where E_0 is the strength of the incident electric field and $\bar{\bar{d}}$ is a dyadic diffraction coefficient. The field of (16) is that due to an elemental length dt of the edge, and the total edge contribution may be obtained by summing all elemental contributions in a contour integral along the edge. The product $\bar{\bar{d}} \cdot \hat{p}$ is

$$\bar{\bar{d}} \cdot \hat{p} = [d_{\perp\perp}\hat{e}_{\perp}^{s} \cos\gamma + d_{\perp\parallel}\hat{e}_{\perp}^{s} \sin\gamma + d_{\parallel\perp}\hat{e}_{\parallel}^{s} \cos\gamma$$
$$+ d_{\parallel\parallel}\hat{e}_{\parallel}^{s} \sin\gamma] \tag{17}$$

and it is necessary to use equations (3-46A) through (3-67) of Mitzner's report to evaluate the components of (16). There is an error in the last term of Mitzner's (3-46A), and the function $\sin(\beta_s + \beta_i)/2$ appearing therein must be replaced with $\sin(\beta_s - \beta_i)/2$.

When the components of (17) are evaluated and inserted in (16), one obtains the expression

$$\bar{E}_d \approx 2E_0\psi_0 dt \left[(D_\perp - D_\perp')\hat{e}_\perp^s \cos\gamma \right.$$

$$- (D_\parallel - D_\parallel')\frac{\sin\beta}{\sin\beta'}\,\hat{e}_\parallel^s \sin\gamma$$

$$\left. - (D_x - D_x')\frac{\sin\beta}{\sin\beta'}\,\hat{e}_\parallel^s \cos\gamma \right] \tag{18}$$

where the primed diffraction coefficients are physical optics terms. It might be noted that the reader will not find (18) written explicitly anywhere in Mitzner's report, nor the diffraction coefficients summarized below.

The unprimed diffraction coefficients in (18) are identically those of Michaeli,

$$D_\perp = D_m, \qquad D_\parallel = D_e, \qquad D_x = D_{em} \sin\beta'. \tag{19}$$

The primed diffraction coefficients are

$$D_\perp' = U^+ \frac{\sin\phi}{\cos\alpha_1 + \cos\phi'} - U^- \frac{\sin(n\pi - \phi)}{\cos\alpha_2 + \cos(n\pi - \phi')} \tag{20}$$

$$D_\parallel' = -U^+ \frac{\sin\phi'}{\cos\alpha_1 + \cos\phi'} - U^- \frac{\sin(n\pi - \phi')}{\cos\alpha_2 + \cos(n\pi - \phi')} \tag{21}$$

$$D_x' = -U^+ \left[\frac{Q\cos\phi}{\cos\alpha_1 + \cos\phi'} - \cos\beta' \right]$$

$$+ U^- \left[\frac{Q\cos(n\pi - \phi)}{\cos\alpha_2 + \cos(n\pi - \phi')} - \cos\beta' \right] \tag{22}$$

where the step functions are

$$U^+ = \begin{cases} 1, & \text{for "plus" face illuminated} \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

$$U^- = \begin{cases} 1, & \text{for "minus" face illuminated} \\ 0, & \text{otherwise.} \end{cases} \tag{24}$$

These step functions toggle the components of (20) through (22) on or off, depending on whether the upper or lower face of the wedge is illuminated by the incident wave.

If it were not for the primed coefficients in Mitzner's result (18), it would be identical to Michaeli's (15). The reason for the difference is that Mitzner's diffraction coefficients give the field due to the edge alone, while Michaeli's includes the contribution from the surface as well as the edge. Thus Michaeli's equivalent current approach extends Keller's theory of diffraction [4] to directions not on the Keller cone, while Mitzner's incremental length diffraction coefficient similarly extends Ufimtsev's theory [5].

As such, the Keller-like coefficients in Michaeli's solution will become singular along the shadow and reflection boundaries, while the Ufimtsev-like coefficients in Mitzner's solution will remain finite there. The singularities in Michaeli's coefficients can be attributed to a surface term which is cancelled in Mitzner's solution by the singularity in the physical optics term. The price one pays for this highly desirable result is that two separate computations are required for any finite edged body. One is the edge contribution as given by (18), and the other is a physical optics integral over the illuminated surface of the body.

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A Simple Derivation of the Basic Design Equation for Offset Dual Reflector Antennas with Rotational Symmetry and Zero Cross Polarization

ROBERT A. SHORE

Abstract—A simple geometric derivation is given of the equation for designing an offset dual reflector antenna with perfect rotational symmetry and zero cross polarization.

The work of several investigators [1]-[3] has established that offset dual reflector antennas can be designed with perfect rotational symmetry and linear polarization to within the geometrical optics approximation. These objectives can be achieved with a paraboloid main reflector and an ellipsoid or hyperboloid subreflector provided that the eccentricity of the subreflector and the relative orientations of the axes of the feed, subreflector, and main reflector satisfy what is called here "the basic design
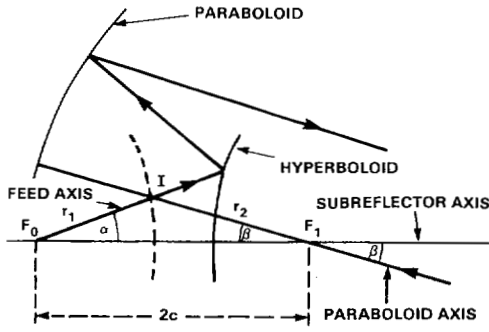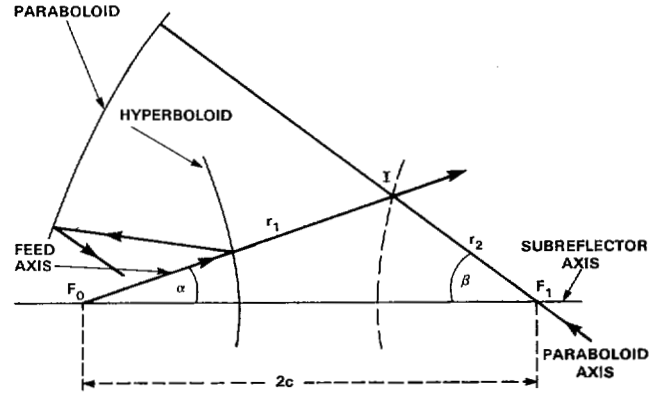
Fig. 1. Geometry of convex hyperboloidal system.

equation." One form of the basic design equation was derived by Mizuguch *et al.* [2], [4] using a somewhat lengthy argument based on obtaining an equation expressing the image on the main reflector aperture of an arbitrary ray from the feed center, while an equivalent and more convenient form was obtained by Dragone [3] using a conformal mapping argument. The purpose of this communication is to show that a simple geometric argument suffices to derive the basic design equation.

We consider the four cases of a 1) convex hyperboloid, 2) concave hyperboloid, 3) concave ellipsoid, and 4) convex ellipsoid subreflector (see Figs. 1-4). In all cases the main reflector focus is at F_1, confocal with the subreflector whose other focus is at F_0. The interfocal distance is denoted by $2c$. The feed axis F_0I is determined by that one of the two points of intersection of the main reflector axis with the parent subreflector surface which is consistent with the placement of the main reflector and subreflector. This method of constructing the feed axis, due to Dragone, is obtained from his principle that for the reflector system to be rotationally symmetric, the direction of the central ray must be unchanged after consecutive reflections at the actual subreflector, the main reflector, infinity (regarding the paraboloid as the limiting case of an ellipsoid with one focus at infinity), and the parent subreflector surface.

Consider first the convex hyperboloidal system of Fig. 1. Applying the law of sines to the triangle F_0F_1I,

$$\frac{\sin(\alpha)}{r_2} = \frac{\sin[\pi - (\alpha + \beta)]}{2c} = \frac{\sin(\beta)}{r_1} \tag{1}$$

with r_1 and r_2 given by a standard polar form for conic sections

$$r_1 = \frac{(c/e)|e^2 - 1|}{e\cos(\alpha) + 1}, \quad r_2 = \frac{(c/e)(e^2 - 1)}{e\cos(\beta) - 1}$$

and where e is the subreflector eccentricity ($e > 1$ for hyperboloid, $0 \leqslant e < 1$ for ellipsoid) and α and β are the angles formed by the subreflector axis F_0F_1 with the feed axis and the paraboloid axis, respectively. The first equality of (1) gives

$$2e\sin(\alpha) - \sin(\alpha + \beta) - e^2\sin(\alpha - \beta) = 0 \tag{2}$$

readily transformable to

$$\tan(\alpha) = \frac{|e^2 - 1|\sin(\beta)}{(e^2 + 1)\cos(\beta) - 2e} \tag{3}$$

while the second equality yields

$$2e\sin(\beta) + \sin(\alpha + \beta) - e^2\sin(\alpha - \beta) \tag{4}$$

equivalent to

$$\tan(\beta) = \frac{|e^2 - 1|\sin(\alpha)}{(e^2 + 1)\cos(\alpha) + 2e}. \tag{5}$$

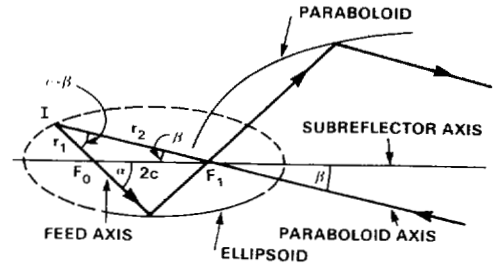Fig. 2. Geometry of concave hyperboloidal system.

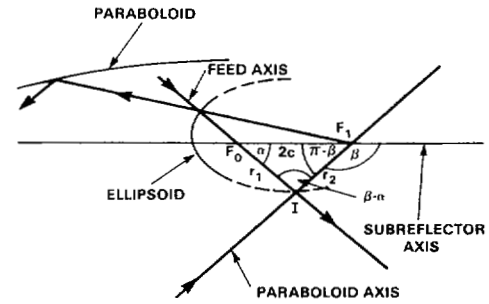Fig. 3. Geometry of concave ellipsoidal system.

Fig. 4. Geometry of convex ellipsoidal system.

Equations (3) and (5) are the Japanese form of the basic design equation. Adding and subtracting (2) and (4) gives

$$\sin(\alpha) + \sin(\beta) - e\sin(\alpha - \beta) = 0 \tag{6}$$

and

$$e[\sin(\alpha) - \sin(\beta)] - \sin(\alpha + \beta) = 0. \tag{7}$$

Taking the difference of (6) and (7) and rearranging gives

$$\tan(\alpha/2) = \frac{e + 1}{|e - 1|}\tan(\beta/2) \tag{8}$$

which is Dragone's form of the basic equation. (Dragone uses 2α and 2β to denote the angles we call α and β, respectively.)

For the concave hyperboloidal system of Fig. 2, the law of sines applied to triangle F_0F_1I again gives (1), now with

$$r_1 = \frac{(c/e)(e^2 - 1)}{e\cos(\alpha) - 1} \quad r_2 = \frac{(c/e)|e^2 - 1|}{e\cos(\beta) + 1}.$$

The resulting equations are thus the same as those for the convex hyperboloid with α and β interchanged.

For the concave ellipsoidal system, the same procedure as used for the convex hyperboloidal system starting with the law of sines applied to triangle F_0F_1I of Fig. 3 again yields (3), (5), and (8). The design equations for the convex ellipsoidal system of Fig. 4

are then readily shown to be the same as those for the convex ellipsoidal system with α and β interchanged.

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Comments on "A New Method of Analysis of the Near and Far Fields of Paraboloidal Reflectors"

HENNING BACH

In the above paper[1] the radiated far field from a reflector antenna is predicted by a "new" method, that determines the far field by a spherical wave expansion of the near field on a sphere enclosing the antenna, once the near field has been found using the geometrical theory of diffraction (GTD). However, this technique of combining a spherical near-field (SNF) transformation and a near-field computation based on the geometrical theory of diffraction is identical to the SNFGTD method originally used by F. Jensen and F. H. Larsen in 1977 [1]. The method was described in detail by H. Bach in the report [2], the contents of which were presented at the NATO Advanced Study Institute in Norwich 1979. In the following years the method was further investigated by several researchers. Thus in 1981 an analysis of its accuracy as compared to physical optics (PO) and moment methods (MM) was performed by Bach, Frandsen, and Larsen. Some of their results are reported in [3], which also contains a description of the near-field calculation and the transformation techniques. Further analyses and applications were presented in [4]-[6], and recently the method has been mentioned in the book by Dr. B. Westcott [7].

In their paper[1] Narasimhan and Christopher claim that "it is evident that the present method gives very good agreement with measured results" and later that the results can be made still more accurate by improving the near-field calculation and by increasing the number of spherical modes in the near field to far-field transformation. Although this has not been done, one can read in the abstract that "it is demonstrated that the technique proposed can predict the fields radiated by the reflector with greater accuracy by comparing the calculated results with the available measured results." Thus it is indicated to the reader that the SNFGTD method is superior to other methods in this respect.

In [3], which was brought to the attention of Narasimhan and Christopher by the referee of their paper, the curves shown in Fig. 1 were presented. These curves demonstrate that for a 20 wavelength reflector antenna fed by a dipole, practically identical H-plane patterns are obtained using SNFGTD, moment methods

[1] M. S. Narasimhan and S. Christopher, *IEEE Trans. Antennas Propagat.,* vol. AP-32, no. 1, pp. 13–19, Jan. 1984.
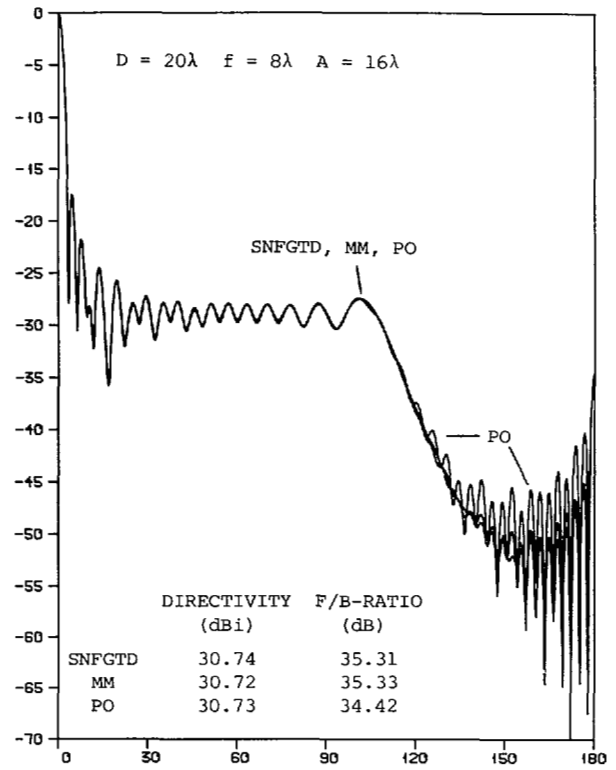

Fig. 1. Comparison of SNFGTD, mm, and PO for 20 wavelength reflector antenna.

and physical optics. The only significant differences occur in the region beyond 120° where physical optics (not using far field GTD) differs from the other curves as could be expected. Thus, in 1981, it had been demonstrated that the method when used to calculate the H-plane pattern of a 20 wavelength antenna excited by a dipole yields results which are practically identical to those of other methods.

In order to restrict this communication as much as possible, I shall comment on only one of Narasimhan and Christopher's results, namely the H-plane radiation pattern shown in Fig. 2 of their paper.[1] There they consider a focused parabolic reflector antenna with a diameter $D = 10.65$ wavelengths, an F/D ratio = 0.25 and illuminated by a dipole, a configuration for which Afifi [8] made measurements in 1967. First of all, serious errors are observed in Narasimhan and Christopher's plot of the PO-GTD results of Koyoumjian [9]. For instance the level of the first sidelobe, as computed by Narasimhan and Christopher, is coincident with Afifi's measurements, but differs from physical optics by 3 dB approximately. This implies that physical optics predicts the level of the first sidelobe with an error of 3 dB approximately, a fact that I feel must be a big surprise to most antenna engineers. Furthermore, while measured and calculated results are almost coinciding on the center part of Kouyoumjian's curves they differ strongly in Narasimhan and Christopher's plot. These discrepancies may be due to bad plotting techniques, but in any circumstance it is not easy to accept the conclusions of Narasimhan and Christopher with regard to the accuracy of the SNFGTD method based on this background.

In Figs. 2(a) and 2(b) are presented patterns for the antenna in question computed at Technical University of Denmark by SNFGTD and physical optics supplemented by far field GTD in the