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A Semiphenomenological Proton-Proton Potential*

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An energy independent potential is constructed which reproduces all available *p-p* data up to 310 Mev. At 310 Mev the potential predicts the phase shift solution 1 of MacGregor et al. The potential includes the central, tensor, linear and quadratic *LS* potentials. The quadratic *LS* potential is manifestly required in the singlet even parity state where the linear *LS* potential vanishes. The linear *LS* potential turns out to be more singular but of shorter range than previously thought. It appears now that the *p-p* data below 310 Mev can be understood in terms of a potential consistent in all respects with the pion theory of nuclear forces.

§ 1. Introduction

There is no *a priori* reason for the existence of the energy independent two nucleon potential in terms of which the two-nucleon data can be understood over the wide energy range. It is nevertheless useful to have a potential picture which reflects, as close as possible, the over-all features of the two-nucleon interaction Not only does such a picture give a basis on which various two-nucleon date can be understood systematically and in simple ways, but also it provides a lead in more fundamental field theoretical studies of nuclear forces. Such a picture can also find useful applications in other branches of nuclear physics.

The aim of the present paper is to report on an attempt to interpret the available proton-proton data up to 310 Mev in terms of a potential of the form

$$V = V_C + V_T S_{12} + V_{LS}(\boldsymbol{L}\cdot\boldsymbol{S}) + V_Q Q_{12}. \tag{1}$$

Here, the subscripts C, T, LS, and Q stand for the central, tensor, linear LS, and quadratic LS potential, respectively. Q_{12} is the operator

$$Q_{12} = 1/2\{(\boldsymbol{L}\cdot\boldsymbol{\sigma}_1)(\boldsymbol{L}\cdot\boldsymbol{\sigma}_2) + (\boldsymbol{L}\cdot\boldsymbol{\sigma}_2)(\boldsymbol{L}\cdot\boldsymbol{\sigma}_1)\}, \tag{2}$$

with eigenvalues

$$\begin{aligned} Q_{12} = 2(\boldsymbol{L}\cdot\boldsymbol{S})^2 + (\boldsymbol{L}\cdot\boldsymbol{S}) - \boldsymbol{L}^2 = (1/4)[J(J^3+2J^2+3J+2) \\ -L(L^3+2L^2+7L+6) - S(S^3+2S^2+3S+2)]. \end{aligned} \tag{3}$$

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The functions V_i ($i=C$, T, LS, Q) depend on the spin and parity of the two-nucleon system. The neutron-proton data will be studied in a separate paper.

It is known that the two-nucleon potential of the form (1) is the most general one consistent with the usual invariance requirements.[1),2)] In general, the functions V_i may depend on the relative momentum p and the angular momentum L as well as on the inter-nucleon distance. In the present work, however, we assume that V_i are the energy independent functions only of the internucleon distance.

Potentials of the form (1) with $V_Q\equiv 0$ have been studied by several authors. Gammel and Thaler[3)] considered a purely phenomenological V_i of Yukawa type. The lack of the pion theoretical foundation in their potential has been criticised elsewhere.[4)] It is remarkable, however, that they have succeeded in reproducing most of the qualitative features of the p-p data up to 310 Mev with such a simple form of V_i. In fact, their work has provided a considerable lead in the course of the present study.

Signell and others[5),6)] have chosen the Gartenhaus potential[7)] together with a phenomenological V_{LS}. Their potential can fit the date up to 150 Mev reasonably well but above this energy only very poorly. It is likely that this shortcoming is due to the unnecessary inflexibility introduced into the potential at small distances by the use of the Gartenhaus potential. No pion theoretical potential can claim its validity in the core region.

The Japanese group, on the other hand, studied the potential (1) with $V_{LS}=V_Q\equiv 0$.[9),10),11)] Following the method proposed by Taketani et al.,[12)] they considered phenomenological inner potentials together with the one-pion-exchange tail. They have been able to reproduce the experimental differential cross section and the polarization below 150 Mev. However, it is very unlikely that this type of potential can be applied successfully to higher energy scattering with only minor modifications so as not to destroy the fit attained at lower energies. Further, the prediction of the potential on the parameter R is in qualitative disagreement with the recent measurement at 150 Mev.[13)] This and perhaps the large negative D (90°) predicted by the potential[11)] indicate that the deviation from the potential expected on the basis of the static pion theory is already significant at 150 Mev. This fact is one of the motivations for the present work.

In its spirit the present work follows the method of Taketani et al.[12)] This is reflected, in particular, in choosing the functional form of V_i. In order to determine parameters specifying thus chosen V_i, we first study the 310 Mev p-p data. We shall find a potential which gives the phase shifts close to the solution 1 of MacGregor et al.[14)] Using this potential we then calculate the phase shifts and the observable quantities at various energies. Wherever the comparison with the experimental data is possible, the agreement is satisfactory.

After the completion of the present work, we were shown a preprint of a recent work by Bryan.[8)] His results as well as the method of approach to the problem are very similar to ours. The only qualitative difference is in the singlet even parity potential, where we include a certain non-static effect whereas Bryan does not.

§ 2. Functional form of V_i

For the V_i in (1) we have chosen the following forms:

singlet even:

$$
\begin{aligned}
{}^1V_C^+(x) &= -(g^2/4\pi)\mu(e^{-x}/x)(1+{}^1a_C^+(e^{-x}/x)+{}^1b_C^+(e^{-x}/x)^2),\\
{}^1V_Q^+(x) &= {}^1G_Q^+\mu(e^{-x}/x)(1+{}^1a_Q^+(e^{-x}/x)+{}^1b_Q^+(e^{-x}/x)^2),
\end{aligned}
\tag{4}
$$

triplet odd:

$$
\begin{aligned}
{}^3V_C^-(x) &= (\mu/3)(g^2/4\pi)(e^{-x}/x)(1+{}^3a_C^-(e^{-x}/x)+{}^3b_C^-(e^{-x}/x)^2),\\
{}^3V_T^-(x) &= (\mu/3)(g^2/4\pi)(e^{-x}/x)(1+3/x+3/x^2)\\
&\quad\times(1+{}^3a_T^-(e^{-x}/x)+{}^3b_T^-(e^{-x}/x)^2),\\
{}^3V_{LS}^-(x) &= {}^3G_{LS}^-\mu(e^{-x}/x)^2(1+{}^3b_{LS}^-(e^{-x}/x)),\\
{}^3V_Q^-(x) &= {}^3G_Q^-\mu(e^{-x}/x)(1+{}^3a_Q^-(e^{-x}/x)+{}^3b_Q^-(e^{-x}/x)^2).
\end{aligned}
\tag{5}
$$

Here μ is the pion mass and $x=\mu r$. We have also assumed a hard core of radius ${}^1x_0^+$ and ${}^3x_0^-$ for (4) and (5), respectively.

As far as V_C and V_T are concerned, the potential reduces, for large x, to the well-known one-pion-exchange potential (OPEP) with the effective pion-nucleon coupling constant $g^2/4\pi$. V_Q has the same range μ^{-1} as V_C and V_T. We shall see later, however, that $|{}^1G_Q^+|$, $|{}^3G_Q^-|\ll g^2/4\pi$. From this and the eigenvalues of Q_{12} given in (3), it is seen that only higher partial waves are appreciably affected by V_Q. At lower energies, where impact parameters of those higher partial waves are large, the tail of the potential (4) and (5) is effectively that of OPEP. This point will be discussed later in more detail.

The parameters a_C and a_T in (4) and (5) determine the deviation of the potential from the OPEP around the pion range $x=1$. In this region we consider the pion theory of nuclear force to be qualitatively reliable.[9),15),16)] Then we expect

$$
{}^1a_C^+\gg 0,\quad {}^3a_C^-\ll 0,\quad {}^3a_T^-<0. \tag{6}
$$

The parameters b_C and b_T will be determined purely phenomenologically by analysing the 310 Mev p-p data.

All the available pion theoretical calculations predict negative ${}^3V_{LS}^-(x)$ with the range $(2\mu)^{-1}$. This is reflected in (5). Since we cannot expect the quantitative validity of the pion theoretical prediction regarding V_{LS} at present, the parameters appearing in ${}^3V_{LS}^-(x)$ will be determined purely phenomenologically.

§ 3. Choice of the phase shift solution at 310 Mev

In order to determine the parameters in (4) and (5), we first study the p-p data at 310 Mev. This will be a natural starting point for the following reasons. First, the experimental data are most abundant at this energy.[17] Consequently, an extensive phase shift analysis has been possible yielding only two solutions.[14]* Second, 310 Mev is sufficiently high so that the scattering, particularly P-wave scattering, is affected by the interaction in the inner-most region. This is necessary for the determination of the parameters b and the hard core radius in (4) and (5). Yet, it may be expected that the non-relativistic potential picture still retains its usefulness at this energy.

We have tried to fit MMS 1. We can enumerate a few reasons for this choice, although none of them is quite convincing. First, apart from the interpretation of phase shifts in terms of potential pictures, MacGregor et al.[14] found in their analysis that MMS 1 is slightly favoured over MMS 2.

Second, it appears that the singlet even phase shifts** of MMS 2 are very hard, if not impossible, to understand in terms of the energy independent potential picture. From Table I we see that, as far as the singlet even phase shifts are

Table I. Singlet even parity phase shifts at 310 Mev. The entries are the nuclear Blatt-Biedenharn phase shifts in radians.

	MMS 1	MMS 2	Bryan[8]	calculated from (4) and (11) with ${}^1V_Q^+=0$	calculated from (4), (11) and (12)
${}^1S_0$	-0.156	-0.506	-0.058	-0.150	-0.150
${}^1D_2$	0.207	0.083	0.216	0.264	0.228
${}^1G_4$	0.013	0.015	0.021	0.035	0.021
${}^1I_6$				0.009	0.002

concerned, MMS 2 is characterised by the large negative ${}^1S_0$ and small positive ${}^1D_2$ phase shift as compared with MMS 1. ${}^1G_4$ phase shifts are much the same for the two solutions as expected from the way the phase shift analysis has been performed.[14] Unlike in GT, the smallness of ${}^1D_2$ phase shift itself does not cause any difficulty here since it is always possible to adjust ${}^1V_Q^+$ to reduce the ${}^1D_2$ phase shift by a suitable amount without affecting the ${}^1S_0$ phase shift. The difficulty lies in the relative magnitudes of ${}^1D_2$ and ${}^1G_4$ phase shifts of MMS 2. We could not find the ${}^1V_Q^+$ which reduces ${}^1D_2$ phase shift to that of MMS 2 without at the same time reducing ${}^1G_4$ phase shift to substantially negative values.

* We shall, in the following, refer to these two solutions as MMS 1 and MMS 2.

** Throughout the present paper the word phase shift refers to the nuclear Blatt-Biedenharn phase shift. More specifically, it is the Blatt-Biedenharn n-p phase shift in the sense that only the nuclear interaction is included in Schrödinger equations and the phase shift is calculated by comparing the solutions with Bessel functions (not with Coulomb functions).

Further, we did not find it possible to reconcile the large negative $^1S_0$ phase shift of MMS 2 with the shape independent parameters known experimentally at low energies.

Finally, as we shall see in §6, the potential which fits MMS 1 can reproduce all other lower energy data remarkably well. This fact may be taken as an indirect evidence in favour of MMS 1. It seems unlikely that the similar situation holds for MMS 2, although such a possibility cannot be entirely excluded.

§ 4. Triplet odd parity potential

In order to facilitate the qualitative understanding of the relation between the potential and phase shifts, we first write down the Schrödinger equation in the triplet odd parity states. Writing

$$\kappa^2 = ME_{CM}/\mu^2 \quad \text{and} \quad U_i(x) = (M/\mu^2)\, V_i(x),$$

with M=nucleon mass, they read

$$\begin{aligned}[d^2/dx^2+\kappa^2-J(J+1)/x^2-U_C(x)-2U_T(x)+U_{LS}(x)\\ +\{J(J+1)-1\}\,U_Q(x)]\,v_J(x)=0,\end{aligned} \tag{7}$$

for the uncoupled ($L=J$) states, and

$$\begin{aligned}&[d^2/dx^2+\kappa^2-J(J-1)/x^2-U_C(x)+2(J-1)/(2J+1)\,U_T(x)\\ &\quad-(J-1)\,U_{LS}(x)-(J-1)^2\,U_Q(x)]\,u_J(x)\\ &\quad-6\sqrt{J(J+1)}/(2J+1)\,U_T(x)\,w_J(x)=0,\\ &[d^2/dx^2+\kappa^2-(J+1)(J+2)/x^2-U_C(x)+2(J+2)/(2J+1)\,U_T(x)\\ &\quad+(J+2)\,U_{LS}(x)-(J+2)^2\,U_Q(x)]\,w_J(x)\\ &\quad-6\sqrt{J(J+1)}/(2J+1)\,U_T(x)\,u_J(x)=0,\end{aligned} \tag{8}$$

for the coupled ($L=J\pm1$) states. Here appropriate superscripts on $U_i(x)$ are understood (see (5)).

The necessity of the forces other than the central and tensor is most clearly seen in the behaviour of $^3P_0$ and $^3P_1$ phase shifts at 310 Mev. From (5), (7) and (8), it may be seen that V_T is much more important than V_C in these states. The strong positive V_T in the OPEP gives rise to a large positive $^3P_0$ phase shift as seen in the second column of Table II. This is in drastic disagreement with MMS 1. On the other hand, the same V_T yields a large negative $^3P_1$ phase shift consistent with MMS 1. If one wants to retain the OPEP tail, as we do here, the V_T has to become strongly negative for $x\lesssim 1$ in order to reduce $^3P_0$ phase shift. This modification in V_T is indeed effective as seen in the third column of Table II. In doing so, however, the agreement of $^3P_1$ phase shift with MMS 1 is destroyed.

Table II. Triplet odd parity phase shifts at 310 Mev. Entries are nuclear Blatt-Biedenharn phase shifts in radians.

	MMS 1	OPEP*	from (5) and (9) but ${}^3V_{LS}^-={}^3V_Q^-\equiv 0$	from (5) and (9) but ${}^3V_Q^-\equiv 0$	from (5) and (9)
${}^3P_0$	−0.197	0.753	0.158	−0.206	−0.213
${}^3P_1$	−0.480	−0.488	−0.262	−0.452	−0.450
${}^3F_3$	−0.062	−0.086	−0.061	−0.068	−0.063
${}^3H_5$	−0.020	−0.025	−0.022	−0.022	−0.019
${}^3P_2$	0.291	0.154	0.071	0.321	0.316
${}^3F_2$	0.020	−0.101	−0.100	0.013	0.007
ϵ_2	−0.051	−0.825	−1.067	−0.091	−0.090
${}^3F_4$	0.056	0.050	0.047	0.078	0.070
${}^3H_4$	0.006	−0.016	−0.009	0.002	−0.002
ϵ_4	−0.215	−0.675	−0.637	−0.391	−0.413
${}^3H_6$	0.004	0.015	0.015	0.015	0.012
${}^3L_6$		−0.007	−0.006	−0.006	−0.008
ϵ_6		−0.733	−0.711	−0.779	−0.788

* $g^2/4\pi=0.08$ and ${}^3x_0^-=0.32$.

Again referring to (7) and (8), it is now easy to see that a strong negative V_{LS} can meet this difficulty. Thus, roughly speaking, the role played by the strong negative V_{LS} is the following. It cooperates with the negative V_T at $x\lesssim 1$ to reduce the ${}^3P_0$ phase shift further down to the MMS 1 value. On the other hand, it also cancels the increase in the ${}^3P_1$ phase shift caused by the negative V_T at $x\lesssim 1$.

The effect of V_{LS} in the coupled states is not so easy to see. Perhaps the most important effect here is the large increase in ϵ_2. This increase is of particular importance in fitting the polarization data.[10]

In Table II we note that the phase shifts for $L\gtrsim 3$ calculated without V_{LS} are in substantial agreement with those of MMS 1. This indicates that V_{LS} must be short ranged.

Based on these qualitative understanding of the effects of various forces, we tried some 150 combinations of parameters appearing in (5). The condition (6) has been imposed on all of them. For some of the phase shifts thus obtained the observable quantities have been calculated and directly compared with the available data. We have finally decided on the set

$$\begin{aligned} &g^2/4\pi=0.08,\ {}^3G_{LS}^-=0.1541,\ {}^3G_Q^-=0.00045,\\ &{}^3x_0^-=0.32,\ {}^3a_C^-=-9,\ {}^3b_C^-=4.6,\ {}^3a_T^-=-1.14,\\ &{}^3b_T^-=0.2,\ {}^3b_{LS}^-=-8.09,\ {}^3a_Q^-=10,\ {}^3b_Q^-=6. \end{aligned} \tag{9}$$

The potential is plotted in Fig. 1 and the phase shifts are given in the last column of Table II.

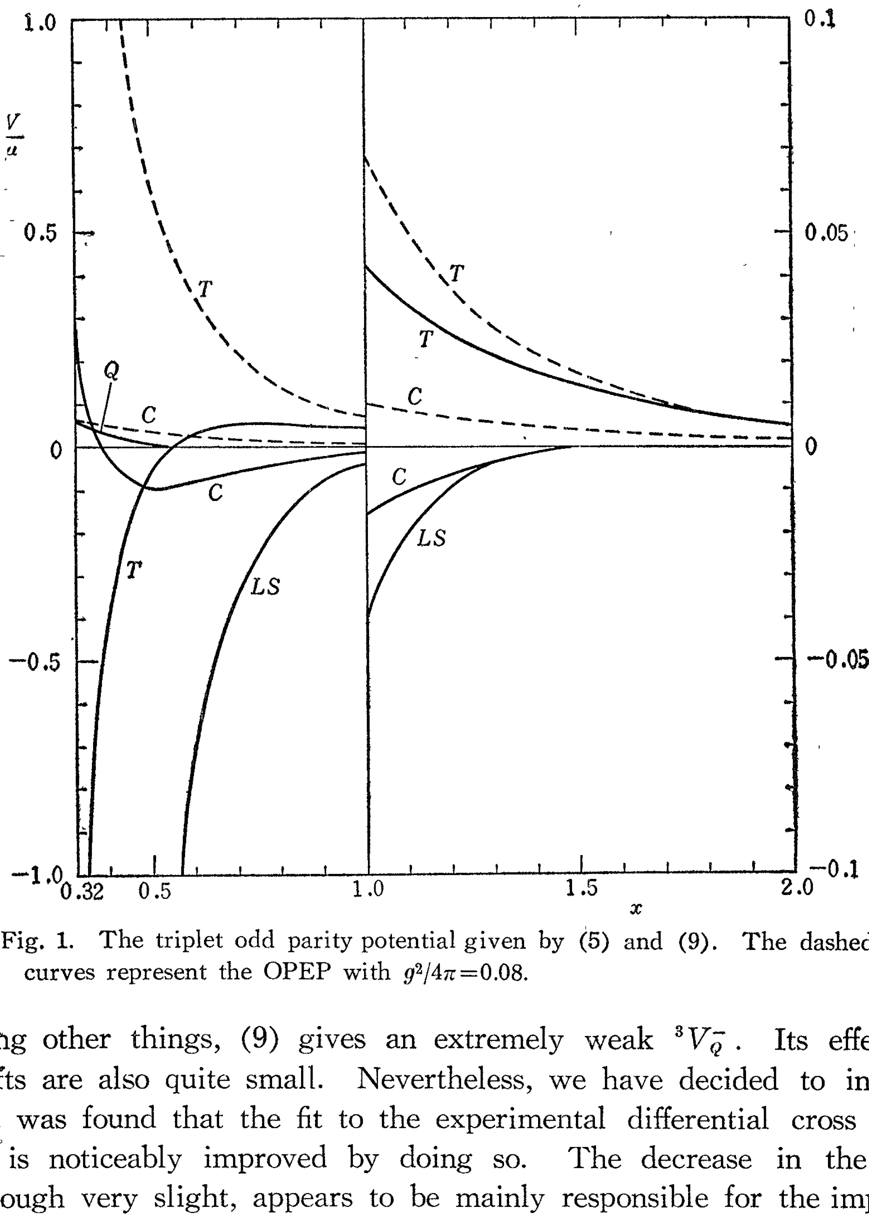

Fig. 1. The triplet odd parity potential given by (5) and (9). The dashed curves represent the OPEP with $g^2/4\pi=0.08$.

Among other things, (9) gives an extremely weak ${}^3V_Q^-$. Its effect on the phase shifts are also quite small. Nevertheless, we have decided to include ${}^3V_Q^-$ because it was found that the fit to the experimental differential cross section at 310 Mev is noticeably improved by doing so. The decrease in the ${}^3F_4$ phase shift, although very slight, appears to be mainly responsible for the improvement. At lower energies, the effect of ${}^3V_Q^-$ is entirely negligible.

(9) shows that our ${}^3V_{LS}^-$ is composed of a weak repulsion of range $(2\mu)^{-1}$ and a strong attraction of range $(3\mu)^{-1}$. The main function of the former is to weaken the latter at $x\gtrsim 1$. This was found necessary mainly in order not to produce too large a ${}^3F_4$ phase shift.

The weak repulsive part of ${}^3V_{LS}^-$ should not be taken literally, since the argument here depends on the particular functional form assumed for ${}^3V_{LS}^-$. Our finding is simply that the strongly attractive part of ${}^3V_{LS}^-$ should not extend too far beyond $x\simeq 1$.

In this connection, we often felt during the course of calculations that the functional form of ${}^3V_{LS}^-$ assumed in (5) might not be quite adequate. Indeed,

Table III. Comparison of various proposed ${}^3V_{LS}^-$. Entries are the ${}^3V_{LS}^-(x)$ in Mev at several internucleon distances $x=\mu r$.

x	GT[3]	SM[5]	present work	Bryan[8]
0.5	−198.5	−88.1	−278.8	−550.0
0.8	−25.8	−17.9	−24.0	−14.4
1.0	−7.53	−7.95	−5.72	−1.62
1.2	−2.12	−3.94	−1.39	−0.25
1.5	−0.35	−1.51	−0.10	−0.02
2.0	−0.03	−0.42	+0.04	−0.00

Bryan[8] finds a monotonic ${}^3V_{LS}^-$ of the form e^{-2x}/x^8 with a straight cutoff at $x=0.54$ in his attempt to fit MMS 1. As may be seen in Table III, Bryan's ${}^3V_{LS}^-$ is substantially weaker for $x\gtrsim 1$ but stronger for $x\lesssim 0.7$ than our ${}^3V_{LS}^-$. For his ${}^3V_{LS}^-$ it is clear from what has been said above that no ${}^3V_Q^-$ is required even at 310 Mev.

Summarizing the discussions on ${}^3V_{LS}^-$, it appears from Bryan's work and what we have learned during the calculations that the ${}^3V_{LS}^-$ should be substantially weaker for $x\gtrsim 1$ but stronger for $x\lesssim 0.7$ than GT, SM or SM1. It is satisfying that this conclusion is in better agreement with the pion theoretical predictions on V_{LS}[10),15),16)] than previously thought.

In Tables IV and V we give the triplet odd parity phase shifts calculated

Table IV. Triplet odd parity uncoupled phase shifts. Entries are the nuclear Blatt-Biedenharn phase shifts in radians calculated from (5) and (9).

E (Mev)	${}^3P_0$	${}^3P_1$	${}^3F_3$	${}^3H_5$
10	0.068	−0.039	−0.001	⋯
18.2	0.117	−0.068	−0.002	⋯
19.8	0.124	−0.073	−0.003	⋯
25.63	0.147	−0.090	−0.004	⋯
30.14	0.160	−0.102	−0.005	⋯
39.4	0.177	−0.124	−0.008	−0.001
50	0.184	−0.146	−0.012	−0.001
66	0.179	−0.174	−0.017	−0.002
80	0.166	−0.196	−0.021	−0.003
95	0.147	−0.218	−0.025	−0.005
120	0.108	−0.251	−0.031	−0.006
145	0.065	−0.282	−0.037	−0.008
180	0.003	−0.321	−0.044	−0.011
200	−0.032	−0.343	−0.047	−0.012
240	−0.100	−0.384	−0.054	−0.015
270	−0.150	−0.413	−0.058	−0.017
310	−0.213	−0.450	−0.063	−0.019

Table V. Triplet odd parity coupled phase shifts Entries are the nuclear Blatt-Biedenharn phase shifts in radians calculated from (5) and (9)

E (Mev)	$^3P_2$	$^3F_2$	ϵ_2	$^3F_4$	$^3H_4$	ϵ_4	$^3H_6$	$^3L_6$	ϵ_6
10	0 011	−0.001	−0.319	…	…	…	…	…	…
18 2	0.028	−0.002	−0 336	…	…	…	…	…	…
19.8	0 032	−0.003	−0 336	..	..	…	…	…	…
25 63	0.045	−0 003	−0.332	0 001	−0 001	−0 759	…	…	…
30.14	0.056	−0 004	−0 326	0 001	−0 001	−0 755	…	…	…
39.4	0.078	−0 004	−0.313	0 003	−0 002	−0 748	…	…	…
50	0 103	−0 003	−0.298	0 004	−0 003	−0 741	…	…	…
66	0.139	−0 002	−0 275	0 007	−0 004	−0 730	…	…	…
80	0.168	0 002	−0 257	0 009	−0 005	−0 720	0 001	−0.001	−0 804
95	0 196	0 003	−0 239	0.012	−0 006	−0 708	0 002	−0 002	−0 792
120	0 234	0 006	−0 213	0.017	−0 008	−0 684	0 003	−0 003	−0.792
145	0 264	0 010	−0 191	0 022	−0 008	−0 656	0.004	−0 004	−0 793
180	0 293	0.013	−0 164	0 031	−0 008	−0 609	0 006	−0.005	−0.795
200	0 304	0 014	−0.151	0 036	−0 008	−0 579	0 007	−0 006	−0 795
240	0.317	0 013	−0 126	0 047	−0 006	−0 514	0 009	−0 007	−0 795
270	0 319	0 011	−0 110	0 057	−0 004	−0 464	0.010	−0 008	−0.793
310	0.316	0 007	−0 090	0 070	−0.002	−0 413	0 012	−0.008	−0.788

from (5) and (9) at various energies. We notice that our potential predicts the phase shift solution type b of MacGregor and Moravcsik at 210 Mev.[23]

§ 5. Singlet even parity potential

The Schrodinger equation for the singlet even parity state reads

$$[d^2/dx^2+\kappa^2-J(J+1)/x^2-U_C(x)+J(J+1)\,U_Q(x)]u(x)=0. \qquad (10)$$

Previous calculations on the zero energy n-p scattering parameters[18] have provided a convenient basis for the determination of ${}^1a_C^+$, ${}^1b_C^+$ and ${}^1x_0^+$ in (4). The combination

$$g^2/4\pi=0.08,\ \ {}^1x_0^+=0.337,\ \ {}^1a_C^+=10,\ \ {}^1b_C^+=8, \qquad (11)$$

was found to give the zero energy scattering length -17.5×10^{-13} cm, the effective range 2.85×10^{-13} cm and at the same time the ${}^1S_0$ phase shift of -0.150 at 310 Mev (see Table I).

The ${}^1D_2$, ${}^1G_4$ and ${}^1I_6$ phase shifts calculated from (11) (with ${}^1V_Q^+\equiv0$) are shown in Table I. It is clear from (10) that a weak attractive (negative) ${}^1V_Q^+(x)$ can improve the fit to MMS 1. Indeed, it was easy to fit the ${}^1D_2$ phase shift of MMS 1 exactly. It was found, however, that the ${}^1D_2$ phase shifts calculated at lower energies, particularly at 145 Mev where accurate data are available, from such a potential turned out too small.* After some readjustment we decided on the set

$$^1G_Q^+ = -0.00155, \quad ^1a_Q^+ = 8 \quad \text{and} \quad ^1b_Q^+ = 6. \tag{12}$$

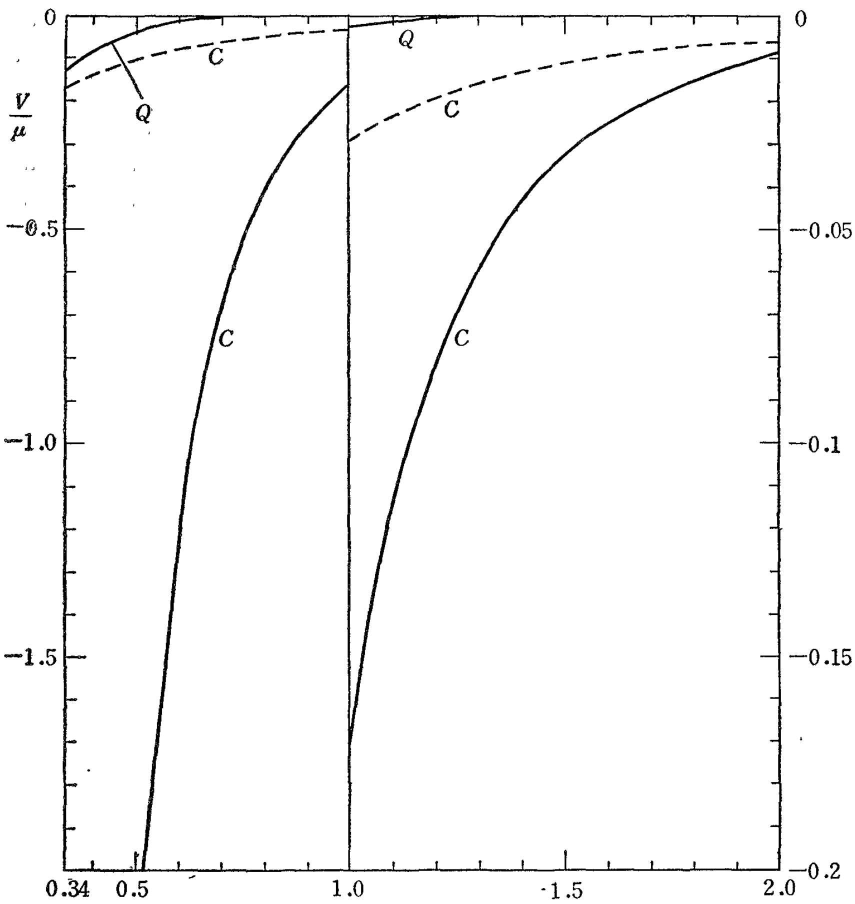

Fig. 2. The singlet even parity potential given by (4), (11) and (12). The dashed curve represents the OPEP with $g^2/4\pi = 0.08$.

The potential determined by (11) and (12) is shown in Fig. 2. The phase shifts are given in Table VI. We again note that our phase shifts at 210 Mev are of type *b* of MacGregor et al.[23]

Unlike in the triplet odd states, the V_Q is really needed in the singlet even state. As already noticed by GT, the $^1D_2$ phase shift at 310 Mev is largely determined by the shape independent parameters at zero energy and the 310 Mev $^1S_0$ phase shift, and must be about 0.26 as long as the interaction is assumed to be the same in $^1S_0$ and $^1D_2$ states. Such a large $^1D_2$ phase shift gives rise to an appreciable forward peak in the differential cross section at 310 Mev. Similar arguments hold for the $^1G_4$ phase shift.

* It is likely that this difficulty is due to an inadequate functional form of $^1V_Q^+$ assumed in (4).

Table VI. Singlet even parity phase shifts. Entries are the Blatt-Biedenharn phase shifts in radians calculated from (4), (11) and (12).

E (Mev)	${}^1S_0$	${}^1D_2$	${}^1G_4$	${}^1I_6$
10	1.000	0.003	…	…
18.2	0.912	0.008	…	…
19.8	0.897	0.009	…	…
25.63	0.844	0.014	…	…
30.14	0.806	0.017	…	…
39.4	0.739	0.024	0.001	…
50	0.670	0.032	0.002	…
66	0.582	0.045	0.003	…
80	0.514	0.057	0.004	…
95	0.448	0.069	0.005	…
120	0.350	0.090	0.007	…
145	0.265	0.114	0.010	0.001
180	0.159	0.139	0.012	0.001
200	0.104	0.154	0.013	0.001
240	0.004	0.184	0.017	0.001
270	−0.065	0.204	0.019	0.001
310	−0.150	0.228	0.021	0.002

In this connection it is interesting to note a remark by Signell[19] regarding the non-static effect in the nuclear forces. Using the formulation in momentum space, he noticed that one of the non-static effects is to reduce (in magnitude) the OPEP tail at high energies. Sugawara[20] argues to the same effect in connection with his recent calculations with Okubo.[16] We see that the introduction of our weak negative ${}^1V_Q^+$ has just such an effect. At low energies when the S-wave scattering predominates and all the higher partial waves have large impact parameters, ${}^1V_Q^+$ has no effect. As the energy increases, ${}^1V_Q^+$ becomes effective but the effect is felt more strongly by higher partial waves (see (10)) which are scattered by the tail of the potential. The Signell effect must also be felt by the S-wave at high energies. However, since the S-wave already feels very strong attraction in the core region (see (11)), the small reduction in the OPEP tail should be relatively unimportant.

We did not find a conclusive evidence for ${}^3V_Q^-$. This probably means that the Signell effect is masked by the strong ${}^3V_{LS}^-$ in the triplet odd parity state. It is quite natural that we found ${}^1V_Q^+$ is really needed since here $V_{LS}=0$.

Bryan[8] has not considered any non-static potential in the singlet even parity states. We believe that an appropriate ${}^1V_Q^+$ added to his potential will improve the fit to the data considerably

§ 6. Observable quantities

Using the phase shifts given in Tables IV, V and VI, we have calculated

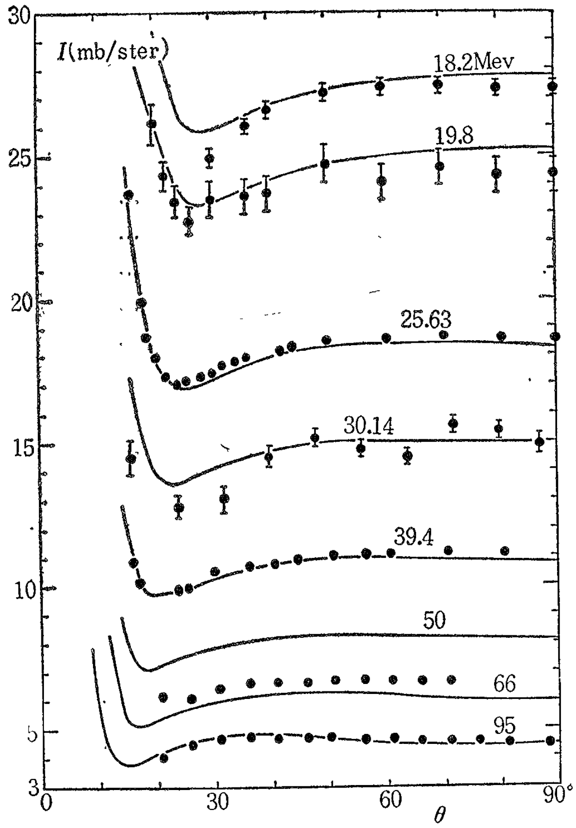

Fig. 3. p-p differential cross sections below 100 Mev. Experimental points are taken from references 24, 25, 26 and 27.

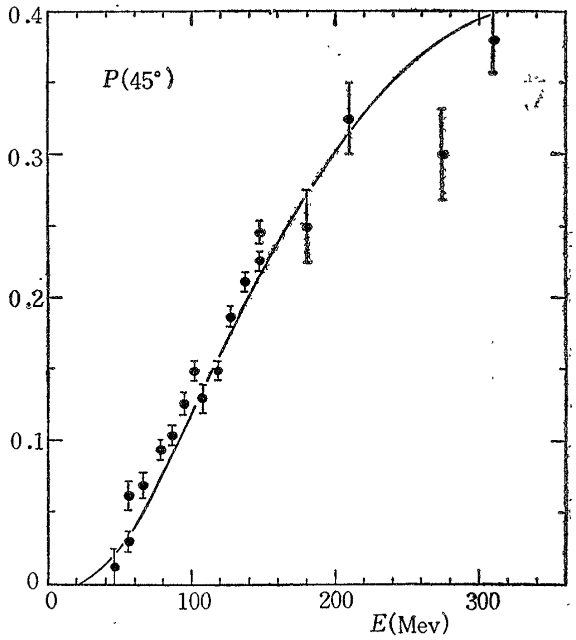

Fig. 5. Plot of $P(45°)$ *vs* energy. Experimental points are taken from references 17, 24, and 25.

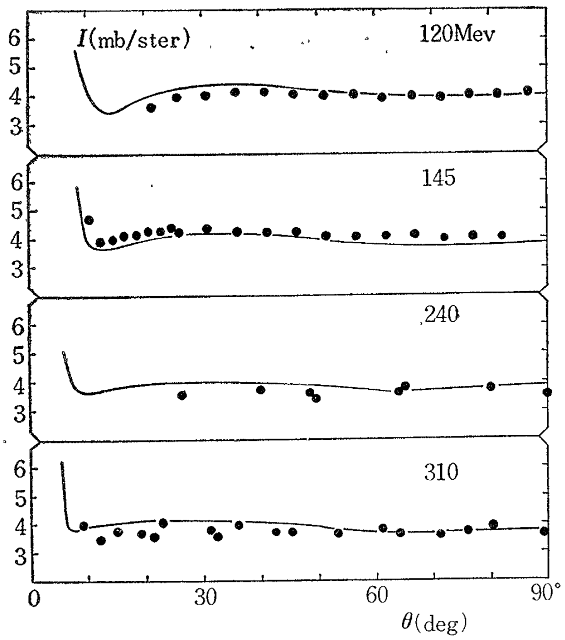

Fig. 4. p-p differential cross sections above 100 Mev. Experimental points are taken from references 17, 24, and 25.

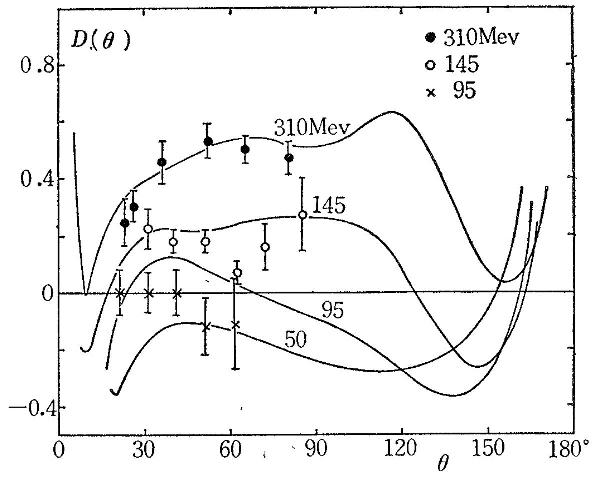

Fig. 6. Plot of D *vs* θ at several energies. Experimental points are taken from references 17, 21, and 28.

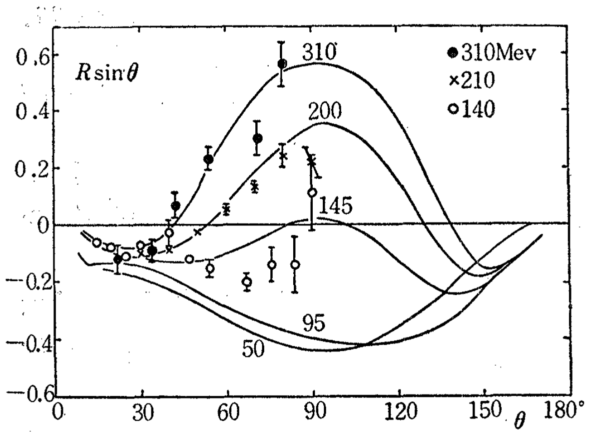

Fig. 7. Plot of $R\sin\theta$ *vs* θ at several energies. Experimental points are taken from references 13, 17, and 29.

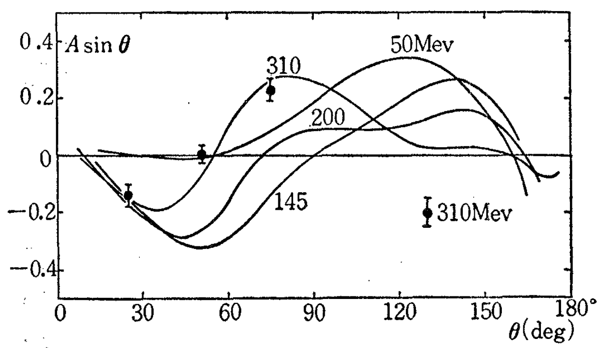

Fig. 8. Plot of $A\sin\theta$ *vs* θ at several energies. Experimental points are taken from reference 17.

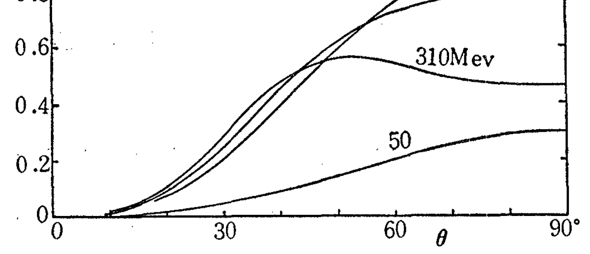

Fig. 9. Plot of $C_{kp}\sin\theta$ *vs* θ at several energies.

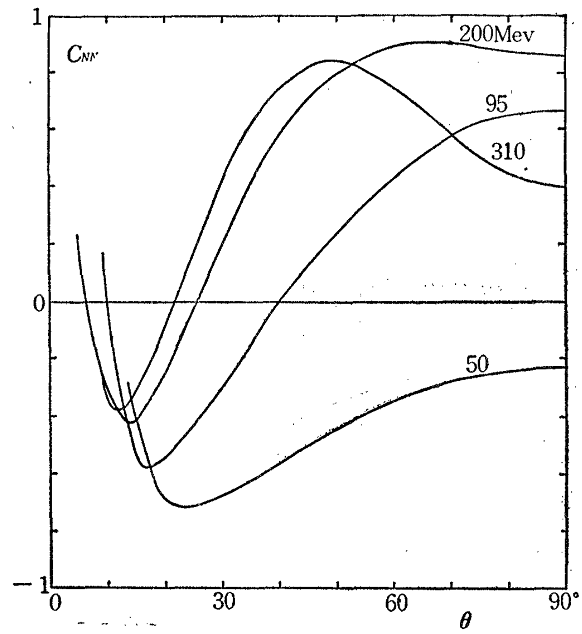

Fig. 10. Plot of C_{nn} *vs* θ. Relevant experimental data are[29]: C_{nn}(315 Mev, 90°) $=0.52\pm0.20$ (Dubna) and C_{nn}(320 Mev, 90°) $=0.75\pm0.11$ (Liverpool).

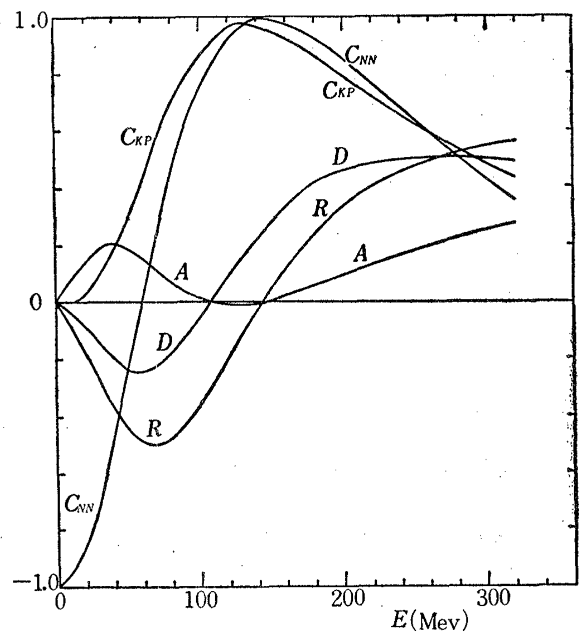

Fig. 11. Plot of triple scattering and spin correlation parameters at 90° *vs* E.

the complete experiment parameters. Some of them are shown in Figs. 3 to 11. In view of the wide energy range covered the fit to the data is remarkable.

Regarding the polarization, we have only shown P (45°) as the function of energy. The angular distribution of P is in general in good agreement with the experimental data The largest deviation from the dada occurs at 145 Mev where the calculated P is smaller than the observed by about 0.04 in the 15°$\sim$35° region.

At 145 Mev, the calculated $D(\theta)$ agrees with the Harvard data [21] According to Nigam,[22] this is due to our small $^3P_0$ phase shift which, in turn, is a consequence of the strong negative $^3V_{LS}^-$ as discussed in §4.

An interesting feature of our potential can be seen from Fig. 11. Our potential predicts $C_{nn}(90°)\simeq 1$ at about 140 Mev indicating that there is no contribution to the scattering from the singlet state at this energy and angle.* Consequently, $R(90°)\simeq A(90°)\simeq 0$ at the same energy. $R(90°, 140\text{ Mev})=0$ is consistent with the recent measurements.[13] Fortunately, it appears that the Harvard and Harwell groups are both working on triple scattering measurements at the energy close to 140 Mev. It will be very interesting to see whether further measurements realize our prediction. If they do, we should be able to get valuable information on the singlet even parity phase shifts at this energy. It may become possible to exclude one or the other of MMS solutions on that ground.

§ 7. Conclusions

We have seen that the singlet even and triplet odd parity potentials defined by (1), (4), (5), (9), (11) and (12) can reproduce all available p-p data up to 310 Mev. At 310 Mev our potential predicts the phase shifts solution 1 of MacGregor et al.[14]

As far as the tail is concerned, our potential has the OPEP tail at low energies. At higher energies it appears that the OPEP tail has to be reduced by small amount. This effect is particularly manifest in the singlet even parity states. Such an effect can be incorporated into the energy independent potential picture in the form of a weak but long ranged quadratic spin-orbit potential.

The LS potential required in the precise fit at 310 Mev is more singular but of shorter range than those of GT, SM and SM1.** It is satisfying that this feature of the LS potential is in better agreement with the predictions of the pion theory of nuclear forces.

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* This can be seen from the expressions for observable quantities in terms of various scattering amplitudes See reference 30)

** In this connection we feel that, if any substantial improvement is to be attempted over our potential, the functional form of $^3V_{LS}^-$ will have to be modified The modification must be in such a way to make the $^3V_{LS}^-$ considerably weaker at $x\gtrsim 1$ without having recourse to the repulsive part in it Such a modification may well lead to the $^3V_{LS}^-$ considered by Bryan [8]

Our central and tensor potentials are consistent with the pion theory of nuclear forces in the sense that they satisfy the conditions (6). The *LS* potential now appears to be not inconsistent with the pion theory. Further, there is some pion theoretical evidence for the weakening of the OPEP tail at higher energy.[19] Thus we are led to the conclusion that the proton-proton scattering up to 310 Mev can be understood in terms of the energy independent potential which is consistent with the pion theory of nuclear forces in every respect. We shall consider the same question in the neutron-proton scattering in a separate paper.

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References

1) L. Puzikov, R. Rydin and J. Smorodinsky, Nucl. Phys. **3** (1957), 436.
2) S. Okubo and R. E. Marshak, Ann. Phys. 4 (1958), 166.
3) J. L. Gammel and R. M. Thaler, Phys. Rev. **107** (1957), 291.
This will be referred to as GT.
4) S. Otsuki, Prog. Theor. Phys. **20** (1958), 171.
R. Tamagaki, Prog. Theor. Phys. **20** (1958), 505.
5) P. S. Signell and R. E. Marshak, Phys. Rev. **109** (1958), 1229.
This will be referred to as SM.
6) P. S. Signell R. Zinn and R. E. Marshak, Phys. Rev. Letters **1** (1958), 416.
This will be referred to as SM1.
7) S. Gartenhaus, Phys. Rev. **100** (1955), 900.
8) R. A. Bryan, Nuovo Cimento **XVI** (1960), 895.
9) J. Iwadare, S. Otsuki, R. Tamagaki and W. Watari, Suppl. Prog. Theor. Phys. No. **3** (1956), 32.
Also see reference 4) and W. Watari, Prog. Theor. Phys. **20** (1958), 181.
10) T. Hamada, J. Iwadare, S. Otsuki, R. Tamagaki and W. Watari, Prog. Theor. Phys. **22** (1959), 566.
11) T. Hamada, J. Iwadare, S. Otsuki, R. Tamagaki, and W. Watari, Prog. Theor. Phys. **23** (1960), 366.
12) M. Taketani, S. Nakamura and M. Sasaki, Prog. Theor. Phys. **6** (1951), 581.
M. Taketani, Suppl. Prog. Theor. Phys. No. **3** (1956), 1.
13) L. Bird, D. N. Edwards, B. Rose, A. E. Taylor and E. Wood, Phys. Rev. Letters. **4** (1960), 302.
E. Thorndike, J. Lefrancois and R. Wilson, Bull. Am. Phys. Soc. **5** (1960), 282.
14) M. H. MacGregor, M. J. Moravcsik and H. P. Stapp, Phys. Rev. **116** (1959), 1284.
15) R. J. N. Phillips, Reports on Progress in Physics **XXII** (1959), 562.
(Physical Society, London).

16) M. Sugawara and S. Okubo, Phys. Rev. **117** (1960), 605, 611.
17) O. Chamberlain, E. Segre, R. D. Tripp, C. Wiegand and T. Ypsilantis, Phys. Rev. **105** (1957), 288.
18) T. Hamada, Prog. Theor. Phys. **24** (1960), 126.
19) P. S. Signell, Prog. Theor. Phys. **22** (1959), 492.
20) M. Sugawara, private communication.
21) C. F. Hwang, T. R. Ophel, E. H. Thorndike, R. Wilson and N. F. Ramsey, Phys. Rev. Letters **2** (1959), 310.
22) B. P. Nigam, Prog. Theor. Phys. **23** (1960), 61.
23) M. H. MacGregor and M. J. Moravcsik, Phys. Rev. Letters **4** (1960), 524.
24) W. N. Hess, Rev. Mod. Phys. **30** (1958), 368.
25) J. N. Palmieri, A. M. Cormak, N. F. Ramsey and R. Wilson, Ann. Phys. **5** (1958), 299.
26) T. H. Jeong, L. H. Johnston and D. E. Young, Phys. Rev. **118** (1960), 1080.
27) L. H. Johnston and D. A. Swanson, Phys. Rev. **111** (1958), 212.
28) E. H. Thorndike and T. R. Ophel, Phys. Rev. **119** (1960), 362.
29) Y. Smorodinsky, Kiev Conference, 1959.
30) H. P. Stapp, T. Ypsilantis and N. Metropolis, Phys. Rev. **105** (1957), 302.