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A simple proof of Heymann's lemma

by

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The Netherlands

A SIMPLE PROOF OF HEYMANN'S LEMMA

of

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Abstract. Heymann's lemma is proved by a simple induction argument .

The problem of pole assignment by state feedback in the system

$$x_{k+1} = Ax_k + Bu_k ; \quad (k = 0, 1, \dots)$$

where A is an $n \times n$ -matrix and B an $n \times m$ -matrix, has been considered by many authors. The case $m = 1$ has been dealt with by Rissanen [3] in 1960. In 1964 Popov [2] showed the pole assignability for complex systems (more generally systems over an algebraically closed field). In 1967 Wonham gave a proof valid for real systems (or more generally for systems over an infinite field). Finally, in 1968, Heymann [1] gave a proof which is valid for systems over an arbitrary field. Heymann's proof depends on the following result.

LEMMA 1. *If (A, b) is controllable and $b = Bv \neq 0$, then there exists F such that $(A + BF, b)$ is controllable.*

By means of this result the multivariable problem can be reduced to the single variable problem.

It is the aim of this correspondence to give a simple proof of this lemma. The result follows immediately from

LEMMA 2. *If (A, B) is controllable and $b = Bv \neq 0$, then there exists u_1, \dots, u_{n-1} such that the sequence defined by*

$$(1) \quad x_1 := b \quad . \quad x_{k+1} := Ax_k + Bu_k ,$$

for $k = 1, \dots, n - 1$ is independent.

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Indeed, if Lemma 2 is shown we can choose u_n arbitrary and define F by $Fx_k = u_k$. Then it is easily seen that $(A + BF)^k b = x_k$, so that $(A + BF, b)$ is controllable.

PROOF OF LEMMA 2. We proceed stepwise. $x_1 \neq 0$ and hence independent. Suppose that x_1, \dots, x_k have been constructed according to (1) and are independent. Denote by \mathcal{L} the linear space generated by x_1, \dots, x_k . We have to choose u_k such that $x_{k+1} = Ax_k + Bu_k \notin \mathcal{L}$. If this is not possible, then

$$(2) \quad Ax_k + Bu \in \mathcal{L}$$

for all u . Choosing in particular $u = 0$ we find

$$(3) \quad Ax_k \in \mathcal{L}$$

and consequently, by the linearity of \mathcal{L} , $Bu \in \mathcal{L}$ for all u . That is, $\text{im}B \subseteq \mathcal{L}$. Also, for $i < k$ we have

$$Ax_i = x_{i+1} - Bu_i \in \mathcal{L}$$

Hence $Ax_i \in \mathcal{L}$ for $i = 1, \dots, k$, and, consequently, \mathcal{L} is A -invariant. From the controllability of (A, B) it follows that \mathcal{L} must be the whole state space, which implies that $k = n$. □

REMARK. In [1] and in [5, Lemma 2.2] proofs of Lemma 1 were given by constructing a particular sequence u_k satisfying the condition of Lemma 2. These constructions may suggest that such a special u_k is essential for the calculation of F , which is not the case as follows from the proof of Lemma 2. It also follows that the u_k 's can be constructed recursively in the following sense: Once $u_1, \dots, u_{\ell-1}$ have been chosen so as to render x_1, \dots, x_{ℓ} independent, one can always continue the construction of the remaining u_k 's.

The may be useful when it comes to an actual numerical computation of F .

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