A Simple Model of the Magnetosphere

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A phenomenological magnetic field model for the earth's magnetosphere is constructed from a dipole field and a uniform field directed sunward in the northern hemisphere and antisunward in the southern hemisphere. The properties of this simple model are compared with those of several other quantitative models. The present model is found to be more suitable for calculations than some other simple models in cases where the distant (>13 R_e) magnetotail configuration is important. Moreover, this model is easily adaptable to changes in the field geometry and to the description of magnetotail asymmetries.

INTRODUCTION

Models of the magnetic field within the magnetosphere have been developed for the purpose of numerical analyses such as particle trajectory tracing and the ordering of satellite data. *Roederer* [1969] reviewed some of the early models. Within the last few years, *Olson and Pfitzer* [1974], *Choe and Beard* [1974], and *Mead and Fairfield* [1975] have developed sophisticated multiple-term models which include magnetospheric currents and the tilt of the earth's dipole axis with respect to the incident solar wind direction. For many purposes, however, simple models are desirable because of their mathematical tractability. Some simplified models have been formulated from truncated versions of the series expansions describing the sophisticated models [e.g., *Williams and Mead,* 1965; *Olson and Pfitzer,* 1974].

In this report an alternative simple model is described. The basic model is constructed from a dipole field and a uniform field directed sunward in the northern hemisphere and antisunward in the southern hemisphere. Some properties of the model are discussed and compared with several other quantitative models. The incorporation of tilt or rotationlike features by the addition of terms is demonstrated to illustrate the flexibility of the model.

THE BASIC MODEL

The magnetic field of the present untilted magnetospheric model is described by the equation

$$\bar{B} = \mu \bar{\nabla}(\cos\theta/r^2) + B_T\hat{x}$$
$$B_T > 0 \qquad 0 < \theta < \pi/2 \qquad (1)$$
$$B_T < 0 \qquad \pi/2 < \theta < \pi$$

in which θ is colatitude, r is the geocentric distance, and $\hat{x}$ is the unit vector directed perpendicular to the dipole axis. The first term in (1) describes the dipole field, and the second term describes a current-sheet field which points in opposite directions in the northern and southern hemispheres. The current sheet extends throughout the magnetosphere, effectively replacing the magnetopause and ring currents in the inner magnetosphere. Two constants, μ and B_T, describe the dipole field and the current sheet field, respectively. At large distances from the dipole, the field in the magnetotail is approximately equal to B_T.

The components of the magnetic field given by (1) are

$$B_r = -2\mu\cos\theta/r^3 + B_T\sin\theta\cos\phi$$

$$B_\theta = -\mu\sin\theta/r^3 + B_T\cos\theta\cos\phi \qquad (2)$$

$$B_\phi = -B_T\sin\phi$$

where ϕ is magnetic longitude measured from midnight and B_T changes sign at the equatorial plane as specified above.

PROPERTIES OF THE BASIC MODEL

The divergence of the field given by (1) vanishes, as it must. It was found that the value $B_T = 0.00015$ produces a magnetosphere which resembles other quantitative models when the dipole field strength $\mu = 0.31$ is invoked and when r is measured in earth radii. Several projected views of the three-dimensional model obtained with these parameters are shown in Figure 1.

Figure 2 shows the field lines in the noon-midnight plane only. The dimpled magnetopause of this model is located at $r \sim 10$. The colatitude θ_c and radial distance r_c of the noon cusp, where the field lines separate to form the tail, can be found by setting $\bar{B}$ (2) equal to zero. The cusp occurs at the constant colatitude

$$\theta_c = \tan^{-1}(2)^{1/2} = 54.7° \qquad (3)$$

For the parameters $\mu = 0.31$ and $B_T = 0.00015$ selected above, the $B = 0$ point is located at the radial distance

$$r_c = [\mu\tan\theta_c/B_T]^{1/3} = 14.3 \qquad (4)$$

The magnitude of $\bar{B}$ along field lines originating at various dipole latitudes is shown in Figure 3 together with similar data given by *Mead* [1964] for his 13-term model. Although the minimum $\bar{B}$ points in the present model lie equatorward of the minima in the Mead model, the behavior of $\bar{B}$ along a field line is similar. A qualitative comparison with several highly sophisticated models based on spacecraft magnetometer data is illustrated in Figure 4, which shows selected field lines in the noon-midnight plane. Aside from the dimple in the field lines near the dayside magnetopause, the geometry of the present simple model appears to agree with the sophisticated models about as well as the sophisticated models agree with each other [e.g., see *Walker,* 1976]. However, the diurnal variation of $|B|$ at constant r that occurs in these other models and in the observed field does not occur in the model described by (1).

Selected field lines of simplified models that are employed in particle trajectory calculations for cosmic ray access studies [*Gall and Orozco,* 1974] are illustrated in Figure 5. One obvious difference between these models and the present model is the configuration of the magnetotail. In the present model the magnetotail is composed of quasi-parallel field lines that gradually converge toward the equatorial plane where they reverse

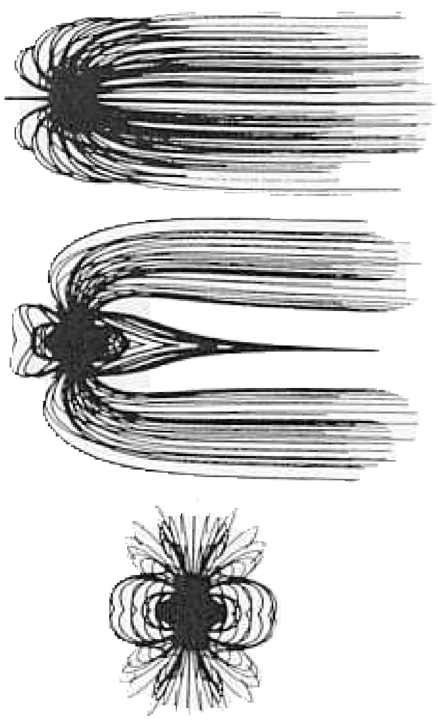

Fig. 1. The basic magnetospheric model described by (1). Projected views from above the equatorial plane (top), in the noon-midnight plane (center), and from the sun (bottom) are shown. Field lines are drawn for colatitudes within 50° of the poles at intervals of 5° and for longitude intervals of 18°.

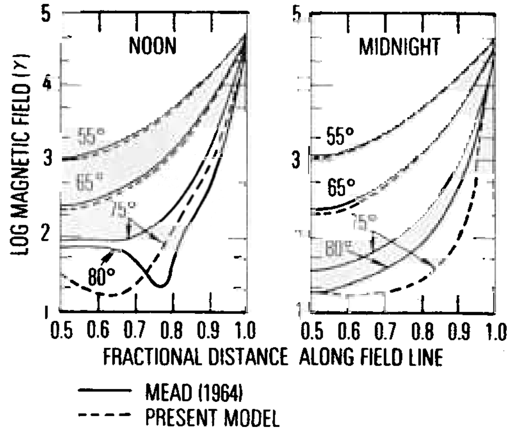

Fig. 3. Comparison of field strengths along field lines originating at various latitudes in the present model (1) and the 13-term Mead model.

direction. Many of the other simple models are actually intended for use only at $r \lesssim 13\ R_e$ because they have tail field lines that diverge or show other unrealistic behavior [e.g., see *Olson and Pfitzer,* 1974]. Since the present model more accurately represents the observed magnetotail [e.g., see *Willis and Pratt,* 1972], it is superior for simulating effects related to the tail configuration.

A primary shortcoming of the present model is the discontinuity of $\vec{B}$ in the equatorial plane. However, this discontinuity is easily smoothed in numerical analyses by introducing a factor $\tanh(r \cos\theta/\delta)$, where δ is a constant, in the tail field term

$$\vec{B} = \mu\vec{\nabla}(\cos\theta/r^2) + B_T \tanh(r\cos\theta/\delta)\hat{x} \qquad (5)$$

(M. Schulz, personal communication, 1978). The effect of this modification, as shown by Figure 6, is essentially that of the addition of a current sheet of 'thickness' δ in the equatorial plane. The value of δ determines the colatitude of the last 'closed' field lines and the shape of the magnetopause near the equator.

TILTED DIPOLE

The dipole in the present model can be tilted with respect to the tail by rotating the coordinate system for the current-sheet field:

$$B_r = -2\mu\cos\theta/r^3 + B_T(\sin\theta\cos\phi\cos\alpha - \cos\theta\sin\alpha)$$
$$B_\theta = -\mu\sin\theta/r^3 + B_T(\cos\theta\cos\phi\cos\alpha + \sin\theta\sin\alpha)$$
$$B_\phi = -B_T\sin\phi\cos\alpha \qquad (6)$$
$$B_T > 0 \qquad \cos\theta > -\sin\theta\cos\phi\tan\alpha$$
$$B_T < 0 \qquad \cos\theta < -\sin\theta\cos\phi\tan\alpha$$

where α is the angle between the dipole axis and the tail field direction $\hat{x}$. Selected field lines for the tilted model with $\alpha = 30°$ are shown in Figure 7. As in other models with a tilted dipole [e.g., *Mead and Fairfield,* 1975], some field lines appear to penetrate the magnetopause. These lines could be eliminated if the appropriate magnetopause currents were included.

ADDITION OF ROTATION

A rotationlike effect can be incorporated in the basic model (2) by adding a term to B_ϕ which describes a cylindrical vortex oriented along the dipole axis:

$$B_\phi = -B_T\sin\phi - K/r\sin\theta \qquad (7)$$

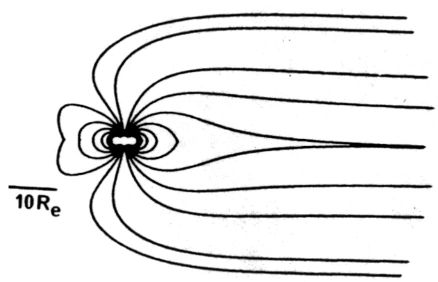

Fig. 2. Field lines described by (1) in the noon-midnight plane, drawn for colatitudes within 50° of the poles at intervals of 5°.

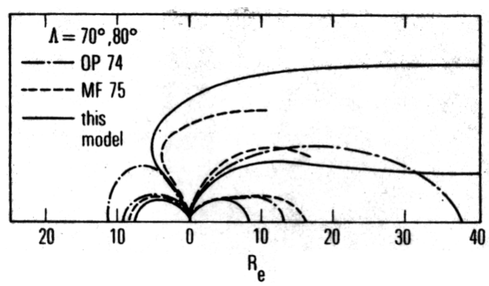

Fig. 4. Comparison of field lines originating at various magnetic latitudes Λ in the *Olson and Pfitzer* [1974] (OP 74), *Mead and Fairfield* [1975] (MF 75), and present magnetospheric field models in the noon-midnight plane.

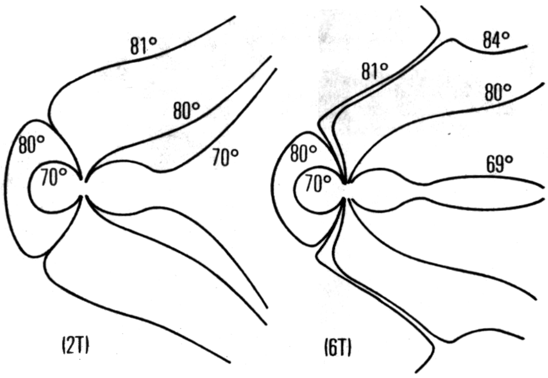

Fig 5. Selected field lines in the noon-midnight plane of the two term (2T) and six term (6T) field models [*Williams and Mead*, 1965] which are used in the calculation of cosmic ray trajectories in the magnetosphere.

Here K characterizes the vortex strength, or rotational velocity. Although this is not a particularly realistic representation of the consequences of the rotation of the magnetosphere, it is qualitatively consistent with the behavior that may be found at latitudes above the region of corotation.

Figure 8 illustrates that the effect of the above addition to B_ϕ is an asymmetric distortion of the field lines and the magnetopause. In fact, a local time asymmetry in the magnetopause shape, similar to the distortion produced by the rotation term in (7), has been observed [*Fairfield and Mead*, 1975; *McDiarmid et al.*, 1976].

CONCLUDING REMARKS

It is noteworthy in the present context that *Mead and Fairfield* [1975], in their report describing a 17-term model obtained by fitting spacecraft magnetometer data, found that the predominant nondipole term in the series expansion of $\vec{B}$ described a field directed sunward in the northern hemisphere and antisunward in the southern hemisphere.

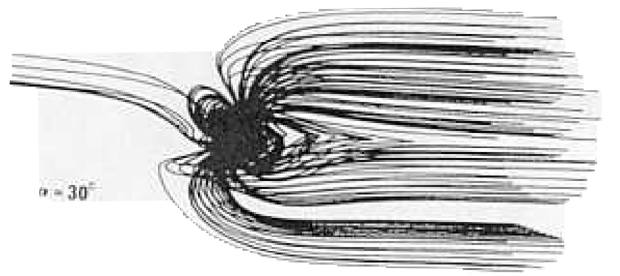

Fig. 7. The tilted magnetosphere model described by (6) with $\alpha = 30°$ as it appears for the same field lines as shown in Figure 1.

The advantages of using the simple model that has been described here are obvious. The step-wise calculation of particle trajectories in a model of the magnetospheric magnetic field with a realistic magnetotail can be carried out using minimal computer time. Moreover, the effects on the magnetosphere of the spatial nonuniformity and temporal variability of the solar wind are simply modeled by adjusting B_T. For example, asymmetric distortions can be described by $B_T = B_T(x, y, z)$. The possibilities afforded by the parameterization of B_T, together with the tilt and rotation options, make this an unusually flexible magnetospheric model for both qualitative and quantitative analyses.

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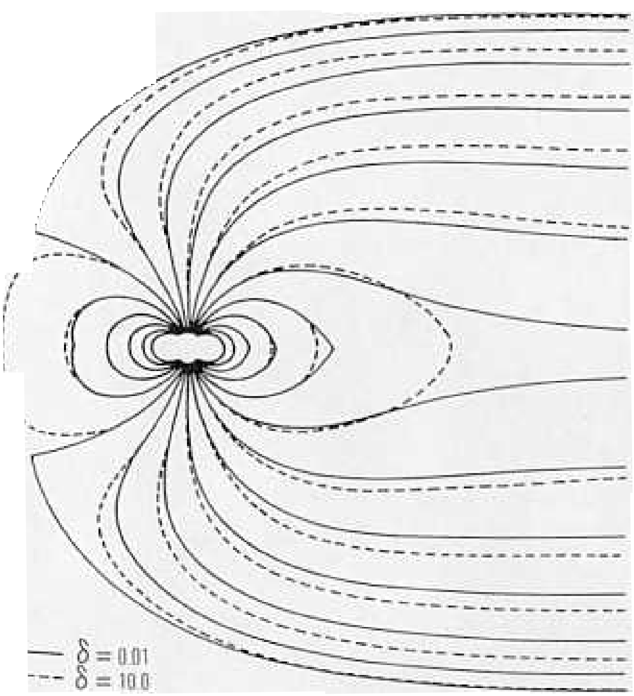

Fig. 6. The magnetosphere model described by (5). Field lines in the noon-midnight plane are shown for two values of δ.

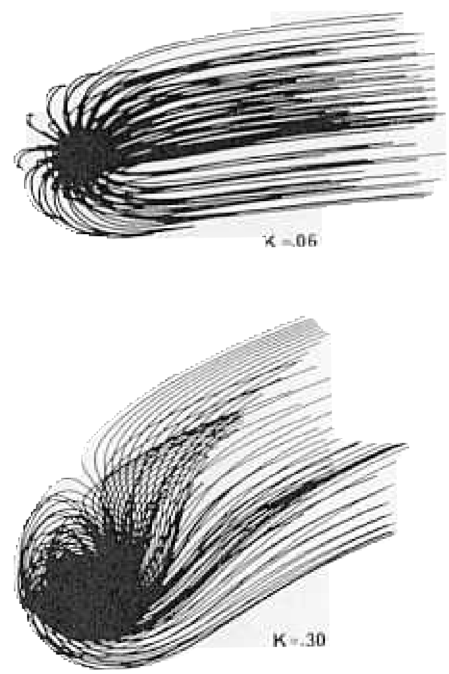

Fig. 8. Illustration of the effect of adding the 'rotation' term $-K/r \sin\theta$ to $\vec{B}$. Equatorial plane projections of the field lines used in Figure 1 are shown for two values of K.

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