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A simple alias-free QMF system with near-perfect reconstruction

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Abstract

This paper presents a design of prototype filters for quadrature mirror filter (QMF) banks. To minimize the value of reconstruction error for near-perfect reconstruction (NPR), linear optimization has been applied. Variable and combinational window functions with high side-lobe-fall-off-rate (SLFOR) have been used to design lowpass prototype filters. Use of window functions resulted in simple implementation. High SLFOR of combinational windows reduced the energy leakage due to aliasing from one sub-band to the other. The proposed optimization algorithm takes a few seconds on Pentium processor.

Keywords: QMF, filterbank, variable window, combinational window.

1. Introduction

Quadrature mirror filter (QMF) bank finds wide application in the area of signal processing, particularly in the sub-band coding of speech, digital audio applications, communication systems, and short-time spectral analysis [1]. A two-band, linear phase, quadrature mirror filter bank was introduced by Johnston [2]. The theory and design of these filterbanks have been dealt with extensively by other researchers [3, 4]. In QMF bank, the input signal $x(n)$ is split into two equally spaced frequency sub-bands by two-band analysis filters $H_0(z)$ and $H_1(z)$, followed by two-fold decimation. At the receiving end, the corresponding synthesis bank has two-fold interpolation in both sub-bands followed by $G_0(z)$ and $G_1(z)$ synthesis filters, and finally, an adder to add both bands. Figure 1 shows the QMF–analysis/synthesis framework. The reconstructed output signal $y(n)$ suffers from three distortions, viz., aliasing, amplitude, and phase distortions [1]. Ideally, these distortions can be completely eliminated, called perfect reconstruction (PR). However, in practice, PR is not possible. To eliminate the aliasing and phase distortions, all the analysis and synthesis filters must be translated to a single lowpass prototype of even-order symmetric FIR linear-phase filter

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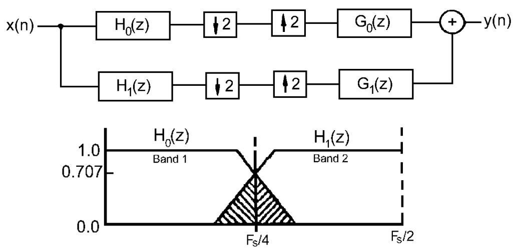

FIG. 1. Two-band QMF system.

[1, 3, 5]. Thus, the design of the whole filterbank reduces to that of the prototype filter. Amplitude distortion still needs to be eliminated. Direct approach to the design of lowpass filters using window technique is not applicable to eliminate the amplitude distortion; this involves optimization techniques [2, 5]. Johnston [2] used Hanning window to design the lowpass prototype FIR filter and proposed nonlinear objective function, which requires nonlinear optimization algorithm to obtain minimum reconstruction error. Creusere and Mitra [4] used a different objective function, which is linear in nature. Linear optimization algorithm has been used to obtain near-perfect reconstruction. Parks–McClellan algorithm has been used to design the lowpass prototype filter. Lin and Vaidyanathan [6] used window method to design lowpass prototype FIR filter. A different objective function linear in nature has been used to minimize the reconstruction error. In both [4] and [6] a single parameter has been optimized.

This work uses the algorithm as proposed in Creusere and Mitra [4] with certain modifications to optimize the objective function. Two combinational window functions [7, 8, 10] having large SLFOR have been used for designing FIR prototype filters. Due to the closed-form expressions of the window functions, the optimization procedure gets simplified. Apart from the combinational windows, Kaiser and Dolph–Chebyshev (DC) windows [11, 13] have also been used. Finally, a comparative evaluation of the window functions used to design the prototype has been done with reconstruction error and far-end attenuation being selected as the main figure of merit.

2. FIR filter design using window method

Impulse response of lowpass prototype filter, $h(n)$, of order (N) designed using window function [11] is of the form $h(n) = h_d(n)w(n)$, where

$$h_d(n) = \frac{\sin\left(f_c\left(n-\frac{N}{2}\right)\right)}{p\left(n-\frac{N}{2}\right)}, \tag{1}$$

is the desired impulse response of the ideal filter with cut-off frequency $f_c = 0.5\ (f_p + f_s)$, with f_s, f_p as the stopband, passband frequencies, respectively, and $w(n)$ is the window function. Window functions used in this work and their filter design relationships are given in Appendix I.

The filter designed using the window is specified by three parameters–cut-off frequency (f_c), filter order (N), and window shape parameter (g_6, g_4, b). For desired stopband attenuation (Δ_s) and transition bandwidth, the order of the filter (N) can be estimated by

$$N = \left\lfloor \frac{D}{\Delta F_S} \right\rfloor + 1, \tag{2}$$

where D is the normalized window width, ΔF_s, the normalized transition width $= (f_s - f_p)/F_s$, and F_s, the sampling frequency in Hertz. The window shape parameter can be determined by the desired stopband attenuation.

3. Design of NPR filters using optimization algorithm

To get the high-quality reconstructed output $y(n)$, the frequency response of lowpass prototype filter, $H(e^{j2pf})$, must satisfy (3) and (4) [4]:

$$|H(e^{j2pf})|^2 + \left|H\left(e^{j\left(2pf - \frac{F_s}{4}\right)}\right)\right|^2 = 1, \quad \text{for} \quad 0 < f < F_s/4, \tag{3}$$

$$|H(e^{j2pf})| = 0, \quad \text{for } f > F_s/4, \tag{4}$$

by assuming that filters have even number of coefficients.

If (4) is satisfied exactly, aliasing is eliminated between nonadjacent bands. In case (3) is satisfied, then amplitude distortion is eliminated [4]. Phase distortion is removed by selecting even-order FIR prototype filter [1, 5]. Constraints (3) and (4) cannot be satisfied exactly for finite length filter order so it is necessary to design a filter which approximately satisfies (3) and (4). Johnston [2] combined the passband ripple energy and out-of-band energies into a single cost function having nonlinear nature and then minimized it using Hooke and Jeaves algorithm [12]. Creusere and Mitra [4] designed filters using Parks–McClellan algorithm that approximately satisfied (3) and (4). The filter length, relative error weighting, and stopband edge were fixed before optimization procedure started, while the passband edge was adjusted to minimize the objective function

$$f = \max_f \left\{ |H(e^{j2pf})|^2 + \left|H\left(e^{j\left(2pf - \frac{F_s}{4}\right)}\right)\right|^2 - 1 \right\}, \text{ for } \quad 0 < f < F_s/4. \tag{5}$$

Lin and Vaidyanathan [6] used different objective function called f_{new} and designed the prototype filters using Kaiser window.

In this work, the objective function (5) has been used to minimize the reconstruction error and to obtain near-perfect reconstruction. High SLFOR combinational window functions have been used to design the prototype lowpass FIR filters. In Creusere and Mitra [4], passband frequency (f_p) is changed in each iteration to minimize the objective function;

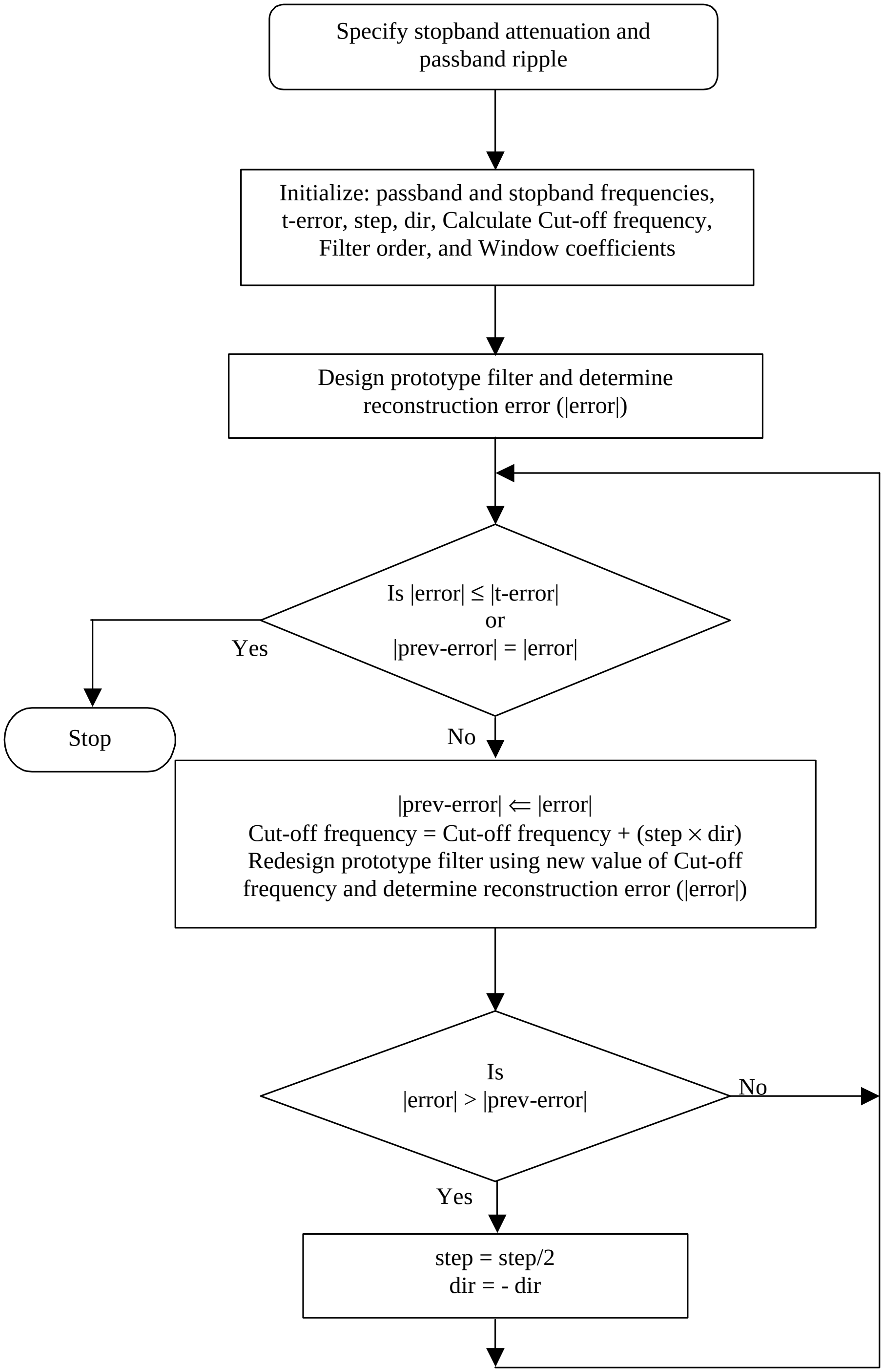

FIG. 2. Proposed flowchart for optimization algorithm.

however, in this algorithm the cut-off frequency (f_c) is varied to get the smallest reconstruction error. The initial window coefficients have been calculated before calling the optimization routine. The modified flowchart of the optimization algorithm is given in Fig. 2. This algorithm adjusts the cut-off frequency in each iteration to minimize the objective function by the stepsize denoted by *step*, calculates the new filter coefficients, and then computes the reconstruction error, f (denoted *error* in the flowchart). Whenever the error increases from the pervious error (depicted as *prev-error* in the algorithm) stepsize is halved and the search direction labeled *dir* is changed. The iterations are stopped either when the error of the current iteration is within the specified tolerance (depicted as *t-error*), which is initialized before the optimization process begins or when *prev-error* equals error.

There is no demarcation between the passband and transition band [2]; however, the approximation in constraint (4) can be achieved by keeping the stopband frequency f_s at $F_s/4$ and then the only constraint is that the response of the filter set must be very close to –3dB at $F_s/4$, so that the sum of $(H^2(f) + H^2(F_s/2 - f))$ must remain close to unity about that point [2]. It is observed that the best way to achieve this is to initialize the passband edge at $F_s/6$ in the optimization algorithm. Depending on the transition width, filter order is obtained and then the window coefficients are calculated. Once the window coefficients have been calculated they are not disturbed, while varying the cut-off frequency which varies the filter coefficients. The magnitude and direction of the cut-off frequency change is varied to obtain filter coefficients, which can minimize the objective function given by (5). Changes in the cut-off frequency are of the magnitude that they do not affect the filter order but the objective function is affected.

For example, Kaiser window has been used to design a lowpass prototype filter with desired stopband attenuation of 88 dB. The values of passband and stopband frequencies are fixed before calling the optimization algorithm and these normalized values are 0.1666 and 0.2500, respectively. The order of the prototype filter is estimated by eqn (2), which is equal to 68. By optimization, the cut-off frequency f_c is adjusted. When $f_c = 0.51549$, the reconstruction error has a minimum value of 0.01098 dB.

4. Comparative performance analysis

QMF banks were designed using window functions described in Appendix I. Results are shown in Tables I–III. In Table I, the value of stopband attenuation was selected as 50 dB, resulting in different filter orders for different window functions. In Table II results

Table I
Performance of QMF filter at 50 dB stopband attenuation

Window function	Reconstruction error (dB)	Filter order (N)	Far-end attenuation (dB)
Kaiser window	0.3208	38	75
DC window	0.1718	32	50
PC4 window	0.1298	66	110
PC6 window	0.1060	50	74

Table II
Performance of QMF filter at N = 80

Window function	Reconstruction error (dB)	Stopband attenuation (dB)	Far-end attenuation (dB)
Kaiser window	0.0116	90	107
DC window	0.0152	82	82
PC4 window	0.0171	64	160
PC6 window	0.0656	52.5	89

Table III
Optimum performance in terms of reconstruction error

Window function	Recon-struction error (dB)	Stopband attenuation (db)	Filter order (*N*)	Far-end attenuation (dB)
Kaiser window	0.0097	88.00	90	107
DC window	0.0086	86.00	36	86
PC4 window	0.0135	59.50	30	110
PC6 window	0.0120	55.00	22	72

corresponding to filter designs with an order (*N*) of 80 are shown. Same value of filter order resulted in different levels of stopband attenuations. Finally, in Table III, a comparison is made of the optimum performance that can be attained with the four window functions.

Apart from the reconstruction error, the far-end attenuation (amplitude of the last ripple in the stopband) is also selected as one of the figures of merits for the comparative study. This parameter is of significance when the signal to be filtered has great concentration of spectral energy. In a sub-band coding, the filter is intended to separate out various frequency bands for independent processing. In the case of speech, e.g. the far-end rejection of the energy in the stopband should be more so that the energy leak from one band to another is minimum. From Tables I and II, it is inferred that as the stopband attenuation increases the value of reconstruction error decreases. The Kaiser window-designed FIR filter gives better performance as compared to the other window functions. Far-end attenuation

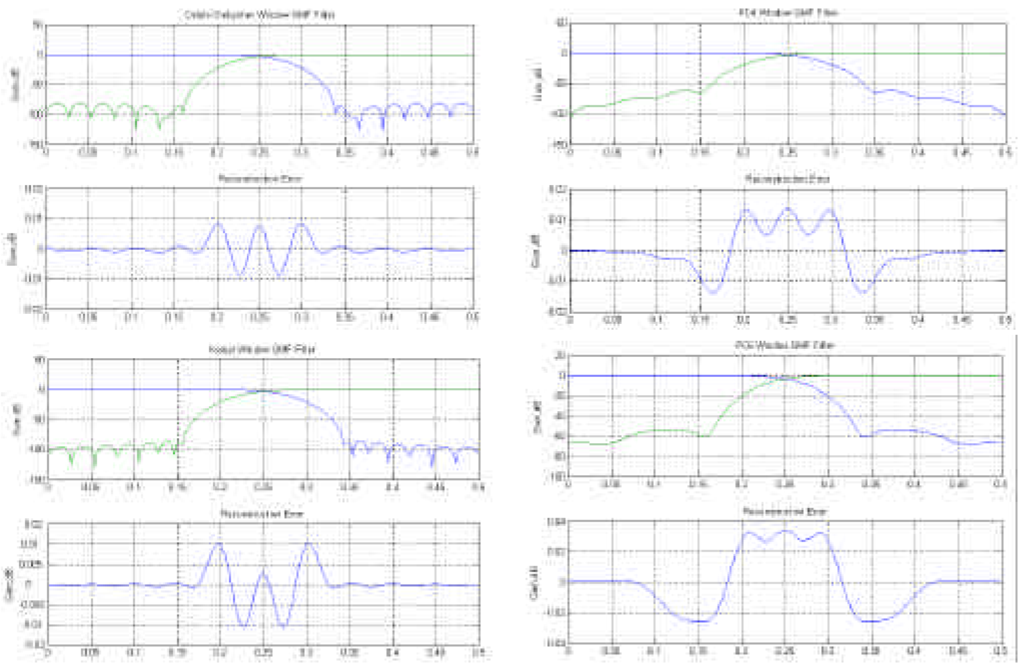

FIG. 3. QMF filter and reconstruction error using combinational and variable windows for N = 36.

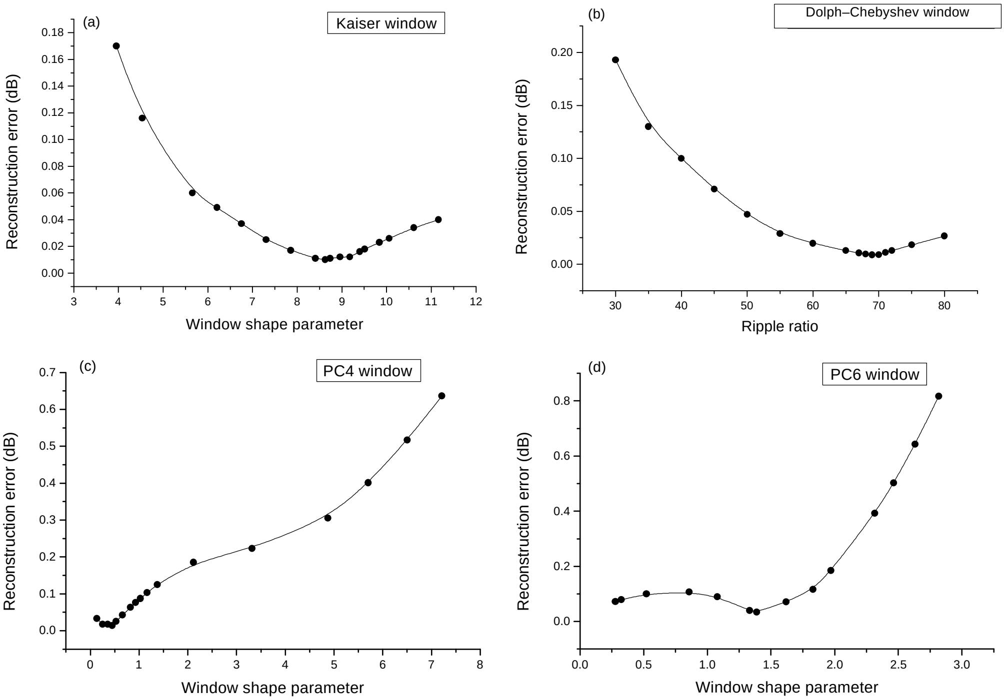

FIG. 4. Variation of reconstruction error with window shape parameter/ripple ratio.

is maximum for PC4 window-based FIR filters. As from Table III, the optimum performance in terms of reconstruction error has been obtained in Dolph–Chebyshev window function. Reconstruction error for the four prototype filters designed with $N = 36$ are shown in Fig. 3 along with their magnitude frequency response. It is also observed practically that the reconstruction error depends on the window shape parameter for Kaiser and combinational windows, whereas for Dolph–Chebyshev window reconstruction error is dependent on the ripple-ratio (r). Plots corresponding to these variations are shown in Fig. 4 for the four windows.

5. Conclusion

A simple algorithm for designing the lowpass prototype filters for QMF banks has been used to optimize the reconstruction error by varying the filter cut-off frequency. Prototype filters designed using high SLFOR combinational window, Kaiser window and Dolph–Chebyshev window functions have been compared. Reconstruction error was found to be dependent on the window shape parameter. Optimum performance with respect to reconstruction error was observed for Dolph–Chebyshev window function. Combinational windows provided better far-end rejection of the stopband energy. This feature helps to reduce the aliasing energy leak into a sub-band from that of the signal in the other sub-band. Simulation studies reveal that the algorithm converges between 270 and 310 iterations and takes a few seconds to optimize the filter coefficients on a Pentium processor.

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Appendix I

Window functions and their filter design relationships

1. *Parzen-cos^6(np/N) combinational window (PC6):* The expression for Parzen-$\cos^6(np/N)$ combinational window with g_6 as window shape parameter is given as [7]:

$$w_{PC6}(n) = \begin{cases} g_6[l_6(n)] + (1-g_6)[d_6(n)], & |n| \le \dfrac{N}{2} \\ 0, & |n| > \dfrac{N}{2} \end{cases}$$

$$0 \le g_6 \le 3.7$$

where,

$$l_6(n) = \begin{cases} 1 - 24\left|\dfrac{n}{N}\right|^2\left(1 - 2\left|\dfrac{n}{N}\right|\right), & |n| < \dfrac{N}{4} \\ 2\left(1 - 2\left|\dfrac{n}{N}\right|\right)^3, & \dfrac{N}{4} \le |n| \le \dfrac{N}{2} \end{cases}$$

$$d_6(n) = \cos^6\left(\frac{np}{N}\right), \quad |n| \leq \frac{N}{2}.$$

FIR filter design relationships are given by the following equations [8]

$$g_6 = a + (b\Delta_s) + (c\Delta_s^2)$$

where,

$$a = 8.15414; \quad b = -0.236709; \ c = 0.00218617, \quad \text{for} \quad 30.32 \leq \Delta_s \leq 51.25$$

$$a = 21.3669; \quad b = -0.605789; \ c = 0.00434808, \quad \text{for} \quad 51.25 < \Delta_s \leq 68.69$$

$$D = a + (b\Delta_s) + (c\Delta_s^2)$$

where,

$$a = 1.82892; \ b = -0.0275481; \ c = 0.00157699, \quad \text{for} \quad 30.32 \leq \Delta_s \leq 43.60$$

$$a = 1.67702; \ b = 0.0450205; \ c = 0.00, \quad \text{for} \quad 43.60 < \Delta_s \leq 49.44$$

$$a = 85.4738; \ b = -3.41969; \ c = 0.035784, \quad \text{for} \quad 49.44 < \Delta_s \leq 57.48$$

$$a = -8.60006; \ b = 0.477004; \ c = -0.00355655, \quad \text{for} \quad 57.48 < \Delta_s \leq 68.69$$

2. *Papoulis-cos^4(np/N) combinational window* (*PC*4): The combinational window of Papoulis-cos^4(*np/N*) with g_4 as window shape parameter is given by [7]:

$$w_{PC4}(n) = \begin{cases} g_4\left[l_4(n)\right] + (1-g_4)\left[d_4(n)\right], & |n| \leq \dfrac{N}{2} \\ 0, & |n| > \dfrac{N}{2} \end{cases}$$

$$0 \leq g_4 \leq 8.235$$

where,

$$l_4(n) = \frac{1}{p}\left|\sin\left(\frac{2p\,n}{N}\right)\right| + \left(1 - 2\left|\frac{n}{N}\right|\right)\cos\left(\frac{2p\,n}{N}\right), \quad |n| \leq \frac{N}{2}.$$

$$d_4(n) = \cos^4\left(\frac{np}{N}\right), \quad |n| \leq \frac{N}{2}.$$

FIR filter design relationships have been established using the method of Prabhu [9], and Sharma *et al.* [10]. These relationships are described as:

$$g_4 = a + (b\Delta_s) + (c\Delta_s^2) + (d\Delta_s^3) + (e\Delta_s^4), \quad \text{for} \quad 26.19 < \Delta_s \leq 61.08$$

where,

$$a = -69.058755; \ b = 8.409918; \ c = -0.321364; \ d = 0.005044; \ e = -0.000028$$

$$D = a + (b\Delta_s) + (c\Delta_s^2) + (d\Delta_s^3) + (e\Delta_s^4), \quad \text{for} \quad 26.19 < \Delta_s \leq 61.08$$

where,

$$a = 8.728537; \ b = -0.412899; \ c = -0.000713; \ d = 0.000355; \ e = -0.000004$$

3. *Kaiser window:* The window function is given by [11]:

$$w(n) = \frac{I_o\left[b\sqrt{1-\left(1-\frac{2n}{N-1}\right)^2}\right]}{I_o\left[b\right]}, \quad 0 \le n \le (N-1)$$

where $I_o[.]$ is the modified zeroth-order Bessel function, and **b**, the window shape parameter. The empirical design equations developed by Kaiser [11] are given by

$$b = \begin{cases} 0, & \text{for } \Delta_s \le 21 \\ 0.5842\,(\Delta_s - 21)^{0.4} + 0.07886\,(\Delta_s - 21), & \text{for } 21 < \Delta_s < 50 \\ 0.1102\,(\Delta_s - 8.7), & \text{for } \Delta_s > 50 \end{cases}$$

$$D = \begin{cases} 0.9222, & \text{for } \Delta_s \le 21 \\ \dfrac{(\Delta_s - 7.95)}{14.36} & \text{for } \Delta_s > 21 \end{cases}$$

4. *Dolph–Chebyshev Window (DC):* The window function is given by [13]:

$$w(n) = \begin{cases} \dfrac{1}{N+1}\left(\dfrac{1}{r} + 2\sum_{i=1}^{N/2} C_N\left[x_o \cos\left(\dfrac{ip}{N+1}\right)\cos\left(\dfrac{2nip}{N+1}\right)\right]\right), & \text{for } |n| \le N/2 \\ 0, & \text{for } |n| > N/2 \end{cases}$$

where,

$$r = \frac{\Delta_s}{\Delta_p},$$

$$x_0 = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{r}\right)\right],$$

$$C_N(x) = \begin{cases} \cos(N\cos^{-1}(x)), & \text{for } |x| \le 1 \\ \cosh(N\cosh^{-1}(x)), & \text{for } |x| > 1 \end{cases}$$

where Δ_p is the desired passband ripple and Δ_s, the desired stopband attenuation.

The empirical design equation developed by Kaiser has been modified by Saramaki [13] for computation of *D*. The modified equation is given by

$$D = \begin{cases} 0.9222, & \text{for } |\Delta_s| \le 21 \\ \dfrac{(\Delta_s - 5.45)}{14.36}, & \text{for } |\Delta_s| > 21 \end{cases}$$