

A Simple Adjustable Window Algorithm to Improve FFT Measurements

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Abstract—The Fourier spectrum of a periodic signal may be obtained by fast Fourier transform algorithms, but, as is well known, special care must be taken to avoid severe distortions introduced by the sampling process. The main problem is the leakage generated by the truncation required to obtain a finite length sampled data.

The usual procedure to reduce leakage is to multiply the sampled signal by a weighting window. Several kinds of windows have been proposed in the literature, and today they are also included in many commercial instruments.

A simple alternative procedure is proposed in this paper. It is implemented with a PC compatible data acquisition board (DAQ) and consists of an algorithm that uses decimation and interpolation techniques. This algorithm is equivalent to the use of an adjustable sampling frequency and correspondingly an adjustable window size.

Results obtained by this method on both harmonic and polyharmonic signals are empirically analyzed and compared with those given by an instrument with built-in FFT capabilities.

Index Terms—Decimation, FFT, IFFT, interpolation, leakage.

I. INTRODUCTION

SPECTRAL analysis of sampled signals is a basic technique used in many scientific disciplines. The signals are usually transformed from the time domain to the frequency domain with the discrete Fourier transform (DFT), which can be very efficiently calculated using a fast Fourier transform algorithm (FFT) [1]. As is well known, aliasing and leakage are introduced in the calculated spectrum if both the sampling frequency and the truncation time are not selected with special care.

Aliasing is produced when the sampling frequency is not high enough, and it is simply reduced by increasing the sampling frequency. Leakage is produced by the unavoidable truncation required to convert the sampled signal into a finite length sequence. To deal with this drawback, special methods, known as interpolated fast Fourier transform (IFFT) techniques, were developed. They calculate the frequency, amplitude, and phase of the original spectral lines from the spectrum disturbed by leakage, and they can be listed along the window that was used to calculate the FFT. The comparison between the different IFFT methods with respect to both their systematic errors and noise sensitivity is presented in [2].

The approach presented in this paper may also be considered as an IFFT technique. The main difference is that it only uses a rectangular window and an algorithm developed in the C language. In spite of its simplicity, the method proves to be very efficient for correcting the leakage drawback.

The main idea of the algorithm is to evaluate the fundamental frequency of the input signal and then to modify the sampling frequency by software, using decimation and interpolation techniques, in order to obtain a rectangular window with a specially suited size. The algorithm is developed for harmonic signals, but the experimental results reported here show a drastic reduction in leakage also in the case of polyharmonic signals.

Today, several laboratory instruments have built-in FFT algorithm and leakage reduction capabilities; the HP5420A oscilloscope, used in this work, is capable of showing “on line” the FFT. It includes three windows: Hanning, Flattop, and Rectangular [3]. A comparison with measurements accomplished with this instrument is also presented.

The organization of this paper is as follows: in Section II, the algorithm is described; in Section III, the experimental setup is presented as well as an empirical analysis of errors for the harmonic case. In Section IV, results obtained with polyharmonic signals are shown; Section V deals with the conclusions.

II. THE ALGORITHM

Let $s(t)$ be a periodic signal with unknown period T_0 . In the following discussion, the case of the harmonic signal $s(t) = A \cos(2\pi t/T_0 + \phi)$ is considered. Let $w(t)$ be a rectangular window of width T_w

$$w(t) = \begin{cases} 1 & 0 \leq t < T_w \\ 0 & t < 0 \text{ or } t \geq T_w \end{cases} \quad (1)$$

and let $x(t) = w(t) \cdot s(t)$. The corresponding Fourier transforms are

$$\begin{aligned} S(f) &= \frac{A}{2} \left[e^{j\theta} \delta \left(f - \frac{1}{T_0} \right) + e^{-j\theta} \delta \left(f + \frac{1}{T_0} \right) \right] \\ W(f) &= T_w e^{-j\pi f T_w} \text{sinc}(f T_w) \\ X(f) &= \frac{A}{2} \left[e^{j\theta} e^{-j\pi T_w (f - (1/T_0))} T_w \text{sinc} \left[T_w \left(f - \frac{1}{T_0} \right) \right] \right. \\ &\quad \left. + e^{-j\theta} e^{-j\pi T_w (f + (1/T_0))} T_w \text{sinc} \left[T_w \left(f + \frac{1}{T_0} \right) \right] \right] \end{aligned} \quad (2)$$

In order to apply the FFT technique the signal is sampled with a sampling period, T_s and N samples are stored. The window

Manuscript received May 5, 2000; revised October 31, 2001. This work was supported in part by the University of Mar del Plata (Project 15/G097) and by CONICET (PEI 272/79).

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Publisher Item Identifier S 0018-9456(02)01379-7.

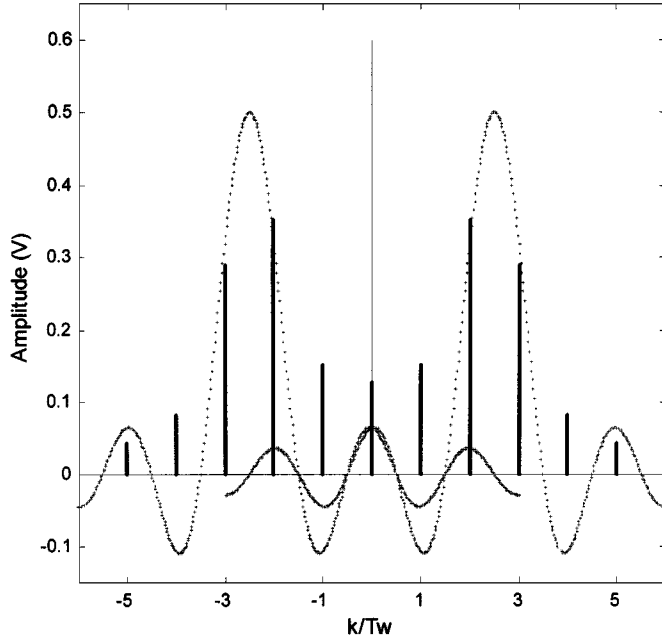


Fig. 1. Standard FFT outcome in the case of a harmonic signal with $T_0 = T_w/2.5$.

$w(t)$ represents a constraint in the number of samples up to a value $N = T_w/T_s$. The discrete spectrum given by

$$\{\tilde{X}(k)\} = \{X(k/T_w), \quad k = 0, \dots, N-1\} \quad (3)$$

will not have artificial spectral components (i.e., leakage) only if T_w/T_0 is an integer, but this is not generally the case because T_0 is not known *a priori*. In Fig. 1, the special case of a sinusoidal waveform with amplitude 1 Vpp and period $T_0 = T_w/2.5$ is considered; the spectrum $X(f)$ is drawn with dotted lines. The typical FFT outcome is a set of samples of this spectrum at frequency intervals $1/T_w$ [1]. Solid lines (bins) in this Fig. 1 represent them. It becomes clear from this graphic that the signal spectrum has been strongly modified: the actual spectral component has been lost, and many spurious spectral components have appeared.

The proposed algorithm consists of the following steps.

- 1) Let A_1 be the amplitude of the highest bin of the discrete spectrum and k_1 its order (i.e., bin 2 in Fig. 1).
- 2) Let $k_2 = k_1 \pm 1$ whichever corresponds to the largest amplitude A_2 (i.e., bin 3 in Fig. 1).
- 3) Assume f_s is high enough to disregard the spectra centered in multiples of f_s . Then, it results that

$$A_1 \propto \text{sinc}(k_1 - C) + \text{sinc}(k_1 + C) = \frac{\cos(k_1\pi)}{\pi} \frac{2C}{(C^2 - k_1^2)} \quad (4)$$

$$A_2 \propto \text{sinc}(k_2 - C) + \text{sinc}(k_2 + C) = \frac{\cos(k_2\pi)}{\pi} \frac{2C}{(C^2 - k_2^2)} \quad (5)$$

with $C = T_w/T_0$. Dividing (4) by (5), one obtains

$$C^2 = \frac{A_1 k_1^2 + A_2 k_2^2}{A_1 + A_2}. \quad (6)$$

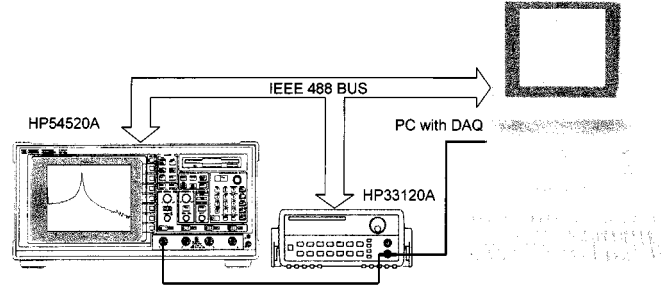


Fig. 2. Experimental setup used in this paper.

- 4) Solve (6) for T_0 and define a new window size T'_w that fits exactly a multiple m of T_0

$$T'_w = mT_0 = \text{int}(C) \frac{T_w}{C} \quad (7)$$

where $\text{int}(C)$ means the integer part of C . Note that this new window size would be obtained by resampling the signal with a different number of samples and/or a different sampling period, but in practice it is not straightforward to make any of these changes. The number of samples is limited to a power of two by any FFT efficient algorithm, and the sampling period is changed by hardware. Consequently, it is not easy to modify it in a continuous way.

To obtain T'_w , we modify the sampling frequency by software to a new value given by

$$f'_s = \frac{N}{T'_w}. \quad (8)$$

Two basic sampling-rate-alteration devices are used to accomplish this task: an *up-sampler* and a *down-sampler* [4].

- 5) Let $L - 1$ be the number of samples inserted by the *up-sampler* between two consecutive samples of the original sequence. Let $M - 1$ be the number of samples removed by the *down-sampler* in between samples. Then

$$f'_s = \frac{L}{M} f_s. \quad (9)$$

From (8) and (9)

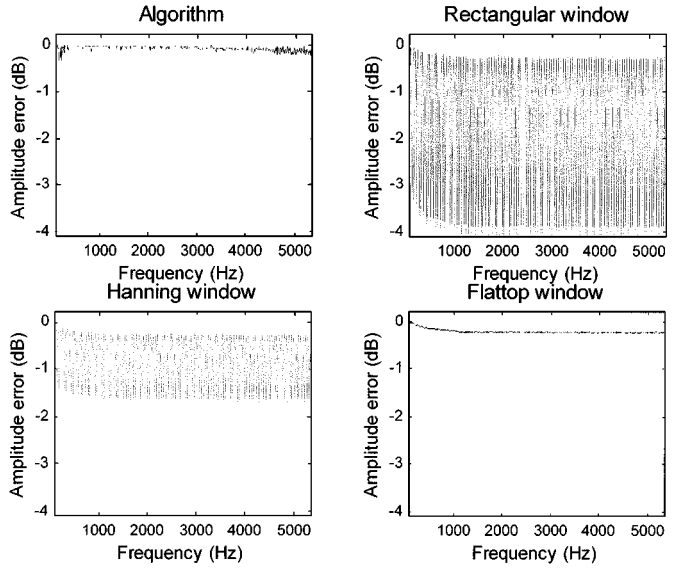
$$\frac{L}{M} = \frac{C}{\text{int}(C)}. \quad (10)$$

Additionally, both L and M must be integer numbers. Throughout this work, we use $M = 1000$ to obtain at least three exact digits in the quotient of (10); the corresponding L value derives from this equation.

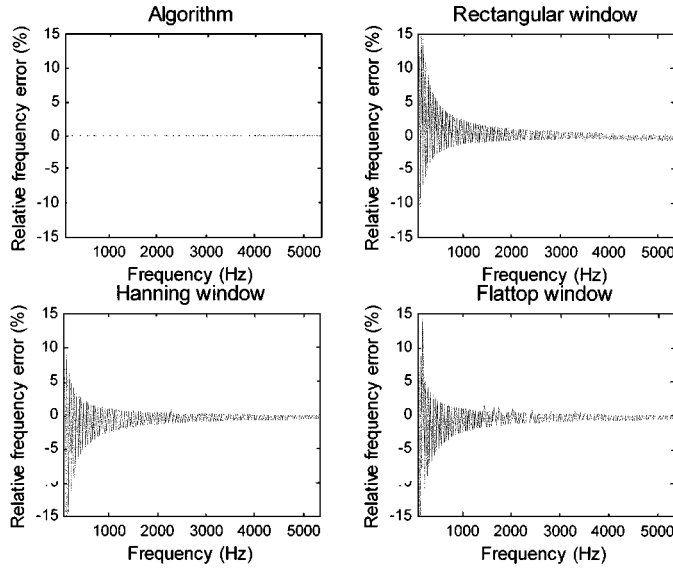
- 6) The last step is to apply the FFT algorithm to the new sequence.

III. EXPERIMENTAL SETUP AND EVALUATION OF THE HARMONIC CASE

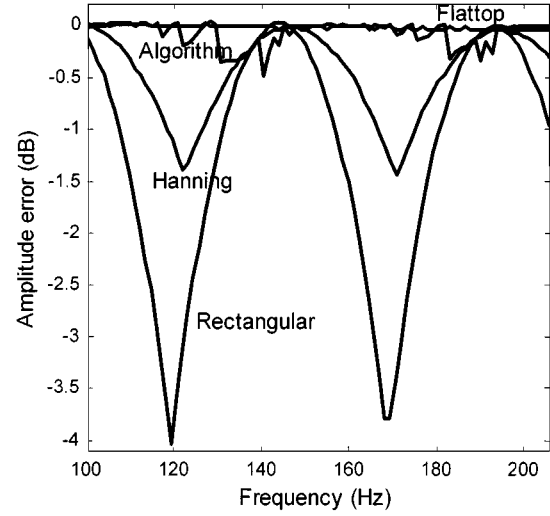
The experimental arrangement is shown in Fig. 2. The Programmable Function Generator (HP33120A) is used to generate the signals to be measured. $N = 1024$ samples at $f_s = 50$ Ks/s are acquired, corresponding to a window size $T_w \cong 20.4$ ms and



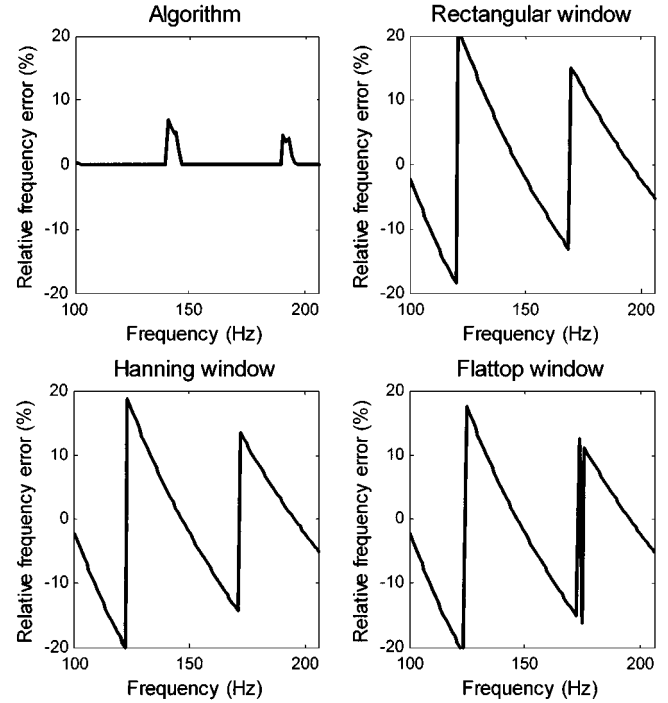
(a)



(b)



(a)



(b)

Fig. 3. Evaluation of systematic errors for different methods in the case of a harmonic signal with variable frequency: (a) amplitude errors; (b) frequency errors.

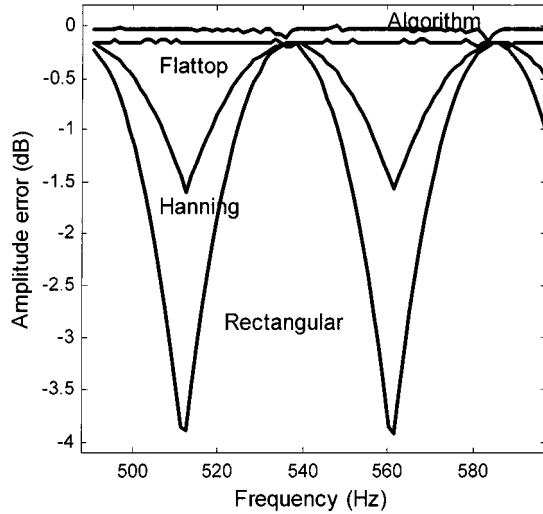
Fig. 4. Enlargement of Fig. 3 for the low frequency range: (a) amplitude errors; (b) frequency errors.

a theoretical frequency resolution of 48.828 Hz. In all cases, we adjust the setup of the scope to measure with its maximum possible resolution (1 dB/div). The HPIB interface is used to control both the measurement and setup procedures.

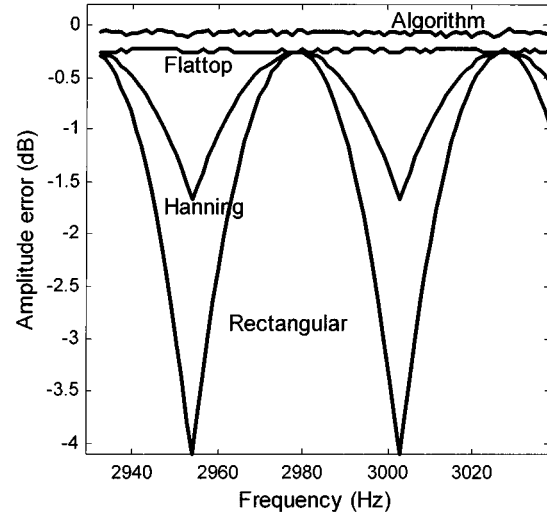
First, we measure a sinusoidal waveform with 1 Vpp (+3.9794 dBm) whose frequency is swept in the range 100 Hz to 5 kHz, approximately. The window size originally used must be greater than the period of the signal in order to obtain good results. In all the measurements reported here, we select a window size at least twice the period of the signal inside the window (this constrains the lowest frequency of 100 Hz). Furthermore, the sampling frequency must be at least ten times the fundamental frequency of the input signal in order to have more than ten samples/period (this constrains our highest frequency to 5 kHz). In fact, these are not unusual restrictions, and they are also a requirement of any FFT technique.

The differences between measured values and the theoretical spectra are shown in Figs. 3–6. In Fig. 3, the complete frequency range is shown. The generator frequency changes in steps of $48.828 \text{ Hz}/4 \cong 12.207 \text{ Hz}$. Figs. 4–6 are the enlargements of Fig. 3 in low, medium and high frequency ranges, respectively, and correspondingly, the generator frequency step is reduced to $48.828 \text{ Hz}/40 = 1.2207 \text{ Hz}$.

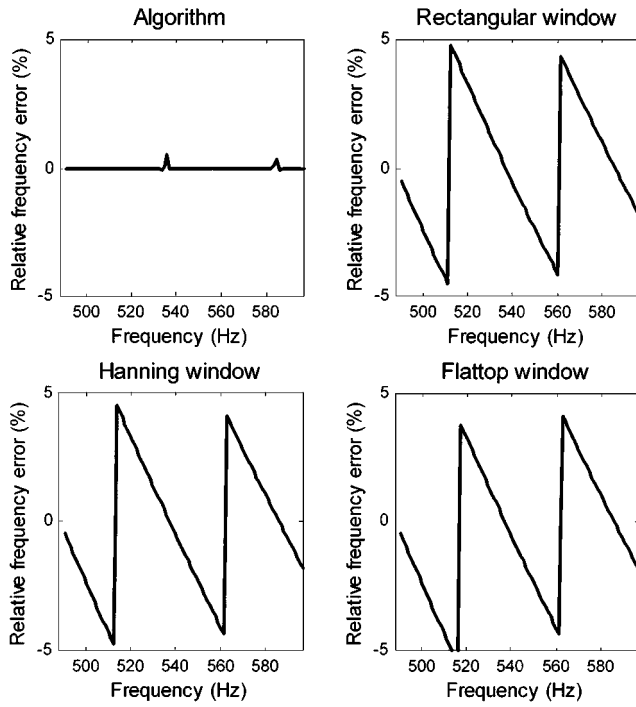
These figures show that the errors in our algorithm increase when the original window size is close to a multiple of the signal period, because for those particular values, the correction in the window size is not really required. Note that this adverse effect diminishes for high frequencies because the number of cycles inside the window increases. On the other side, for these same



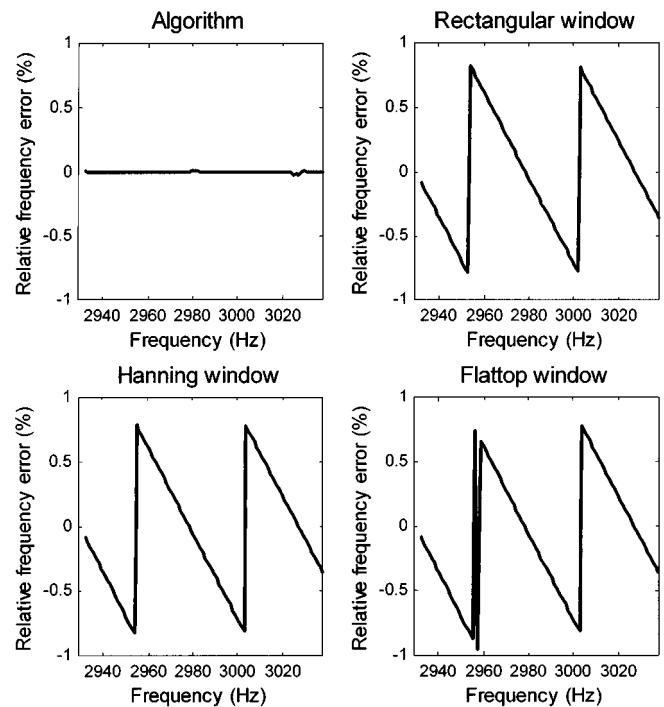
(a)



(a)



(b)



(b)

Fig. 5. Enlargement of Fig. 3 for the medium frequency range: (a) amplitude errors; (b) frequency errors.

Fig. 6. Enlargement of Fig. 3 for the high frequency range: (a) amplitude errors; (b) frequency errors.

values of frequency, the errors produced by the weighting windows of the scope are minimum. The error obtained with our algorithm is considerably lower than that produced by the scope even in these cases. Only in the very low frequency range, the flattop window gives the spectrum with a lower amplitude error [see Fig. 4(a)].

The error curves labeled “algorithm” are the mean value of 100 measurements in order to take into account the window starting time influence. The corresponding frequency and amplitude dispersions have upper limits σ_f and σ_A , respectively, with the following values: in Fig. 3 $\sigma_f = 0.421$ Hz and $\sigma_A = 0.29$ dB; in Fig. 4 $\sigma_f = 0.3327$ Hz and $\sigma_A = 0.27$ dB; in Fig. 5 $\sigma_f = 0.315$ Hz and $\sigma_A = 0.04$ dB; finally in Fig. 6 $\sigma_f = 0.0289$ Hz and $\sigma_A = 0.028$ dB.

In the case of scope measurements, the data acquisition process always started at the same trigger level (0 V), and then an average was not required.

IV. POLYHARMONIC CASES

The second set of measurements is made on a square wave with period $T_w/9.5$. This is an especially difficult measurement for the scope, but our algorithm works well, as it is reported in Table I. The column labeled *Exact* corresponds to the theoretical Fourier transform. The column *Algorithm* corresponds to our results. The remaining three columns labeled, respectively, *Rectangular*, *Hanning*, and *Flattop*, report the measurements performed by the HP5420A.

TABLE I
FREQUENCY AND AMPLITUDE ERRORS IN THE FFT OF A SQUARE WAVEFORM USING A COMMERCIAL INSTRUMENT AND THE ALGORITHM HERE PRESENTED

	Exact	Algorithm		Rectangular		Hanning		Flattop	
	Value	Value	Error	Value	Error	Value	Error	Value	Error
Freq. (Hz)	463.87	463.37	-0.50	488.28	24.41	488.28	24.41	439.45	-24.42
Amplitude	dBm	dBm	Δ dB	dBm	Δ dB	dBm	Δ dB	dBm	Δ dB
1 st harm.	6.078	6.062	-0.016	2.188	-3.890	4.688	-1.390	5.937	-0.141
3 rd harm.	-3.465	-3.520	-0.055	-6.563	-3.098	-5.000	-1.535	-3.438	0.027
5 th harm.	-7.902	-8.028	-0.126	-11.563	-3.661	-9.375	-1.473	-7.813	0.089
7 th harm.	-10.824	-10.831	-0.007	-14.063	-3.232	-12.188	-1.375	-10.930	-0.099
9 th harm.	-13.007	-13.075	-0.068	-16.625	-3.618	-14.688	-1.681	-13.125	-0.118

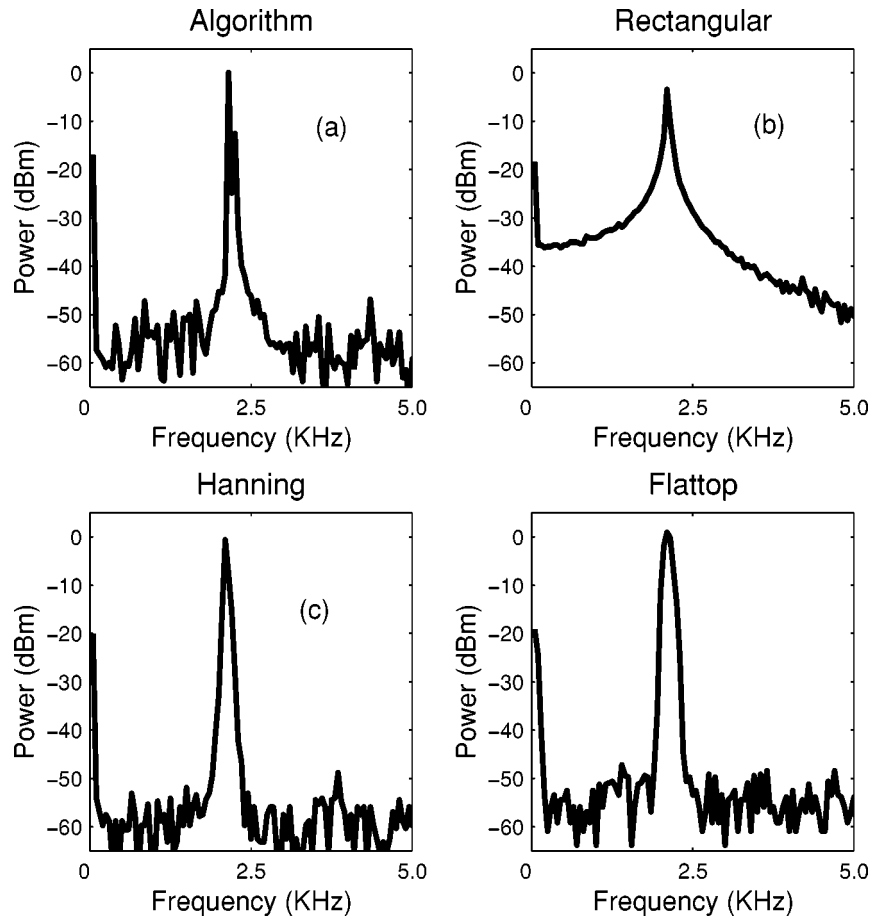


Fig. 7. Case of a signal consisting of two sinusoidal waveforms, one with 2077 Hz and 0.66 Vpp and the other with 2167 Hz and 0.17 Vpp. (a) Spectra obtained by our algorithm; (b)-(d) are the spectra measured by the HP54520A with its different built-in windows.

Using our algorithm, the errors in both amplitude and frequency determinations are greatly reduced, and in the case of frequency determinations, it is always the best choice. This is a consequence of the automatic adjustment of the window size.

The last case we study is a compound signal consisting of two sinusoidal waves of 2077 Hz, 0.66 Vpp and 2167 Hz, 0.17 Vpp, respectively. The FFT is shown in Fig. 7(a)–(d), and the measured values are reported in Table II. This signal is especially in-

teresting because its theoretical spectrum has two spectral components so close to each other, that a very sensitive frequency resolution is required to single out any of them. The spectra obtained with the HP scope and its different windows are shown in Fig. 7(b)–(d). Note that the scope is not capable of solving each spectral component and gives us only one peak. On the other hand, in Fig. 7(a) the spectrum obtained with our algorithm is shown: both spectral components are clearly identified. This ability is confirmed in Table II.

TABLE II
FREQUENCY AND AMPLITUDE ERRORS IN THE FFT OF A SIGNAL CONSISTING OF TWO CLOSE SINUSOIDAL COMPONENTS USING A COMMERCIAL INSTRUMENT
AND THE ALGORITHM HERE PRESENTED

	Exact	Algorithm		Rectangular		Hanning		Flattop	
	Value	Value	Error	Value	Error	Value	Error	Value	Error
First sine wave:									
Freq. (Hz)	2077	2075	-0.09%	2020	-2.74%	2050	-1.3%	2050	-1.3%
Amp (dBm)	0.37	0.0844	-0.29 db	-3.75	-4.12 db	-1.25	-1.62 db	0.9375	0.57 db
Second sine wave:									
Freq. (Hz)	2167	2171.4	0.2%	N.D.*	N.D.*	N.D.*	N.D.*	N.D.*	N.D.*
Amp (dBm)	-11.41	-12.54	-1.13 db	N.D.*	N.D.*	N.D.*	N.D.*	N.D.*	N.D.*

N.D.*: not detected.

V. CONCLUSIONS

The simple strategy presented in this paper may be implemented on any PC with a DAQ. Leakage is greatly reduced when compared with traditional windowing methods.

The worst values are obtained when the size of the rectangular window is close to a multiple of the signal period, i.e., when $\text{int}(C)/C$ is very close to an integer. In these cases, it would be better to eliminate the adaptive mechanism. In spite of this drawback, the method is more efficient than the use of weighting windows even in these cases.

It is remarkable that, in order to obtain frequency errors of the same order as that produced by the algorithm (i.e., lower than 1 Hz), a window length fifty times larger would need to be used with the corresponding increase in both processing time and memory capabilities. Note that, in the case of frequency determinations, the usual notion of frequency resolution as f_s/N is no longer valid in evaluating the capabilities of the measuring system. In fact, the algorithm changes dynamically the sampling frequency, diminishing the error well over the original frequency resolution.

It is also possible to combine our algorithm with modern instruments including built-in data acquisition and FFT capabilities, to improve the spectral analysis in a very simple and inexpensive way.

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