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A SIMPLIFIED THERMAL CONTINUUM FUNCTION FOR THE X-RAY EMISSION FROM CORONAL PLASMAS

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SUMMARY

At temperatures below $30 \cdot 0 \times 10^6$ °K, the free–bound process makes an increasingly significant contribution to the total continuum flux. While the free–free spectrum may be represented by a relatively simple expression, a detailed calculation of the free–bound spectrum requires a lengthy summation process over all the ionization stages and levels of the abundant coronal ions.

A simple empirical expression is presented which gives the magnitude and spectral slope of the free–bound contribution over the energy interval 1·5–15 keV for the temperature range $4 \cdot 0$–$20 \cdot 0 \times 10^6$ °K. This expression gives results that agree to better than 15 per cent with more detailed calculations of the thermal continuous spectrum.

1. INTRODUCTION

Calculations of the free–free and free–bound continuous spectra emitted by coronal plasmas have been presented previously by one of us (Culhane 1969). This work will be referred to hereafter as Paper I. The expression which gives the flux of free–free radiation as a function of photon energy (or wavelength) is relatively straightforward. In cases where the free–free process is responsible for most of the continuum flux the expression given in equation (2) of Paper I may be used to describe the continuous spectrum emitted by a coronal plasma whose electrons have a Maxwellian velocity distribution. This expression is based on the use of the classical cross section for the free–free process, corrected by a multiplicative Gaunt factor. In Paper I, the Gaunt factors used were obtained from the work of Karzas & Latter (1961). An alternative approach, employed at higher photon energies and electron temperatures by Chodil *et al.* (1968), is to use the non-relativistic Born approximation for the free–free cross section. The calculation of the free–bound flux is more laborious since summations over the abundant elements, their ionization stages and principal shells are required (see equations (8) and (9) of Paper I). In this paper, a simple approximation is used for the Gaunt factor and an average value of Z^2 is employed.

It has been shown in Paper I, however, that at temperatures in the range below $20 \cdot 0 \times 10^6$ °K, that are commonly encountered in the solar corona during flares and in the cores of coronal active regions, the free–bound process contributes a significant part of the total continuum flux, particularly at lower temperatures. If emission integrals and temperatures are derived by comparing observational data with calculated free–free continuous spectra, the neglect of the free–bound process can lead to errors in the estimation of both plasma temperature and emission integral.

The errors in temperature are not usually large. Nevertheless, in the soft X-ray region small temperature changes have a significant effect on the flux emitted; particularly if the plasma temperature is relatively low to begin with. Emission integrals, obtained by comparisons with calculated free–free spectra, can be overestimated by factors of two or more.

Several studies of the continuum emission from the solar corona have recently been carried out (cf, Hudson, Peterson & Schwartz 1969; Culhane & Phillips 1970) and work of this kind will continue in the future. In order to avoid the need for repeated computation or tabulation of the free–bound spectrum, we have derived an empirical analytic expression which takes account of both free–bound and the free–free flux. This expression is presented below and its use in the derivation of plasma temperatures and emission integrals is demonstrated. The temperature and energy ranges over which this expression may be reliably used are discussed. Although these ranges are somewhat limited, they include temperature values that are commonly encountered in flare plasmas and in coronal active regions.

2. THE SIMPLIFIED CONTINUUM EMISSION FUNCTION

The expression for the thermal continuous spectrum is of the form

$$N(E) = 3\cdot 6\times 10^{-39}\,\overline{Z^2}T^{-0\cdot 5}E^{-1\cdot 0}\exp\left(-\frac{E}{kT}\right)(\bar{g}(E,\,T)+f(E,\,T))\int N_e^2\,dV$$

$$\text{photons cm}^{-2}\,\text{s}^{-1}\,\text{keV}^{-1}\text{ at Earth distance} \qquad (1)$$

where T is the plasma temperature in degrees Kelvin, N_e is the electron density, E is the photon energy in keV, $\bar{g}(E, T)$ is the temperature averaged free–free Gaunt factor for a Hydrogen plasma, k is Boltzmann's constant, dV is an element of volume in the emitting region of the solar corona and $\int N_e^2\,dV$ is the emission integral. In equation (2) of Paper I, the product $Z^2\bar{g}(E, T, Z)$ was averaged for the twelve most abundant coronal elements. In the present work, the expression for the continuum flux has been simplified by taking a mean value of Z^2, $(\overline{Z^2})$, obtained from a weighted average over a 'coronal' set of element abundances. This set of abundances included a value of 0·20 for the helium to hydrogen ratio. This procedure leads to a value of 1·6 for $\overline{Z^2}$. A simplified expression of the form

$$\bar{g}(E,\,T) = \left(\frac{E}{kT}\right)^{-0\cdot 3} \qquad (2)$$

has been used for the free–free Gaunt factor. This approximate form was used previously by Rappaport *et al.* (1969) and is appropriate for low Z plasmas.

The function $f(E, T)$ is an empirical function which has been fitted to the free–bound spectra that were calculated in Paper I. We have found that an expression of the form

$$f(E,\,T) = \bar{g}(E,\,T)\left[\left(\frac{88}{E}\right)^{0\cdot 33\,kT} - 1\right]^{-1} \qquad (3)$$

may be used to express the free–bound contribution to the total continuum flux. The complete expression for the thermal continuum flux at Earth distance may therefore be obtained by substituting equations (2) and (3) into equation (1) and letting $\bar{Z}^2 = 1 \cdot 6$. The complete continuum emission function then becomes

$$N(E) = 4 \cdot 4 \times 10^{-4 \cdot 1} E^{-1 \cdot 3} T^{-0 \cdot 2} \exp\left[-\frac{E}{kT}\right]\left[1 - \left(\frac{E}{88 \cdot 0}\right)^{0 \cdot 33\, kT}\right]^{-1 \cdot 0} \int_V N_e^2\, dV \quad (4)$$

photons $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{keV}^{-1}$ at Earth distance.

Multiplication of equations (1) or (4) by the factor 4π (astronomical unit)^2 will yield the flux emitted by a volume of plasma $V = \int dV$, with electron density N_e, located in the solar corona.

If equation (4) is restricted in its application to the photon energy range 1·5–15 keV and the temperature range $4 \cdot 0$–$20 \cdot 0 \times 10^6$ °K, the agreement with the results presented in Paper I will be to better than 15 per cent. The nature of the agreement may be seen in the data shown in Fig. 1 where continuous spectra from Paper I and spectra obtained with equation (4) are plotted. In order to facilitate the comparison, equation (4) has been expressed in units of erg $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{Å}^{-1}$. At temperatures less than $4 \cdot 0 \times 10^6$ °K and at low photon energies ($E < 1 \cdot 5$ keV), the spectra obtained with equation (4) diverge significantly from spectra calculated using the more basic formalism of Paper I. This happens because of the increasing importance of the free–bound process at low temperatures; at

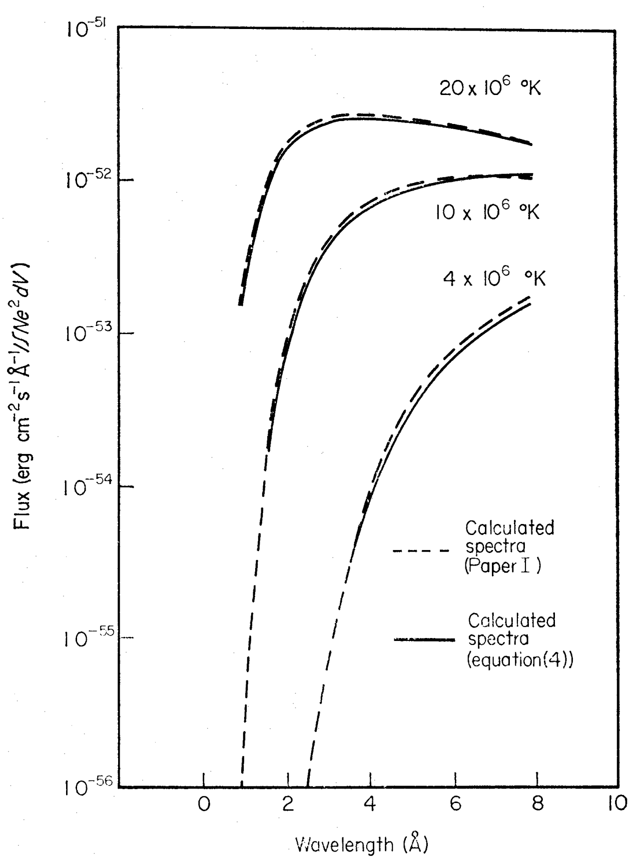

FIG. 1. *Thermal continuous spectra from Paper I are compared with spectra calculated using equation (4) from the present paper.*

temperatures above $30 \cdot 0 \times 10^6$ °K, the free–free process becomes dominant. In the temperature and energy ranges quoted above, equation (4) provides a good representation of the thermal continuous spectrum. The form of the expression is such that it may be conveniently fitted to experimental data by varying temperature and emission integral or folded into a detector response function.

3. THE USE OF CONTINUUM SOURCE FUNCTIONS TO FIT EXPERIMENTAL DATA

At least two simpler versions of equation (1) have been used previously with X-ray data to obtain values of plasma parameters for both the Sun and other soft X-ray sources. The values of these parameters may be obtained in two ways. In one approach (e.g., that of Gorenstein, Gursky & Garmire 1968) the plasma parameters (or, in addition, the parameters of the intervening medium) are varied until a calculated set of data agrees in the sense of a 'best fit' with the data set obtained by an instrument in flight. The best fit may be characterized by a minimum in the value of χ^2. All the significant detector parameters (quantum efficiency, resolution, photon escape probability) are included in the calculation of data sets. An alternative approach (Boldt, Holt & Serlimitsos 1968; Fritz *et al.* 1969) involves the calculation of a detector's response to a particular emission function for all the likely values of the plasma parameters. The parameter values leading to a detector response that is identical to the response observed in flight are then identified.

In either case, it is preferable to have available a simple analytic form of the emission function. Two commonly used simplifications of equation (1) are

$$\bar{g}(E, T) = 1 \cdot 0 \text{ and } f(E, T) = 0 \tag{1a}$$

and

$$\bar{g}(E, T) = \left(\frac{kT}{E}\right)^{0 \cdot 3} \text{ and } f(E, T) = 0 \tag{1b}$$

though of course other approximate forms or a tabulation of exact values may be used for the free–free Gaunt factor.

It is of interest to examine the different values of plasma parameters that are obtained depending upon whether equation (1a), equation (1b) or the more complete equation (4) is used.

If a calculated and a measured continuous spectrum are available for an isothermal plasma, the emission integral may be established from the relation

$$\int N_{\text{meas}}(E)dE = \int_V N_e^2 dV \int N_{\text{calc}}(E)dE \tag{5}$$

where the $N(E)$'s are differential photon fluxes and the calculated flux is for the case of unit emission integral. The use of the assumptions stated in equations (1a) and (1b) will lead to an overestimate of the plasma emission measure since the contribution of the free–bound flux is ignored in both of these assumptions. The magnitude of this error may be seen in the data of Fig. 2. Here the ratio of the emission integral obtained by using equation (1a) to that obtained with the more accurate equation (4) is plotted as a function of temperature. A similar curve is plotted for emission integrals obtained from equation (1b). Both curves refer to a photon energy of $5 \cdot 0$ keV.

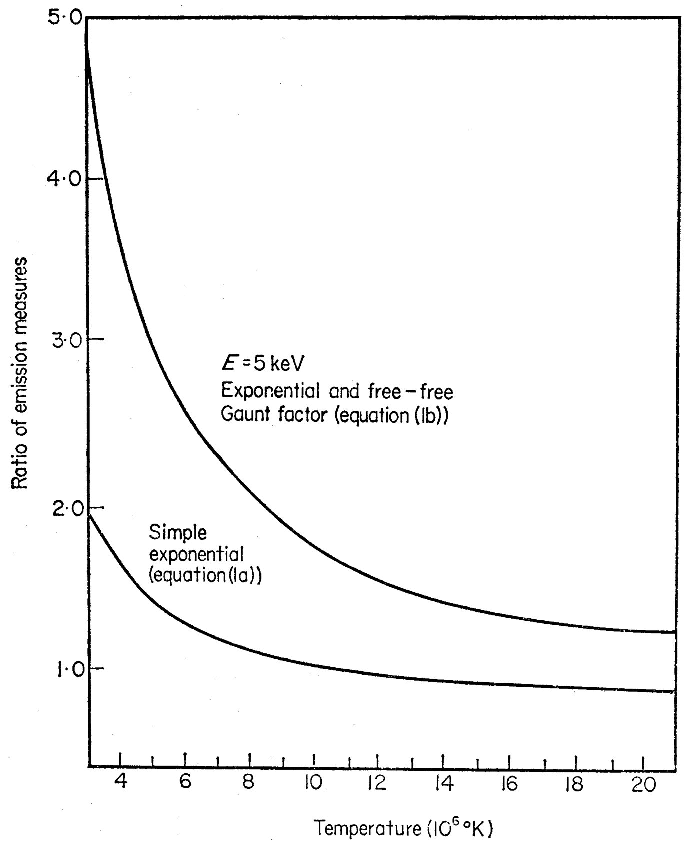

FIG. 2. *The ratio of emission measure, determined with a continuum flux from equations (1a) and (1b), to that obtained by using equation (4) is plotted as a function of temperature. All the ratios were determined at a photon energy of 5 keV.*

Although equation (1b) more correctly represents the free–free spectrum, emission integrals that are large by a factor of four or more can be obtained because of the neglect of the free–bound contribution. Emission integrals derived with the aid of the simple exponential emission function (equation (1a)) are closer to the values that would be obtained by using equation (4). The lower plot in Fig. 2 shows that the emission integrals, obtained by using the simple exponential expression, are within 10 per cent of the correct value over the temperature range $5 \cdot 0 \times 10^6$–$20 \cdot 0 \times 10^6$ °K.

It is, however, somewhat artificial to evaluate the usefulness of the various expressions in the manner just described. When an emission function is being fitted to experimental data, the fitting will probably be carried out by varying both temperature and emission integral. In order to provide a more realistic evaluation, we have fitted all three equations (1a), (1b) and (4) to data obtained from a pair of solar rocket flights. The rockets carried large area proportional counter detectors, sensitive in the energy range 3–30 keV. The instrument has already been described (Reed *et al.* 1968) and flight results will be presented in later publications. All three expressions were fitted to the data by iterating the temperature and emission integral parameters until agreement was obtained between calculated and measured spectra.

Values of temperature and emission integral so obtained are listed in Table I. The emission integrals have been determined in all cases by fitting the emission functions to data obtained over a significant energy range while the data shown

TABLE I

Values of temperature and emission measure

Date of flight	Solar conditions	Continuum source function	Temperature (10^6 °K)	Emission measure
1968 Feb.	Small X-ray event in progress	Simple exponential (Equation (1a))	10.6 ± 0.2	2.1×10^{47}
		Thermal free–free (Equation (1b))	11.0 ± 0.2	2.8×10^{47}
		Thermal free–free and free–bound (Equation (4))	10.6 ± 0.2	2.3×10^{47}
1967 April	No X-ray events in progress	Simple exponential (Equation (1a))	7.0 ± 0.3	3.6×10^{46}
		Thermal free–free (Equation (1b))	7.3 ± 0.3	5.0×10^{46}
		Thermal free–free and free–bound (Equation (4))	7.0 ± 0.3	2.8×10^{46}

in Fig. 2 were obtained on the assumption that the emission integral was determined at a single value of photon energy. When the correct free–free expression (1b) is used, the derived values of temperature and emission integral differ significantly from the values obtained with the more complete free–free and free–bound expression (equation (4)). However the use of the simple exponential approximation (equation (1a)) produces quite good agreement with equation (4) for both temperature and emission integral.

4. CONCLUSIONS

Equation (4), Section 2, has been found to give results that agree to within 15 per cent with the more detailed calculations of the thermal continuous spectrum that were presented in Paper I.

This expression uses a simple function to represent the free–free Gaunt factor and an average value of Z^2 appropriate to the solar corona. The contribution of the free–bound process to the total continuous spectrum is also represented by a simple analytic function. The complete expression may be used over the temperature range 4.0–20.0×10^6 °K and the energy range 1.5–15 keV.

Results obtained with the complete expression (4) are compared with those obtained using the simplifying assumptions given in equations (1a) and (1b). Substantial errors can occur in deriving emission integrals, particularly at lower temperatures. However, the simple exponential source function (equation (1) with assumptions (1a)) more correctly represents the total continuous spectrum than does the expression for the free–free process alone with Gaunt factor included.

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