# A SHADINg MODEL FOR ATMOSPHERIC SCATTERING CONSIDERINg LUMINOUS INTENSITY DISTRIBUTION OF LIGHT SOURCES 

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## Abstract

Studio spotlights produce dazzling shafts of light, while light scattered from fog illuminated by automobile headiights renders driving difficult. This is because the particles in the illuminated volume become visible by scattering light. A shading model for scattering and absorption of light caused by particles in the atmosphere is proposed in this paper. The method takes into account luminous intensity distribution of light sources, shadows due to obstacles, and density of particles. The intensity at a viewpoint is calculated by integration of light scattered by particles between the viewpoint and a given point on an object. The regions to be treated in this manner are localized by considering illumination volumes and shadow volumes caused by obstacles in the illumination volumes.

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## 1. INTRODUCTION

Many papers have been published which enhance the realism of calculated images by considering properties of objects, such as reflection, refraction, and transparency, and properties of light sources such as point sources, linear sources, area sources, polyhedron sources [1-3], and sky light [4]. In order to produce realistic images, the phenomena in the space between light sources and objects must also be considered; that

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is, the effect of atmospheric particles along the path of the light. Light scattered from particles such as water and dust in the air causes illuminated volumes to glow. In particular, understanding this effect makes it possible to simulate such real-world phenomena as: 1) shafts of light caused by headlights of automobiles, street lamps, beacons, or searchlights at night (these effects are important for drivers), 2) light beams caused by spotlights in studios and on stages, 3) shafts of light pouring through windows, and 4) visibility reduction of traffic signals or traffic signs in fog or haze.

Representative of methods considering particles in the air is the fog effect in day time, commonly used in flight simulators. This method simply calculates attenuation as a function of depth from the viewpoint. Some light scattering models have been developed: Blinn first developed a method adapted to displaying Saturn's rings or cloud layers [5], but this method is limited to thin layers. Kajiya developed a method taking account of densities of particles [6]. Recently Max made it possible to display light beams passing through gaps of clouds or trees [7]. In all these methods, however, light sources are limited to a simple, single parallel or isotropic source, ignoring the distribution of luminous intensity of the light source. In order to display illumination effects in fog, we must take into account the luminous intensity characteristics of actual light sources, in particular spotlights and headlights, and also must consider effects due to multiple light sources. This paper presents a method considering these factors.

In this paper we use the following assumptions: 1) light sources treated here are point sources with variable angular intensity distributions, and a parallel source. 2) the light from multiple scattering and the illumination onto objects due to scattering can be neglected. 3) objects treated here are composed of convex polyhedra.

## 2. CALCULATION FOR LIGHT SCATTERING

Illuminated volumes in the air glow by light scattered from particles in the space, while light traversing the space is attenuated by absorption. These effects are modelled as follows: for scattering, particles are considered to be an infinite number of point sources; for attenuation, the atmosphere is supposed to be a semi-transparent
object.
First, we discuss the effects of light scattering due to a point source whose angular range of illumination is limited (e.g. a spotlight).

As shown in Fig. 1, the intensity of light reaching a viewpoint $P_{v}$ from a point $P_{i}$ on an object can be obtained by summing the reflected intensity, attenuated by traversing $\mathrm{P}_{i} \mathrm{P}_{\mathcal{V}}$ through the absorbing medium, with the intensity scattered in the direction of the viewpoint from each point along $P_{i} P_{V}$, The attenuation varies exponentially with distance (e.g., from a point on $\mathrm{P}_{i} \mathrm{P}_{v}$ to $\mathrm{P}_{v}$ ), as described by Bouguer's law (e.g., [8]). The total contribution from scattering is obtained by integrating over all points along $\mathrm{P}_{i} \mathrm{P}_{v}$. For example, at any point $P$ on $P_{i} P_{V}$ at distance $s$ from $\mathrm{P}_{v}$, the intensity scattered in the direction of the viewpoint is $I_{p}(s)$, and will be attenuated by traversing distance $s$ to the viewpoint. Thus, the intensity of light at the viewpoint is given by

$$
\begin{equation*}
I=I_{i} e^{-\tau(L)}+\int_{0}^{L} I_{p}(s) e^{-\tau(s)} \sigma d s \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
\tau(s)=\int_{0}^{s} \operatorname{\sigma dt} \tag{2}
\end{equation*}
$$

$\sigma$ is the extinction coefficient per unit length and depends on particle density (a function of position), ${ }^{\tau}(s)$ expresses the optical depth of $\mathrm{PR} v$, and $\tau(L)$ that of $\mathrm{P}_{i} \mathrm{P}_{v}$.

Consider the direct illumination due to a point light source with luminous intensity $I(\theta, \phi)$ ( $\theta$ is the angle from illumination axis; $\phi$ is the revolution angle; see Appendix), and uniform ambient light $I_{a}$, which is due to interreflection of light from objects' surfaces and to multiple scattering. The intensity of light reaching the point $P$ from the light source is also attenuated due to absorption by particles and subject to the inverse square law of distance. Then $I_{p}$ is given by the following equation.

$$
\begin{equation*}
\mathrm{I}_{\mathrm{p}}(\mathrm{~s})=\frac{I(\theta, \phi)}{\mathrm{r}^{2}} w \mathrm{~F}(\alpha) \mathrm{e}^{-\tau(\mathrm{r})}+\mathrm{I}_{\mathrm{a}} \tag{3}
\end{equation*}
$$

Here $w$ is the reflectance, $r$ is the distance between the light source $Q$ and the point $P, \tau(r)$ is the optical depth of $P Q$ (see equation (2)), $F(\alpha)$ is the scattering phase function, and $\alpha$ is the angle between the forward scattering direction and the scattered ray (see Fig.1). The phase function is a function of particle radius, index of refraction, and wave length. The simplest example of the phase function is that for isotropic scattering, $F(\alpha)=$ constant. For extremely small particles such as molecules of the air, Rayleigh scattering theory is used; $F(\alpha)=K\left(1+\cos ^{2} \alpha\right)$ ( $K$ is a constant). For relatively small particles such as fog, Mie scattering theory is used. Mie theory is very complicated, so this paper uses the following experimental approximation [9] for foggy atmosphere,

$$
F(\alpha)= \begin{cases}K\left(1+9 \cos ^{16}(\alpha / 2)\right) & \text { : hazy atmosphere }  \tag{4}\\ K\left(1+50 \cos ^{64}(\alpha / 2)\right) & \text { : murky atmosphere }\end{cases}
$$

where $K$ is a constant.


Fig. 1 Intensity of light reaching a viewpoint from a point $\mathrm{P}_{i}$ on an object.

In equation (1), $I_{i}$ is the intensity of light arriving at point $P_{i}$ on an object, and is also subject to attenuation due to the distance from the light source just as in equation (3) (i.e., $I_{i}$ depends on $\left.\exp (-\tau(r)) / r^{2}\right)$. Calculation of illuminance due to point sources with variable angular intensity distributions is performed by the method given in Reference [2]. If the density of particles is uniform, equation (1) can be expressed as follows

$$
\begin{equation*}
\mathrm{I}=\mathrm{I}_{i} \mathrm{e}^{-\sigma \mathrm{L}}+\mathrm{I}_{\mathrm{a}}\left(1-\mathrm{e}^{-\sigma \mathrm{L}}\right)+\int_{0}^{\mathrm{L}} \mathrm{wI}(\theta, \phi) \mathrm{e}^{-\sigma(\mathrm{r}+\mathrm{s})} F(\alpha) / \mathrm{r}^{2} \sigma \mathrm{ds} \tag{5}
\end{equation*}
$$

In applying this equation, the following properties should be noted:
a) The third term refers to the shaft of light. Thus, if the light ray doesn't pass through an illuminated volume, the intensity is given by the first two terms.
b) Even for uniform density, we may consider many types of luminous intensity distributions, therefore numerical integration is useful because of the difficulty of obtaining an analytical solution.
c) Outdoors, during the day, $I_{a}$ corresponds to the intensity of the atmosphere at infinite distance, that is, the first two terms of equation (5) are equivalent to the equation for fog effect [10].
d) For multiple light sources, the total intensity is obtained by summing up the intensities yielded by equation (5) for each light source.
e) In the case of a parallel source, $I_{p}$ of equation (3) becomes:

$$
\begin{equation*}
I_{p}(s)=w I_{0} F(\alpha) e^{-T(r)}+I_{a} \tag{6}
\end{equation*}
$$

where $I_{0}$ is the intensity of light and $\tau(r)$ the optical depth from the reference plane (e.g., a


Fig. 2 Calculation of scattered light using illumination and shadow volumes.
window plane for a shaft of light pouring through a window).

## 3. ILLUMINATION VOLUMES AND INTEGRATION SEGMENTS

Light sources treated here are point sources with variable angular intensity distributions, and parallel sources. In order to minimize the application of equation (1) (requiring a lengthy calculation), illumination volumes are localized by the following principles: (1) For point sources, directions of radiation are limited by reflectors and lenses. (2) For axisymmetric luminous distributions, the illumination volume is defined as a circular cone whose vertex is the light center, and whose central axis coincides with the illumination axis (see Fig.2). (3) For nonaxisymmetric luminous intensity distributions such as headlights, the illumination volume is an elliptical cone. (4) For a parallel light source entering through a window, the illumination volume is a prism of which the base is the window and the base is swept in the direction of light (see Fig. 3).

Shadows on particles in the atmosphere are as important as those on the surfaces of objects; when there are objects in an illumination volume, non-illuminated parts arise within it.

In order to detect shadows on objects, we use shadow volumes [11] defined by the light source and the contour lines of each polyhedron as viewed from the light source [2]. These shadow volumes are also used for detecting shadows in the atmosphere.

The shadows on each particle along the line between the viewpoint and a point on an object (hereafter called the ray) can be obtained by using the ray tracing algorithm (in this case, the ray tracing algorithm is applied for each sampling point on the ray). However, this is very time-consuming and gives rise to sampling errors. The following method is more efficient and accurate. The shadow segments on the ray are obtained by using intersection points between the


Fig. 3 Illumination volume for a parallel source.
ray and both illumination volumes and shadow volumes, which are pre-calculated before intensity calculation at each pixel. No aliasing problems caused by shadow calculation arise because shadow regions are accurately calculated.

The segments in which scattered light must be calculated (e.g., $s_{1} s_{2}$ and $s_{3} s_{L_{4}}$ in Fig.2) are extracted by the following processes.

The following preprocessing is executed:
i) Extract illumination volumes for each light source.
ii) Extract polyhedra intersecting illumination volumes.
iii) Extract shadow volumes formed by the light sources and those polyhedra.

Then the following steps are executed for each pixel on every scan line.
iv) Extract illumination volumes intersecting the ray, and calculate the intersection points. If there is no intersection, no calculation is required for scattered light.
v) Calculate the intersections between the ray and all shadow volumes within each illumination volume.
vi) Extract illuminated segments on the ray. (These segments are visible from the light source.)
vii) Integrate intensities of the scattered light on the illuminated segments of the ray.

In step ii), all polyhedra are given a spherical bounding volume, and the polyhedra intersecting illumination volumes are extracted using these bounding spheres. Steps i), ii) and iii) need not be recalculated if the viewpoint is changed, as they are independent of the viewpoint. They are also used for detection of shadows on faces.

For steps iv) and $v$ ), in order to calculate intersections between the ray and illumination and shadow volumes efficiently, the regions visible


Fig. 4 Pre-calculation of intersections between the ray and shadow and illumination volumes on the screen.
from the screen are pre-calculated; the contour lines of shadow and illumination volumes viewed from the viewpoint are projected onto the sereen as shown in Fig. 4. Since the projected contour lines are convex polygons, they can be extracted by means of polygon scan conversion. Then the illumination and shadow volumes intersecting with the ray are easily obtained. In this way, the regions in which scattered light is calculated are localized (see dotted parts in Fig. 4). When the light source is a point source, the illumination volume projects onto the screen as a triangle whose vertex is the projected light center. The projected shadow volume is a polygon (see shaded area in Fig. 4) bounded by a triangle with vertex at the projected light center. The shadow volumes intersecting the ray can be extracted efficiently by means of these bounding triangles. When an illumination volume includes the viewpoint, the illumination volume can't be projected onto the screen, and the intersection test must be executed in 3-D space, even though computation time increases.

The extraction of integration segments (i.e., the visible parts when viewed from a light source) in step $v i$ ) is perfomed by using the notion of
quantitative invisibility, as in hidden line elimination [12]. For example, in Fig. 5, as we traverse the ray from $P_{v}$ to $P_{i}$, the quantitative invisibility is incremented each time the ray enters a shadow volume ( $s_{2}, s_{3}$, and $s_{6}$ in Fig.5), and decremented when it leaves ( $s_{4}$ and $s_{5}$ ). Segments for which quantitative invisibility is zero are visible ( $s_{1} s_{2}$ and $s_{5} s_{6}$ in Fig.5).

## 4. DENSITY DISTRIBUTION OF PARTICLES

In many cases, the effect of spotlights in studio or stage lighting is intensified by using dry ice. Dry ice, smoke, dust, and fog or haze exhibit non-uniform particle densities. We discuss calculation of scattered light under such conditions.

In a dense volume, the extinction of light increases due to the increased optical depths both from the light source to a particle and from the particle to the viewpoint. On the other hand, because the intensity of scattered light also increases, dense volumes in most cases are bright. In order to calculate the intensity at the viewpoint $P_{v}$ due to scattered light from a point $P$ on the ray, we must know the luminous intensity of light source $Q$ in the direction of $P$, the particle density at P , and the optical depths of QP and $\mathrm{PP}_{v}$. Therefore, to take into account the distribution of density, we establish a model for density distribution and a method for calculating optical depths:

## 1) Modeliing of density distribution

In this paper, we do not consider arbitrary density distributions over the whole space, but rather the simpler case in which certain regions of the atmosphere are occupied by smoke or fog of uniform density. One researcher has represented volume density using a 3-D grid, with numerical densities for each element [6], but this method requires a great deal of memory. We use a model in which the boundaries of layers of different densities are represented by curved surfaces.

For a foggy medium due to dry ice, the difference of densities between the fog and the atmosphere is large, and because the fog is heavy,


Fig. 6 Calculation of optical depths from the light source to a particle and from the particle to the viewpoint.
it lies below the atmosphere. The boundary surface between the layers can be expressed by mapping height functions onto a horizontal plane (see Fig. 6). Properly speaking, this boundary surface must be calculated by physical simulation, but here we use a Fourier series composed of a short sum of sine waves, as used to represent clouds in reference [13]. For smoke, we map thickness functions onto a number of ellipses.

## 2) Calculation of optical depth

As shown in equation (2), the optical depth can be obtained by integration of densities. We numerically integrate the density along the ray from the viewpoint to the object. By integrating in this direction, the optical depth of intermediate points is easily available for calculating attenuation of scattered light. When density is sampled only at uniform intervals along the ray, aliasing problems may occur. To overcome this problem, we pre-calculate the intersections between the ray and the boundary surface, detect the intervals in which sampling errors may occur, and add intermediate sampling points in those intervals.

For the case with upper and lower layers of different densities, as shown in Fig. 6, the ray can traverse at most one density boundary, so the calculation can be simplified as follows: we assume that the extinction coefficients for upper and lower layers are $\sigma$ and $\sigma^{\prime}$, and the boundary surface of the layer is $g(x, y)$. If the point $P_{i}$ on an object exists above $g$, equation (5) can be used without any modification. If not, the optical depth, $\tau=\sigma(r+s)$, of equation (5) must be replaced by $\tau=\sigma(r+s)-\left(\sigma-\sigma^{\prime}\right)\left(r_{1}+s_{1}\right)$ (see Fig. 6 for $r_{1}$, $s_{1}$ ).

## 5. EXAMPLES

Fig. 7 shows examples of the proposed method. Pictures (a), (b) and (c) are examples for studio lighting: a scene of dancing pencils from the animation entitled "Feast of Light". Picture (a) shows the image illuminated by four spotlights and a special effects light that combines a daisy shape
and polka dots. The front-left spotlight exhibits a soft edge, while the front-right exhibits a sharp edge. In picture (b), we see shadow effects due to obstacles; the shape of the shaft of light is clipped. Pictures (c) and (d) depict dry ice and smoke, respectively.

Pictures (e), (f) and (g) are examples of night scenes outdoors. Picture (e) shows a building illuminated by searchlights, street lamps and headlights; as analytical descriptions of the luminous intensities of street lamps and headlights are not available, actual numerical data are used (see Appendix). Note that due to the phase function, searchlights appear brighter when directed toward the observer than when directed away. Pictures ( $f$ ) and ( $g$ ) show the effect of the phase function (see equation (4)): the beams in (g) are more glaring than those in (f), where the phase function is neglected. Picture ( $h$ ) shows a shaft of light pouring through a window.

As shown in these examples, the proposed method produces realistic images.

## 6. CONCLUSION

This paper describes a method of representing illumination effects considering particles in the atmosphere. The following conclusions can be stated from the results.

1) The proposed method could be used for lighting designs for selecting and arranging lights with consideration of atmospheric conditions, and for estimating illumination effects in studios or stages.
2) Shafts of light due to light sources such as spotlights and headlights, taking account of luminous intensity distributions, can be modelled by considering scattered light from particles in the atmosphere.
3) By considering illumination volumes for light sources, rendering efficiency can be increased by localizing regions in which light scattering is computed.
4) For shadows, intersections of rays with illumination volumes and with shadow volumes are

(a)

(b)

Fig. 7 Examples.

(c)

(e)

(g)

(d)

(f)

(h)

Fig. 7 Examples (continued).
obtained efficiently by using their projected contour lines. The shadow segments on the ray are extracted using the hidden line technique, with the ray viewed from the light source.

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Fig. 8 Illustration of angles for non-axisymmetric luminous intensity.

## APPENDIX

Calculation of Luminous Intensity Characteristics

1) Axisymmetric distribution of luminous intensity

In this case the intensity can be expressed by a function of angle $\theta$ from the illumination axis (see Fig. 8). In most cases the distribution curves of the intensity can be expressed as a polynominal of $\cos \theta$. Here we consider the sharpness of the illuminated edge. In order to control the illumination range, the expression $I(\theta)=I o(\cos \theta)^{C}$ has been proposed [1] (For this expression, the illumination range decreases as $c$ increases). However, because sharp edges cannot be produced, and because of the computational expense of the exponent in the cosine term, this function is not well suited to modelling spotlights. Therefore we propose the following function for spotlights:

$$
I(\theta)=I_{0}\{(1-q)(\cos \theta-\cos \gamma) /(1-\cos \gamma)+q\},(7)
$$

where $I_{0}$ is the luminous intensity at $\theta=0, \gamma$ is the beam spread, and $q$ is a parameter which controls the sharpness of the edge $\langle q=1$ : very sharp, $q=0$ : very soft) (see Fig. 2 for $\gamma$ ).
2) Non-axisymmetric distribution of luminous intensity

In this case the luminous intensity is expressed by $I(\theta, \phi)$, where $\phi$ is the revolution angle from the reference plane which includes the illumination axis (refer to Fig. 8). In the figure the light center is $Q_{e}$, the reference point is $Q_{0}, Q_{c}$ is a point on the illumination axis, and $P$ is an arbitrary point. Then cos $\phi$ can be obtained by the inner product of the normal vectors of triangles, $Q_{e} Q_{c} Q_{0}$ and $Q_{e} P Q_{C}$.

First we discuss luminous intensity expressed by functions. For example, luminous intensities for the daisy shape and for polka dots like those produced by a mirror ball in a disco are calculated as follows. In these sources, the intensity $I(\theta, \phi)$
can be expressed as the product of $I(\theta)$ of equation (7) and the following function $k(\phi)$.

$$
k(\phi)=\max \{p+(1-p) \mid 2 \bmod (\phi, \Delta \phi)-\Delta \phi) \mid / \Delta \phi, 0\} \text { ( } 8 \text { ) }
$$

where the symbol "mod" means modulo, $\Delta \phi$ and $p(p<1)$ are the period of luminous intensity and the ratio of the minimum intensity to the maximum intensity, respectively: When $p$ is small (including negative), the radius of the polka-dots becomes smali. For the daisy shape, the distribution of luminous intensity is given by

$$
I(\theta, \phi)=I(\theta) \mathbf{k}(\phi) .
$$

For the polka-dot pattern, the luminous intensity can be obtained by adding the variation in the $\theta$ direction to equation (9). That is,

$$
\begin{equation*}
I(\theta, \phi)=I(\theta) \mathbf{k}(\phi) \mathbf{k}(\theta) \mathbf{v} \tag{10}
\end{equation*}
$$

where $v=\bmod ([\phi / \Delta \phi]+[\theta / \Delta \theta], 2)(i . e ., v=0$ or $v=1): v$ is used to decrease the number of polka dots.

For arbitrary distributions, numerical 2 uminous intensity data should be used, for example, every 5 degrees on $\theta$ and $\phi$. Intervening luminous intensities are calculated by linear interpolation. For headlights, however, more detailed data, for example every 2 degrees, are necessary because of the radical change of intensities (see Fig. 9).


Fig. 9 Distribution curve of luminous intensity for a headlight.

