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A set of decentralized PID controllers for an *n* – link robot manipulator

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Abstract. A class of stabilizing decentralized proportional integral derivative (PID) controllers for an n-link robot manipulator system is proposed. The range of decentralized PID controller parameters for an n-link robot manipulator is obtained using Kharitonov theorem and stability boundary equations. Basically, the proposed design technique is based on the gain-phase margin tester and Kharitonov's theorem that synthesizes a set of PID controllers for the linear model while nonlinear interaction terms involve in system dynamics are treated as zero. The stability analysis of the composite system with the designed set of decentralized PID controllers is investigated by incorporating bounding parameters of interconnection terms in LMI formulation. From the range of controller gains obtained by the proposed method, a genetic algorithm is adopted to get an optimal controller gains so that the tracking error is minimum. Simulation results are shown to demonstrate the applicability of the proposed control scheme for solution of fixed as well as time-varying trajectory tracking control problems.

Keywords. Kharitnov theorem; decentralized controller; robot manipulator; tracking error; linear matrix inequality; interconnected system.

1. Introduction

To improve the tracking performance of an interconnected system, the decentralized controller has been widely accepted in industry due to ease of implementation, low cost hardware set-up and tolerance to failure. Decentralized control schemes have been shown to be robust to a wide range of parametric and nonlinear time-varying uncertainties with an added benefit of being possible to implement with parallel processors. Thus, giving enhanced computational speed. The solution of decentralized tracking control problem for robot manipulator is slightly complex since we cannot split the overall system into several subsystems whose states and input

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torques are not isolated from each other due to the coupling caused by nonlinear inertial terms. Among the works reported in this area are Narendra & Oleng (2002), Tang *et al* (2000), Tarokh (1996), Tang & Guerrero (1998), Wang & Wend (1999), Liu (1999) and Seraji (1989). Narendra & Oleng (2002) have shown that in strictly decentralized adaptive control systems, it is theoretically possible to track desired outputs with zero error. Tang *et al* (2000) and Tarokh (1996) have presented an asymptotically stable decentralized adaptive control scheme to enable accurate trajectory tracking. Tang & Guerrero (1998) obtained an extremely simple controller, consisting of a linear state-feedback with an additional signal designed to compensate for the coupling among the joints, parameter uncertainty and bounded disturbances. In Wang & Wend (1999) the dynamics of the subsystems are divided into two parts: a nominal system and uncertainties. Based on the nominal system and the bounds on the uncertainties, the Riccati equation approach is used to control the motion of robot manipulators. A set of nonlinear decentralized tracking error for each subsystem is formulated so that the passivity property of robot dynamics can be used in the controller design (Liu 1999). For manipulator tracking tasks, decentralized approaches are not that straight forward since the overall system cannot be decomposed into subsystems whose states and control inputs are not totally decoupled from one another because of the inherent coupling such as moment of inertia and Coriolis force. Several attempts have been made to control independently each robot joint attached to actuator by processing local measurements available from that joint. As a result, how to improve the tracking performance of robots through decentralized control is still an interesting topic in control literature.

In this paper, a class of decentralized stabilizing controllers is designed for an n-link robot manipulator using Kharitonov's theorem and boundary stability condition for an interaction free subsystem. The control objective is to achieve accurate tracking of desired joint trajectories. The significant results of Siljak & Stipanovic (2000) demonstrate how the Linear Matrix Inequalities (LMIs) formulation can be used to quadratically stabilize nonlinear interconnected system via decentralized linear constant feedback laws. Motivated by the work of Siljak & Stipanovic, we tried to establish based on LMI approach, how the designed set of decentralized controllers for each subsystems can be utilized to stabilize the interconnected nonlinear systems.

The paper is organized as follows. In section 2, problem formulation and a set of decentralized PID controller for interaction free subsystems have been developed using the basic principles of Kharitonov's theorem and stability boundary condition. The stability analysis of composite system (with interaction terms) using the designed decentralized controller has been studied in section 3 based on LMI formulation. Simulation results are presented in section 4 to demonstrate the effectiveness of the proposed control. Concluding remarks are given in section 5.

2. Controller design based on stability boundary equation and Kharitonov's theorem

The Kharitonov's theorem for linear interval plants is exploited for the purpose of synthesizing a set of stabilizing PID controller to meet design specification in terms of gain margins and phase margins (Huang & Wang 2000). Controller is designed to simultaneously stabilize the four Kharitonov's vertex polynomials. A specific Kharitonov region can be obtained in the parameter plane using the parameters of the controller as the axes. This region constitutes the whole admissible stabilizing PID controllers. The proposed method not only provides a necessary and sufficient condition for a set of interval polynomials, but also fulfills several specifications simultaneously. The concept of stability equation method by Lii *et al* (1993) is employed to plot stability boundary of a system and simultaneously the gain margins (equal to 1) and phase margins (equal to zero) are maintained along the stability boundary in a controller parameter plane or parameter space.

2.1 *Problem formulation*

Let us consider the dynamics of an n-link robot manipulator described by the nonlinear equation

$$\tau = M(\theta)\,\ddot{\theta} + V(\theta, \dot{\theta}) + G\,(\theta)\,, \tag{1}$$

where $M(\theta)$ is an $n \times n$ symmetric positive definite inertia matrix, $V(\theta, \dot{\theta})$ is an $n \times 1$ Coriolis and centrifugal vector, $G(\theta)$ is an $n \times 1$ gravity vector of the manipulator, θ is the $n \times 1$ vector representing joint angular positions, and τ is the $n \times 1$ vector of applied joint torques. For simplicity, it is denoted that

$$N(\theta, \dot{\theta}) = V(\theta, \dot{\theta}) + G\,(\theta)\,. \tag{2}$$

There are uncertainties in $M(\theta)$ and $N(\theta, \dot{\theta})$ due to unknown load on the manipulator and unmodelled frictions. The following bounds are assumed on the uncertainties (Qu & Dawson 1996).

(i) There exist positive definite matrices $M_u(\theta)$ and $M_l(\theta)$ such that

$$M_u\,(\theta) \geq M\,(\theta) \geq M_l\,(\theta) > 0.$$

(ii) There exist $N_u(\theta, \dot{\theta})$ and a nonnegative function $n_{\max}(\theta, \dot{\theta})$ such that

$$\|N_u(\theta, \dot{\theta}) - N(\theta, \dot{\theta})\| \leq n_{\max}(\theta, \dot{\theta}).$$

The state variables to be $x_1 = \theta$, $x_2 = \dot{\theta}$ and the control to be

$$u = M_u\,(\theta)^{-1}\,(\tau - N_u(\theta, \dot{\theta})).$$

Then

$$\begin{aligned} \dot{x}_1 &= \dot{\theta} = x_2 \\ \dot{x}_2 &= \ddot{\theta} = M(\theta)^{-1}(\tau - N(\theta, \dot{\theta}) = M(\theta)^{-1}M_u(\theta)u + M(\theta)^{-1}(N_u(\theta, \dot{\theta}) - N(\theta, \dot{\theta})) \\ &= M(x_1)^{-1}M_u(x_1)u + M(x_1)^{-1}(N_u(x_1, x_2) - N(x_1, x_2)). \end{aligned} \tag{3}$$

The state space representation of (3) is given by

$$\dot{x} = Ax + B(u + f(x)u) + Bh(x), \tag{4}$$

where $f(x) = M(x_1)^{-1}M_u(x_1) - I$, and $h(x) = M(x_1)^{-1}(N_u(x_1, x_2) - N(x_1, x_2))$

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix} \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

$Bf(x)$ is the uncertainty in the input matrix and in order to make its effect maximum $f(x)$ is taken as

$$f(x) = M_l(x_1)^{-1}M_u(x_1) - I \geq 0, \tag{5}$$

where M_l and M_u are the lower and upper bounds of inertia matrix that is found out as explained in Lin & Brandt (1998). In order to make the system in the input decoupled form equation (5) is written as

$$f(x) = daig(\lambda_{\max}(f(x)), \tag{6}$$

which is obtained from the following expression

$$\lambda_{\min}(f)x^T x \leq x^T f x \leq \lambda_{\max}(f)\, x^T x. \tag{7}$$

The uncertainty $h(x)$ has the following bounds (Meressi *et al* 1993):

$$\begin{aligned} \|h(x)\| &= \|M(x_1)^{-1}(N_u(x_1, x_2) - N(x_1, x_2))\| \\ &\leq \|M(x_1)^{-1}\| \times \|N_u(x_1, x_2) - N(x_1, x_2))\| \\ &\leq \|M_l(x_1)\|^{-1} n_{\max}(x_1, x_2). \end{aligned} \tag{8}$$

Although for the $h(x)$ given in (4), $\|h(x)\|$ may not be quadratically bounded, in many cases, we can find out the largest physically feasible region of x and determine a quadratic bound for $\|h(x)\|^2$ such that

$$h(x)^T h(x) \leq x^T Q x, \tag{9}$$

where Q is a positive definite matrix. For the development of the decentralized control scheme, it is convenient to view each joint as a subsystem of the entire manipulator system and state variables are rearranged in (4) and are rewritten as

$$\begin{aligned} \dot{x}_i &= A_{si} x_i + B_{ni} u_i + B_i h_i(x); \\ y_i &= C_{si} x_i, \qquad i = 1, 2, \ldots, n, \end{aligned} \tag{10}$$

where

$$B_{ni} = B_i + B_i\, \lambda_{\max}(f(x)),\ f(x) = M_l(z)^{-1} M_u(z) - I \geq 0,$$

$$u = M_u(z)^{-1}(\tau - N_u(x))$$

$$\|h(x)\| = \left\| M(z)^{-1}\,(N_u(x) - N(x)) \right\|,\ x_i = \begin{bmatrix} x_{i1} \\ x_{i2} \end{bmatrix} = \begin{bmatrix} \theta_i \\ \dot{\theta}_i \end{bmatrix} \text{ and } z = [x_{11} x_{21} \ldots\ldots x_{n1}]^T$$

$$A_{si} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_{si} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

If the variation of $f(x)$ is considered, its minimum value is zero. So the input matrix B_{ni} in equation (10) varies from B_i to B_{ni}. Hence, the transfer function of the ith joint with nonlinear interaction terms $h_i(x) = 0$, i – 1,2,…n becomes

$$G_i(s) = \frac{1}{a_i s^2}, \tag{11}$$

where $a_i \in \left[a_i^- a_i^+\right]$ is the interval parameter associated with the jth joint having $a_i^- = 1/b_{ni}$, $a_i^+ = 1/b_i$. The parameters b_i and b_{ni} are the elements of B_i and B_{ni}, respectively, i.e., $B_i = \begin{bmatrix} 0 \\ b_i \end{bmatrix}$ and $B_{ni} = \begin{bmatrix} 0 \\ b_{ni} \end{bmatrix}$.

Since from equation (11) the plant is an interval plant, a parameter plane method based on the gain-phase margin tester method and Kharitonov's theorem has been adopted in designing a set of stabilizing PID controllers.

2.2 *Design a set of stabilizing PID controller*

Figure 1 shows decentralized control scheme for the ith joint of system (10). A decentralized PID controller for the ith joint is described by

$$\tau_i(t) = K_{pi}e_i(t) + K_{di}\dot{e}_i(t) + K_{ii}\int_0^t e_i(t)dt, \tag{12}$$

where $e_i(t) = \theta_{di}(t) - \theta_i(t)$ is the position tracking error of ith joint, K_{pi}, K_{ii} and K_{di} are respectively the proportional, integral and derivative gains of the ith joint controller, θ_{di} denotes the ith joint desired constant position and θ_i be the actual joint position.

The control problem is to provide a complete solution to the constant gain stabilizing control parameters K_{pi}, K_{ii}, K_{di} such that the position error $e_i(t)$ reduces to zero with time, i.e., $\lim_{t\to\infty} e_i(t) = 0$.

Figure 2 shows the s-domain representation of the ith subsystem given in equation (10) with $h_i = 0\,(w_i = B_i h_i = 0)$. The open loop transfer function can be written as

$$G_{oi}(s) = G_i(s)C_i(s). \tag{13}$$

For $s = j\omega$, we have

$$G_i(j\omega)C_i(j\omega) = \alpha_i e^{j\beta_i}, \tag{14}$$

where $|G_i(j\omega)C_i(j\omega)| = \alpha_i$ and $\angle G_i(j\omega)C_i(j\omega) = \beta_i$.

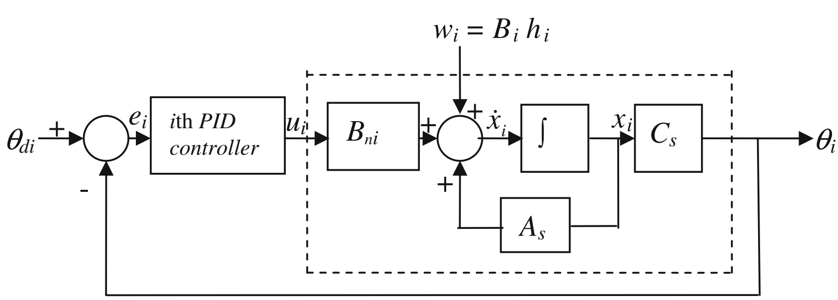

Figure 1. Decentralized control scheme for ith joint.

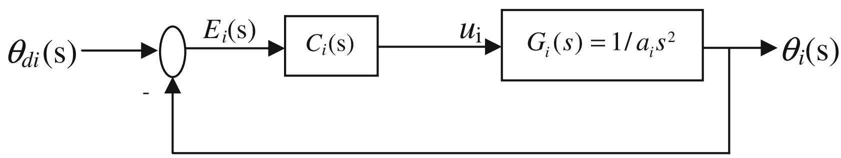

Figure 2. Transfer function model for interaction free ith joint.

Equation (14) can be written in the following form

$$\begin{aligned} 1 + \frac{1}{\alpha_i} e^{-j(180+\beta_i)} G_i(j\omega) C_i(j\omega) = 0, \\ 1 + A_i e^{-j\gamma_i} G_i(j\omega) C_i(j\omega) = 0, \end{aligned} \tag{15}$$

where $A_i = \frac{1}{\alpha_i} = \frac{1}{|G_i(j\omega)C_i(j\omega)|}$ and $\gamma_i = 180 + \beta_i$.

It is noted that A_i is the gain margin of the ith subsystem when $\gamma_i = 0$ and γ_i is the phase margin when $A_i = 1$. More specifically, one can easily determine the gain margin and phase margin of the system by adopting the gain-phase margin tester $A_i e^{-j\gamma_i}$, which can be represented by an additional block in cascade with $G_i(s)C_i(s)$ and shown in figure 3.

The controller $C_i(s)$ is designed to simultaneously stabilize the ith subsystem (9) with $h_i(x) = 0$. Let

$$P_i(s) = 1 + A_i e^{-j\gamma_i} C_i(s) G_i(s). \tag{16}$$

Using the expression for $G_i(j\omega)$ given (11) and $C_i(s) = K_{pi} + K_{ii}/s + K_{di}s$ in characteristic equation (16) we get

$$as^3 + (-K_{pi}s + K_{ii} + s^2 K_{di})(A_i \cos\gamma_i - jA_i \sin\gamma_i) = 0. \tag{17}$$

This is a third order interval polynomial, and it is sufficient to check the stability of one Kharitonov polynomial $P^{+-}(j\omega)$ (Meressi *et al* 1993), which is given below.

$$a^+ s^3 + (-K_{pi}s + K_{ii} + s^2 K_{di})(A_i \cos\gamma_i - jA_i \sin\gamma_i) = 0. \tag{18}$$

The objective here is to find all possible sets of K_{pi}, K_{ii} and K_{di} that make the characteristic equation (18) to be stable. Equating the real and imaginary part of (18) to zero we get two expressions as

$$\omega^3 a^+ - \omega K_{pi} A_i \cos(\gamma_i) + A_i \sin(\gamma_i)(K_{ii} - \omega^2 K_{di}) = 0, \tag{19}$$

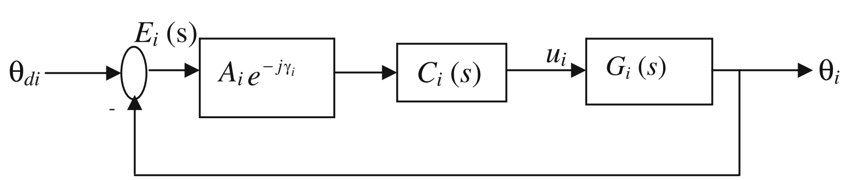

Figure 3. ith joint with controller and gain-phase margin tester.

$$\omega K_{pi} A_i \sin(\gamma_i) + A_i \cos(\gamma_i)(K_{ii} - \omega^2 K_{di}) = 0. \tag{20}$$

It can be noted that the number of controller parameters are more than the number of equations and it is necessary to assign one of the controller parameter (say K_{di}) and the remaining controller parameters are solved from (19) and (20) with conditions of marginal stability i.e., $A_i = 1$ and $\gamma_i = 0$. To have the stable region in the parametric plane, one generally finds the stability boundary first, and then determines the stable region by the sign of $J_j = \partial\mathrm{Re}/\partial K_{pi} \cdot \partial\mathrm{Im}/\partial K_{ii} - \partial\mathrm{Re}/\partial K_{ii} \cdot \partial\mathrm{Im}/\partial K_{pi}$, where Re and Im are real and imaginary parts of equation (18). If the sign of J_j is positive (negative) facing the direction in which ω is increasing, the left (right) side of the stability boundary is the stable region (Siljak 1969). Thus, one can obtain the range of controller parameters (K_{pi}, K_{ii}) for a fixed value of K_{di} from the boundary of the stability region for the system.

The controller is designed based on the linear system, i.e., with $h_i = 0$. A stability analysis for the composite nonlinear system (10) based on Lyapunov method is investigated by transforming quadratic terms into an equivalent LMI framework (Boyd *et al* 1994).

3. Stability analysis of an *n*-link robot manipulator

Recall the system equation (10) that is rewritten as

$$\dot{x}_i = A_{si} x_i + B_{ni} u_i + w_i(t, x) \quad i = 1, 2, \ldots, n, \tag{21}$$

where $w_i = B_i h_i$, and it is required that nonlinear term w_i satisfies the quadratic constraints.

$$w_i^T(t, x) w_i(t, x) \leq \alpha_i^2 x^T W_i^T W_i x, \quad i = 1, 2, \ldots, n, \tag{22}$$

where $\alpha_i > 0$ are interconnection parameters and W_i are constant matrices of appropriate dimensions. The input to the system (21) with the PID controller is

$$u_i(t) = K_{pi} e_i(t) + K_{di} \dot{e}_i(t) + K_{ii} \int_0^t e_i(t) dt. \tag{23}$$

Assume θ_d (desired Position) = 0.

$$u_i(t) = K_{pi}(-y_i) + K_{di}(-\dot{y}_i) + K_{ii} \int_0^t (-y_i) dt,$$

where $y_i = C_{si} x_i$ is the output of the system. Let $x_{ai} = -K_{ii} \int_0^t y_i dt$, so

$$\dot{x}_{ai} = -K_{ii} y_i = -K_{ii} C_{si} x_i. \tag{24}$$

To study the stability of the interconnected system substitute for u_i in (21) and augment it with (24) to have the following form.

$$E_{ni} \dot{x}_{wi} = A_{ni} x_{wi} + w_{ni}(t, x_w), \tag{25}$$

where

$$E_{ni} = \begin{bmatrix} I + B_{ni}K_{di}C_{si} & 0 \\ 0 & 1 \end{bmatrix}, \; A_{ni} = \begin{bmatrix} A_{si} - B_{ni}K_{pi}C_{si} & B_{ni} \\ -K_{ii}C_{si} & 0 \end{bmatrix},$$

$$w_{ni}(t, x_w) = \begin{bmatrix} w_i(t, x) \\ 0 \end{bmatrix}, \quad \text{and } x_{wi} = \begin{bmatrix} x_i \\ x_{ai} \end{bmatrix}.$$

In compact form equation (25) can be rewritten as

$$E_{new}\dot{x}_w = A_{new}x_w + w_n(t, x_w), \tag{26}$$

where $A_{new} = diag\{A_{n1}A_{n2}, \ldots, A_{nn}\}$ and $E_{new} = diag\{E_{n1}, E_{n2}, \ldots, E_{nn}\}$ are matrices of appropriate dimensions and the nonlinear term $w_n = \left(w_{n1}^T, w_{n2}^T, \ldots, w_{nn}^T\right)^T$ is a function of $x_w = \left[x_{w1}^T, x_{w2}^T, \ldots, x_{wn}^T\right]^T$. Since the set of stabilizing PID controllers is obtained, the matrices E_{new} and A_{new} are, in turn, of interval form. In (26), the nonlinear function is constrained as

$$w_n^T(t, x_w)w_n(t, x_w) \le x_w^T \left(\sum_{i=1}^{n} \alpha_i^2 W_{ai}^T W_{ai}\right) x_w, \tag{27}$$

where W_{ai} is a constant matrix with appropriate dimension.

Theorem 1: The nonlinear system (21) is robustly stabilizable with degree α_i by the control law (23) if for matrices P_1, P_2, P_3 of compatible dimensions, and $\gamma_1, \gamma_2, \ldots, \gamma_n > 0$ and there exists a feasible solution for the following LMI problem for all the corner matrices of A_{new} and E_{new}.

Minimize $\sum_{i=1}^{n} \gamma_i$,

subject to $P_1 > 0$, and

$$\begin{bmatrix} {A_{new}^{r_1}}^T P_2 + P_2^T A_{new}^{r_1} & P_1 - P_2^T E_{new}^{r_2} + {A_{new}^{r_1}}^T P_3 & P_2^T & W_{a1}^T & \cdots & W_{an}^T \\ P_3^T A_{new}^{r_1} + P_1 - {E_{new}^{r_2}}^T P_2 & -{E_{new}^{r_2}}^T P_3 - P_3^T E_{new}^{r_2} & P_3^T & 0 & \ldots & 0 \\ P_2 & P_3 & -I & 0 & \ldots & 0 \\ W_{a1} & 0 & 0 & -\gamma_1 I & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ W_{an} & 0 & 0 & 0 & \ldots & -\gamma_n I \end{bmatrix} < 0, \tag{28}$$

where $r_1, r_2, \ldots, 2^{k^2}$, $\gamma_i = 1/\alpha_i^2$, k is the size of the matrices A_{new} and E_{new}, $i = 1,2,\ldots,n$.

Proof:

The constraint (27) is equivalent to the quadratic inequality

$$\begin{bmatrix} x_w^T & w_n^T(x) \end{bmatrix} \begin{bmatrix} -\sum_{i=1}^{N} \alpha_i^2 W_{ai}^T W_{ai} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x_w \\ w_n(x) \end{bmatrix} \le 0. \tag{29}$$

For the descriptor system (26), we introduce an augmented system (Lin *et al* 2005) to get the following equation for the augmented vector $z(t) = \left[x_w^T(t)\dot{x}_w^T(t)\right]^T$,

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_w \\ \ddot{x}_w \end{bmatrix} = \begin{bmatrix} 0 & I \\ A_{new} & -E_{new} \end{bmatrix}\begin{bmatrix} x_w \\ \dot{x}_w \end{bmatrix} + \begin{bmatrix} 0 \\ w_n(x) \end{bmatrix}. \tag{30}$$

For simplicity, it is denoted that

$$F = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \overline{A} = \begin{bmatrix} 0 & I \\ A_{new} & -E_{new} \end{bmatrix}, z = \begin{bmatrix} x_w \\ \dot{x}_w \end{bmatrix} \text{ and } \overline{w}(x) = \begin{bmatrix} 0 \\ w_n(x) \end{bmatrix}.$$

Let us choose a Lyapunov function candidate (Cao & Lin 2004) for the descriptor system (30) as

$$V = z^T F P z, \tag{31}$$

where $P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$ is nonsingular with $P_1 = P_1^T > 0$, and $FP = (FP)^T$ due to special structures of F and P. We compute

$$\dot{V} = z^T\left(\overline{A}^T P + P^T\overline{A}\right)z + \overline{w}^T(x)\,Pz + z^T P^T\overline{w}(x).$$

In order that the descriptor system (30) is stable, it is required that

$$P_1 > 0, \qquad z^T\left(\overline{A}^T P + P^T\overline{A}\right)z + \overline{w}^T(x)\,Pz + z^T P^T\overline{w}(x) < 0. \tag{32}$$

Equation (32) is equivalently can be written as

$$\begin{aligned} & P_1 > 0, \\ & x_w^T\left(A_{new}^T P_2 + P_2^T A_{new}\right)x_w + \dot{x}_w^T\left(-E_{new}^T P_3 - P_3^T E_{new}\right)\dot{x}_w \\ & + \dot{x}_w^T\left(P_1 - E_{new}^T P_2 + P_3^T A_{new}\right)x_w + x_w^T\left(A_{new}^T P_3 + P_1 - P_2^T E_{new}\right)\dot{x}_w \\ & + + w_n^T(x)\,P_2 x_w + w_n^T(x)\,P_3\dot{x}_w + x_w^T P_2^T w_n(x) + \dot{x}_w^T P_3^T w_n(x) < 0. \end{aligned} \tag{33}$$

These inequalities can be rewritten as,

$$P_1 > 0,$$

$$\left[x_w^T\ \dot{x}_w^T\ w_n^T(x)\right]\begin{bmatrix} A_{new}^T P_2 + P_2^T A_{new} & A_{new}^T P_3 + P_1 - P_2^T E_{new} & P_2^T \\ P_3^T A_{new} + P_1 - E_{new}^T P_2 & -E_{new}^T P_3 - P_3^T E_{new} & P_3^T \\ P_2 & P_3 & 0 \end{bmatrix}\begin{bmatrix} x_w \\ \dot{x}_w \\ w_n(x) \end{bmatrix} < 0. \tag{34}$$

By using S-procedure (Yakubovich 1977) it is possible to combine quadratic inequalities (29) and (34) into one single linear matrix inequality (LMI) form as

$$\begin{bmatrix} A_{new}^T P_2 + P_2^T A_{new} + \beta \sum_{i=1}^{N} \alpha_i^2 W_{ai}^T W_{ai} & A_{new}^T P_3 + P_1 - P_2^T E_{new} & P_2^T \\ P_3^T A_{new} + P_1 - E_{new}^T P_2 & -E_{new}^T P_3 - P_3^T E_{new} & P_3^T \\ P_2 & P_3 & -\beta I \end{bmatrix} < 0, \quad (35)$$

where $P_1 > 0$ and a number $\beta > 0$. By repeatedly applying the Schur complement formula (Boyd *et al* 1994) to equation (35) with $\beta = 1$, the above equation can be rewritten as

$$\begin{bmatrix} A_{new}^T P_2 + P_2^T A_{new} & A_{new}^T P_3 + P_1 - P_2^T E_{new} & P_2^T & W_{a1}^T & \cdots & W_{an}^T \\ P_3^T A_{new} + P_1 - E_{new}^T P_2 & -E_{new}^T P_3 - P_3^T E_{new} & P_3^T & 0 & \ldots & 0 \\ P_2 & P_3 & -I & 0 & \ldots & 0 \\ W_{a1} & 0 & 0 & -\gamma_1 I & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ W_{an} & 0 & 0 & 0 & \ldots & -\gamma_N I \end{bmatrix} < 0, \quad (36)$$

where $\gamma_i = 1/\alpha_i^2$.

The matrices A_{new} and E_{new} of (26) are interval matrices (obtained from (25)). As discussed by Mansour (1988), Jiang (1987) and Garofalo *et al* (1993), a sufficient condition for the stability robustness of interval matrices, i.e., matrices having the elements varying within given bounds, requires that the derivative of Lyapunov function be negative definite when evaluated at the so-called corner matrices. The corner matrices of an $n \times n$ interval matrix A are defined as $A^r = \left\{a_{ij}^r\right\}, r = 1, 2, \ldots, 2^{n^2}$ with $a_{ij}^r = al_{ij}$ or $au_{ij}, i, j = 1, 2, \ldots, n$, where al_{ij} and au_{ij} are minimum and maximum values, respectively of *ij*th element of interval matrix. Hence equation (36) should be satisfied for all the corner matrices of A_{new} and E_{new} for composite system (30) to be asymptotically stable. The matrix W_{ai} is assumed such that constraint (27) is satisfied and the bounding parameter α_i is to be maximized. Hence (36) can be reformulated as an LMI optimization problem as stated in (28). In other words, system (21) is robustly stabilizable by the set of designed decoupled stabilizing PID controllers provided the LMI (28) has a feasible solution for all corner matrices. This completes the proof.

4. Simulation results for two-link robot manipulator

Consider a two-link manipulator as shown in figure 4 and its dynamics can be described by nonlinear equation (1). The matrices $M(\theta)$, $V(\theta, \dot{\theta})$ and $G(\theta)$ for this two-link robot are

$$M(\theta) = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix} = \begin{bmatrix} a_1 + a_2 + 2a_3 \cos\theta_2 & a_2 + a_3 \cos\theta_2 \\ a_2 + a_3 \cos\theta_2 & a_2 \end{bmatrix},$$

$$V(\theta, \dot{\theta}) = \begin{bmatrix} -(a_3 \sin\theta_2)\left(\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\right) \\ (a_3 \sin\theta_2)\,\dot{\theta}_1^2 \end{bmatrix},$$

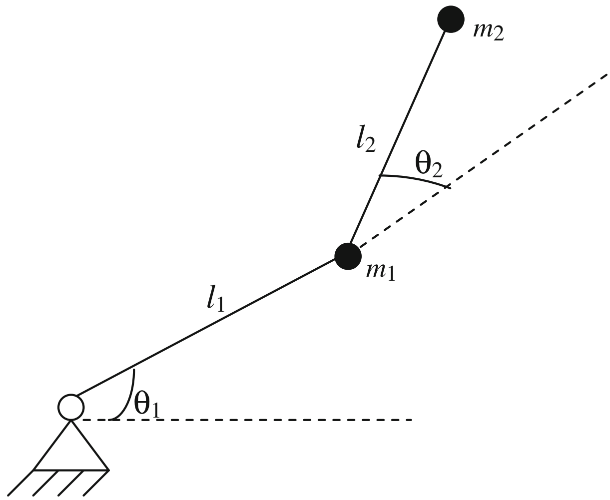

Figure 4. Schematic of a two-link revolute robot.

$$G(\theta) = \begin{bmatrix} a_4 \cos\theta_1 + a_5 \cos(\theta_1 + \theta_2) \\ a_5 \cos(\theta_1 + \theta_2) \end{bmatrix}. \tag{37}$$

In the above expression $a_1, a_2, \ldots, a_5$ are constant parameters obtained from mass $(m_1,\ m_2)$ and length $(l_1,\ l_2)$ of robot links $\left[a_1 = (m_1 + m_2)\, l_1^2,\ a_2 = m_2 l_2^2,\ a_3 = m_2 l_1 l_2,\ a_4 = (m_1 + m_2)\, l_1 g, a_5 = m_2 l_2 g\right]$. The parameters are $m_1 = m_2 = 1.0\,\text{kg}$, $l_1 = l_2 = 1.0\,\text{m}$ and $g = 9.81\,\text{m/s}^2$.

For the system (10) with the expression of (37) we have the following numerical values.

$$A_{s1} = A_{s2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_{n1} = B_{n2} = \begin{bmatrix} 0 \\ 5.8 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_{s1} = C_{s2} = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$h(x) = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} 1.72x_{12}^2 + 1.68x_{22}^2 + 3.36x_{12}x_{22} \\ -\left(5.12x_{12}^2 + 1.72x_{22}^2 + 3.44x_{12}x_{22}\right) \end{bmatrix},$$

$$\text{and } x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \end{bmatrix}, x_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = \begin{bmatrix} \theta_2 \\ \dot{\theta}_2 \end{bmatrix}. \tag{38}$$

The stabilizing set of gains for links 1 and 2 for system (38), obtained by solving equations (19), (20) assuming values for K_{di} from 1 to 100 and ω varying from 0.01 to 25 Hz are shown in figure 5. The shaded region in figure 5 is the stabilizing controller parameter space of joints 1 and 2. The set of controller gains for joints 1 and 2 are taken as

$$\begin{aligned} K_{p1} &\in [\ 10.1\ \ 500\ ],\ K_{i1} \in [\ 10.1\ \ 500\ ],\ K_{d1} \in [\ 10\ \ 100\ ], \\ K_{p2} &\in [\ 10.1\ \ 500\ ],\ K_{i2} \in [\ 10.1\ \ 500\ ],\ K_{d2} \in [\ 10\ \ 100\ ]. \end{aligned} \tag{39}$$

Knowing the ranges of controller gains (31) of the joints 1 and 2, genetic algorithm based optimization technique (Goldberg 1989) is used to maximize the fitness function J_f given by

$$J_f = \frac{1}{1 + J}, \tag{40}$$

where $J = \int_0^t \sum_{i=1}^n e_i^2 dt$ and $e_i(t) = \theta_{di}(t) - \theta_i(t)$.

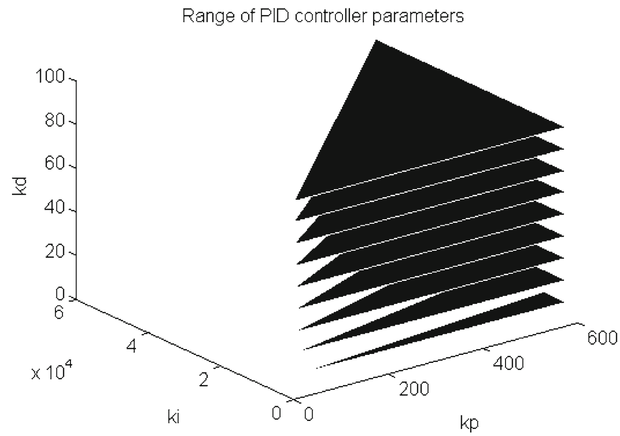

Figure 5. Range of K_p, K_i, K_d gains for joints 1 and 2.

The optimal controller parameters are obtained for fixed as well as time-varying desired positions. The genetic operations used are arithmetic crossover, uniform mutation and ranking selection. The population size of 50 is taken and GA is run for 25 generations.

4.1 *Case 1: Fixed desired positions*

Suppose the desired positions for joints 1 and 2 are $\theta_{d1} = 30^\circ$ and $\theta_{d2} = 45^\circ$. The combined optimal control parameters using genetic algorithm based optimizing technique for each joint are obtained as

$$\begin{aligned} K_{p1}^* &= 176.83,\ K_{i1}^* = 150.32,\ K_{d1}^* = 86.84, \\ K_{p2}^* &= 127.99,\ K_{i2}^* = 128.4,\ K_{d2}^* = 42.09. \end{aligned} \tag{41}$$

The combined optimal control law for each joint is given by

$$\begin{aligned} u_1(t) &= K_{p1}^* e_1(t) + K_{i1}^* \int_0^t e_1(t)dt + K_{d1}^* \dot{e}_1(t) \\ &= 176.83 e_1(t) + 150.32 \int_0^t e_1(t)dt + 86.84 \dot{e}_1(t), \\ u_2(t) &= K_{p2}^* e_2(t) + K_{i2}^* \int_0^t e_2(t)dt + K_{d2}^* \dot{e}_2(t) \\ &= 127.99 e_2(t) + 128.4 \int_0^t e_2(t)dt + 42.09 \dot{e}_2(t). \end{aligned} \tag{42}$$

Figures 6 and 7 show the desired position and actual position of joints 1 and 2 with the designed control laws (42). The position errors of each joint are also plotted in figures 8 and 9, respectively.

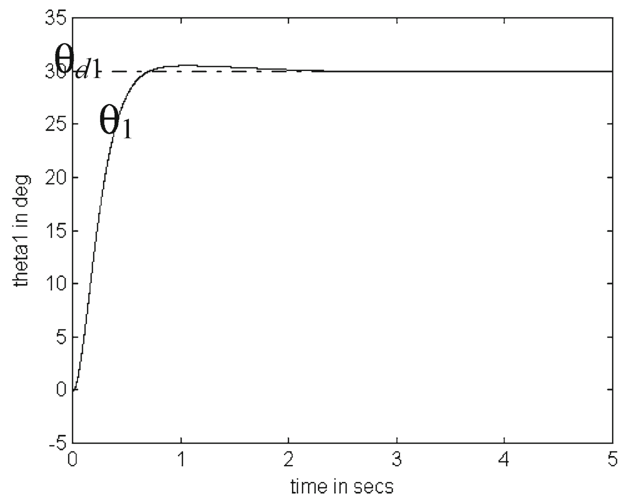

Figure 6. θ_{d1} and θ_1 for joint 1with optimal PID gains.

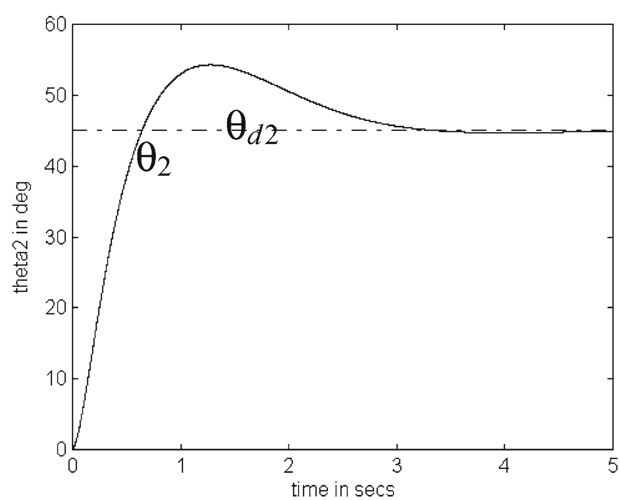

Figure 7. θ_{d2} and θ_2 for joint 2 with optimal PID gains.

4.2 *Case 2: Time-varying desired positions*

Consider time-varying desired positions for joints 1 and 2 as $\theta_{d1} = (1 - \cos t)$ and $\theta_{d2} = (1 - \cos t)$. The combined optimal control parameters using genetic algorithm based optimizing technique for each joint are obtained as

$$\begin{aligned} &K^*_{p1} = 129.11,\ K^*_{i1} = 43.29,\ K^*_{d1} = 56.31,\\ &K^*_{p2} = 77.99,\ K^*_{i2} = 20.4,\ K^*_{d2} = 17.04. \end{aligned} \tag{43}$$

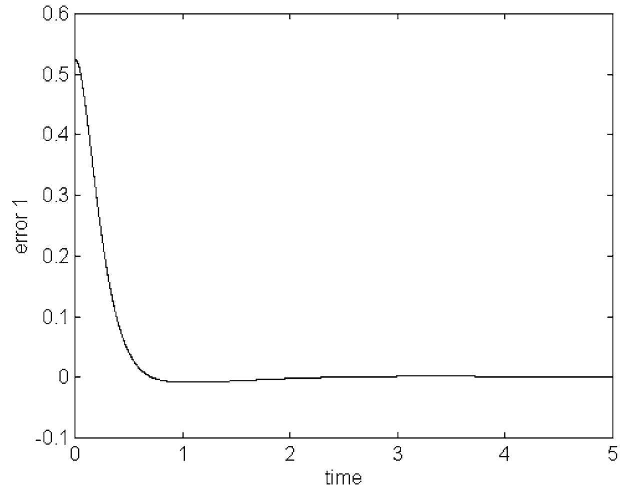

Figure 8. Position error of joint 1 with optimal PID gains.

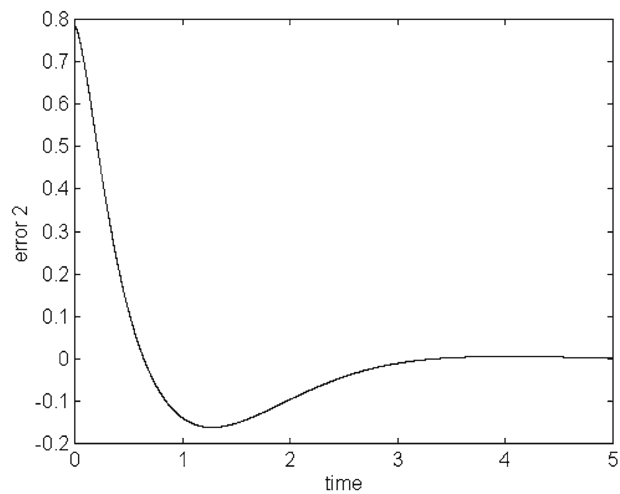

Figure 9. Position error of joint 2 with optimal PID gains.

The control laws with optimal controller parameters for both joints are obtained as

$$u_1(t) = K_{p1}^* e_1(t) + K_{i1}^* \int_0^t e_1(t)dt + K_{d1}^* \dot{e}_1(t) = 129.11 e_1(t) + 43.29 \int_0^t e_1(t)dt + 56.31 \dot{e}_1(t),$$
$$u_2(t) = K_{p2}^* e_2(t) + K_{i2}^* \int_0^t e_2(t)dt + K_{d2}^* \dot{e}_2(t) = 77.99 e_2(t) + 20.4 \int_0^t e_2(t)dt + 17.04 \dot{e}_2(t). \tag{44}$$

Figures 10 and 11 show the desired and actual positions of joints 1 and 2 with the designed optimal control laws (44). The position errors of each joint are plotted in figures 12 and 13.

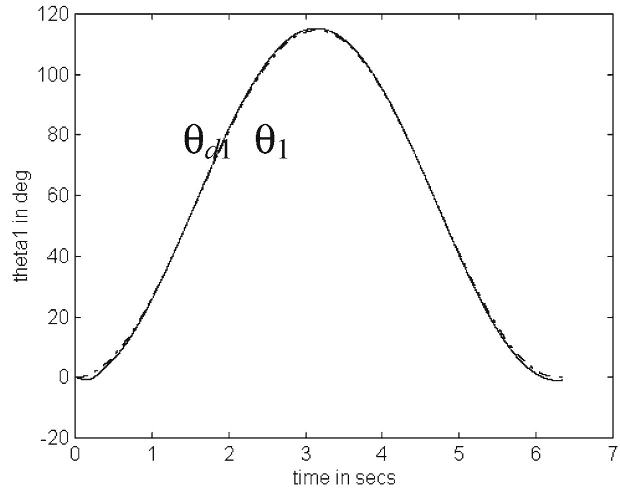

Figure 10. θ_1 and θ_{d1} with optimal PID gains.

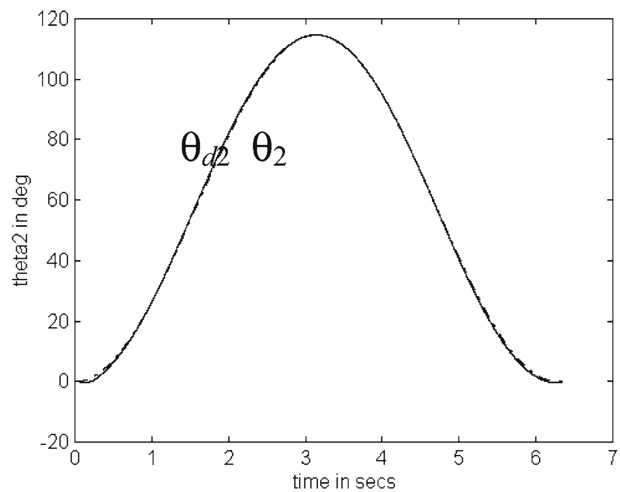

Figure 11. θ_2 and θ_{d2} with optimal PID gains.

Figures 6–13 reveal the effectiveness of the proposed decentralized PID control scheme and further it ensures tracking errors converge to zero asymptotically.

4.3 *Stability analysis of two-link robot manipulator*

The stability analysis of the two-link robot manipulator (38) with the designed set of controllers (39) was studied by solving the LMI optimization problem (28) for all the corner matrices of A_{new} and E_{new}. The designed ranges of A_{new} and E_{new} are calculated using equations (25)–(26) with the controller gains (39) and are given by

$$\begin{aligned} A_{new} &= diag\{A_{n1}, A_{n2}\}, \\ E_{new} &= diag\{E_{n1}, E_{n2}\}, \end{aligned} \tag{45}$$

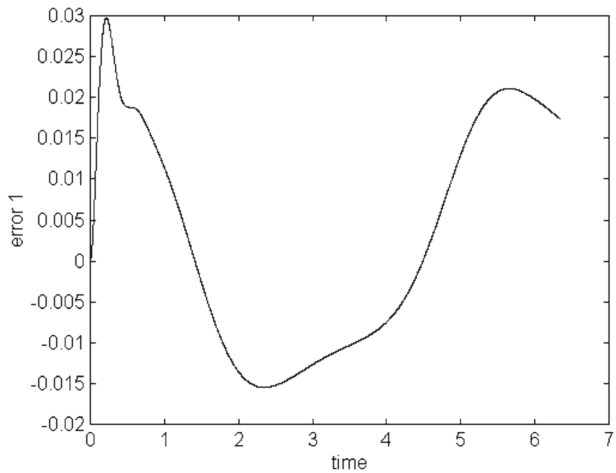

Figure 12. Position error of joint 1 with optimal PID gains.

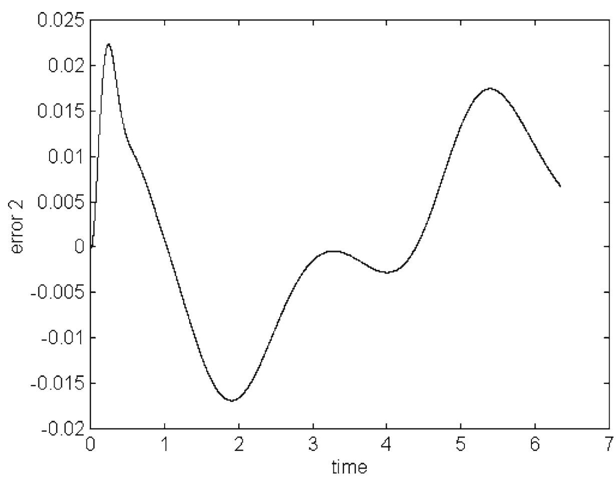

Figure 13. Position error of joint 2 with optimal PID gains.

where

$$A_{n1} = \begin{bmatrix} 0 & 1 & 0 \\ [\,-2900 \;\; -58.6\,] & 0 & 5.8 \\ [\,-500 \;\; -10.1\,] & 0 & 0 \end{bmatrix}, \qquad E_{n1} = \begin{bmatrix} 1 & 0 & 0 \\ [\,58 \;\; 580\,] & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$A_{n2} = \begin{bmatrix} 0 & 1 & 0 \\ [\,-2900 \;\; -58.6\,] & 0 & 5.8 \\ [\,-500 \;\; -10.1\,] & 0 & 0 \end{bmatrix} \text{ and } \quad E_{n2} = \begin{bmatrix} 1 & 0 & 0 \\ [\,58 \;\; 580\,] & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

As given in equation (27), the term $w_n(t, x_w)$ can be bounded by a quadratic inequality and is constrained as

$$\begin{aligned} w_{n1}^T(t, x_w) w_{n1}(t, x_w) &= \left(3.36 x_{12} x_{22} + 1.72 x_{12}^2 + 1.68 x_{22}^2\right)^2 \\ &\leq 17.07 x_{12} x_{22} \leq 8.54 \left(x_{12}^2 + x_{22}^2\right) \leq x_w^T \alpha_1^2 W_{a1}^T W_{a1} x_w, \\ w_{n2}^T(t, x_w) w_{n2}(t, x_w) &= \left(-3.44 x_{12} x_{22} - 5.12 x_{12}^2 - 1.72 x_{22}^2\right)^2 \\ &\leq 29.44 x_{12} x_{22} \leq 14.72 \left(x_{12}^2 + x_{22}^2\right) \leq x_w^T \alpha_2^2 W_{a2}^T W_{a2} x_w \end{aligned} \quad (46)$$

(since x_{12} and x_{22} are much less than unity and the terms associated with the power of x_{12} and x_{22} equal to three or more than three are neglected), where $\alpha_1, \alpha_2 > 0$ and W_{a1} and W_{a2} are found out as

$$W_{a1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.92 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.92 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad W_{a2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.87 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.87 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (47)$$

Two elements each of A_{n1}, A_{n2} and E_{new} are of interval form, i.e., four corner matrices for each of A_{n1}, A_{n2} and E_{new} are obtained. Table 1a shows the corner matrices of A_{n1}, A_{n2} and E_{new}. Thus sixteen corner matrices are possible for A_{new} with the eight corner matrices of A_{n1}, A_{n2}. These corner matrices of A_{new} are given in table 1b. This sixteen combinations of A_{new} and four corner matrices of E_{new} are considered, thus there are sixty-four combinations for which optimization problem (28) is solved using LMI control toolbox (Gahinet *et al* 1995) with W_{a1}, W_{a2} taken as equation (47). As the number of links increases the number of LMIs to be solved increases exponentially (2^{3n} where n is the number of links of the manipulator). Table 1c shows the values of α_1, α_2 obtained by solving the LMI problem (28) with the designed range of controller parameters given by (39). It is seen that a feasible solution exists for all the corner matrices. Hence, it is concluded that the set of decentralized PID controllers based on

Table 1a. Corner matrices of A_{n1}, A_{n2} and E_{new}.

$A_{n1}^1 = \{A_{n1}\underline{(2,1)},\ A_{n1}\underline{(3,1)}\}$	$A_{n2}^1 = \{A_{n2}\underline{(2,1)},\ A_{n2}\underline{(3,1)}\}$	$E_{new}^1 = \{E_{n1}\underline{(2,1)},\ E_{n2}\underline{(2,1)}\}$
$A_{n1}^2 = \{A_{n1}\underline{(2,1)},\ A_{n1}\overline{(3,1)}\}$	$A_{n2}^2 = \{A_{n2}\underline{(2,1)},\ A_{n2}\overline{(3,1)}\}$	$E_{new}^2 = \{E_{n1}\underline{(2,1)},\ E_{n2}\overline{(2,1)}\}$
$A_{n1}^3 = \{A_{n1}\overline{(2,1)},\ A_{n1}\underline{(3,1)}\}$	$A_{n2}^3 = \{A_{n2}\overline{(2,1)},\ A_{n2}\underline{(3,1)}\}$	$E_{new}^3 = \{E_{n1}\overline{(2,1)},\ E_{n2}\underline{(2,1)}\}$
$A_{n1}^4 = \{A_{n1}\overline{(2,1)},\ A_{n1}\overline{(3,1)}\}$	$A_{n2}^4 = \{A_{n2}\overline{(2,1)},\ A_{n2}\overline{(3,1)}\}$	$E_{new}^4 = \{E_{n1}\overline{(2,1)},\ E_{n2}\overline{(2,1)}\}$

where $A_{ni}\underline{(2,1)}$ and $A_{ni}\overline{(2,1)}$ denote the lower and upper limits of (2, 1)th element of matrix A_{ni}.

Table 1b. Corner matrices of A_{new}.

$A_{new}^1 = \{A_{n1}^1 A_{n2}^1\}$	$A_{new}^5 = \{A_{n1}^1 A_{n2}^2\}$	$A_{new}^9 = \{A_{n1}^1 A_{n2}^3\}$	$A_{new}^{13} = \{A_{n1}^1 A_{n2}^4\}$
$A_{new}^2 = \{A_{n1}^2 A_{n2}^1\}$	$A_{new}^6 = \{A_{n1}^2 A_{n2}^2\}$	$A_{new}^{10} = \{A_{n1}^2 A_{n2}^3\}$	$A_{new}^{14} = \{A_{n1}^2 A_{n2}^4\}$
$A_{new}^3 = \{A_{n1}^3 A_{n2}^1\}$	$A_{new}^7 = \{A_{n1}^3 A_{n2}^2\}$	$A_{new}^{11} = \{A_{n1}^3 A_{n2}^3\}$	$A_{new}^{15} = \{A_{n1}^3 A_{n2}^4\}$
$A_{new}^4 = \{A_{n1}^4 A_{n2}^1\}$	$A_{new}^8 = \{A_{n1}^4 A_{n2}^2\}$	$A_{new}^{12} = \{A_{n1}^4 A_{n2}^3\}$	$A_{new}^{16} = \{A_{n1}^4 A_{n2}^4\}$

Table 1c. α_1, α_2 values obtained solving LMI problem (28).

	E^1_{new}		E^2_{new}		E^3_{new}		E^4_{new}	
	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
A^1_{new}	0.1585	0.1374	0.1584	0.1376	0.1584	0.1376	0.2226	0.1934
A^2_{new}	0.1584	0.1376	0.1583	0.1376	0.2184	0.1897	0.2227	0.1935
A^3_{new}	0.1566	0.1360	0.1584	0.1376	0.1566	0.1361	0.2227	0.1935
A^4_{new}	0.0323	0.0281	0.1584	0.1376	0.0323	0.0281	0.2044	0.1772
A^5_{new}	0.1584	0.1376	0.2184	0.1898	0.1583	0.1376	0.2227	0.1935
A^6_{new}	0.2184	0.1896	0.2186	0.1898	0.2186	0.1898	0.2229	0.1936
A^7_{new}	0.1566	0.1360	0.2185	0.1898	0.1566	0.1360	0.2227	0.1935
A^8_{new}	0.0323	0.0281	0.2043	0.1773	0.0323	0.0281	0.2042	0.1774
A^9_{new}	0.1566	0.1360	0.1566	0.1361	0.1584	0.1376	0.2227	0.1935
A^{10}_{new}	0.1566	0.1360	0.1565	0.1360	0.2185	0.1898	0.2227	0.1935
A^{11}_{new}	0.1565	0.1361	0.1567	0.1361	0.1567	0.1361	0.2226	0.1934
A^{12}_{new}	0.0323	0.0281	0.1566	0.1361	0.0323	0.0281	0.2042	0.1774
A^{13}_{new}	0.0323	0.0281	0.0323	0.0281	0.1584	0.1376	0.2044	0.1772
A^{14}_{new}	0.0323	0.0281	0.0323	0.0281	0.2043	0.1772	0.2042	0.1774
A^{15}_{new}	0.0323	0.0281	0.0323	0.0281	0.1566	0.1361	0.2042	0.1774
A^{16}_{new}	0.0323	0.0280	0.0323	0.0281	0.0323	0.0281	0.2041	0.1772

Kharitonov's theorem and stability boundary equation stabilizes the two-link manipulator system (38) with the numerical values of local controller parameters (39). The finite numerical values of α_i, $i = 1,2$ indicate that the decentralized robust stability analysis of interconnected nonlinear system with its maximum nonlinear perturbations.

5. Conclusion

A class of stabilizing decentralized PID controllers was designed for each link of a two-link manipulator using parameter plane method and Kharitonov's theorem. A Kharitonov region was obtained graphically such that a PID controller with coefficients selected from this region stabilizes the whole uncertain nonlinear system. Even though the design of PID controllers was done based on linear system, the linear controller stabilizes the nonlinear system, which is proved by solving LMI optimization problem and thereby obtaining the bounding parameter of the interconnection terms. From the simulation results shown, it is concluded that the proposed controllers closely tracks the constant as well as time-varying desired positions.

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