

A SIMPLE CHARACTERIZATION OF THE TRACE-CLASS OF OPERATORS

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ABSTRACT. The trace-class (τc) of operators on a Hilbert space is characterized in terms of existence of certain centralizers.

KEY WORDS AND PHRASES. Trace-class of Operators, H^* -algebra.

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1. INTRODUCTION.

Not long ago Saworotnow [1] characterized the trace-class τA (see [2]) associated with an arbitrary H^* -algebra A (see [3]) as well as the trace-class (τc) of operators (see [4]). Now, we shall show that, in the second case, there is a much simpler characterization.

We shall use the terminology and the notation of Saworotnow [1]. In particular, a trace algebra is a Banach $*$ -algebra with a trace tr and with the following properties:

$$(1) \text{tr}(xy) = \text{tr}(yx), \quad (2) \text{tr}(x^*x) = n(x^*x), \quad (3) n(x^*) = n(x)$$

$$(4) |\text{tr } x| \leq n(x) \text{ and } (5) x \neq 0 \text{ implies } x^*x \neq 0$$

where $x, y \in B$ and $n(\)$ denotes the norm of B . It is also assumed that $n(xy) \leq n(x)n(y)$ for all $x, y \in B$.

2. MAIN RESULT.

THEOREM. Let B be a simple trace-algebra (see [1]). Assume that for each

$a \in B$ there exists a (linear) centralizer U such that $Ua \geq 0$ ($\text{tr } x^*Uax \geq 0$ for each $x \in B$), $(Ux)^2 = a^*a$ and $n(a) = \text{tr}Ua$. Then there exists a Hilbert space H such that B is isometric to the trace-class (τ) (see [4]) of operators on H .

PROOF. Let A be the H^* -algebra associated with B (see [1]) and let Tr denote the trace on τA induced by A (see [2], p. 97). It follows from simplicity of B that the ideal

$$I = \left[\sum_{i=1}^n x_i y_i : x_i, y_i \in B \right]$$

is dense in B . Also the norm $n(\cdot)$ of B coincides on I with the norm $\tau(\cdot)$ induced by A (see [2], p. 99):

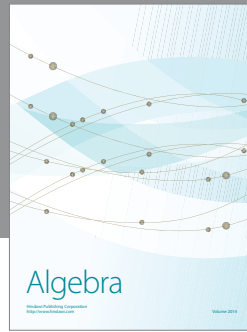
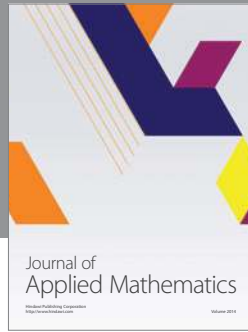
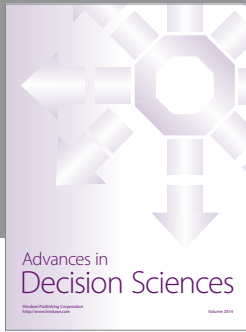
$$\begin{aligned} n\left(\sum x_i y_i\right) &= \text{tr}(U \sum x_i y_i) = \sum \text{tr} U x_i y_i = \sum (y_i^*, U x_i) \\ &= \sum \text{Tr}(U x_i y_i) = \text{Tr} U \left(\sum x_i y_i\right) = \text{Tr} \left[\sum x_i y_i\right] = \tau\left(\sum x_i y_i\right) \end{aligned}$$

where U denotes the centralizer associated with $\sum x_i y_i$. The equality $U \sum x_i y_i = \left[\sum x_i y_i\right]$ follows from the fact that the positive square root of the member $\left(\sum x_i y_i\right) * \sum x_i y_i$ of A is unique. Thus we may conclude that B is identifiable with τA .

Now we can complete the proof as in Saworotnow [1], the proof of the corollary to Theorem 2.

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