenstein entropy. In the beginning, we will touch the problem of the stability of isolated BH when it is surrounded by a bath of thermal radiation, the BHs can also be in stable equilibrium with the thermal bath at a fixed temperature, on contrary to the standard Boltzmann description. We will investigate a possible phase transitions in the system, we display a transition from the Hawking–Page and the change of the small-BH/large-BH of the first order, similarly to the model of Schwarzschild BH in the anti-de Sitter (AdS) space. These results confirm the resemblance between Tsallis-asymptotically flat in LQG and Boltzmann-AdS in the BH thermodynamics in the case $q > 1$.

In this letter, we will essentially study some of the thermodynamic characteristic of the Schwarzschild BHs that is based on the Tsallis entropy instead of the Bekenstein

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entropy. After processing some general observations on the non-extensive Tsallis entropy of Schwarzschild BHs and quantum gravity theory in the Sect. 2. In Sect. 3, we study the temperature and the thermodynamic stability of Schwarzschild BHs, where the heat capacity and Gibbs free energy has also been examined. In Sect. 4, a semi-classical approach is used to find an approximate relation between the q and γ parameters of Schwarzschild BHs. Finally, in Sect. 5, we use the Stefan–Boltzmann law to get an expression for the decay time (i.e. lifetime) of the Schwarzschild BHs. The final section deals with some concluding remarks.

2 Tsallis approach for black holes in loop quantum gravity

Loop quantum gravity presents quantum discreteness of the spectrum of kinematic geometry operators such as the area operator and the volume operator. despite the fact that these results are very important, one of the results of this idea is the statistical mechanical explanation of the thermodynamic properties of the BH, which has been discussed in the references [3,4].

In the framework of LQG , we consider the statistical mechanical properties of the quantum isolated horizon [18]. A basis is given by spin networks for the Hilbert space of canonical gravity. These are graphs whose edges are assorted by representations of the gauge group of the theory. This group is $SU(2)$ in the case of gravity, and the corresponding representations are therefore labeled by j_k , taking the positive half-integers values $\{1/2, 1, 3/2, \dots, s/2\}$, corresponding to the spin associated with the k^{th} puncture and s is the maximum number of spins ($s = 2j_{max}$). The A surface acquires the quantum area $a(j_k)$ of the horizon if it has an intersection by an edge of such a spin network carrying the label j_k [19,20], the BH quantum area is

$$a(j_k) = 8\pi\gamma\ell_p^2\sqrt{j_k(j_k+1)}, \quad (1)$$

where γ being the dimensionless Immirzi parameter and $\ell_p = \sqrt{\hbar G/c^3}$ the Planck length. A path for the determination of the BH entropy is provided by loop quantum gravity where the statistical mechanical properties of quantum isolated horizons are studied in [18]. In particular, they are specified by means of quantum states that are constructed by the association of spin variables with punctures on the horizon. More precisely, a special eigenvalue of the area operator. We will deal with a horizon made up of N punctures that are situated on $s = 2j_{max}$ different surfaces of quantum area $a(j_k)$ with spin j_k , we obtain n_k number of punctures in the spin j_k . we can conclude that the area of a BH horizon can be formed by a large number of spin network edges puncturing the surface.

The incorporate, at the level of statistical mechanical, of the effect of a bias in the probabilities of the microstates of the underlying quantum mechanical system was the idea behind the introduction of the notion of non-extensive entropy called q -entropy of Tsallis, also named q -statistics [13]. This opens the door to the generalization of usual Boltzmann–Gibbs statistics by the introduction of adequate generalized q -logarithm defined on \mathbb{R}^{+*} by the formulas

$$\ln_q(x) \equiv \int_1^x t^{-q} dt = \frac{x^{1-q} - 1}{1-q}. \quad (2)$$

Let us observe that the inverse function of the q -logarithm is the q -exponential, defined as follows:

$$\exp_q(x) \equiv [1 + (1-q)x]_+^{\frac{1}{1-q}}, \quad (3)$$

with $[z]_+ = z$ if $z \geq 0$, and zero otherwise. We verify that $\ln_q[\exp_q(z)] = \exp_q[\ln_q(z)] = z$, $\forall z \in \mathbb{R}$.

According to Eqs. (2 and 3) we can see that q -parameter gives us a new classical definitions of the exponential and logarithm functions. The q -entropy in this framework statistics, can be written as

$$S_q = \frac{(1 - \sum_{i=1}^{\Omega} p_i^q)}{(q-1)} \quad (4)$$

where p_k is the probability to occupy k^{th} micro-state and Ω is the total microstates number of the considered system and q is known as the q -parameter, at the limit $q \rightarrow 0$ we must recover the BG entropy. Then assuming that all the possible microstates occurring with equal probability, on uses $p_k = 1/\Omega$ for all k in Eq. 4 to arrive at the formula

$$S_q = \frac{\Omega^{1-q} - 1}{1-q}. \quad (5)$$

Since the estimate of this is now known from the knowledge of the dimension of the Hilbert space, the remaining is just a mathematical procedure. The q -statistics has prospered when applied to many experimental scenarios (i.e. cosmic rays, astrophysical models, quark-gluon plasma [21], gas of interacting atoms and photons, BH s ...). Utilizing the definition of micro-canonical ensemble, where all the states have the same probability, Tsallis' entropy is reduced to what is presented in paper [13].

In formula 1, the quantity of contributions of all area edge with spin j_l given to the total area. We can consider the horizon area of BH to be the result of the surface of the horizon puncturing by the edges of a large number of spin network [11]. This punctures that exist on the boundary of horizon also increases the dimensions of the Hilbert space of the boundary theory. If the edge puncture has a label j_l the dimension

of the spin j_l representation increases by a factor of $2j_l + 1$. In large number N of edges with spins $j_l = 1, \dots, N$. The dimension of the Hilbert space is thus [11]

$$\Omega = \prod_{l=1}^N (2j_l + 1) \tag{6}$$

It can be explained that statistically the most important contribution to 6 are those in which the minimum possible value for the spin dominates. Let us indicate the lowest spin value by j_{\min} . One obtains [11]

$$\Omega = (2j_{\min} + 1)^N, \tag{7}$$

where N can be calculated from the area A of the BH and from one puncture of area $a(j_k)$ resulting from each edge [11], which is given by

$$N = \frac{A}{a(j_{\min})} = \frac{A}{8\pi\gamma\ell_p^2\sqrt{j_{\min}(j_{\min} + 1)}}. \tag{8}$$

The BH entropy with area A is given by the logarithm of the dimension of the Hilbert space of the boundary theory. In classical BG statistics and using Eq. (7) the entropy is thus

$$S = N \ln(2j_{\min} + 1), \tag{9}$$

we will use the entropy of Tsallis (we can view a method analogous to that described in ref. [1,14]) to generalize Boltzmann–Gibbs statistics. Using Eq. (7) and Tsallis entropy, Eq. (5), we have

$$S_q = \frac{(2j_{\min} + 1)^{(1-q)N} - 1}{1 - q}. \tag{10}$$

The micro-states in large BH s which attribute to every puncture the smallest quantum area $j_{\min} = 1/2$ dominate the counting as they maximize the punctures number prescribed for a given area of horizon, so replace $j_{\min} = 1/2$ in Eq. (10) the entropy then become

$$S_q = \frac{2^{(1-q)N} - 1}{1 - q}, \tag{11}$$

which is the result that was reported in ref. [14] for Tsallis statistics in LQG . Putting Eq. (8) and $j_{\min} = 1/2$ in Eq. (11). The expression become

$$S_q = \frac{2^{(1-q)A/4\pi\gamma\sqrt{3}} - 1}{1 - q} = \frac{1}{1 - q} \left[\exp \left((1 - q) \Lambda(\gamma) \frac{A}{4\ell_p^2} \right) - 1 \right], \tag{12}$$

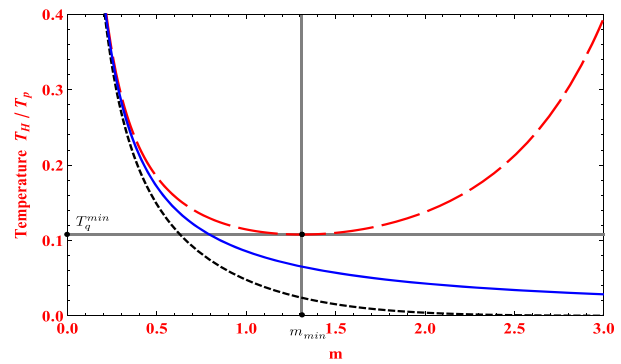


Fig. 1 The figure shows the temperature of a Schwarzschild BH as a function of masses in the asymptotically flat case with Boltzmann ($q = 1$, blue-continuous) and Tsallis entropies ($q > 1$, red-dashed and $q < 1$, black-dotted)

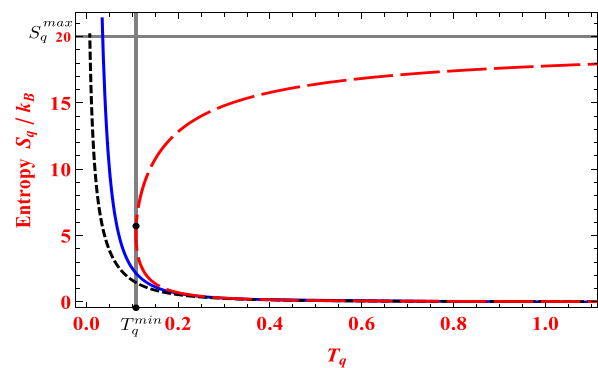


Fig. 2 The figure shows the temperature of a Schwarzschild BH as a function of masses in the asymptotically flat case with Boltzmann ($q = 1$, blue-continuous) and Tsallis entropies ($q > 1$, red-dashed and $q < 1$, black-dotted)

where

$$\Lambda(\gamma) = \frac{\ln 2}{\pi\sqrt{3}\gamma} = \frac{\gamma_0}{\gamma}.$$

A complete agreement with Bekenstein–Hawking result (i.e. $S_{BH} = A/4\ell_p^2$) is obtained if we set $q \rightarrow 1$ and $\gamma = \gamma_0$ see [22]. In the simplest case of a Schwarzschild, the mass of the BH is related to the area of the Horizon by the formula

$$A = 4\pi R_s^2 = 16\pi \frac{G^2 M^2}{c^4} = 16\pi \ell_p^2 m^2. \tag{13}$$

where $m = M/M_P$ and $M_P = \sqrt{\hbar c/G}$.

Using this result in Eq. (12) to get the entropy of Tsallis– BH in LQG

$$S_q = \frac{k_B}{1 - q} \left[\exp \left(4\pi (1 - q) \Lambda(\gamma) m^2 \right) - 1 \right]. \tag{14}$$

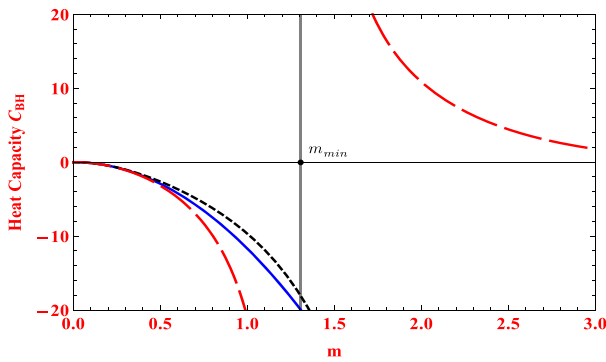


Fig. 3 The figure shows the heat capacity of a Schwarzschild BH as a function of masses in the asymptotically flat case with Boltzmann ($q = 1$, blue-continuous) and Tsallis entropies ($q > 1$, red-dashed and $q < 1$, black-dotted)

In the following sections, we will fix the BI -parameter that appears in the figures with $\gamma \simeq 0.274$ see [23,24].

3 Thermodynamic stability of black holes

On the basis of the Tsallis model with $q > 1$, we studied the properties of thermodynamic for the Schwarzschild BH , if (14) is correct, one obtains the thermodynamic quantities following [1]:

$$S_q(m) = \frac{k_B}{1-q} [\exp(m^2/2m_{\min}^2) - 1], \tag{15}$$

$$T_q(m) = \left(\frac{1}{k_B} \frac{\partial S_q}{\partial E} \right)^{-1} = \left(\frac{1}{k_B M_P c^2} \frac{\partial S_q}{\partial m} \right)^{-1},$$

$$= (q-1) T_P \frac{\exp[m^2/2m_{\min}^2]}{m/m_{\min}^2}, \tag{16}$$

$$C_q(m) = T_q \frac{\partial S_q}{\partial T_q}$$

$$= \frac{m^2/m_{\min}^2 \exp[-m^2/2m_{\min}^2]}{(q-1)(m^2/m_{\min}^2 - 1)}, \tag{17}$$

$$G_q(m) = E - T_q \times S_q$$

$$= M_P c^2 \frac{[m^2/m_{\min}^2 + (1 - \exp[m^2/2m_{\min}^2])]}{m/m_{\min}^2}, \tag{18}$$

where T_P is the Planck temperature such as $T_P = M_P c^2/k_B$ and

$$m_{\min} = [8\pi (q-1) \Lambda(\gamma)]^{-1/2}, \tag{19}$$

it is the minimum mass [1] of the temperature functions at $q > 1$, i.e. $\partial_m T_q(m)|_{m=m_{\min}} = 0$. Therefore the minimum value of T_q is $T_q^{\min} = T_P \sqrt{\frac{(q-1)e}{8\pi \Lambda(\gamma)}}$, then there exists a lower value of temperature for the BH see Fig. 1.

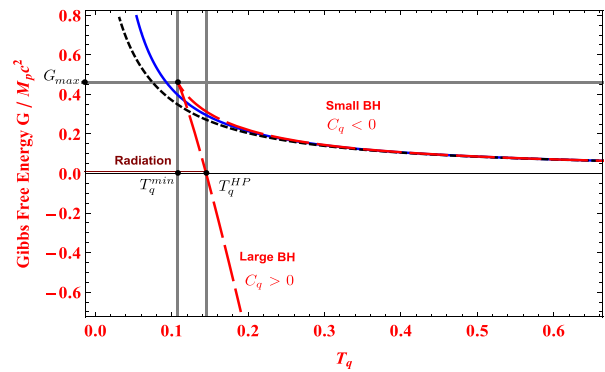


Fig. 4 The figure shows the free energy of a Schwarzschild BH in the Tsallis model as a function of the temperature in the standard with Boltzmann entropy ($q = 1$, blue-continuous) and the Tsallis entropy ($q > 1$, red-dashed and $q < 1$, black-dotted)

We clearly observe a thermodynamic behavior different from the case of the Schwarzschild BH asymptotically flat.

In Fig. 2 illustrates how Tsallis $q > 1$ and Boltzmann $q = 1$ Entropy are related to the temperature of the Schwarzschild BH s. Thus, entropy function $S_q(T)$ has two horizontal asymptotes, $S_q = 0$ and $S_q = k_B/(q-1)$. More specifically in Fig. 3, the heat capacity C_q is not always negative: it becomes positive for the large BH s with $m > m_{\min}$ while it is negative for $m < m_{\min}$ and not defined at $m = m_{\min}$.

The behaviors of the Gibbs free energy [1] G_q for $q > 1$ is depicted in Fig. 4, We observe a minimum temperature T_q^{\min} , at $m = m_{\min}$, no BH solution can exist below the minimal temperature and the space is filled with pure radiation.

There exist two branches above T_q^{\min} , the upper branch describes small BH s $m < m_{\min}$ (and Schwarzschild type $q \leq 1$) with negative specific heat; which are thermodynamically unstable and can not be in thermal equilibrium, while the lower branch have positive specific heat and are therefore thermodynamically stable locally for large BH s for $m > m_{\min}$.

Hence, When we have $T_q^{\min} < T_q < T_q^{HP}$, We observe that Gibbs free energy G_q of those BH s is always positive, This also confirms the results of [25] for the Schwarzschild- AdS - BH . in general, BH s with $T_q > T_q^{HP}$ (that is to say $m > m_{HP}$ in Fig. 5), have negative Gibbs free energy and represent the globally preferred state.

Furthermore, at $T_q = T_q^{HP}$, the intersection point of the bottom branch and $G_q = 0$ indicates the well known phase transition between thermal radiation and large BH s, this is completely analogous to the *Hawking-Page* [25] phase transition of Schwarzschild BH s in AdS space see [26]. This phase transition can be interpreted as confinement/deconfinement phase transition by checking the different configurations of a quark and anti-quark in AdS/QCD .

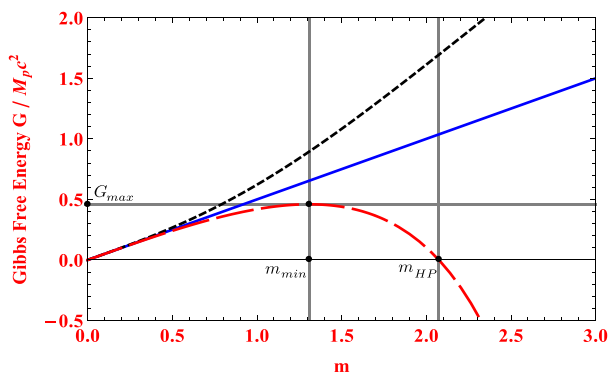


Fig. 5 The figure shows the Gibbs free energy of a Schwarzschild BH as a function of masses in the asymptotically flat case with Boltzmann ($q = 1$, blue-continuous) and Tsallis entropies ($q > 1$, red-dashed and $q < 1$, black-dotted)

We can see from Eq. (18) that this point occurs at :

$$m_{HP} = m_{\min} \sqrt{B} \simeq 1.58520 \times m_{\min} \tag{20}$$

$$T_q^{HP} = T_q^{\min} \sqrt{e^{B-1}/B} \simeq 1.34409 \times T_q^{\min} \tag{21}$$

where $B = -1 - 2W_{-1} \left[-\frac{1}{2\sqrt{e}} \right]$ and $W_{-1}(z) \equiv W(x)$ is the branch for $-1/e \leq x < 0$ of the Lambert W function is defined by $W(z)e^{W(z)} = z$ [27,28].

4 A semi-classical estimation of the q -parameter of the black holes

In particle physics, and remarkably, for astronomical BHs, one speculates small quantum BHs that can not be detected in experimental terms, so q -parameter may deviate from unity (i.e. $q = 1 + \epsilon$). In what follows, we treat an estimate of Schwarzschild BH whose temperature is the minimum value of 19 leading to $M_{\min} = M_P [8\pi (q - 1) \Lambda(\gamma)]^{-1/2}$, if the energy of such a state, $E_{\min} = M_{\min}c^2$, is situated near the ground state energy of quantum mechanics [29,30]. Of course, we can have an approximate for this limit state from semi-classical considerations. We see that the ground state of the harmonic oscillator is $E_0 = \omega_0 \hbar/2 = kc\hbar/2 = \pi \hbar c/2\lambda_0$ (the boundary condition is results in the reality that $\sin(k\lambda_0) = 0$ this is satisfied when $k\lambda_0 = \pi$). Here, λ_0 and ω_0 represents the fundamental wavelength and frequency respectively of the system, and $D_{\min} = 2R_{\min}$ (the BH diameter) is an appropriate method λ_0 i.e. $\lambda_0 \approx D_{\min}$, yielding to $E_0 = \pi \hbar c/2D_{\min} = \pi \hbar c^3/8GM_{\min}$. The energy of the corresponding ground state quantum oscillator must be close to the minimum point of the temperature, it means that $E_0 \geq E_{\min}$, and taking into account the relation 19, we

obtain

$$\begin{aligned} \frac{\pi \hbar c^3}{8GM_{\min}} &\geq M_{\min}c^2 \\ \frac{\pi}{8} M_P^2 &\geq M_{\min}^2 \end{aligned} \tag{22}$$

Now, once we can determine q , then Eqs. (19) and (22) will gives a relation between q and γ by

$$q \geq \frac{\sqrt{3}\gamma}{\pi \ln 2} + 1, \tag{23}$$

as an approximate relation between q and γ . In fact, when we fixed the value $\gamma \approx 0.274$ see [23,24]. We find $q \approx 1.218$, is how well approximated by cosmic ray observations ($q = 11/9$) [31] and by the quark coalescence fit to Relativistic Heavy Ion Collider RHIC ($q \approx 1.2$) [30].

5 Decay time for the black holes

The radiation recorded by distant observer, the Hawking radiation turns out to be black body radiation of the temperature T_q [32]. This process results in a reduction of mass of the BH-evaporation. It is assumed that the BH radiates according to the Stefan–Boltzmann law [33], It is applied at the BH surface area A , according to Hawking’s temperature T_q and relativistic equivalence relation $E = Mc^2$, reads as

$$-\frac{dE}{dt} = -c^2 \frac{dM}{dt} = \sigma_{SB} A T_q^4 \tag{24}$$

where $\sigma_{SB} = \pi^2 k_B^4/60\hbar^3 c^2$ is the Stefan–Boltzmann constant [33]. This law enables us to calculate the lifetime of the BH [34] by an analog approach to Hawking’s work on the time of the BH’s evaporation [35]. From the Eqs. 16 and 24, we obtain the modified Stefan–Boltzmann law for the BH radiation power law derivation as follows

$$-\frac{dm}{dt} = \frac{4\pi^3 (q - 1)^4 \exp [2m^2/m_{\min}^2]}{15t_P \frac{m^2}{m_{\min}^8}}, \tag{25}$$

where $t_P = \sqrt{\hbar G/c^5}$ is the Planck time. So we can write

$$dt = -\frac{15t_P}{4\pi^3 (q - 1)^4 \exp [2m^2/m_{\min}^2]} \frac{m^2/m_{\min}^8}{dm}. \tag{26}$$

Therefore the evaporation time of a BH of initial mass m_0 can be expressed by the formula

$$\begin{aligned} t_{life}(m_0) &= -\frac{15 (q - 1)^{-4} t_P}{4\pi^3 m_{\min}^5} \\ &\times \int_{m_0}^0 \frac{m^2/m_{\min}^2}{\exp [2m^2/m_{\min}^2]} dm/m_{\min}. \end{aligned} \tag{27}$$

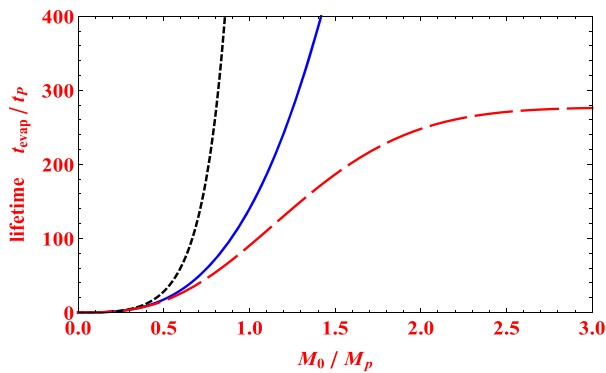


Fig. 6 The scaled life time of *BH* against black-body radiation in the Tsallis-statistic ($q > 1$, red-dashed and $q < 1$, black-dotted) and in the Boltzmann-statistic ($q = 1$, blue-continuous) as functions of the initial masse of *BH*

After the change of variable as

$$x = m/m_{\min}, \quad x_0 = m_0/m_{\min},$$

the last integral becomes

$$t_{life}(m_0) = \frac{15(q-1)^{-4} t_P}{4\pi^3 m_{\min}^5} \int_0^{m_0/m_{\min}} x^2 \exp(-2x^2) dx. \tag{28}$$

The finite decay time, during which an initial radius, R , shrinks to zero, can be analytically obtained in both cases for the Tsallis. The analytic results of the integrations are

$$t_{life}(m_0) = \frac{15(q-1)^{-4} t_P}{32\pi^3 m_{\min}^4} \times \left[\sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{\sqrt{2}m_0}{m_{\min}}\right) - \frac{2m_0}{m_{\min}} \exp\left(-\frac{2m_0^2}{m_{\min}^2}\right) \right], \tag{29}$$

where $\operatorname{erf}(z)$ is the error function defined by $\operatorname{erf}(z) := \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$.

For $q \rightarrow 1$ or For small initial mass $m_0 \ll m_{\min}$ and $\gamma = \ln 2/\pi\sqrt{3}$, the life time $t_{life}(m_0)$ can be approximated by the classical Schwarzschild result

$$t_{life}(m_0) \simeq 5120 \frac{\pi G^2}{\hbar c^4} M_0^3. \tag{30}$$

We see that the result is very close to Hawking’s time for *BH* evaporation [35].

Though, for *BH* with large initial mass $m_0 \gg m_{\min}$, $t_{life}(m_0)$ can be approximated by the limit

$$t_{life}(\infty) \simeq 60t_P \sqrt{\frac{\Lambda^5(\gamma)}{(q-1)^3}}, \tag{31}$$

very large *BH*s do not live forever [30] in this approach: they also have a finite decay time see Fig. 6.

6 Conclusion

In this letter we studied the Bekenstein–Hawking entropy as a non-extensive entropy of Schwarzschild *BH* horizons, and by considering their equation of state based on the Tsallis non-extensive entropy using the *LQG* theory [1, 14]. In this framework, our major concentricity will be to study some characteristics of a Schwarzschild *BH* gathering the Tsallis entropy. This feature (and the whole $T_q(M)$ curve at values of $q > 1$) is of the same form as the result from a *BH* in anti-de Sitter space within the original of the *BG* entropy formula [25]. The stability analysis also show that, if its mass is bigger than M_{\min} which is the mass of a Schwarzschild *BH* with temperature T_q^{\min} , in the Tsallis formalism, a Schwarzschild *BH* has positive heat capacity and is stable, if its mass is smaller than M_{\min} the *BH* will have a negative specific heat and will be unstable. Where it is also proved that a *Hawking–Page BH* phase transition results at a critical temperature T_q^{\min} which relies on the q -parameter of the Tsallis formula.

Therefore, A semi-classical estimate analysis of the energy E_{\min} at this minimum M_{\min} leads to a Bekenstein bound on the entropy parameters q and γ in the Tsallis entropy of micro *BH*s with $q \simeq \frac{\sqrt{3}\gamma}{\pi \ln 2} + 1$, and when we fixed the value of $\gamma \simeq 0.274$ [23, 24], we surprisingly concluded that this estimate for q -parameter ($q \approx 1.218$) is very close to the observed value of cosmic ray and distribution of the power law of coalescing quark in hadrons in experiments with high energy accelerators ($q \approx 1.2$). In this approach, the *BH*s with very large masses do not live forever and the Hawking radiation becomes important, they also have a finite decay time, and the maximum age is associated with two parameter q and γ . In our next work we will focus on the study of the thermodynamic properties of Schwarzschild *BH*s as well as those of Schwarzschild–de Sitter to relate their entropy to gravitation (coupling) and to geometry (horizon surface) in the framework of non-extensive statistical mechanics with the quantification of the horizon surface by means of the *LQG*, and all this by relying on the Sharma–Mittal entropy [9].

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: This manuscript is not associated with any of the additional data, so this data will not be deposited. All information relating to this manuscript is contained in the theoretical formalism developed in this work and can be extracted from the equations illustrated in this manuscript.]

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Funded by SCOAP³.

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