A SIMULATION ANALYSIS OF CASH FLOW ATTENUATION UNDER ALTERNATIVE FINANCIAL OPERATING STRUCTURES

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Abstract

This paper presents the results of a simulation analysis of the stochastic processes which determine cash flow attenuation under alternative financial operating structures. The environment of the study is that of a rapidly expanding retailing firm, which must consider a myriad of alternative operating structures during its embryonic development. Simulation is employed to determine risk-return profiles for the various alternatives. Efficiency frontiers are next computed for each of five measures of profitability, over a time adjusted horizon. An operating structure which produces minimal attenuation, and hence maximal profitability, is then determined by analyzing the efficiency frontiers of the various alternatives.

Introduction

The framework within which financial decisions are made in the modern corporation is extremely broad and complex. As Beranek[1] has suggested, the finance function is concerned with: (1) determining both the attractiveness and costs involved in the various uses of the firm's resources; (2) locating and establishing costs of sources of supply of funds; and (3) making a choice of both sources of supply and uses of resources which seek to maximize the attainment of organizational goals. Financial analysis is thus directed towards determining projects which produce benefits and demand financial resources, finding the sources of financial resources, and then making financial decisions involving combinations of projects and sources of supply of funds.

In recent times remarkable developments have occurred in the field of financial analysis. One major development has been the revolution of information-processing techniques, in which high speed digital computers have been employed to acquire, store, retrieve, and analyze a myriad of financial information. Another major development in financial analysis has been the accelerated utilization of management science techniques -- mathematical programming, game theory, search methods, and simulation -- in dealing with the complex financial interrelationships of the modern corporation.

This paper presents the results of the application of a computer-based management science technique, digital simulation, to the basic financial problem of determining an "optimal financial operating structure." In a manner similar to Weston and Brigham[2], we define financial operating structure to refer to the right hand side of the balance sheet -- the financing of the resources required by the firm. In the study described herein, the major concern was that of securing intermediate term (three to five year) debt financing. An optimal financial operating structure was therefore defined as being one that produced the minimal cash flow attenuation over this intermediate time horizon.

Statement of the Problem

The environment of the simulation study involved a rapidly expanding mobile home retailing company located in a Midwestern state. Incorporated in 1968, its sales volume had risen dramatically during its brief history. Because the unit value of a mobile home was relatively substantial, about 95% of the purchases of mobile homes were debt financed by the consumer. Additionally, about 80% of all the consumers who financed the purchase of their mobile home expected the retailing company to provide them with convenient access to a debt source of funds. Consequently, the greatest hinderance to the company's growth during its embryonic development was its inability to secure and maintain an adequate source of debt financing for its customers. Over the years, the company had used a variety of financial institutions in attempting to alleviate this problem; including commercial banks, savings and loan institutions, insurance companies, and credit subsidiaries of other corporations. None of these arrangements had proven satisfactory.

Since the primary objective of the owners of the company was to profitably achieve financial independence from external sources, they were interested in ways of vertically integrating so that they could exercise self control in the financing channel. After considering several alternatives, they arrived at a tentative agreement with a savings and loan institution that allowed the retail sales company to set up its own captive finance company, which was financed by the savings and loan institution, with a strict reinsurance requirement provided by a third party insurance company. Faced with this set of conditions, the owners then sought to negotiate for a combination of three financial factors which would produce minimal cash flow attenuation, and hence would create the greatest expansionary impetus for the company. The three critical factors were:

1. <u>Borrowing Rate</u>. The interest rate at which the retailing company could borrow from the

savings and loan institution. (The feasible range was thought to be 7% - 10%.)

2. Credit Line. The total amount of money the retailing company could borrow from the savings and loan institution to satisfy its annual financing arrangements. (The feasible range was thought to be $300,000 - $1,500,000.)

3. Contract Percentage. The percentage of each sales contract which the retailing company would be allowed to finance through its captive finance company by the savings and loan institution. (The feasible range was thought to be 50% - 70%.)

Obviously, there were a large number of combinations of these three factors which could result from the negotiation process between the retailing company and the savings and loan institution. For example, if a large credit line was sought, it was likely to result in a higher borrowing rate for the retailing company. A similar tradeoff was likely to occur if the retailing company sought a relatively high contract percentage. Conversely, a lower borrowing rate could be obtained with a lower credit line and/or contract percentage. As a result, some procedure was required to determine a priority list which could be utilized in the bargaining process to determine a "starting condition" combination of these three factors. Specific issues of concern were:

1. What starting condition combination of factors results in the maximum net cash inflow (minimal cash flow attenuation)?

2. What starting condition combination of factors results in the most profitable earnings per common share?

3. What starting condition combination results in the most efficient utilization of operating assets?

4. What starting condition combination will offer the greatest inducement for potential equity investors?

5. What starting condition combination will afford the greatest risk of insolvency?

6. What is the relative ranking in importance between the three factors which constitute a starting condition?

Research Methodology

The most difficult part of any financial analysis is determining and evaluating the numerous alternatives that may exist, due to the uncertainty and hence risk that surround these alternatives. Two rather traditional approaches to solving this dilemma include:

1. "Best guess estimates" - the analyst specifies his best assumptions about the key variables affecting future costs, revenues, and investment requirements in terms of single point estimates. The outcome of the investment decision, based on these "best guesses" is then considered acceptable if it exceeds a specified criterion of return or payback.

2. "Forced-fit forecasts" - the analyst specifies the expected financial outcome and the actual outcome is then forced to fit the original estimate. For example, if sales fall short of prediction, heavier advertising may then be employed as a corrective maneuver.

The superficiality of these approaches need no further discussion.

Risk Profile Analysis

Recently, a third method of dealing with uncertainty and risk which is much more sophisticated, but much less readily understood, had gained credence. The idea was first suggested by Hertz[3] in his classic article which dealt with the use of computer simulation in treating the uncertainty involved in financial decision making. The basic concept has been extended considerably by Hertz[4], Salazar and Sen[5], and Hess and Quigley[6].

This new method of dealing with uncertainty has been named "risk profile analysis" by Hertz[4]. Using risk profile analysis the first step is to isolate the key variables which will affect future costs and revenues. The next step is to weigh all of the available information about each of these key financial variables and then, from this information, develop a probability distribution for future revenues which is termed the "uncertainty profile". This probability distribution may be determined objectively if the decision maker has the pertinent data. Realistically, however, in practice it will often be subjectively assigned. Once an uncertainty profile has been developed for each key financial variable, one can then sample repeatedly from the distributions of these variables as summarized in their uncertainty profiles. The most efficient and convenient sampling procedure is, of course, computer simulation. From the results of these simulations, a probability distribution, or "risk profile", can be ascertained. This risk profile produces an "expected value" and definite probabilities are associated for all possible outcomes. In addition, the spread of the distribution of the potential returns about the expected values of all outcomes is a useful risk measurement.

Decision Criterion - Efficiency Frontiers

Having determined a risk profile, decision criteria must next be established for choosing between capital structures with alternative risk profiles. Clearly, any decision criterion must consider both maximization of economic return and minimization of economic risk, two factors which are not necessarily compatible. Herein, an efficiency frontier criterion, based on the work of Markowitz[7] was utilized. Assuming that one can simulate the financial results from the choice of a particular operating structure, the expected return -- coupled with the standard deviation of the financial results obtained from that policy -- will indicate the "efficiency" of the operating structure under that policy. (Note that within this simulation we are linearly combining the uncertainty profiles of revenues, costs, and investments, and, hence, can expect the results to be normally distributed.)

The expected return and standard deviation can then be plotted on a graph to illustrate the effectiveness of any policy, and a line can then be drawn through the points of greatest yield for a given standard deviation. This line is called the efficiency frontier because it represents the best return management can obtain, given a particular variance, unless it can find: (1) a policy that will yield a greater return on investment for no more variance, or (2) develop financial structures with different uncertainty profiles that provide project choices with less variance for equivalent returns.

Model Formulation

The basic structure of the simulation model used to test cash flow attenuation under differing operating structures is shown in Figure 1. The model was modular in nature, and had four major components.

Forecast Module

The initial exogenous input to the simulation model was an estimate of potential mobile home sales developed from historical and forecasted annual sales of mobile homes in the United States, as provided by the U.S. Department of Commerce. A least square trend equation, based on historical data, was developed for forecasting total national sales of mobile homes. This trend equation was then utilized in conjunction with the company's three year average share of the market to forecast annual, and monthly, average sales of mobile homes. These monthly forecasts were then seasonally adjusted, based on indices developed from historical company data. A five year forecasting horizon was employed.

Demand Simulation Module

All product characteristics which influenced the cash flow of the ABC Finance Company were considered as exogenous variables. Based on an analysis of sales data, the company's entire product line was categorized on the basis of four major factors:

1. The seven (7) basic physical sizes of mobile homes and their associated prices.

2. The consumers' desire to either purchase the mobile home outright or finance its purchase.

3. The consumers' desire to purchase comprehensive and credit life insurance on a cash sale. (Note: The consumer was required by the external savings and loan company to purchase this insurance on financed units.)

4. The consumers' possibility of default on a financed mobile home purchase contract.

A detailed analysis of the company's historical sales data was then undertaken, and probability distributions for each of these four factors were determined. This process was extremely rigorous, and its details are omitted here for the sake of brevity. The analysis of the four major product characteristic factors resulted in the creation of twenty-eight (28) separate " product categories", with each product category being some combination of the four factors described above. A cumulative frequency distribution for these product categories was derived and became the basis for the subsequent Monte Carlo simulation of product demand. Within each simulation run, all product categories were effectively simulated, i.e., each simulation run encompassed a five year time horizon and resulted in sales of 300-1000 mobile homes.

Cash Flow Attenuation Module

As each mobile home sale was simulated, its consequences upon cash flow attenuation were measured and accumulated. The product characteristics of each mobile home sale determined the set of cash flow attenuation characteristics, as can be seen by referring to Figure 1. below. For example, a mobile home, purchased on credit would require the additional purchase of comprehensive and credit life insurance policies from the external insurance company. This would create a net cash inflow to the ABC Finance Company equal to 35% of the total premium on comprehensive insurance policies and 45% of the total premium on credit life insurance policies. In addition, the ABC Finance Company would have to reinsure the financed contract at 0.75% of the loan principal times the number of years for which the loan was financed. Another series of events affecting cash flow would result as the contract loan was repaid, or in certain instances was terminated in default. Obviously, the number of possible cash flow attenuation effects which could result from various combinations and interaction of the status and exogenous variables of this module is enormous. In total, there were twenty-six major assumptions, limitations, and constraints which were utilized to create the internal status and exogenous variables of the cash flow attenuation module. These factors are explained in detail in Appendix A.

Performance Criteria Module

The performance criteria module attempted to measure cash flow attenuation and its affects upon profitability. Five performance criteria (endogenous variables) were employed:

1. Return on Equity (ROE) = earnings after taxes divided by the equity at the beginning of the year.

2. Present Worth of the Common Stockholder (PWCOM) = the equity position of future years discounted back to 1970.

3. Future Net Cash Inflow (FCF) = the difference between unavoidable cash inflows and outflows.

4. Return on Assets (ROA) = earnings before

interest and taxes divided by average net assets.

5. Return on Operating Assets (ROOA) = income before interest and taxes earned on operating assets divided by average net operating assets.

Model Implementation

The basic model described above was programmed in FORTRAN IV, because of the obvious need for great programming flexibility, and because of the discrete nature of the problem being analyzed. It was run on an IBM 360/50, under a HASP operating system, and required approximately 125 K bytes of storage. Simulation runs were made for thirty sets of "starting conditions" (replicates), where a starting condition (e.g., 7/70/1.5) referred to a specific borrowing rate (i.e., 7%), a specific contract percentage (i.e., 70%), and a specific line of credit (i.e., $1.5 million). Running times (CPU) varied by starting conditions, from a low of 161 seconds to a high of 1,111 seconds and were for a simulated time horizon of five years. Refer to Table 1 for a summary of a sample of ten starting conditions and their associated running times.

Simulation Test Results

Table 2 presents a set of typical test results for three starting conditions. The results are stated in terms of the expected value, standard deviation, and coefficient of variation (expressed in terms of percent) for each of the endogenous variables, for each of the five years covered by the study. For example, the present worth of the common stockholder (PWCOM) distribution in year #3 for a borrowing rate of 7%, a contract percentage of 60%, and a credit line of $500,000 has a mean expected value of $22.2 million, a standard deviation of $1.716 million and a coefficient of variation of 8.10%. Note additionally that some starting conditions had a substantial probability for the occurrence of insolvency. The probability of bankruptcy was used to adjust the raw data, and adjusted data for the same three sets of starting conditions are shown in Table 3. Adjustment of the expected values was achieved by dividing the raw data by [1.0-P(Bankruptcy)]. The probability of bankruptcy was determined by pretesting the various starting conditions with the simulation model under a set of arbitrary demand patterns.

Efficiency Frontier Decision Rules

The expected values and coefficients of variability were next used to construct "gross" efficiency frontiers, by year, and by performance criterion. A typical gross efficiency frontier is shown in Figure 2. As can be seen from Figure 2, starting conditions 8/70/0.3, 8/50/0.3, and 7/60/0.5 lie on the gross efficiency frontier because each produces the maximum return on equity for a given degree of risk.

Gross efficiency frontiers were derived for both the raw data, and the data which had been adjusted by the probability of bankruptcy. Table 4 summarizes the starting conditions which lay on the gross efficiency frontiers, for each year, and each performance criterion, for the three starting conditions whose performance criteria were summarized previously in Tables 2 and 3.

A refinement to the "gross" efficiency frontiers described above was next made, by determining the first and second "floors" under the gross efficiency frontiers. The first floor was E - S, where E was the mean expected value and S was the standard deviation. Similarly, the second floor was E -2S. Data of this variety, for the three starting conditions shown previously in Table 4(b), is presented in Table 5.

The rationale underlying the need for this refinement can be explained by referring to Figure 3. Assume that starting conditions A and B both lie on the gross efficiency frontier for some particular year and performance criterion. Note that E_a is less than E_b which intuitively would lead the decision maker to prefer starting condition B to starting condition A. However, if he has no confidence in expecting the likelihood of E_a occurring any more than E_b, and is a risk averter, he would now prefer starting condition A to starting condition B, since $E_a - S_a$ is greater than $E_b - S_b$. The higher the value of K (number of standard deviations from E) that is chosen by the decision maker, the more conservative will be his decisions since he will be taking into account less likely possibilities of loss. The distributions of the various performance criteria cannot necessarily be assumed to be normally distributed. However, by use of Chebyshev's inequality any value of $K \geq 1$ can be given a probabilistic interpretation in terms of the likelihood that observations will fall beyond $E \pm KS$.

The efficiency floor concept was next applied to the adjusted performance criteria data. These results for a set of seven starting conditions are shown in Table 6. The seven starting conditions presented in Table 6 were those that produced the most meaningful and relevant results from the sample of thirty replicates tested in the study. Obviously, many more replicates could be tested.

Interpretation of Results

The interpretation of the results of the simulation analysis are somewhat apparent from an examination of the preceding Tables and Figures. However, these results must also be interpreted in terms of the basic issues of concern previously stated.

Issue 1 may be answered by directly considering the "Future Cash Flow" column of Table 6. Starting conditions 8/60/0.5 or 8/70/0.5 are preferred in year #1, depending on the amount of certainty that the decision maker requires, but starting condition 8/70/0.5 is decidedly the better choice in year #2. In year #3, the risk attitude of the decision maker will again determine whether 8/70/0.5 or 7/70/0.3 is preferred. In both year #4 and year #5, starting condition 7/70/0.3 is preferred. Viewing the first issue broadly, the answer to the question: "Which set of starting conditions is best?", is largely dependent upon the risk attitude and the time perspective of the decision maker. However, the value of the computer simulation approach to the resolution of this issue is apparent in that the decision maker has been provided the basic information that will enable him to subjectively make his decisions.

The next three issues can be evaluated in the same manner as was first issue. The second issue would be analyzed by considering the "Present Worth of the Common Stockholder" column in Table 6. The third issue would be analyzed by considering the "Return on Operating Assets" column in Table 6. The fourth issue would be analyzed by considering both the "Return on Equity" column and the "Return on Asset" column in Table 6. For the sake of brevity, these further analyses are left to the more interested reader.

The fifth issue, that of potential insolvency, can be answered by referring to Table 7. As evidenced from Table 7, there is a significant risk of insolvency for starting conditions 7/60/0.5 and 8/50/0.3.

The final issue entails consideration of all the factors mentioned in regard to deciding the previous issues. However, examination of Table 6 allows some insight into the relative ranking of the importance among the three factors of a starting condition. Note that most of the entries are towards the top of Table 6 for all years and all performance measures. Additionally, the entries for most of the criteria show a transition from condition 8/70/0.5 in the first two years to 7/70/0.3 in the last two years. This led to the conclusion that contract percentage was the factor which had the most impact on performance and that a larger credit line was more important than a lower borrowing rate in the initial two years. This ranking was then reversed in the later years in that lower borrowing rates became more important than larger credit lines.

Conclusion and Suggestion for Future Research

The use of computer simulation, as reported herein, offered the management of the mobile home retailing company a powerful tool for analyzing the risk consequences of various financial operating structures. Specifically, the simulation methodology entailed:

1. The simulation of risk profiles for a number of alternative operating structures.

2. The use of various performance criteria for evaluating the merits of the alternative operating structures, as evidenced by their risk profiles.

3. The establishment of a decision rule for choosing among the alternative operating structures, based on their respective performance criteria.

Its major benefit was that management, after deciding what level of risk and what time horizon it wished to examine, was able to effectively negotiate with the savings and loan institution to arrive at a set of starting conditions which would enable it to meet its corporate objective. The empirical results of the research attest to the soundness of the simulation methodology using risk profile analysis and efficiency frontiers. The pragmatic value of the results of study were primarily those accruing to management from being able to examine alternative operating structures at various levels of risk and over various time horizons.

Extensions of the present model are both obvious and numerous. As with any simulation model, more replicates can be tested, and further insights into the nature of the problem gained. Additionally, the large number of exogenous variables employed in the simulation model could be refined or expanded on the basis of further analysis.

Acknowledgement

The author of this paper wishes to acknowledge the contribution of two very capable graduate students at the University of Missouri - St. Louis, Mr. William Dey and Mr. Edward Brandt, in developing the simulation model which was the basis for this paper. The cooperating firm in the study has asked to remain anonymous.

Biographical Sketch

Robert E. Markland is an Associate Professor - Management Science at the University of Missouri (St. Louis). Dr. Markland has published articles in Financial Executive, Naval Research Logistics Quarterly, Production and Inventory Management, and several other academic journals. He serves as a consultant to the Ralston Purina Company, and to other profit and non-profit organizations. His current research interests are in large scale simulation modeling, mathematical programming, and in the application of operations research techniques to police science.

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Figure 1. BASIC STRUCTURE OF THE SIMULATION MODEL

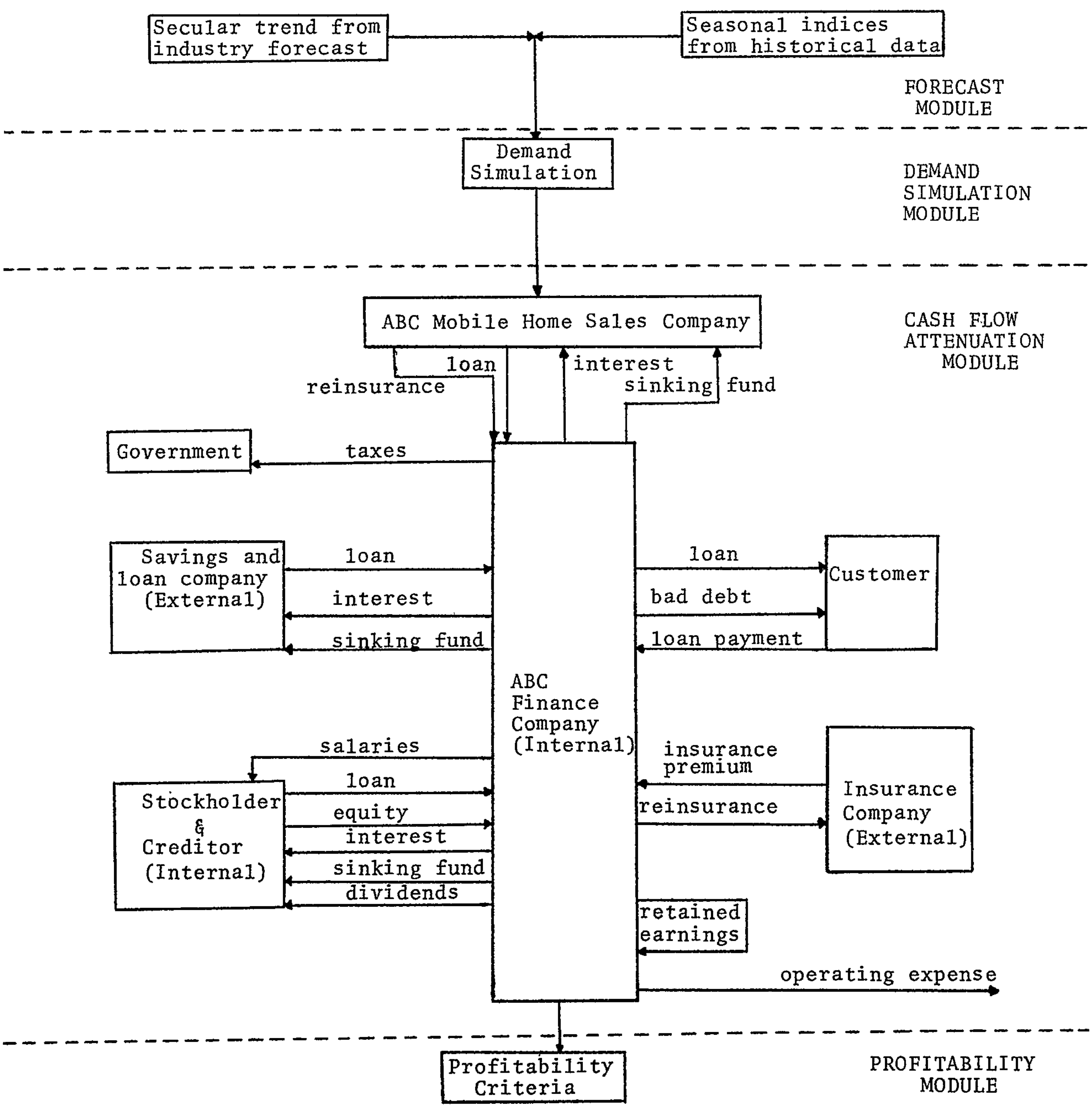

Figure 2. GROSS EFFICIENCY FRONTIER-ROE YEAR #1 (Unadjusted)

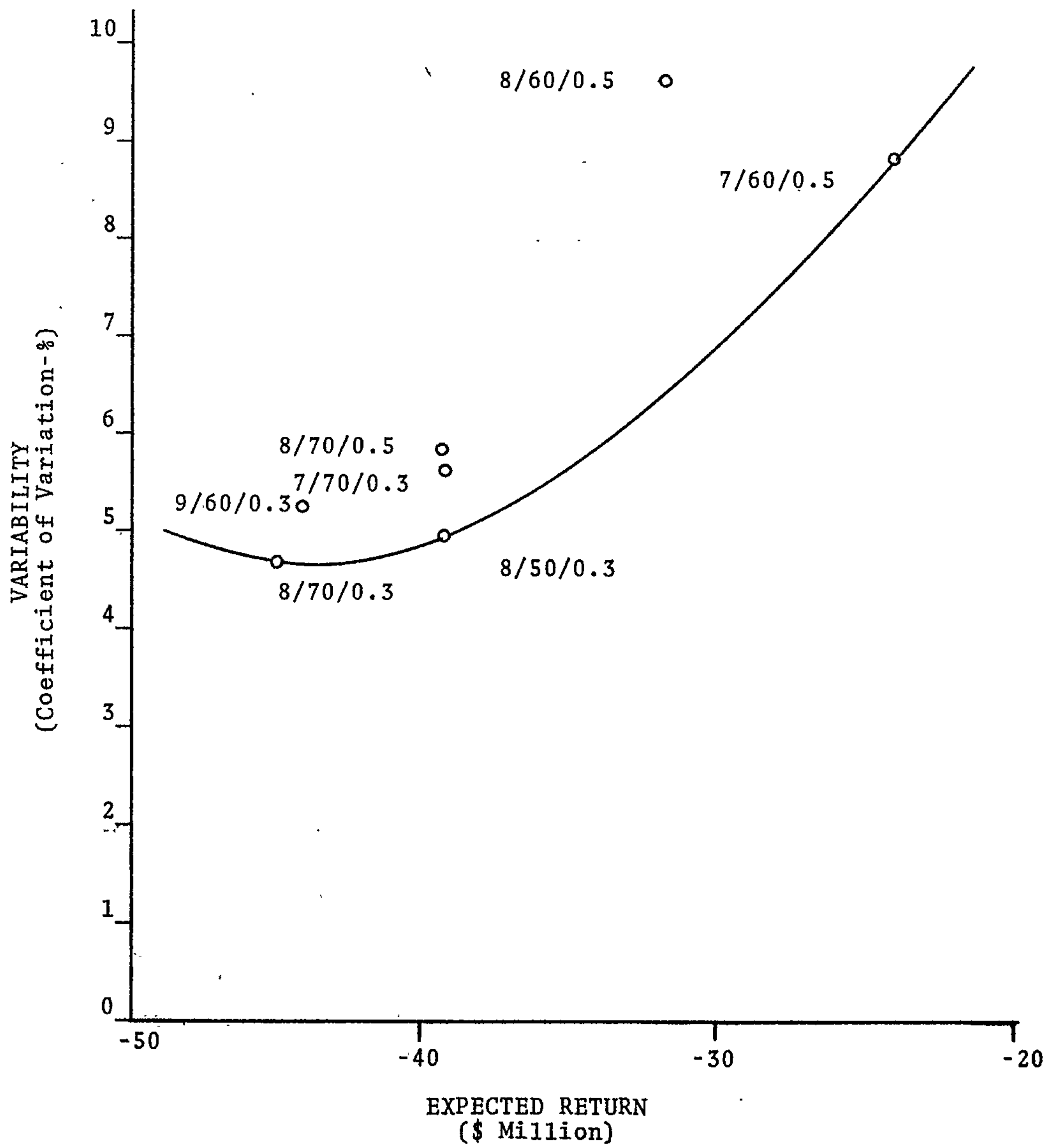

Figure 3. EXAMPLE OF THE EFFICIENCY FLOOR CONCEPT

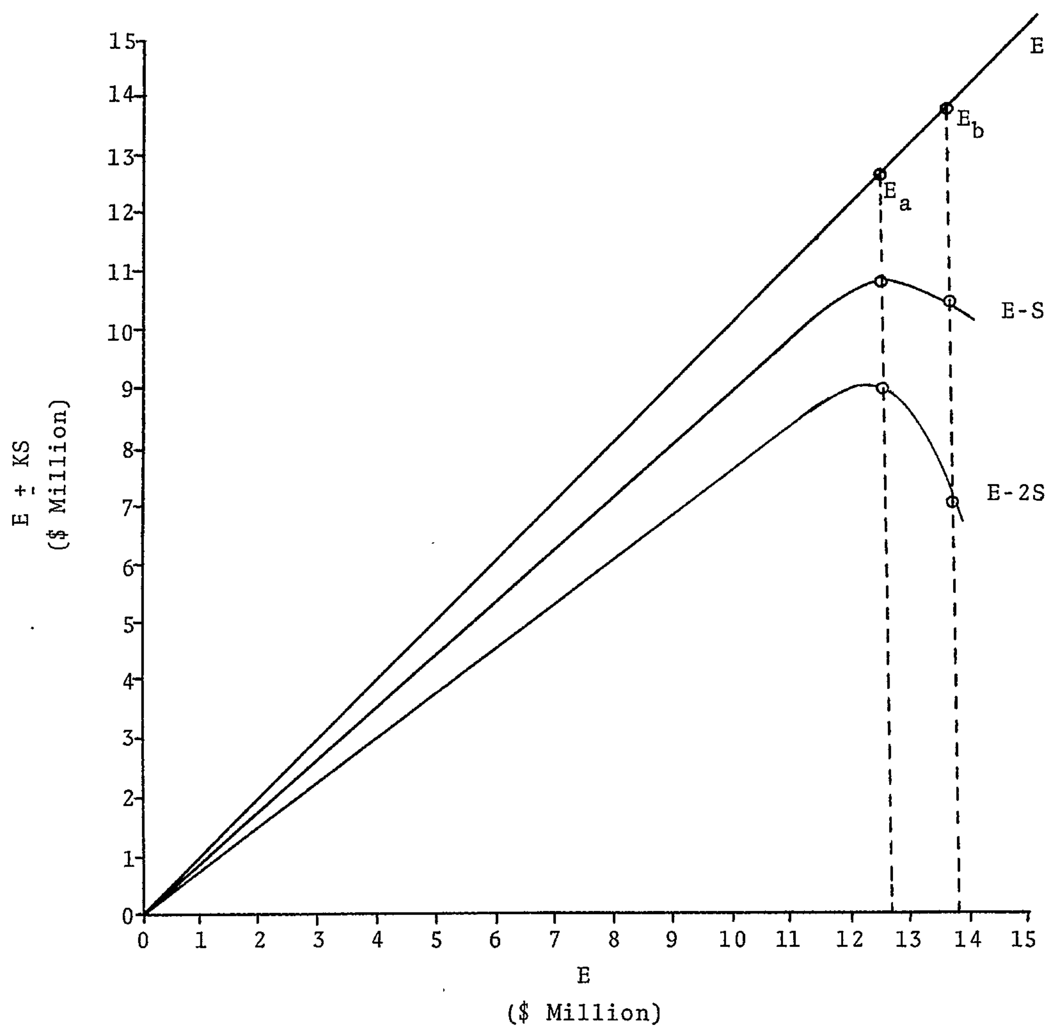

Table 1. SUMMARY OF STARTING CONDITIONS AND RUNNING TIMES

STARTING CONDITION	RUNNING TIME (CPU Seconds)	
7/60/0.5	211	
7/70/1.0	1,111	
7/70/0.3	193	Average Running Time = 328 CPU Seconds
8/50/0.3	371	
8/60/0.5	196	
8/70/0.5	241	
8/70/0.3	195	
9/50/0.3	161	
9/60/0.3	192	
9/70/1.5	409	

Table 2. PERFORMANCE CRITERIA SUMMARY (UNADJUSTED)

STARTING CONDITION		YEAR #1 ROE %	YEAR #1 PWCOM $ Mill.	YEAR #1 FCF $ Mill.	YEAR #1 ROA %	YEAR #1 ROOA %	YEAR #2 ROE %	YEAR #2 PWCOM $ Mill.	YEAR #2 FCF $ Mill.	YEAR #2 ROA %	YEAR #2 ROOA %
7/60/0.5	MEAN	-23.5	-30.8	106	4.3	0.7	44.2	5.0	164	8.4	4.7
	STD DEV	2.056	3.465	5.8	0.447	0.486	4.152	2.419	8.6	0.292	0.386
	CVAR	8.74	11.24	5.44	10.23	66.72	9.39	48.29	5.24	3.45	8.07
8/60/0.5	MEAN	-30.9	-45.1	91	4.4	0.7	29.7	-5.9	136	8.4	4.6
	STD DEV	2.998	6.601	7.2	0.751	0.755	7.058	5.188	10.2	0.298	0.397
	CVAR	9.68	14.62	7.98	17.00	106.76	23.72	87.81	7.55	3.54	8.48
9/60/0.3	MEAN	-42.6	-74.5	53	2.8	-1.9	5.4	-29.0	99	8.9	4.6
	STD DEV	2.221	6.882	1.7	0.605	0.351	8.444	7.123	3.5	0.518	0.437
	CVAR	5.21	9.24	3.21	21.61	18.47	156.37	24.52	3.54	5.76	9.50

STARTING CONDITION		YEAR #3 ROE %	YEAR #3 PWCOM $ Mill.	YEAR #3 FCF $ Mill.	YEAR #3 ROA %	YEAR #3 ROOA %	YEAR #4 ROE %	YEAR #4 PWCOM $ Mill.	YEAR #4 FCF $ Mill.	YEAR #4 ROA %	YEAR #4 ROOA %
7/60/0.5	MEAN	61.1	21.1	22.2	10.3	6.3	53.5	28.6	280	10.6	6.9
	STD DEV	2.122	1.716	11.9	0.183	0.209	1.711	1.157	13.9	0.088	0.151
	CVAR	3.47	8.10	5.37	1.78	3.30	3.19	4.04	4.97	0.83	2.17
8/60/0.5	MEAN	66.7	14.2	184	10.2	6.2	53.2	23.0	230	10.7	6.9
	STD DEV	6.898	2.265	11.1	0.207	0.246	2.475	1.487	12.7	0.162	0.143
	CVAR	10.33	15.89	6.04	2.01	3.92	4.65	6.46	5.54	1.51	2.06
9/60/0.3	MEAN	81.9	3.1	140	10.5	6.1	57.7	14.7	178	10.9	6.8
	STD DEV	5.970	3.046	7.1	0.274	0.323	4.537	1.986	8.6	0.138	0.179
	CVAR	7.29	97.98	5.07	2.61	5.30	7.86	13.16	4.83	1.27	2.63

STARTING CONDITION		YEAR #5 ROE %	YEAR #5 PWCOM $ Mill.	YEAR #5 FCF $ Mill.	YEAR #5 ROA %	YEAR #5 ROOA %	PROBABILITY OF BANKRUPTCY
7/60/0.5	MEAN	44.6	31.7	352	10.7	7.4	0.5333
	STD DEV	1.992	0.787	15.7	0.116	0.147	
	CVAR	4.46	2.48	4.46	1.08	1.98	
8/60/0.5	MEAN	43.4	26.8	278	10.7	7.2	0.0333
	STD DEV	1.708	1.101	12.8	0.126	0.112	
	CVAR	3.93	4.09	4.58	1.17	1.53	
9/60/0.3	MEAN	45.5	20.3	212	10.9	7.2	0.0000
	STD DEV	2.631	1.258	10.8	0.123	0.115	
	CVAR	5.78	6.19	5.09	1.13	1.60	

Table 3. PERFORMANCE CRITERIA SUMMARY (ADJUSTED)

STARTING CONDITION		YEAR #1 ROE %	YEAR #1 PWCOM $ Mill.	YEAR #1 FCF $ Mill.	YEAR #1 ROA %	YEAR #1 ROOA %	YEAR #2 ROE %	YEAR #2 PWCOM $ Mill.	YEAR #2 FCF $ Mill.	YEAR #2 ROA %	YEAR #2 ROOA %
7/60/0.5	MEAN	-11.0	-14.4	50	2.0	0.3	20.6	2.3	77	3.9	2.2
	STD DEV	11.81	15.56	53.	2.200	0.490	22.23	2.995	82.	4.221	2.398
	CVAR	107.4	108.2	107.	110.0	163.3	107.9	128.0	107.	108.2	109.0
8/60/0.5	MEAN	-29.9	-43.6	88	4.3	0.7	28.8	-5.7	132	8.1	4.5
	STD DEV	6.293	10.38	18.	1.088	0.752	8.752	5.209	26.5	1.533	0.929
	CVAR	21.05	23.79	20.2	25.30	107.43	30.39	91.23	20.1	18.93	20.64
9/60/0.3	MEAN	-42.6	-74.5	53	2.8	-1.9	5.4	-29.0	99	8.9	4.6
	STD DEV	2.221	6.882	1.7	0.605	0.351	8.444	7.123	3.5	0.513	0.437
	CVAR	5.21	9.24	3.21	21.61	18.47	156.37	24.52	3.54	5.76	9.50

STARTING CONDITION		YEAR #3 ROE %	YEAR #3 PWCOM $ Mill.	YEAR #3 FCF $ Mill.	YEAR #3 ROA %	YEAR #3 ROOA %	YEAR #4 ROE %	YEAR #4 PWCOM $ Mill.	YEAR #4 FCF $ Mill.	YEAR #4 ROA %	YEAR #4 ROOA %
7/60/0.5	MEAN	28.5	9.9	104	4.8	3.0	25.0	13.4	131	5.0	3.2
	STD DEV	30.55	10.63	111	5.136	3.163	26.73	14.29	140.0	5.299	3.456
	CVAR	107.2	107.6	107.	107.0	105.4	106.9	107.0	106.9	106.0	108.0
8/60/0.5	MEAN	64.5	13.8	178	9.9	6.0	51.4	22.2	223	10.3	6.7
	STD DEV	13.77	3.393	34.9	1.854	1.151	9.854	4.380	43.2	1.927	1.247
	CVAR	21.35	24.62	19.61	18.73	19.17	19.17	19.70	19.37	18.71	18.61
9/60/0.3	MEAN	81.9	3.1	140	10.5	6.1	57.7	14.7	178	10.9	6.8
	STD DEV	5.070	2.046	7.1	0.274	0.323	4.537	1.936	8.6	0.138	0.179
	CVAR	7.29	97.98	5.07	2.61	5.30	7.86	13.16	4.83	1.27	2.63

STARTING CONDITION		YEAR #5 ROE %	YEAR #5 PWCOM $ Mill.	YEAR #5 FCF $ Mill.	YEAR #5 ROA %	YEAR #5 ROOA %
7/60/0.5	MEAN	20.8	14.8	165	5.0	3.5
	STD DEV	22.32	15.82	176.0	5.349	3.706
	CVAR	107.3	107.0	132.1	107.0	105.9
8/60/0.5	MEAN	42.0	26.0	270	10.4	7.0
	STD DEV	7.980	4.945	51.6	1.929	1.312
	CVAR	19.00	19.04	19.11	18.55	18.74
9/60/0.3	MEAN	45.5	20.3	212	10.9	7.2
	STD DEV	2.631	1.258	10.8	0.123	0.115
	CVAR	5.78	6.19	5.09	1.13	1.60

Table 4. STARTING CONDITIONS ON GROSS EFFICIENCY FRONTIERS

a.) Unadjusted Data

STARTING CONDITION	ROE					PWCOM					FCF					ROA					ROOA				
	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5
7/60/0.5	X	X	X	X		X	X	X	X	X	X	X	X	X	X		X	X	X						
8/60/0.5																									
9/60/0.3			X																X						

b.) Adjusted Data

STARTING CONDITION	ROE					PWCOM					FCF					ROA					ROOA				
	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5
7/60/0.5	X					X	X																		
8/60/0.5	X	X				X	X	X			X	X													
9/60/0.3	X		X			X	X												X						

KEY: Y1 - Year One

X - Indicates that starting condition lies on the gross efficiency frontier for the particular year and performance criterion.

Table 5. FIRST AND SECOND FLOORS UNDER THE GROSS EFFICIENCY SET

(Adjusted Data)

STARTING CONDITION	FLOOR	ROE Y1	ROE Y2	ROE Y3	ROE Y4	ROE Y5	PWCOM Y1	PWCOM Y2	PWCOM Y3	PWCOM Y4	PWCOM Y5
7/60/0.5	E	-11.0					-14.4	2.3			
	E-S	-22.8					-30.0	-0.7			
	E-2S	-34.6					-45.5	-3.7			
8/60/0.5	E	-29.9	28.8				-43.6	-5.7	13.8		
	E-S	-36.2	20.1				-54.0	-10.9	10.4		
	E-2S	-42.5	11.3				-64.4	-16.1	7.0		
9/60/0.3	E	-42.6		81.9							
	E-S	-44.8		75.9							
	E-2S	-47.0		70.0							

STARTING CONDITION	FLOOR	FCF Y1	FCF Y2	FCF Y3	FCF Y4	FCF Y5	ROA Y1	ROA Y2	ROA Y3	ROA Y4	ROA Y5
7/60/0.5	E										
	E-S										
	E-2S										
8/60/0.5	E	88	132								
	E-S	70	106								
	E-2S	52	79								
9/60/0.3	E									10.9	
	E-S									10.8	
	E-2S									10.6	

STARTING CONDITION	FLOOR	ROOA Y1	ROOA Y2	ROOA Y3	ROOA Y4	ROOA Y5
7/60/0.5	E					
	E-S					
	E-2S					
8/60/0.5	E					
	E-S					
	E-2S					
9/60/0.3	E					
	E-S					
	E-2S					

Table 6. STARTING CONDITIONS ON REFINED EFFICIENCY FRONTIERS
(Adjusted Data)

STARTING CONDITION	ROE					PWCOM					FCF					ROA					ROOA				
	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5
7/70/0.3		2		2				C	A	A			2	A	A			A		A			2	A	A
8/70/0.5											2	A	1			A	A				A	A	1		
8/70/0.3			A	1	A														A						
7/60/0.5	A					A	A																		
8/60/0.5		1						B			1														
9/60/0.3																									
8/50/0.3																									

KEY:

A - preferred for all values of K
B - preferred for K = 0
C - preferred for K > 0
1 - preferred for K ≤ 1
2 - preferred for K ≥ 2

Table 7. PROBABILITY OF BANKRUPTCY BY STARTING CONDITIONS

STARTING CONDITION	PROBABILITY OF* BANKRUPTCY
7/60/0.5	0.5333
8/50/0.3	0.6000
8/60/0.5	0.0333

*All other starting conditions had a P(Bankruptcy)=0.0000

Appendix A. Status and Exogenous Variables of the Cash Flow Attenuation Module

1.) The capital structure restriction for the ABC Finance Company was 90% debt. Equity was defined as the sum of the capital stock, retained earnings, and deferred insurance commission accounts.

2.) No state income tax or licensing fees were included in the costs of the ABC Finance Company. Moreover, the state sales tax and the Federal corporate tax structure were assumed to be unchanged over the period of the forecast.

3.) There was no distribution of earned surplus of the ABC Finance Company at any time. Instead, all earnings were "plowed back" into the ABC Finance Company operations.

4.) Each set of starting conditions (borrowing rate, contract percentage, and credit line) remained constant over the forecast period.

5.) All installment contracts between the ABC Finance Company and the consumer, and between the ABC Finance Company and the savings and loan company were 84 months in length. The loan was obtained from the savings and loan company instantaneously, so that the due datesfor the first payment on both loans were on the same day.

6.) The lending rate for the ABC Finance Company on consumer loans remained constant at 7% add-on over the forecast period.

7.) All units which were purchased outright and whose purchasers desired insurance, generated a one year comprehensive insurance policy.

8.) No provisions were made to allow for the occurrence of early retirement of the entire contract on the part of the consumer or for late payments. All payments were assumed to be received and paid when they were due.

9.) All cash transactions occurred instantaneously at the end of every month. All cash payments updates occurred prior to calculation of the cash reserves necessary to finance the next month's sales requirements.

10.) All units which were financed by the consumer required the purchase of 3 years of comprehensive insurance and 7 years of credit life insurance. Even if the ABC Finance Company was unable to make the installment loans and an alternative financial source was utilized, the insurance was still written by ABC Finance Company.

11.) The total price for any particular mobile home was approximately equal to the invoice price with state sales tax added, plus the comprehensive insurance premium, which was approximately equal to the down payment that was required by the ABC Sales Company. Credit life insurance premiums were not part of the price structure and were added at the time of purchase. The premium was $0.60/$100 for each year of the contract.

12.) Both the comprehensive and credit life insurance premium schedules remained unchanged over the forecast period.

13.) The basic invoice price of each of the product categories remained unchanged over the forecast period.

14.) There was always a two month lag between the time that the consumer in a default status ceased to make his payments and the time when the ABC Finance Company recognized that a default had occurred.

15.) The commission percentages allowed the ABC Finance Company for acting as the agent for the insurance company were 35% of total premium on comprehensive insurance policies and 45% of total premium on credit life insurance policies. These percentages remained constant over the forecast period.

16.) The ABC Finance Company was required to reinsure every contract that it borrowed against at the savings and loan company. The reinsurance charge was 0.75% of the loan principal times the number of years for which the loan was financed. This amount was remitted to the external insurance company at the time that the loan was negotiated and there were no rebates allowed. The charge remained constant over the forecast period.

17.) If a consumer contract was terminated via default, the matching loan from the savings and loan company was not prepaid. Rather, the ABC Finance Company continued to make the regular monthly payments and terminated its loan in the normal manner.

18.) Salaries of all personnel employed by the ABC Finance Company were considered to be constant and unavoidable. The monthly salary cost was $3,000.

19.) Monthly operating expenses were a constant percentage of the month ending balance in the Accounts Receivable account. The percentage was 0.0833%.

20.) Old Age and Survivors Insurance payments were 5.2% of the first $7,800 of the annual salary. Rate base and percentage were constant over the forecast period.

21.) The total of monthly payments from the ABC Finance Company to the savings and loan company were added back to the available credit line so that the funds could be borrowed again at some time in the future. Additionally, at the end of each year it was assumed that the savings and loan company expanded the credit line by the same increment that was originally agreed upon prior to the start of business.

22.) The minimum cash operating balance was $5,000 or 0.33% times the amount of Accounts Receivable at the end of any quarter of operation, whichever was higher.

23.) The cash balance was monitored only on a spot basis at the end of each month's transactions.

24.) If a consumer defaulted on his contract, the ABC Finance Company had to rebate the unearned comprehensive and credit life insurance commission. The rebate was determined by multiplying the total commission by the ratio of the remaining months in the contract to the total contract length. At the same time, the insurance company paid the balance of the unpaid principal carried on the books at the time of default.

25.) The cash balance was allowed to remain idle unless it exceeded $100,000 at any time. If an excess above this amount existed, it was used to finance a consumer loan and there was no equivalent borrowing from the savings and loan company.

26.) If the minimum cash balance constraint, capital structure constraint, or credit line constraint was violated, the ABC Finance Company was not allowed to finance any loans until the violative posture was corrected. The company was allowed however, to continue to act in the capacity of insurance agent on all sales.