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A Simple and Effective Precoding Scheme for Noise Whitening on Intersymbol Interference Channels

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TR 92-2 r1

A Simple and Effective Precoding Scheme for Noise Whitening on Intersymbol Interference Channels†

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Abstract

A precoding scheme for noise whitening on intersymbol interference channels is presented. This scheme is compatible with trellis-coded modulation and unlike Tomlinson precoding allows constellation shaping. It can be used with almost any shaping scheme (including the optimal SVQ shaping) as opposed to trellis precoding which can only be used with trellis shaping. The implementation complexity of this scheme is minimal — only three times that of the noise prediction filter, and hence effective noise whitening can be achieved by using a high-order predictor.

Index Terms: Precoding, equalization, ISI channels, noise whitening, shaping, trellis coding.

† The first and the third authors are affiliated with the Systems Research Center at the University of Maryland; their work was supported in part by National Science Foundation grants NSFD MIP-9109109, MIP-86-57311 and NSFD CDR-85-00108.

I. Introduction

The need to send data at high bit-rates over band-limited channels has led to transmission at high baud-rates, thus making use of almost all of the available bandwidth. Most practical channels however have a non-ideal frequency response, especially near the band edges where the transmitted signal can be considerably attenuated leading to intersymbol interference (ISI). To compensate for this attenuation, an equalizer can be used at the channel output. But this has the effect of boosting and coloring the noise. Alternatively, decision feedback equalization (DFE) can be used to eliminate ISI without noise enhancement [1]. When used in coded modulation systems, DFE results in high complexity decoding algorithms [2]-[5]. Tomlinson-Harashima precoding [6],[7] equalizes the signal before transmission, is relatively simple to implement and can be used with coded modulation. But this precoding scheme does not realize any shaping gain that results from having a spherical constellation boundary rather than a cubic boundary. Recently, Eyuboglu and Forney [8] have proposed a trellis precoding scheme that whitens the noise at the equalizer output. This scheme combines precoding and trellis shaping [9] and achieves 0.7-0.9 dB shaping gain with a 4-state trellis. There are however two drawbacks of trellis precoding: (i) the complexity is dependent on the number of states in the shaping-trellis and (ii) it is compatible only with trellis shaping and cannot be combined with other shaping schemes such as the optimal shaping scheme described in [10],[11]. This paper describes a new precoding scheme[1] that is simple to implement, is transparent to shaping and can be used in place of Tomlinson-Harashima precoding to realize both coding and shaping gains over ISI channels.

II. Precoding for Noise Whitening

Consider the block-diagram of a simple pre-equalization system shown in Fig. 1. This

[1] Shortly after this work was submitted to the *IEEE Transactions on Communications*, similar precoding schemes developed independently by Motorola Information Systems [12] and General DataComm [13] were presented at the TIA meeting in Jan. 1992.

system uses a noise prediction-error filter $H(z)$ to whiten the noise at the equalizer output. The ISI introduced by the prediction-error filter is compensated for at the transmitter by pre-equalizing the signal with the filter $1/H(z)$ before transmission. Here it is assumed that the noise prediction-error filter $H(z)$ is minimum phase and hence the pre-equalizer $1/H(z)$ is stable. It can be shown that the optimal (minimum mean squared-error) linear predictor of a given order is always minimum phase (Chapter 7 in [14]). The problem with the above system is that the output x_n of the pre-equalizer can have large peaks even when the pre-equalizer input a_n is peak-limited. Also, x_n can have a significantly higher average power than a_n, eliminating the shaping gain that could have been realized if the channel were ideal. To solve these problems, we propose the system shown in Fig. 2. In this system the pre-equalizer has been replaced by a nonlinear precoder (shown in Fig. 4). The ISI introduced by the noise prediction-error filter is not directly compensated for by the precoder but is totally removed after Viterbi trellis decoding. The nonlinear precoder only modifies its input signal a_n slightly, just enough to ensure that the input r_n to the Viterbi trellis decoder is always an additive white noise affected version of a sequence consistent with the trellis. The sequence r_n can therefore be decoded using the Viterbi trellis decoder and then the ISI can be removed. This is much simpler than DFE schemes for coded modulation such as reduced-state sequence estimation and parallel decision feedback decoding, where the ISI is removed within the trellis decoder leading to a complex decoding algorithm.

The precoding scheme of Fig. 2 is now described. Fig. 3 shows the constituent translated 2D lattice $\Lambda + (1/2, 1/2) = \mathbf{Z}^2 + (1/2, 1/2)$ of a trellis-coded multidimensional constellation. This lattice is the union of 2^u cosets of a sublattice Λ', where u is the number of coded bits per 2D ($u = 2$ and $\Lambda' = 2\mathbf{Z}^2$ in Fig. 3). Let V denote the Voronoi region of the sublattice Λ'. The points on the boundary of V are either included or excluded from V in such a manner that V contains no two different points that are modulo Λ' equivalent. Note that in Fig. 3, if a_n is a sequence of points in $\mathbf{Z}^2 + (1/2, 1/2)$ that is consistent with the trellis, then the sequence $a_n + 2(i_n, j_n)$ is also consistent with the trellis for $i_n, j_n \in \mathbf{Z}$. In general, any sequence that is equivalent to a_n modulo Λ' is consistent with the trellis. We shall use this fact later to establish that the input to the Viterbi trellis decoder in Fig. 2 is the noise affected version of a sequence that is consistent with the trellis.

Next, consider the nonlinear precoder shown in Fig. 4. The feedback f_n in this precoder can be expressed as the sum of two components, m_n and q_n, where $m_n = f_n$ mod Λ' and $q_n = f_n - m_n$ is the quantized feedback. The quantity q_n is the result of quantizing f_n to the nearest point in Λ' such that the quantization error $m_n \in V$. The nonlinear precoder of Fig. 4 is equivalent to the linear precoder of Fig. 5 with the input a_n replaced by $a_n + q_n$.

Assuming that the linear equalizer eliminates the ISI introduced by the channel, the output of the noise prediction-error filter in Fig. 2 is $r_n = a_n + q_n + z_n$, where z_n is the whitened noise sequence. Since a_n and $a_n + q_n$ are mod Λ' equivalent, $r_n - z_n$ is a sequence that is consistent with the trellis. Hence r_n can be decoded using the Viterbi trellis decoder resulting in $v_n = a_n + q_n$ (assuming no decoding error) at the output of the Viterbi decoder. To remove the ISI caused by q_n, the sequence v_n is passed through the linear filter $1/H(z)$ of Fig. 5 to obtain the sequence $w_n = a_n - m_n$ from which a_n can be recovered by picking the point closest to w_n (quantizing w_n) that belongs to the same coset as v_n. This follows since a_n is in the same coset as v_n and $m_n \in V$. The sequence a_n can be converted to binary data just as in the ideal channel case. Since the ISI removing filter $1/H(z)$ is stable, any decoding errors that occur do not propagate indefinitely.

Note that the precoding filter (and the quantization operation) in the transmitter should be exactly identical to the ISI removing filter (and the following quantizer) in the receiver. Even a slight difference (perhaps in the precision of the arithmetic used) can cause a significant increase in the error probability.

The transmitted sequence x_n is the sum of two independent components, $-m_n$ and the trellis encoder output a_n. The average power of x_n is therefore the sum of the average powers of a_n and m_n. For an ideal channel, only a_n needs to be transmitted ($x_n = a_n$ when $H(z) = 1$), the additional power $E\{m_n^2\}$ is hence the price paid for transmitting over ISI channels. The transmission of additional power results in a reduction in the effective shaping gain of the constellation. For non-ideal channels ($H(z) \neq 1$), it is reasonable to assume that m_n is uniformly distributed in V and the additional power needed is the average power of V.

When the rate of the constellation is large, the average power of a_n is considerably larger than the average power of V and hence only a small reduction in shaping gain results. As an example, consider a trellis-coded constellation with 2 coded bits/2D, a coding redundancy of 0.5 bits/2D (as in 4D 16-state Wei code [15]) and a rate of 7.5 bits/2D. If no shaping is used, the average power of this constellation (normalized to 2D) is $(16^2 - 1)/6 = 42.50$. Even if optimal shaping is used (1.53 dB shaping gain) the transmitted power is no less than $42.50/1.422 = 29.88$. Since the average power of the region V (in this case a 2×2 square) is 0.667, the resulting loss in shaping gain in no more than 0.095 dB. Together with the fact that a very reasonable complexity 64-dimensional SVQ shaping[2] results in 1.26 dB shaping gain on ideal channels, this suggests that a net shaping gain of 1.16 dB can be achieved on ISI channels by using the proposed precoding scheme on SVQ-shaped constellations. This is significantly higher than the shaping gains possible with trellis precoding of comparable complexity. According to [8], most of the prediction gain in voice-band telephone line channels can be achieved by using as few as 2 to 3 taps in the noise prediction filter. Since the complexity of the above precoding scheme is small — only three times the complexity of the noise prediction filter — a higher-order predictor can be easily implemented making the noise almost white and resulting in larger prediction gains than trellis precoding.

Although, as mentioned before, any decoding errors that occur do not propagate

[2] This is a shaping technique presented in [10],[11]. It represents an optimally shaped constellation as the codebook of a structured vector quantizer and uses the quantizer encoding/decoding algorithms to index the constellation points.

indefinitely because the ISI removing filter $1/H(z)$ is stable, they can propagate for a long duration if $H(z)$ has zeros close to the unit-circle. If this is the case, the filter $1/H(z)$ in the precoder and the identical ISI removing filter in the receiver can be implemented as FIR filters with an impulse response that is the truncated version of the response of $1/H(z)$. The 'feedback' f_n (see Fig. 5) is then generated from the past values of the filter input $a_i + q_i$, $(i < n)$ rather than the output x_i. This will ensure that the effect of any error is limited to a fixed-duration. It might appear at first that the FIR filter will require a large number of taps to cancel the ISI effects of the noise prediction-error filter (especially if $H(z)$ has zeros near the unit-circle), but as the following argument suggests, this may not be essential. If the FIR filter has a small order, implementing an approximation of $1/H(z)$, the channel equalizer (which adapts based on the output of the noise prediction-error filter) will interpret this as an exact implementation of $1/H(z)$ but a slightly modified channel response. The equalizer will then adapt to compensate for this modified channel response. The above approximation is effectively equivalent to a slight mismatch between the precoder and the channel, possibly leading to a small increase in the noise variance. Exactly how much the noise variance increases is a function of the extent of mismatch. Another cause of this mismatch between the precoder and the channel is a drift in the channel characteristics with time while the precoder is kept fixed. This issue has been addressed in [8].

An interesting observation is that with this scheme, it is even in principle not possible to transmit at channel capacity. This is because for transmitting at or near capacity, almost all of the coding gain must be realized by using powerful (possibly very complex) trellis codes. For such codes however, the region V will become large, and hence for a given transmission rate, the shaping gain will eventually disappear.

III. Conclusion

We have presented a simple but effective precoding scheme for noise whitening on ISI channels. This scheme permits any type of shaping and is compatible with trellis coding. Its implementation complexity is significantly less than that of trellis precoding. The resulting reduction in shaping gain is small and becomes negligible as the rate per 2D of the constellation increases.

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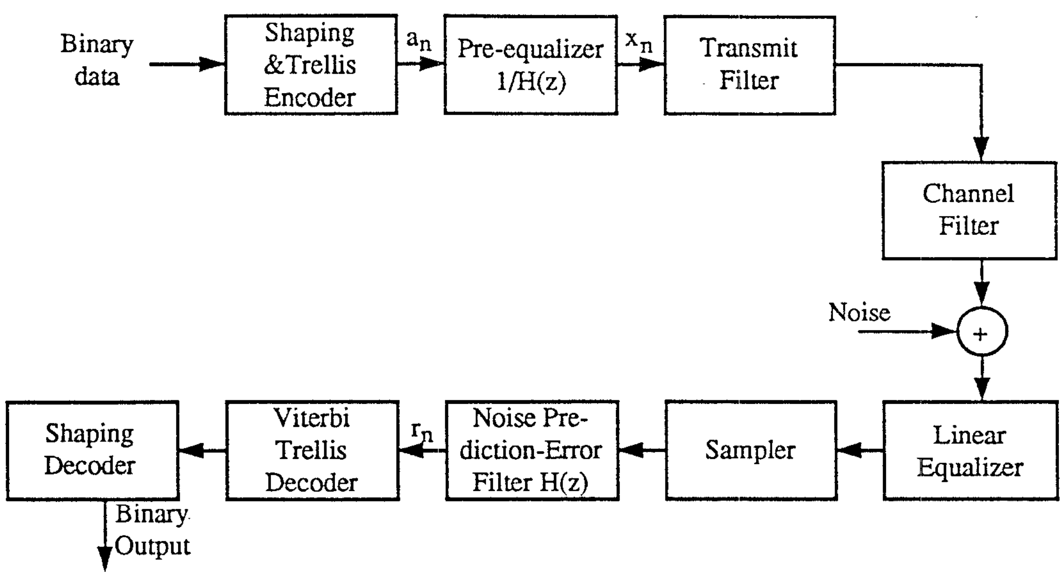

Fig. 1: A Simple Pre-equalization System for Noise Whitening

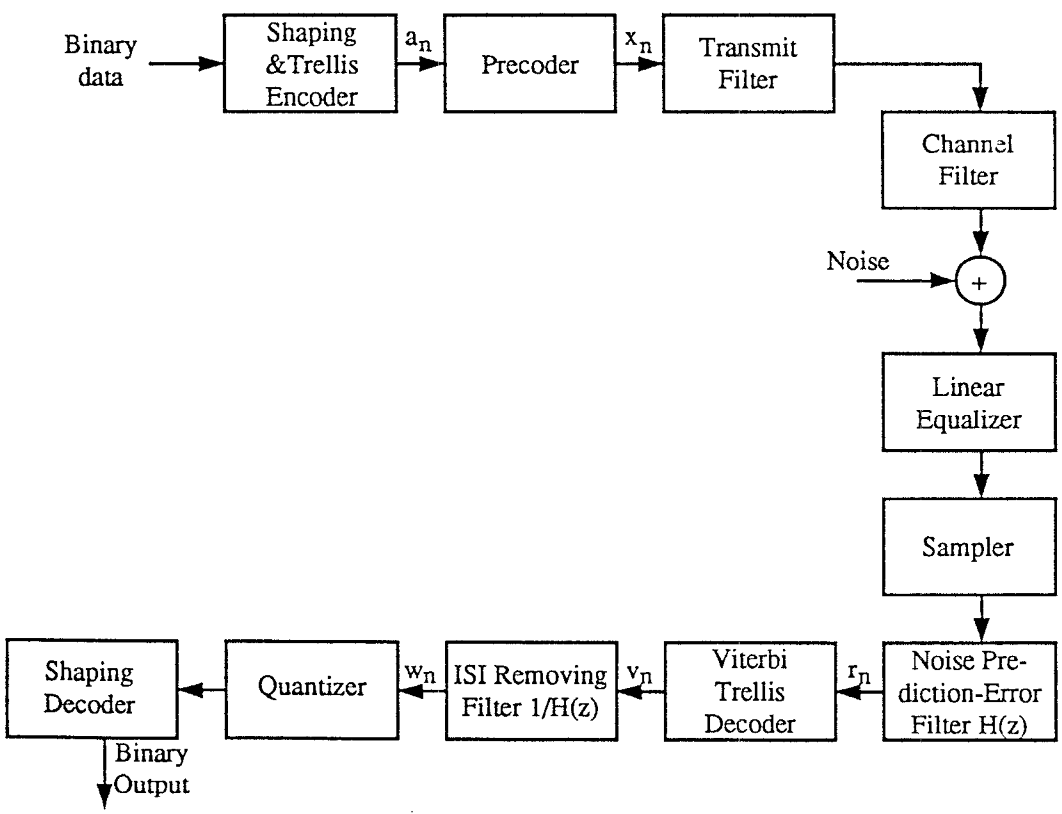

Fig. 2: The Proposed Precoding Scheme

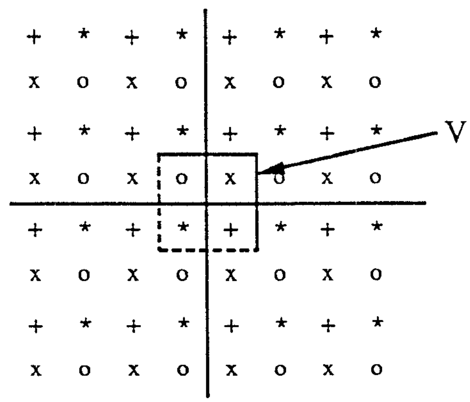

Fig. 3: Constituent 2D Lattice of a Trellis-Coded Constellation with 2 Coded Bits/2D (4 Cosets)

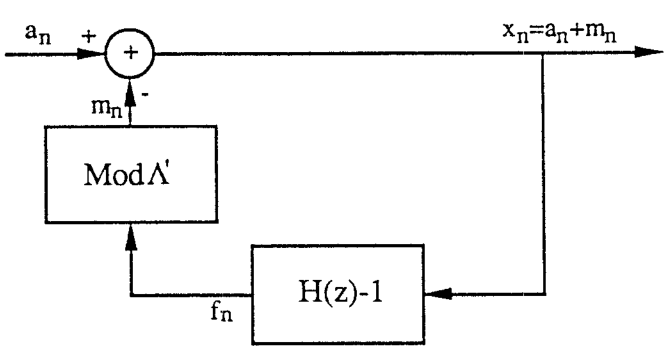

Fig. 4: The Precoder in Fig. 2

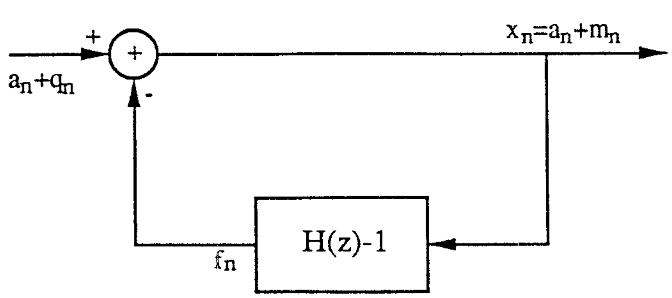

Fig. 5: Equivalent Linear Precoder