A sequential algorithm for testing climate regime shifts

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[1] Empirical studies of climate regime shifts typically use confirmatory statistical techniques with an a priori hypothesis about the timing of the shifts. Although there are methods for an automatic detection of discontinuities in a time series, their performance drastically diminishes at the ends of the series. Since all the methods currently available require a substantial amount of data to be accumulated, the regime shifts are usually detected long after they actually occurred. The proposed sequential algorithm allows for early detection of a regime shift and subsequent monitoring of changes in its magnitude over time. The algorithm can handle the incoming data regardless whether they are presented in the form of anomalies or absolute values. It can be easily used for an automatic calculation of regime shifts in large sets of variables. *INDEX TERMS:* 1620 Global Change: Climate dynamics (3309); 1694 Global Change: Instruments and techniques; 1635 Global Change: Oceans (4203); 3309 Meteorology and Atmospheric Dynamics: Climatology (1620); 3394 Meteorology and Atmospheric Dynamics: Instruments and techniques. **Citation:** Rodionov, S. N. (2004), A sequential algorithm for testing climate regime shifts, *Geophys. Res. Lett.*, *31*, L09204, doi:10.1029/2004GL019448.

1. Introduction

[2] The notion that climate variations often occur in the form of "regimes" began to become appreciated in the 1990s. This paradigm was inspired in large part by the rapid change of the North Pacific climate around 1977 [e.g., *Kerr*, 1992] and the identification of other abrupt shifts in association with the Pacific Decadal Oscillation (PDO) [*Mantua et al.*, 1997]. A number of methods have been developed to detect a regime shift, or discontinuity, in time series. Typically, these methods employ standard statistical techniques, such as the Student's or Mann-Kendall tests, or their modifications. A review of those methods is provided by *Easterling and Peterson* [1995], who developed their own method. *Lanzante* [1996] discusses two difficult problems in regime shift detection caused by the existence of multiple shift points and trends in time series and appears to have found solutions for both. Having compared his method (hereafter referred to as the L method) with that of *Easterling and Peterson* [1995], he concludes that the two methods are comparable overall, although the L method is a bit more sensitive (can detect weaker discontinuities) and performs better if the discontinuities are well separated in time. *Lanzante* [1996] notes, however, that his method cannot be used within 10 points of the ends of time series.

[3] This problem—deterioration of the test statistics toward the ends of time series—is common for all the methods reviewed. Practically it means that in order to detect a regime shift with a certain degree of confidence it is necessary to accumulate enough data (at least 10 or more years) to apply a formal statistical test. By the time these data are available, the system may be close to shifting once again to the opposite state. Therefore it is critically important to find a tool that allows for estimating the probability of a regime shift with a minimum delay and then monitoring how this probability changes over time.

[4] One possible approach to this problem is to use a sequential data processing technique. In sequential analysis the number of observations is not fixed. Instead, observations come in sequence. For each new observation a test is performed to determine the validity of the null hypothesis H_0 (in our case the existence of a regime shift). There are three possible outcomes of the test: accept H_0, reject H_0, or keep testing. A simple algorithm based on this very general idea is described below.

2. Algorithm

[5] *Step 1*. Set the cut-off length l of the regimes to be determined for variable X. The parameter l is similar to the cut-off point in low-pass filtering. More information on how to choose l will be given below.

[6] *Step 2*. Determine the difference *diff* between mean values of two subsequent regimes that would be statistically significant according to the Student's t-test:

$$diff = t\sqrt{2\sigma_l^2/l},$$

where t is the value of t-distribution with $2l - 2$ degrees of freedom at the given probability level p. Here we assume that the variances for both regimes are the same and equal to the average variance σ_l^2 for running l-year intervals in the time series of variable X.

[7] *Step 3*. Calculate the mean $\bar{x}_{R1}$ of the initial l values of variable X as an estimate for regime $R1$ and the levels that should be reached in the subsequent l years to qualify for a shift to regime $R2$, $\bar{x}'_{R2} = \bar{x}_{R1} \pm diff$.

[8] *Step 4*. For each new value starting with year $i = l + 1$ check whether it is greater than $\bar{x}_{R1} + diff$ or less than $\bar{x}_{R1} - diff$. If it does not exceed the $\bar{x}_{R1} \pm diff$ range then it is assumed that the current regime has not changed. In this case, recalculate the average $\bar{x}_{R1}$ to include the new value x_i and $l - 1$ previous values of variable X and wait for the next value to come. If the new value x_i exceeds the $\bar{x}_{R1} \pm diff$ range, then this year is considered as a possible start point j of the new regime $R2$.

[9] *Step 5*. After the shift point is established, each new value of x_i, where $i > j$, is used to confirm or reject the null

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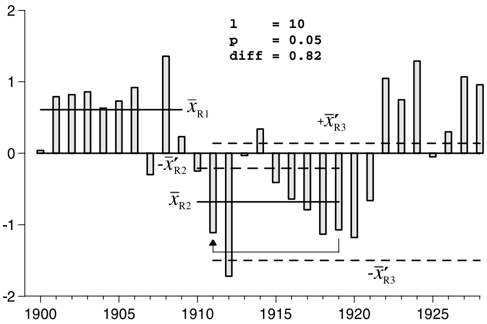

Figure 1. Example illustrating the work of the regime shift detection algorithm for the January PDO index. See text for details.

hypothesis of a regime shift at year j. If the anomaly $x_i - \bar{x}'_{R2}$ is of the same sign as the one at the time of a regime shift, it would increase the confidence that the shift did occur. The reverse is true if the anomalies have the opposite signs. This change in the confidence of a regime shift at $i = j$ is reflected in the value of the regime shift index (RSI), which represents a cumulative sum of the normalized anomalies:

$$\mathrm{RSI}_{i,j} = \sum_{i=j}^{j+m} \frac{x_i^*}{l\sigma_l}, m = 0, 1, \ldots, l-1.$$

Here $x_i^* = x_i - \bar{x}'_{R2}$ if the shift is up, or $x_i^* = \bar{x}'_{R2} - x_i$ if the shift is down. If at any time from $i = j + 1$ to $i = j + l - 1$ the RSI value turns negative, proceed to step 6, otherwise proceed to step 7.

[10] *Step 6*. The negative value of RSI means that the test for a regime shift at year j failed. Assign zero to RSI. Recalculate the average value $\bar{x}_{R1}$ to include the value of x_j and keep testing the values of x_i starting with $i = j + 1$ for their exceedence of the range $\bar{x}_{R1} + diff$ as in step 4.

[11] *Step 7*. The positive value of RSI means that the regime shift at year j is significant at the probability level p. Calculate the actual mean value for the new regime $\bar{x}_{R2}$. At this point, it becomes the base one, against which the test will continue further. The search for the next shift to regime $R3$ starts from year $i = j + 1$. This step back is necessary to make sure that the timing of the next regime shift is determined correctly even if the actual duration of regime $R2$ was less than l years. The calculations continue in a loop from step 4 through step 7 until all the available data for variable X are processed. If there are several variables, the final RSI is the average of RSIs for each variable.

3. An Example

[12] Figure 1 illustrates how the algorithm works for the January PDO index. In this example, the cut-off regime length $l = 10$ and the probability level $p = 0.05$. For this probability level and $2l - 2 = 18$ degrees of freedom the critical value of the Student's t-distribution $t = 2.1$ (two-tailed test). Based on the entire available data set, 1900–2003, the average variance for running 10-year intervals $\sigma_{10}^2 = 0.76$. From this, $diff = 2.1\sqrt{2 \cdot 0.76/10} = 0.82$. The mean value of the PDO index for the first 10 years (1900–1909) is $\bar{x}_{R1} = 0.61$. Therefore, the mean value for regime $R2$, $\bar{x}'_{R2}$, should be either greater than $0.61 + 0.82 = 1.43$ or less than $0.61 - 0.82 = -0.21$ to qualify for a significant regime shift. In 1910 the PDO index was -0.25 and this year is considered as the starting point of a new regime with $\mathrm{RSI}_{1910,1910} = (0.25 - 0.21)/0.87/10 = 0.004$. Due to strongly negative values of the PDO index in 1911 (-1.11) and 1912 (-1.72), the regime shift index increases to $\mathrm{RSI}_{1912,1910} = 0.28$. Although in the next two years (1913 and 1914) the RSI somewhat decreases, it remains positive and the calculation continue until 1919. The final index value for regime $R2$ is $\mathrm{RSI}_{1919,1910} = 0.54$.

[13] The search for a shift to regime $R3$ starts from 1911 using the mean value for regime $R2$, $\bar{x}_{R2} = -0.68$, as the base level. The values of the PDO index are checked whether they are greater than $0.82 - 0.68 = 0.14$ or lower than $-0.68 - 0.82 = -1.50$. Since the index value for 1911 (-1.11) is not outside this range, the test of this year fails. In 1912 the value of the PDO index (-1.72) is below the negative threshold; hence, this year is marked as the beginning of a new regime with $\mathrm{RSI}_{1912,1912} = (1.72 - 1.50)/0.87/10 = 0.02$. In 1913, however, the regime shift index becomes negative, $\mathrm{RSI}_{1912,1913} = 0.02 + (0.03 - 1.50)/0.87/10 = -0.15$, and 1912 is no longer considered as the year of a regime shift. A similar situation occurs in

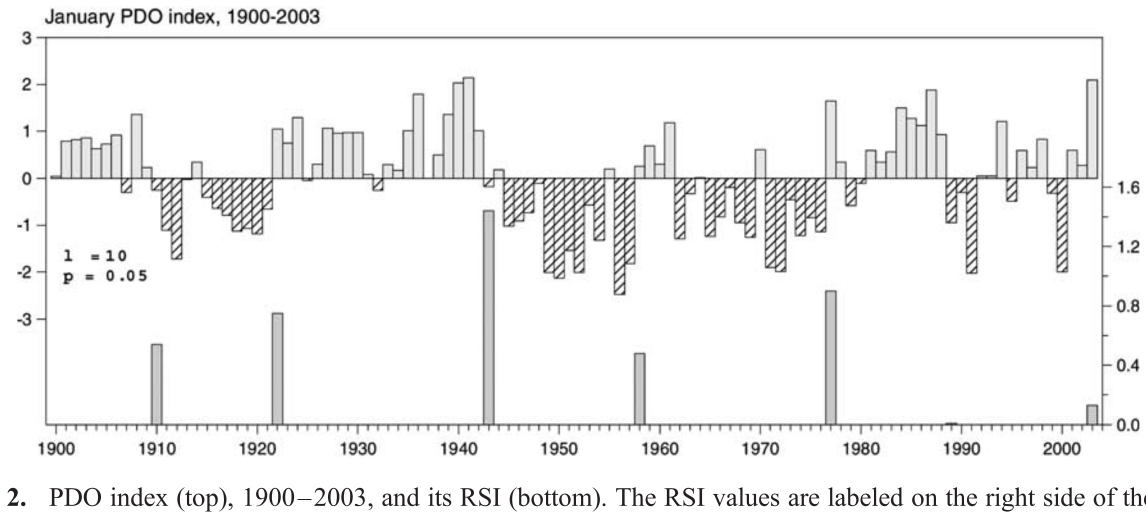

Figure 2. PDO index (top), 1900–2003, and its RSI (bottom). The RSI values are labeled on the right side of the figure.

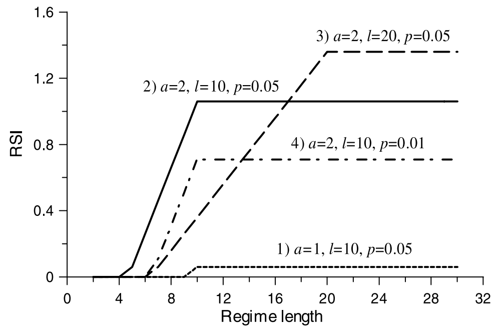

Figure 3. Theoretical RSI values for a shift of magnitude a in a random $N(0, 1)$ time series as a function of the regime length and different cut-off length l and probability level p.

1914. This year is first marked as a potential regime shift upward, but the test fails in 1915 when the RSI becomes negative. A sustainable positive shift occurs in 1922, for which the final value of the regime shift index is $RSI_{1931,1922} = 0.75$. Figure 2 shows the result of the calculations for the entire time series of the January PDO index.

[14] Since the algorithm keeps track of directions of regime shifts, the sign of the final RSI values can be changed accordingly. In some situations, however, it is preferable to know just the magnitude of the shifts regardless of their signs. For example, examining regime shifts in a set of 100 physical and biological time series, *Hare and Mantua* [2000] had to reverse some series so that they all changed in the same direction around 1977, and then did it again for the shift of 1989. Using the regime shift index eliminates the necessity in such manipulations with time series, which may be quite subjective.

4. Parameters Affecting the RSI Value

[15] A correct interpretation of the results obtained by this method requires an understanding of how such variable parameters as the cut-off length l and probability level p can affect the RSI values. The cut-off length l determines the minimum length of the regimes, for which the magnitude of the shifts remains intact. Figure 3 shows the RSI theoretical values for shifts of magnitude a in normally distributed random time series with the zero mean and unit standard deviation. If the cut-off length $l = 10$ and probability level $p = 0.05$, the critical difference between the regimes $diff = 2.1\sqrt{0.2} = 0.94$. For a regime shift of one standard deviation, RSI = 1 − 0.94 = 0.06 for all the regimes lasting 10 years or longer (Figure 3, line 1). If the actual regime length L is less than l, then RSI is reduced by $(l - L) * diff$. For a situation described by line 1 it means that RSI becomes zero for all $L < 10$. In other words, all the regime shifts with a magnitude of one standard deviation or less will be filtered out if they last less than 10 years.

[16] Line 2 characterizes a similar situation, but for regime shifts with a magnitude of two standard deviations. As one can expect, when the magnitude of a shift increases, the regimes shorter than l years also can pass the test (in this case as short as 5 years). It is worth noting that, although the magnitude of the shifts for those shorter regimes is reduced, their timing is determined correctly.

[17] As cut-off length l increases, the degrees of freedom also increases, which means that the statistically significant difference between the mean values of two successive regimes becomes smaller. This translates into higher values of the RSI for the regimes of l years or longer, as can be seen in Figure 3 by comparing lines 2 and 3. In contrast, the lower the probability level, the higher the difference *diff*, and hence, the lower the RSI value (compare lines 2 and 4).

[18] It is important to note that the algorithm is designed to detect abrupt shifts and may not work if a transition from one regime to another is more gradual. For example, a shift from negative to positive values in the PDO index was detected in 1958 (Figure 2). The subsequent transition back to negative PDO values, however, was too gradual, and there was no year for which the t-test was passed during that transition. Reducing l from 10 to 7 or fewer years would allow for detecting a shift in 1962, whereas increasing l to 15 or more years would result in eliminating the shift from 1958–1961.

5. Regime Shift Dynamics

[19] Working with observed time series, it is important to know the dynamics of those regime shifts that are deemed to have had an appreciable impact in the past. Figure 4 illustrates changes in the RSI values m years after the regime shifts in the January PDO index, where m varies from 1 to l = 10 years. Out of 94 years examined, 32 (or 34%) were marked as possible start years of new regimes, which is close to what can be expected by chance. In our Monte Carlo experiment with 10000 random N (0, 1) time series of the same length as the PDO index, 35% of the years were marked as possible start years at $m = 1$. As m increases, the number of those initial regime shifts in the PDO index that survived the test declines, but slower than the exponential decline in the case of random numbers. By year $m = 10$, six regime shifts (or 6.4%) passed the test versus 0.3% expected by chance. These six major shifts were the only ones for which the RSI values were 0.2 or greater by $m = 3$ (Figure 4). Therefore, if a future regime shift in the PDO index reaches this level by $m = 3$, there is a good chance that it will be as important as the other six major regimes. The probability to reach this level in a Gaussian, white noise process is less than 0.05.

[20] It should be emphasized that the method proposed here cannot tell whether the observed regimes are realizations of a Gaussian, red noise process or "true" regimes

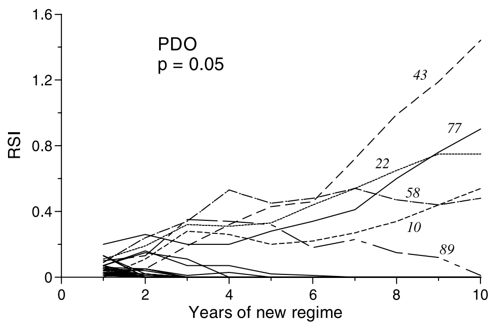

Figure 4. RSI values as a function of years after the regime shifts in the January PDO index. Numbers at the lines are years of the shifts.

Table 1. Years of Regime Shifts Detected by the L and R Methods in the January PDO Index With an Imposed Trend Measured in the Index Units Per Decade

Trend	L Method	R Method
0	43, 77, 22, 10, 58, 39	43, 77, 22, 10, 58, 89
0.1	77, 45, 22, 58, 10, 39, 89, 84, 53	43, 77, 22, 58, 10
0.2	77, 22, 43, 10, 58, 35, 39, 89, 84, 94, 65	43, 77, 22, 58, 11
0.3	58, 77, 22, 45, 11, 35, 83, 89, 94, 65, 13	77, 43, 22, 58, 11
0.4	58, 77, 22, 45, 10, 35, 84, 62, 89, 93, 13	77, 22, 45, 58, 11
1.0	58, 27, 77, 45, 94, 62, 21, 35, 39, 53, 83	77, 22, 58, 35, 45

with different statistics. This is a general property of the currently available methods of regime shift detection. For example, *Rudnick and Davis* [2003] have demonstrated that the composite method used by *Hare and Mantua* [2000] to identify regime shifts is likely to find them in red noise as well. *Percival et al.* [2001] applied two models to the North Pacific index, which exhibits the same regime shifts as the PDO index: (1) first-order autoregressive, or red noise, model and (2) fractionally differenced (FD) model. They found that, although the FD model tends to have even more of a regime-like behavior than the red noise, statistical tests cannot distinguish the superiority of one model over the other.

6. Comparison With the L Method

[21] *Lanzante* [1996] developed a robust, non-parametric procedure of regime shift detection. In particular, his L method is designed to be less sensitive to the presence of trends than other methods. If a trend is present in a time series, it may create a serious problem because it is easy to falsely identify as a shift point the center of this time series. The method described above (hereafter referred to as the R method) and the L method were compared in their performance by applying them to the January PDO index with an imposed trend of an increasingly steep slope. This test makes sense because the PDO index represents the leading principal component of sea-surface temperature anomalies in the North Pacific north of 20°N with the removed global warming signal. It would be interesting to see, therefore, whether the methods are able to detect the same shifts in the PDO index when they occur on the background of a warming trend.

[22] Table 1 lists the years of regime shifts detected by these two methods. The shifts for the L method are given in the order they were chosen, and for the R method in the order of the RSI value, from the highest to the lowest. The R method was applied using $p = 0.05$ and $l = 10$. The trend is positive, measured in the units of the PDO index per decade. In the stationary case (no trend added), both methods produce almost the same sequence of shifts, except the last one. As the trend gets steeper, the upward shifts become more prominent than the downward shifts. When the trend is 0.1 units per decade, the primary shift detected by the L method occurs in 1977. When the trend reaches 0.3 per decade, another upward shift of 1958 becomes the primary one. In the R method, the shift of 1977 also becomes the primary one, but it occurs only when the trend reaches 0.3 units per decade. The shift of 1958 does not reach the primary status at all for the trends tested. Also, the L method detects a large number of shifts that are not present in the stationary series, which is not the case with the R method. Even when the trend is 1 unit per decade, which means that it accounts for about 90% of the total variance of the time series, the R method performs reasonably well.

7. Summary

[23] Statistical studies of climate regime shifts typically involve some sort of confirmatory or hypothesis-driven analysis. Such an analysis requires an a priori hypothesis that a regime shift has occurred at a certain time. Then using a statistical test (e.g., the Student's t-test) this hypothesis is either rejected or confirmed. In contrast, the sequential analysis described here belongs to the category of exploratory or data-driven analysis that does not require an a priori hypothesis on the timing of regime shifts. This greatly facilitates an application of the algorithm for automatic computations, when the number of variables processed can be practically unlimited. Another advantage of the algorithm is that it can handle the incoming data regardless whether they are presented in the form of anomalies or absolute values. This eliminates the necessity to select the base period to calculate anomalies, which is a source of ambiguity that affects the timing and scale of the regimes.

[24] A comparison with a previous method for an automatic detection of multi-point discontinuities, the L method [*Lanzante*, 1996], shows that the two methods produce approximately equal results when the time series contain no trend. The R method, however, appears to yield more consistent results than the L method when the regime shifts occur on the background of a long-term trend.

[25] The most important feature of the proposed method may be its ability to detect a regime shift relatively early and then monitor how its magnitude changes over time. The user can adjust the setting parameters so that the method will detect those shifts that have meaningful environmental and biological implications.

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