A SIMPLER PROOF OF THE BOROS–FÜREDI–BÁRÁNY–PACH–GROMOV THEOREM

ROMAN KARASEV

The main topic of this talk is:

Problem. Let $d+1$ random points $x_0, \dots, x_d$ be distributed independently in $\mathbb{R}^d$. Show that one point $c \in \mathbb{R}^d$ is covered by the simplex $\operatorname{conv}\{x_0, \dots, x_d\}$ with probability p_d with largest possible value of p_d.

Endre Boros and Zoltán Füredi (1984) established the best constant $p_2 = 2/9$ when the points are distributed with the same discrete distribution.

Imre Bárány (1982) considered arbitrary dimension and random points distributed by the same discrete distribution. The constant was roughly $p_d = (d+1)^{-d}$. This result was obtained by partitioning the N distribution points into $\sim \frac{N}{d+1}$ groups of $d+1$ each by the Tverberg theorem and then applying the colorful Carathéodory theorem to every $(d+1)$-tuple of $(d+1)$-tuples.

János Pach (1998) considered arbitrary dimension and points distributed with different discrete distributions. The constant p_d was approximately $\frac{1}{(5d)^{d^2}(d+1)}$.

In case of the same discrete distribution for all points Uli Wagner (2003) has improved the bound to $p_d = \frac{d^2+1}{(d+1)^{d+1}}$.

Recently Mikhail Gromov (2010) has developed a topological approach to estimating multiplicity of maps, in particular, giving a better bound $p_d = \frac{1}{(d+1)!}$ for the probability of covering by the convex hull, which improves to $p_d \geq \frac{2d}{(d+1)!(d+1)}$ when some two points have the same distribution.

Gromov actually proved a much stronger result: Instead of several finite point sets in $\mathbb{R}^d$ one can consider a continuous map of the join of $d+1$ finite sets to $\mathbb{R}^d$ (or the d-skeleton of large enough simplex) and study covering by the images of faces of maximal dimension under this map. But we will not consider such generalizations here.

The proof of Gromov is not easy to understand. It used an abstract notion of the space of cocycles. Moreover, the space of cocycles was defined as a simplicial set, which is even harder to imagine.

In this talk we are going to decipher Gromov's proof and make it almost elementary and very short.

2000 *Mathematics Subject Classification.* 52C35,52C45,60D05.

Key words and phrases. multiplicity of map, simplicial depth.