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A SIMPLE MODEL OF HERD BEHAVIOR*

ABHIJIT V. BANERJEE

We analyze a sequential decision model in which each decision maker looks at the decisions made by previous decision makers in taking her own decision. This is rational for her because these other decision makers may have some information that is important for her. We then show that the decision rules that are chosen by optimizing individuals will be characterized by herd behavior; i.e., people will be doing what others are doing rather than using their information. We then show that the resulting equilibrium is inefficient.

I. INTRODUCTION

There are innumerable social and economic situations in which we are influenced in our decision making by what others around us are doing. Perhaps the commonest examples are from everyday life: we often decide on what stores and restaurants to patronize or what schools to attend on the basis of how popular they seem to be. But it has been suggested by Keynes [1936], for example, that this is also how investors in asset markets often behave (the famous “beauty contest” example).¹ In the literature on fertility choices it has frequently been suggested that various fertility decisions (how many children to have, whether or not to use contraception, etc.) are heavily influenced by what other people in the same area are doing.² It has also been suggested that the same kind of factor also influences the decision to adopt new

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* I thank Aniruddha Dasgupta, Mathias Dewatripont, Bob Gibbons, Tim Guinnane, Sandy Korenman, Eric Maskin, Andreu Mas-Colell, Barry Nalebuff, Klaus Nehring, Avner Shaked, Lin Zhou, and two anonymous referees for helpful comments and suggestions.

¹ See Scharfstein and Stein [1990] for some evidence suggesting that this is indeed how managers behave.

² See Cotts Watkins [1990].

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The Quarterly Journal of Economics, August 1992
technologies. Voters are known to be influenced by opinion polls to vote in the direction that the poll predicts will win; this is another instance of going with the flow. The same kind of influence is also at work when, for example, academic researchers choose to work on a topic that is currently “hot.”

The aim of this paper is to develop a simple model in which we can study the rationale behind this kind of decision making as well as its implications. We set up a model in which paying heed to what everyone else is doing is rational because their decisions may reflect information that they have and we do not. It then turns out that a likely consequence of people trying to use this information is what we call herd behavior—everyone doing what everyone else is doing, even when their private information suggests doing something quite different.

But this suggests that the very act of trying to use the information contained in the decisions made by others makes each person’s decision less responsive to her own information and hence less informative to others. Indeed, we find that in equilibrium the reduction of informativeness may be so severe that in an ex ante welfare sense society may actually be better off by constraining some of the people to use only their own information.

A common real world example may make our basic argument clearer. Most of us have been in a situation where we have to choose between two restaurants that are both more or less unknown to us. Consider now a situation where there is a population of 100 people who are all facing such a choice.

There are two restaurants A and B that are next to each other, and it is known that the prior probabilities are 51 percent for restaurant A being the better and 49 percent for restaurant B being better. People arrive at the restaurants in sequence, observe the choices made by the people before them, and decide on one or the other of the restaurants. Apart from knowing the prior probabilities, each of these people also got a signal which says either that A is better or that B is better (of course the signal could be wrong). It is also assumed that each person’s signal is of the same quality.

Suppose that of the 100 people, 99 have received signals that B is better but the one person whose signal favors A gets to choose first. Clearly, the first person will go to A. The second person will now know that the first person had a signal that favored A, while

3. See, for example, Kislev and Shchori-Bachrach [1973].
4. See, for example, Cukierman [1989].
5. I am indebted to an anonymous referee for this very transparent example.
her own signal favors B. Since the signals are of equal quality, they effectively cancel out, and the rational choice is to go by the prior probabilities and go to A.

The second person thus chooses A regardless of her signal. Her choice therefore provides no new information to the next person in line: the third person's situation is thus exactly the same as that of the second person, and she should make the same choice and so on. Everyone ends up at restaurant A even if, given the aggregate information, it is practically certain that B is better.

To see what went wrong, notice that if instead the second person had been someone who always followed her own signal, the third person would have known that the second person's signal had favored B. The third person would then have chosen B, and so would have everybody else.

The second person's decision to ignore her own information and join the herd therefore inflicts a negative externality on the rest of the population. If she had used her own information, her decision would have provided information to the rest of the population, which would have encouraged them to use their own information as well. As it is, they all join the herd.

The identification of this externality, which we call "herd externality," and the investigation of what it implies, is the main contribution of this paper. The model we present is extremely simple and does not aspire to capture any specific institutional detail. There is a set of options represented by a line segment, and within this set there is one correct option. The aim of the game is to find the correct option. All those who find the correct option get z, while all others get 0. There is a population of N people who take their decisions in a fixed order; each person moves knowing the choices made by those before her but not the information these choices were based on. Each individual may either be uninformed, in which case she has no signal, or informed, in which case she has a signal about what the right option is. This signal, however, may not be correct. It is correct only with probability β; otherwise it is completely uninformative. Everybody is rational in the Bayesian sense, and the equilibrium we look at is a Bayesian-Nash equilibrium.  

Because of the extreme simplicity of this model, using quite

6. This formulation is somewhat more complicated and somewhat less natural than the two-restaurant setting discussed above, but it turns out that deriving a general result about the equilibrium decision rule which holds for all parameter values is actually simpler in this setting.
elementary arguments we are able to derive a number of rather striking results that derive directly from the presence of the herd externality. These results are summarized below.

1. The equilibrium pattern of choices may be (and for a large enough population, will be) inefficient in the ex ante welfare sense. Before people know the order in which they are choosing, they may all agree to prevent the first few decision makers from observing the choices made by anyone else. This is a direct consequence of the herd externality and suggests why herd behavior may be undesirable from the social point of view.

2. The probability that no one in the population chooses the correct option is bounded away from zero for any size of the population. Indeed, by making the probability $\beta$ small, we can make this probability as large as we like. This contrasts with the case where the decision makers choose without looking at each other (i.e., they follow their own information). Since the information they have is independent, as long as the population is large enough, someone must choose the right option in this case.

3. Since the herd externality is of the positive feedback type (if we join the crowd, we induce others to do the same), the equilibrium pattern of choices will be very volatile across several plays of the same game. The signals (which are partly random and need not be correct) that the first few decision makers have will determine where the first crowd forms, and from then on, everybody joins the crowd. This may shed some light on observations of "excess volatility" made in the context of many asset markets\(^7\) and the frequent and apparently unpredictable changes in fashions.

The emphasis on the herd externality also distinguishes our work from two other explanations of clustering behavior that have been suggested before. One is an explanation based on strong complementarities: some things are more worthwhile when others are doing related things.\(^8\) Examples of such complementarities are fashions in consumption (see, for example, the analysis of fashions in Karni and Schmeidler [1989]) and network externalities in production (see Arthur [1989], Farrell and Saloner [1985], and Katz and Shapiro [1985]). Whether or not it is as important in

\(^7\) The idea that informational externalities may explain observations of excess volatility is also discussed in Banerjee [1988].

\(^8\) As far as we know, there is actually no formal model that tries to explain herd behavior in these terms; what exists in the literature is the idea that if complementarities are sufficiently large, then people will do what the crowd is doing even if left to themselves they would have done something else. Under suitable conditions this could clearly lead to herd behavior.
other contexts where we observe herd behavior is an open question. In any case, there is no contradiction between this view and ours; our point is that many aspects of herd behavior can be explained quite plausibly without invoking these kinds of gains from association.

A different explanation of herd behavior, which, like the present work is based on informational asymmetries, was suggested in an interesting recent paper by Scharfstein and Stein [1990]. The key difference between their explanation and the one suggested here is that their explanation is based on an agency problem; in their model the agents get rewards for convincing a principal that they are right. This distortion in incentives plays an important role in generating herd behavior in their model. By contrast, in our model agents capture all of the returns generated by their choice so that there is no distortion in incentives.9

In any case, this approach is not inconsistent with our approach. This kind of principal agent problem (trying to convince someone else into believing that you know something) seems common enough, especially in the context of asset markets. On the other hand, in many of the other potential instances of herd behavior, such as fertility choices, adoption of innovations, voting etc., there is no obvious principal agent problem. It is therefore useful to establish that inefficient herd behavior can arise even when the individuals themselves capture the rewards from their decisions.

The strategy of this paper is as follows: the basic model is presented in Section II and analyzed in Section III. The results are discussed in Section IV. Several extensions and modifications of the basic model are presented in Section V. We conclude in Section VI.

The essential ideas in this paper are generally quite straightforward. Nonetheless, a fully rigorous treatment of the results derived here will certainly be very cumbersome and repetitive. To prevent the paper from being too unreadable, we have omitted some proofs

9. There are two other important differences between the two models. In the Scharfstein and Stein model unlike in ours, all agents get a signal. However, only some of these signals are potentially informative (though not necessarily correct); the rest are duds; and the agents cannot distinguish between the two types of signals. In our model this would amount to assuming that agents do not know whether they have a signal or not. The decision to herd and not use one's signal is therefore somewhat less significant (since they actually may not have a signal) than it is in our model (where they know they have a signal). On the other hand, it is possible to generate instances of herd behavior in their model even if there are only two agents. In the model given here the equilibrium is always (second-best) optimal if there are two agents.
II. The Basic Model

There is a population of agents of size $N$ each of whom maximizes the identical risk-neutral utility function $VNM^{10}$ defined on the space of asset returns. For convenience we shall just assume that this utility is the same as the monetary amount received by the person.

There is a set of assets indexed by numbers in $[0,1]$. Call the $i$th asset $a(i)$. The physical return to the $i$th asset to the $n$th person investing in that asset is $z(i) \in R$. Let us assume that there is a unique $i^*$ such that $z(i) = 0$ for all $i \neq i^*$ and $z(i^*) = z$, where $z > 0$. This is essentially the assumption that the excess return on one asset to the people investing in it is strictly greater than that on all other assets.

Of course, everybody, given these payoffs, would want to invest in $i^*$. The trouble is no one knows which one it is. We assume uniform priors so there is not even a likely candidate for $i^*$. However, some people have an idea of which one it might be. Formally, there is a probability $x$ that each person receives a signal telling her that the true $i^*$ is $i'$. The signal need not, of course, be true, and the probability that it is false is $1 - \beta$. If it is false, then we assume that it is uniformly distributed on $[0,1]$ and therefore gives no information about what $i^*$ really is.

The decision making in this model is sequential; one person chosen at random takes her decision first (she cannot decide to delay her decision). The next person, once again chosen at random, takes her decision next but she is allowed to observe the choice made by the previous person and can benefit from the information contained in it. However, she is not allowed to find out whether or not the person before her actually got a signal.\textsuperscript{11}

The rest of the game proceeds in the same way, with each new decision maker making her decision on the basis of the history of the past decisions and their own signal if they have one. After everybody has made her choice, all the alternatives that have been

\textsuperscript{10} It will be evident that for much of what we say it is irrelevant whether we take our utility function to be risk neutral or risk averse. The only point where we need the assumption is when we measure ex ante welfare.

\textsuperscript{11} And a fortiori she cannot observe the signal observed by her predecessor.
chosen are tested, and if any of these turn out to work, those who have chosen it receive their rewards. If no one has chosen an option that works, the truth remains undiscovered, and no one gets rewards.

It will be assumed that the structure of the game and Bayesian rationality are common knowledge. Each person’s strategy is a decision rule that tells us for each possible history what that person will choose. We are looking for a Bayesian Nash equilibrium in these strategies. The nature of the equilibrium play, however, turns out to depend on certain critical tie-breaking assumptions. Some of these assumptions may be dispensed with by strengthening the equilibrium concept, but it seems more natural to introduce these as explicit assumptions. These assumptions are listed below; the relevance of these assumptions will become clear in the appropriate context. It should also be possible to see that each of these assumptions is made to minimize the possibility of herding.

**ASSUMPTION A.** Whenever a decision maker has no signal and everyone else has chosen $i = 0$, she always chooses $i = 0$.

**ASSUMPTION B.** When decision makers are indifferent between following their own signal and following someone else’s choice, they always follow their own signal.\(^{12}\)

**ASSUMPTION C.** When a decision maker is indifferent between following more than one of the previous decision makers, she chooses to follow the one who has the highest value of $i$.

**III. THE EQUILIBRIUM DECISION RULE**

The first decision maker’s decision will clearly depend on whether or not she has a signal. If she has a signal, she will certainly follow her signal. While if she has no signal, by our Assumption A she will choose $i = 0$. This choice minimizes

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\(^{12}\) This assumption is not entirely innocuous. If the third decision maker decides to follow the second independently of her informational situation, the fact that she follows provides no information to the next decision maker. The next decision maker who has a signal, now faces the same choice; she is indifferent between following her own signal and joining the second and the third decision maker. If she too ignores her own signal, the next decision once again has the same choice and so on. There could, for example, be an equilibrium in which almost everybody follows either the first decision maker or the first deviant. Since this involves even more herding than the equilibrium we describe above, its welfare properties will typically be worse; and in this sense by making the above tie-breaking assumption, we are choosing to focus on the best of the set of possible equilibria.
misinformation: the only case where this will cause confusion is when \( i^* = 0 \), but since this happens with probability 0, we can ignore this possibility.\(^{13}\)

If the second decision maker has no signal, then she will of course imitate the first decision maker and invest in the same asset. However, if she has a signal and the first person has not chosen \( i = 0 \), she has a problem. She knows that the first decision maker had a signal and this signal is as likely to be right as her own signal. She is therefore indifferent between following the first decision maker’s signal and following her own signal. In this situation our Assumption B becomes relevant.\(^{14}\) By invoking this assumption, we determine that the second person will, in this case, follow her own signal.

The third decision maker can observe four possible histories: one or both of her predecessors may have chosen \( i = 0 \), and if neither of them had chosen \( i = 0 \), they could have still either agreed or disagreed. If they both chose \( i = 0 \), the third person should follow them if she has no signal and follow her signal otherwise. In all the other cases, if she does not have a signal, she should follow the person who has not chosen \( i = 0 \). If both the others have chosen \( i \neq 0 \) but have not agreed with each other, of course this does not determine a course of action. Since she is indifferent, however, we can invoke our third tie-breaking rule, Assumption C, which tells us that she should follow the person with the highest \( i \).

On the other hand, if the third decision maker does have a signal \( i' \), she will follow her own signal, unless both people before her have chosen the same option and this option is neither \( i = 0 \) nor \( i = i' \). When only one of them has chosen something different from \( i = 0 \) and \( i = i' \) and the other has chosen \( i = 0 \), this is a consequence of our Assumption B. And, of course, when the third person’s signal matches the choices made by one or both of her predecessors, she must choose to follow her own signal since this could not happen unless her signal was correct.

The last point is much more general than this specific situation and deserves to be emphasized. Whenever some person’s signal matches the choice made by one of her predecessors, she should always follow her signal. This follows from the fact that the probability that two people should get the same signal and yet both

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13. Of course, there is nothing special about the point 0. The person could just as well choose some other point \( i \) as long as everybody knows what \( i \) is.

14. Once again, this assumption is not entirely innocuous.
be wrong is zero. To deal with the remaining case, we prove the following simple lemma.

**Lemma 1.** If the first and the second decision makers have both chosen the same \( i \neq 0 \), the third decision maker should choose to follow them.

**Proof of Lemma 1.** Note that

\[
\text{prob}[i^* = i | H] = \frac{\alpha^2 \beta^2 (1 - \beta) + \alpha^2 \beta (1 - \beta)(1 - \alpha)}{\text{prob}[H]}
\]

\[
\text{prob}[i^* = i' | H] = \frac{\alpha^2 \beta (1 - \beta)(1 - \alpha) \beta}{\text{prob}[H]},
\]

where \( H \) represents the event in which the first two people have both chosen \( i \) and the third person has the signal \( i' \).

Clearly, the first term is greater than the second. The third person should therefore choose \( i \).

Q.E.D.

This has a simple intuitive explanation: the third person knows that the first person must have a signal, since otherwise she would have chosen \( i = 0 \). The first person's choice is therefore at least as good as the third person's signal. Further, the first person has someone who has followed her. This is some extra support for the first person's choice, since it is more likely to happen when the first person is right than when she is wrong. It is therefore always better to follow the first person.

The same intuition tells us what should happen in any situation when several options other than \( i = 0 \) have been chosen but only one of them has been chosen by two people. Assume that the next person does not have a signal that matches any of the options that have already been chosen (if she has a signal that matches someone else's choice, she should, of course, follow her own signal). In the situation where this option is not the one with the highest \( i \), it is of course clear (by our Assumption C) that both those people must have the same signal and therefore they must be right. In the situation where it is the one who has the highest \( i \), the argument in the previous paragraph applies, and therefore once again this option is the best. In either case, therefore, the next person should choose the option that has been chosen by two people.

Once one option has been chosen by two people, the next person should always follow that option unless her signal matches one of the options that have been already chosen; in that case she
should follow her own signal. A combination of this decision rule with Assumption C tells us that the next person will observe one of three alternative histories.

i) One option (other than \(i = 0\)) has been chosen by more than one person, and this is the one that has the highest \(i\).

ii) One option (other than \(i = 0\)) has been chosen by more than one person, and this is not the one with the highest \(i\).

iii) Two options (other than \(i = 0\)) have been chosen by more than one person, one of which is the one with the highest \(i\).

In the second and the third scenarios, it is clear that the option which is not the highest value of \(i\) is the correct option, and all subsequent decision makers should choose it. The argument in the first case is very similar to the argument for Lemma 1. The next decision maker should decide to follow the option that has already been chosen by more than one person.

The same argument can now be extended to all subsequent decision makers. This yields the following proposition which summarizes all the arguments we make above.\(^{15}\)

**Proposition 1.** Under Assumptions A, B, and C, the unique (Nash) equilibrium decision rule that everyone will adopt is decision rule D\(^{16}\) given below.

1. The first decision maker follows her signal if she has one and chooses \(i = 0\) otherwise.

2. For \(k > 1\), if the \(k\)th decision maker has a signal, she will choose to follow her own signal either if and only if (a) holds or if (a) does not hold, (b) holds, where (a) and (b) are given below.
   (a) Her signal matches some option that has already been chosen.
   (b) No option other than \(i = 0\) has been chosen by more than one person.

3. Assume that the \(k\)th decision maker has a signal. If any option (among those already chosen) other than the one with

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\(^{15}\) A formal proof is available from the author on request.

\(^{16}\) Actually this is not the full decision rule. Technically we have to specify the optimal decision in other parts of the decision tree. In particular, we have to specify the decision rule in the following scenario: one person finds that his option matches someone else's and chooses that option. In the equilibrium specified it is known that he would not choose that option unless his signal matches that option. Therefore, everybody else follows him expecting that no one else will deviate. However, now someone else deviates and chooses another of the options that have already been chosen.

It is true that this can happen only with probability zero, but when it happens, we need to say what the subsequent decision makers will choose. However, it turns out that agents are actually indifferent between sticking to the original path and deviating to the new one, and we can make any assumption we like.
the highest \( i \) has been chosen by more than one person, the \( k \)th decision maker will choose this option, unless her signal matches one of the other options that has already been chosen. In this case she chooses the latter option.

4. Assume that the \( k \)th decision maker has a signal. If the option with the highest \( i \) (among those already chosen) has been chosen by more than one person and no other option (except \( i = 0 \)) has been chosen by more than one person, she will choose this option unless her signal matches one of the options already chosen. In this case she chooses the latter option.

5. Assume that the \( k \)th decision maker does not have a signal. Then she will choose \( i = 0 \) if and only if that is what everyone else has chosen. Otherwise, she chooses the option with the highest value of \( i \) that has already been chosen unless one of the other options (excluding \( i = 0 \)) has been chosen by more than one person. In this case, she chooses the latter option.

The only part of the statement of this proposition that is not explained above is the uniqueness. Because each person’s payoff is completely independent of the choices made by everyone coming after her in the decision process, there are no strategic elements here, and we can solve this game by moving forward in the game tree. The uniqueness of the solution is therefore automatically guaranteed.

The equilibrium decision rule as stated above is somewhat complicated and potentially confusing. In Figure I the decision rule for decision maker \( k \) (for \( k > 2 \)) is presented in a schematic form that may be easier to follow.

IV. DISCUSSION OF RESULTS

A. Description of the Equilibrium

The equilibrium decision rule in the above model is characterized by extensive herding; agents abandon their own signals and follow others even when they are not really sure that the other person is right. The first person always follows her own signal if she has one, and so does the second person, but we cannot guarantee that even the third person follows her own signal. If the first person chooses \( i \neq 0 \) and the second person follows her, the third person will always follow them. All subsequent decision makers will also choose the same option.
Herding can also happen when the first and second person, and for that matter the third and fourth person, choose different options. After $k$ different options have been chosen, for any positive $k$, if the next decision maker does not have a signal, she will choose the option with the highest value of $i$ (among those already chosen). Following this, all subsequent decision makers will choose the same option unless one of their signals matches one of the options already chosen. This can happen only if the correct option has already been chosen. So, there will be herding at an incorrect option unless the first decision maker to have a signal or someone coming after her but before the first subsequent decision maker without a signal, made the correct choice.

We can actually calculate the expression for the probability that no one in the population chooses the right option, however large the population. A simple calculation establishes that this probability is

$$[1 - \alpha(1 - \beta)]^{-1}(1 - \alpha)(1 - \beta).$$

This probability is clearly decreasing in both $\alpha$ and $\beta$ which makes intuitive sense. Further, if $\beta$ is sufficiently small, this probability will be very close to one.

To see why this is worth a remark, note that if all the decision makers took their decisions without observing the choices made by others, some people will always end up choosing the correct option (in fact, for a large enough population, the proportion of the
population who will choose the correct option will almost certainly be close to $\alpha \beta$).

It is also important to note that just the fact that people observe the decisions made by others does not guarantee that there will be herd behavior. Consider, for example, the following modified version of the Normal Learning Model: there is a Normal (Gaussian) distribution with known variance but unknown mean. The distribution of the mean is known. In each period a different agent gets a signal that is a random drawing from this distribution. The agent then chooses her best guess for the mean of the distribution (he is minimizing a loss function that is quadratic in distance from the mean) given her signal and the choices made by her predecessors. It is quite easy to show that this sequence of choices converges to the mean for almost every sequence of signals. Therefore, the result we get contrasts sharply with the result from this superficially quite similar model.

The key reason why we get a different result is that in our model the choices made by agents are not always sufficient statistics for the information they have. If the choices are always sufficient statistics, future agents always know what information their predecessors had acted upon, and therefore there is no herd externality and no inefficiency. It is when the choices made by some agents affect the information that subsequent decision makers have that there is a potential for herd externality.

The fact that the choices in our model do not have this sufficient statistic property clearly has to do with how we specify the payoffs. It is evident that, for a wide range of signals, the agents in our model will always choose the same option; this lack of invertibility is what causes the sufficient property to fail. We conjecture that typically whenever the space of choices and the space of signals are of comparable dimension and the payoff function is continuous, we shall have this kind of invertibility. So, for example, in the model presented above, we are more likely to have invertibility if the agents could (and would want to) vary the size of their investment in different informational settings. On the other hand, there are many real reasons why it may not be possible to vary one’s choice enough to register all the information one has—machines, for example, come in only a small number of sizes.

B. Welfare Properties

The welfare question here is motivated by the herd externality. While joining the herd is optimal for the current agent given
the play of the game, it reduces the chance that future agents may discover the truth. So if we consider an ex ante measure of welfare, imagining that in some primeval state before the game begins all agents have an equal chance of being at any position in the sequence of arrivals, we may be able to show that social welfare is lower in the equilibrium we have described than in other plays of the game.

This is in fact what we find. Consider another play of the game in which agents follow the decision rule $D^*$ given below.

1. If an agent has a signal, she follows that signal, unless someone before her has already followed someone else. In that case she follows suit.

2. If an agent does not have a signal, she picks some option that has not been picked by anyone else, unless someone before her has already followed someone else. In that case she follows suit.

This play of the game is set up to ensure that the right choice always gets revealed as long as there are enough people in the population. With a very large population it is easy to see that this would mean that an arbitrarily large fraction of the population will always make the right choice. Formally, the probability that at least two people have not received the true signal by the time we get to the $n$th person is

$$1 - (1 - \alpha\beta)^{n-1} - (n - 1)(1 - \alpha\beta)^{n-2}\alpha\beta.$$ 

For any $\epsilon > 0$, it is easy to see that we can choose an $n(\epsilon)$ large enough that this probability is at least $1 - \epsilon$. Now this tells us that a lower bound for the ex ante expected utility for agents following this rule is

$$z[N - n(\epsilon)](1 - \epsilon)/N,$$

where $N$ is the size of the population. Notice that by making $N$ large we can make this arbitrarily close to $N(1 - \epsilon)$.

By contrast, the probability that no one will discover the right choice in the herding equilibrium we described before is

$$\Pi = [1 - \alpha(1 - \beta)]^{-1}(1 - \alpha)(1 - \beta),$$

and therefore the ex ante expected utility is bounded above by

$$zN[1 - \Pi].$$

Since we can choose $\epsilon$ to be as small as we would like to by making
N large, it is easy to see that we can make the above expression larger than this expression. There is at least one decision rule that for large enough N does better than the equilibrium we described.

It may be objected here that we have simply described a strategy and not explained how it will be implemented and the equilibrium may yet be constrained Pareto optimal. However, this is not true. This decision rule may be implemented by using a number of different incentive schemes. One way that works is to punish heavily anybody who is a follower at any option that turns out to be the incorrect one, while equally rewarding everybody who chooses the right option. Given these rewards, no one will choose to be a follower unless they were absolutely sure that the option that they were choosing was the right one. But this is, of course, exactly what we want.

Indeed, even if D* cannot be implemented, some of its advantages can be captured simply by not allowing the first n agents to observe anybody else's choice when they are making their own choice. The rest of the population is then allowed to choose sequentially, with each person observing the choices made by all her predecessors.

The first n people will, of course, choose to follow their own signals if they have one and choose at random if they have no signal. The only way that more than one of these people will choose the same option is if they are both right. Thus, as long as at least two of the first n people have chosen the same option, the rest of the population will realize that this must be the correct option and choose it.

But an argument exactly paralleling the one given above in the case of D* can now be used to establish that for a large enough population, by choosing n suitably, we can make the fraction of the population who do not choose the correct option arbitrarily small. In other words, in terms of ex ante welfare, the economy may be better off if the early decision makers are not allowed to observe the choices made by the other decision makers than in our original equilibrium. In other words, destroying information (in this limited sense), can be socially beneficial.

17. This question is discussed in more detail in an earlier version of this paper which is available from the author.
18. This is, of course, only in terms of ex ante welfare; in terms of ex post welfare this is certainly not true.
A. Alternative Payoff Structures

In the model we analyzed in the last section, everyone who chooses the right outcome gets the same reward irrespective of how many others chose this option before and after them. This may be approximately the correct model of the rewards we get for choosing the right restaurant, but in many other real world examples the rewards will depend on the number of people who have chosen this option and our rank among them. The total amount of the reward may be fixed, or at least it may not increase as fast as the number of people who choose the correct option. And in many instances there are extra rewards for being first or second to choose the correct option.

To the extent that we have ignored these possibilities, our analysis may overstate the extent to which there will be herding. Both of these possibilities suggest that it pays more to choose the correct option when most others have chosen something else. This kind of reward for originality clearly discourages herd behavior.

To make this idea precise, consider a model that is identical to the previous one except that in this case the first person to choose the right option gets a bigger output than all the others, who are all assumed to get the same amount. A real world institution where exactly these rewards are not too implausible is academia; the emphasis is on being first to do something. If you are not first, the rank does not matter very much.

A little reflection should persuade the reader that this model is not really that different from our basic model. It is true that the relative incentives for being first versus being second are greater now, which should discourage agents from choosing to be second but people without signals and people with signals that exactly match that of someone else, will still be followers. Consequently, it is possible that the number of people behind someone becomes so large and the evidence of their numbers so convincing, that informed agents will decide to ignore their information and the attraction of the large first prize and join the herd.

Formally, we have Proposition 2.

**Proposition 2.** If the return to the first person choosing the right option is $z_1$ and the return to everybody else who chooses that option is $z_2$, and $0 < z_2/z_1 \leq 1$, then the unique equilibrium
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decision rule 19 under Assumptions A, B, and C, is described by an integer $k^* > 0$ and the following rules.

1. The first decision maker follows her signal if she has one and chooses $i = 0$ otherwise.

2. For $k > 1$, if the $k$th decision maker has a signal, she will choose to follow her own signal either if and only if (a) holds or if (a) does not hold, (b) holds, or if (b) does not hold, (c) holds, where (a), (b), and (c) are given below.
   (a) Her signal matches some option that has already been chosen.
   (b) No option other than $i = 0$ has been chosen by more than one person.
   (c) The option with the highest $i$ (among those already chosen) has not been chosen by more than $k^*$ people for some $k^* \geq 1$, and no other option other than $i = 0$ has been chosen by more than one person.

3. Assume that the $k$th decision maker has a signal. If any option (among those already chosen) other than the one with the highest $i$ has been chosen by more than one person, the $k$th decision maker will choose this option, unless her signal matches one of the other options that have already been chosen. In this case she chooses the latter option.

4. Assume that the $k$th decision maker has a signal. If the option with the highest $i$ (among those already chosen) has been chosen by more than $k^*$ people and no other option (except $i = 0$) has been chosen by more than one person, she will choose this option unless her signal matches one of the options already chosen. In this case she chooses the latter option.

5. Assume that the $k$th decision maker does not have a signal. Then she will choose $i = 0$ if and only if that is what everyone else has chosen. Otherwise she chooses the option with the highest value of $i$ that has already been chosen unless one of the other options (excluding $i = 0$) has been chosen by more than one person. In this case she chooses the latter option.

The proof of this result is omitted since it is a straightforward extension of arguments used in Section III. In fact, it should be evident that the result in Section III is a special case of this result.

19. Once again we have not actually provided the complete decision rule here; for this would need to say what happens at all the unreached information sets. However, this can be done in a straightforward way.
for the case where $z_2 = z_1$. What is striking about that special case is that $k^*$, which we leave as an undetermined integer in this result, is one in that special case.

What is more, if $z_2/z_1$ is not much smaller than one, it is easy to see that the value of $k^*$ will still be one. As $z_2/z_1$ goes to zero, $k^*$ of course increases, but at least for values of $z_2/z_1$ relatively close to one, there will be a substantial degree of herding. Therefore, some of the basic welfare intuitions are going to be quite similar to the previous case. For $N$ large enough, it will still be true that the decision rule $D^*$ will do better than this rule though the margin of gain will be less substantial.

While this exercise suggests that the results we got from our basic model are robust, it also suggests a criticism of our approach. What we have done is to take an exogenously given payoff structure and then argue that as long as the payoff structure falls within a certain class we will get socially inefficient herding. It may be argued that this is misleading because if the social costs of herding were large enough, they would automatically bring into place mechanisms that will modify the payoff structures and reduce herding. In fact, a message of the first part of this section is that one can always reduce and even eliminate herding by having very high rewards for originality. One might then take the presence of the institution of patent laws (which rewards being first) as proof that society can always find ways of preventing inefficient herding.

In our opinion, however, this would be going too far. We feel that in many of the cases we consider there are substantial informational and transactions costs as well as institutional constraints which prevent the use of the appropriate incentives to eliminate herding. In some cases, like the restaurant example we considered in the introduction, it is difficult to think of how one could put any incentive scheme into place. But even in cases where there is a mechanism for rewarding originality, as patent protection for example, the degree to which the incentives can be changed may be quite limited. Anecdotal evidence suggests, for example, that the amount of protection patents provide varies a lot from case to case.

If decreasing returns (average payoffs decline as the number of people who choose it increases) tends to reduce herding, one would expect increasing returns, which rewards doing what a lot of others are doing, to increase the tendency to herd. This is indeed what we find. In an earlier version of this paper (which is available from the author), we present a detailed analysis of this case which shows
among other things that if the increasing returns are very strong, the unique equilibrium decision rule chosen by backward programming in the game tree involves everyone choosing the same option.

B. Alternative Information Structures

Another possible criticism of our basic model is that it is based on rather demanding informational assumptions. We assumed that each person knows the entire history of choices made by people before her. This is clearly a strong assumption that may be valid in some cases (like academic research) but not in others. Examples where one would like to make a weaker assumption include the restaurant example suggested in the introduction. In that case we can usually observe how many people have chosen each restaurant but not in which order they have made these choices.

However, inspection of the equilibrium decision rules suggests that in fact they do not make use of any information about the order of choice so that the same results apply as long as the distribution of choices is observable. It should be noted, however, that this is possible only under an assumption like our Assumption C, which is specified in terms of a location ("choose the highest among the options already chosen") rather than in terms of some order information ("follow the last decision maker").

C. Endogenizing the Order of Choice

In this model we have assumed that the order of choice is exogenously fixed: a more natural assumption (but one which complicates matters immensely) is to assume that choose when to move taking into account the fact that waiting is costly. When this cost is high, we shall have a game that is similar to the one we analyzed; when it is low, however, a new and interesting set of possibilities arises. A key question here is whether, if the waiting costs are low enough, all the agents with signals choose before all the agents without signals and whether this results in an efficient outcome. It turns out, somewhat surprisingly perhaps, that it is possible to have situations with low waiting costs where some of the uninformed move before some of the informed and the outcome is inefficient.20 However, the overall analysis is rather complicated, and we do not yet have very precise ideas about what happens in the general case.

20. To see the intuition behind this result, note that it is the marginal and not the absolute value of information that matters in the decision of whether or not to wait.
VI. CONCLUSIONS

We conclude with remarks on deficiencies of our model and possible directions of research.

The most serious departure of our model from reality is probably our assumption that signals to the agents are essentially free; a more realistic analysis would combine the question of incentives for obtaining these signals with the kinds of considerations we discuss. However, it would seem that dropping this assumption will encourage people to try to “free ride” on other people’s ideas and this would only exacerbate the herding problem. In this direction, at least, our results seem robust.

Our assumption that there is a continuum of options and payoffs exhibit a discontinuity at the true value, is defensible but somewhat unorthodox. Notice, however, that this assumption would be quite standard if there were only a large but finite number of options (then there would no discontinuity, for example). Preliminary investigations show that the results we get for the case where there are a large but finite number of options are much more complicated but quite similar. We therefore feel justified in working with this much more tractable model which we see as an approximation to the other case.

However, it may still be objected that what is missing from our model is the fact that options which are in some sense close to the true option are often better than the other options. By assuming that all options other than the right one get the same return, we have not allowed for this possibility. This is clearly an important direction for future research.

We have implicitly assumed that the agents in this model cannot actually trade in signals. This may be partly due to the problem of enforcing contracts describing the exchange of an idea, and partly due to the presence of transactions costs. Also, if there are some (perhaps very small) gains to having others choose what you are choosing, everybody will have the incentive to claim that they had a signal and that it matched the option they had chosen. However, since the absence of this kind of trade has serious welfare consequences, it is probably worthwhile to examine this assumption more closely.

Also, since people gain information by choosing later than others, there may be strategic aspects of timing that may be worth investigating. Some current work by the author attempts to extend this analysis in this direction.
The assumption that there are only two types of decision makers—those who have a signal and those who do not—though quite strong, can be easily relaxed to allow for decision makers with signals of different quality. As long as there are only a few different types whose signals differ substantially in quality, the results are quite similar.

References