

A Simulation Model for
Job Shop Scheduling

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ABSTRACT

Production scheduling is concerned with the allocation of resources and the sequencing of tasks. Sequencing problems, except for special cases, are very difficult to solve analytically. Consequently, heuristics are used frequently to solve this problem.

A popular class of heuristics is referred to as dispatching rules. A dispatching rule is a discipline by which jobs are assigned priorities at different work stations. Competing rules in a multi-stage, multi-job problem are generally evaluated on the basis of their performance in simulation tests.

The purpose of this paper is to present an analysis of dispatching rules using a job shop simulation model. The analysis involves 20 different dispatching rules in a 9-machine shop, for 4 sets of 10000 jobs.

1. INTRODUCTION

Production scheduling is concerned with the allocation of resources and the sequencing of tasks to produce goods and services. Although allocation and sequencing decisions are closely related, it is very difficult to model mathematically the interaction between them. However, by using a hierarchical approach, the allocation and the sequencing problems can be solved separately. The allocation problem is solved first and its results are supplied as inputs to the sequencing problem.

The resource allocation problem can sometimes be solved using aggregate production planning techniques. To specify completely the input to the sequencing problem, the resulting detailed or item plan (also referred to as the master schedule) has to be disaggregated. A breakdown by component parts can be obtained in a straightforward way by using Material Requirements Planning (MRP) systems. Although MRP continues to be popular in practice, many issues still need to be resolved to make it an effective production planning tool. (For reviews of aggregate production planning and materials requirement planning, see Bitran and Hax [5] and Smith [36] respectively).

The sequencing problem, except for special cases, falls into a category of combinatorially-difficult problems known as NP-complete (Garey and Johnson [15]). Consequently, there has been a concentration on the use of heuristics to solve this problem. A class of heuristics that has been found to work well and is in popular use, is referred to as dispatching rules. A dispatching rule is a discipline by which jobs are assigned priorities at individual work stations. The job with the "highest" priority is always processed first. The priorities of the

remaining jobs in the queue may change over time as other jobs enter the queue.

There is a wide variety of dispatching rules. These are based on information about due-dates, processing times, status of jobs and status of queues. Depending on the information used, dispatching rules can be classified as local or global, simple or composite and, static or dynamic. For an extensive survey of dispatching rules, see Panwalker and Iskander [33].

Competing dispatching rules are evaluated on the basis of their performance in simulation tests. Most of these tests have been conducted in the context of a manufacturing job shop. Although there have been many simulation studies, most research has concentrated on jobs which only require fabrication. Some research has examined assembly operations. However, a lot of work remains to be done involving fabrication and assembly jobs for which MRP systems are appropriate.

The purpose of this paper is to present an analysis of dispatching rules using a job shop simulation model. Section 2 presents a classification of job shops and dispatching rules and a review of job shop simulation research. Section 3 contains a description of the general simulation model. Section 4 reports on the results of the simulation runs involving 20 different dispatching rules in a 9-machine job shop for 4 sets of 10000 jobs which do not require assembly. Finally, section 5 presents concluding remarks.

2. A REVIEW OF JOB SHOP SIMULATION RESEARCH

The first section explained that dispatching rules are used to solve sequencing problems. The purpose of this section is to present a survey of the research literature on job shops. The survey focuses on simulation models for multi-stage job shops. As an introduction to the survey, the section begins with a discussion of job shops, dispatching rules, and the simulation methodology.

2.1. Job Shops

In both theory and practice, much of the work on sequencing problems has been related to manufacturing. Consequently, the job shop has become a favorite theoretical construct to study the various components and interactions of complex sequencing problems.

2.1.1. Components of a Job Shop

A job shop model may include the following components:

- a. operations - elemental tasks.
- b. jobs - one or more related operations that comprise a basic task module.
- c. events - occurrences corresponding to the movement of jobs in the shop.
- d. machines - the facilities which perform the operations
- e. workers - the resources which operate the machines
- f. queues - the sets of jobs waiting at machines.
- g. routes - lists of the order and the corresponding machines in which the operations of a job have to be performed.
- h. bill of materials - lists of parts and their quantities that

are required for different operations of a job.

i. time

arrival time - the time at which the job is ready for processing at the shop

processing time - the time it takes for a machine to perform an operation for a particular job.

due-date - the time by which the job is supposed to be finished.

j. dispatching rules - the methods that specify how machine operators choose which job in their queue to process next.

k. schedule - the order in which the jobs are processed by the machines.

l. performance measures - the criteria by which the schedule is evaluated.

2.1.2. Classification of Job Shops

Sequencing problems have been studied in a variety of job shop settings. Since conditions vary, different solution approaches have been required. The following classification illustrates the diversity of settings and common solution approaches. For a broad classification of various scheduling problems and a review of important theoretical developments of the different classes of problems, see Graves [19].

Job shops can be defined in the following terms:

a. The time environment.

The time environment of the shop can be deterministic or stochastic. In a deterministic shop, all times (i.e. arrival,

processing, due-date) are known and fixed. In a stochastic shop, any of the times may be random variables with a specified probability distribution.

b. The job arrival process.

In a static shop, all the jobs arrive simultaneously and thus are ready for processing at the same time. On the other hand, in a dynamic shop, jobs arrive at different times.

In the static case, since all jobs are completely known and available, a fixed schedule can be made. Graphical, enumerative or mathematical programming methods are commonly used. However, in the dynamic case, the schedule can change whenever a new job arrives. Thus, heuristic methods are preferred for scheduling a dynamic shop.

c. The machine configuration.

The shop can have one to several different machines. The number of distinct machines describes an n-stage shop, where n is the number of distinct machines. There can also be machines with identical functions and which are grouped into machine centers. This configuration is termed parallel machines.

Shop configurations with one or two distinct machines can be studied with algebraic and probabilistic methods. Optimum schedules and dispatching rules have been found for this class of problems for some performance measures [2].

Optimal solutions have been found for some highly restrictive problems involving three machines. In general, however, for shops with more than two distinct machines, analytical methods have failed to find optimal solutions. This is due to the computational complexity of such problems. For an example of computational complexity, see Conway, et

al., p. 95 [12]. As a result, these configurations have been studied using simulation methods which use dispatching rules. The rules have been found to vary in performance relative to one another, depending on job and shop characteristics and on performance criteria.

d. The operation flow process.

If, at one extreme, all jobs follow the same route through the machines, the shop is termed as a pure flow shop. If, at the other extreme, each job has a unique route, the shop is termed as a pure job shop. In the pure flow shop, most solution approaches involve permutation schedules and heuristic procedures. In the pure job shop, branch and bound procedures, enumeration and sampling methods for static and deterministic conditions, and simulation experiments with dispatching rules are used [2].

If each job may have only one of a fixed set of routings (which implies a fixed line of products), the shop is called a closed job shop. On the other hand, if a job may have any arbitrary route, the shop is termed an open job shop. Open shops are concerned with sequencing jobs through the machines and use similar solution approaches as pure job shop problems. Closed shops involve the additional problem of lot-sizing because jobs are generated by inventory replenishment decisions. In addition to the approaches used in open shop problems, closed shop problems also use lot-sizing methods.

2.2. Dispatching Rules

One of the earliest and most extensive studies on dispatching rules was done by Conway [7, 8, 9]. The 92 dispatching rules (some which change in relative weights) in his RAND study [7] are classified on the

basis of information requirements into local rules and global rules. Local rules only require information on the jobs waiting at a machine, while global rules require additional information about jobs or machines in other parts of the shop. The classification appears in table 2.1.

TABLE 2.1

Conway's Classification of Priority Rules

a. Local, operation: Rules based on the attributes of imminent operation of jobs in a particular queue. Attributes of the imminent operation include processing time, arrival time in queue and due-date.

b. Local, job: Rules based on the attributes of the jobs in a particular queue. Attributes of the job include arrival time in shop, due-date, total or remaining number of operations, and total or remaining sum of processing times.

c. Global, current status: Rules based on current value of attributes of any queues or machines and of any jobs in the shop. Attributes include number of jobs in next queue and sum of imminent processing time in next queue.

d. Global, predicted status: Rules based on predicted values of attributes of any queues or machines and of any jobs in the shop. Attributes include expected additions to current queues.

Another way to classify the rules used in Conway's study are as simple rules and composite rules. Simple rules are based on only one kind of information (local or global) while composite rules are combinations of simple rules. These combinations appear as sums, products, ratios, or differences of different rules or weighted versions of them.

Others have attempted to classify rules in different ways. Gere [16], closely following the simple-composite dichotomy, categorizes rules as dispatching rules, heuristics and scheduling rules.

Dispatching rules are techniques by which a value is assigned to each waiting job and the job with the minimum value is selected. Heuristics are some "rule of thumb" while a scheduling rule is a combination of one or more priority rules and/or one or more heuristics. Jackson [23] distinguishes between static rules and dynamic rules. Static rules are those in which job priority values do not change with time while dynamic rules are the juxtaposition of static rules. Finally, Moore and Wilson [27] have combined Conway's local-global classification with Jackson's static-dynamic classification into a two-dimensional classification.

2.3. The Simulation Methodology

Dispatching rules in multi-stage open job shops are evaluated using simulation. A brief overview of the simulation methodology explains its appropriateness for this class of sequencing problems.

Simulation is a strategy evaluation technique which uses an abstract representation of reality (i.e. a model) and studies its behavior through time. The behavior may be influenced by certain or uncertain factors. For models which consider uncertainties, the technique involves the following steps:

- a. Describe the system to be studied.
- b. Formulate simplifying assumptions about the system.
- c. Under the set of assumptions, identify:
 - c.1. Parameters. System attributes which are held constant during the period that the system is being studied.
 - c.2. Exogenous variables. System attributes which are subject to random variations through time. The variations are represented by appropriate probability

distributions.

c.3. Endogenous variables. System attributes whose values are generated by changes in the exogenous variables.

d. Develop a model which embodies the interrelationships among the parameters, the exogenous and the endogenous variables.

e. Use a random number generator to generate a set of intertemporal events based on the random variation of the exogenous variables.

f. Run the model.

g. Collect statistics on the resulting values of the endogenous variables.

h. Analyze results with descriptive and inferential statistical methods.

i. Draw conclusions and/or propose new sets of model attributes to observe and analyze.

Simulation is an alternative for problems where analytical solutions are practically impossible to compute. Gonzalez and Macmillan [18] explain the use of simulation in a succinct manner. They say:

Alternatives to the use of simulation are mathematical analysis, experimentation with either the actual system or a prototype of the actual system, or reliance upon experience and intuition. All, including simulation, have limitations. Mathematical analysis of complex systems is very often impossible; Experimentation with actual or pilot systems is costly and time consuming, and relevant variables are not always subject to control. Intuition and experience are often the only alternatives to (computer) simulation available but can be very inadequate.

Simulation problems are characterized by being mathematically intractable and having resisted solution by analytical methods. The problems usually involve many variables, many parameters, functions which are not well-behaved mathematically, and random variables. Thus simulation is a technique of last resort. Simulation applies to the design and analysis of system behavior.

For systems design, alternative designs can be generated by using

different sets of parameters and exogenous variables, and their results compared to some norm. In systems analysis, the object of study is how real transformations take place. Simulation involves developing a model which embodies the hypothesis for the transformation, supplying the model with real data and comparing the model's results with real outcomes to verify the model's hypothesis.

Since simulation may involve random variables, a large number of inputs and several similar runs are required to establish a statistical basis for the results. Likewise, simulation models may involve many mathematical expressions that define the relationships of various system variables. Consequently, simulation involves a lot of computation, and only with the advent of the computer, has it become a practical method. Today, simulation models are almost always developed as computer programs.

2.4. Multi-stage Job Shop Simulation Models

The previous sections have reinforced the notion that mathematical approaches are limited to the study of shops with two, perhaps three, machines. In larger shop configurations, simulation is the only practical alternative approach. The discussion will now focus on past research in multi-stage job shop scheduling using simulation models, progressing from simple to more complicated models.

2.4.1. Initial Assumptions

Most of the simulation models which are included in this survey have been based on or have extended from the following initial assumptions.

a. Shop.

a.1 The shop has only one limiting resource -- its machines.

b. Machines.

b.1 Each machine is continuously available for assignment without significant division of the time scale into shifts or days and without consideration of temporary unavailability for causes such as breakdown or maintenance.

b.2 There is only one machine of each type in the shop.

b.3 Each machine can handle at most one operation at a time.

c. Jobs.

c.1 Jobs are strictly ordered sequences of operations, without assembly or partition.

c.2 Jobs arrive in a random manner derived from an exponential distribution.

c.3 There is no splitting or combination of jobs.

c.4 The job scrap rate is zero.

d. Operations.

d.1 Each operation can be performed by only one machine in the shop.

d.2 An operation may not begin until its predecessors are complete.

d.3 Preemption is not allowed -- once an operation is started on a machine, it must be completed before another operation can begin on that machine.

d.4 The processing times of successive operations of a particular job may not be overlapped. A job can be in-process on at most one operation at a time.

d.5 Set-up and processing times are randomly generated from exponential distributions and are sequence independent.

d.6 Transit time between machines is zero.

d.7 Processing time as well as due-dates are known upon arrival in the shop.

Simulation research in job shop scheduling focuses on developing effective dispatching rules for given operating conditions. The conditions studied have been varied, as the following discussion suggests.

2.4.2. Machine-Limited Systems

Most research has assumed machine limited job shops. This means that labor is assumed to be always available and so, waiting time occurs only when a machine is busy processing another job.

The prominent managerial concerns of a job shop are minimizing shop congestion and meeting due-dates. A common measure of shop congestion is the mean flowtime [2]. Flowtime is defined as the difference between the finishing or completion time of a job and its arrival time. Another measure of shop congestion is mean lateness. Lateness is defined as the difference between the completion time of a job and its due-date. Common measures for meeting due-dates are percentage of jobs tardy and mean job tardiness. Tardiness is a derivative of lateness. It is defined as positive lateness, or the difference between the completion time and the due-date of a job, whenever the former exceeds the latter. A more rigorous mathematical definition of the measures is presented in section 3.

Many studies have been concerned with finding rules that minimize

mean flowtime and mean job tardiness. For the one-machine problem, Smith [37] has shown that sequencing jobs in order of nondecreasing processing time minimizes mean flowtime. This rule, which is also called the shortest processing time (SPT) rule, has been shown by Conway [8] to be among the best performing rules when minimizing the mean flowtime in a machine-limited job shop is the objective.

Conway [9] has also shown that the SPT rule is among the best rules that minimize mean job tardiness. However, by its nature, the SPT rule favors jobs whose tasks have short processing times, and postpones jobs with longer processing times. As a result, jobs with longer processing times tend to be tardy. Consequently, the SPT rule suffers from a high job tardiness variance compared to such benchmark rules as the first-come-first-served (FCFS). Among the rules which use due-date information, Conway found that the shortest slack per operation (SOPN) rule exhibits one of the lowest mean and variance tardiness measures. Slack is defined as the amount of time remaining before the job becomes due less the time required to complete the job processing. Operations refer to the remaining operations.

Researchers have been concerned with whether or not additional information significantly improves the performance of the dispatching rules. Findings have been mixed. In their study of critical ratio rules for shops coordinated with inventory systems, Berry and Rao [4] have found that more information does not significantly increase scheduling performance. In fact, their dynamic rules, which involve changes in due-dates corresponding to inventory updates, caused a significant reduction in shop performance. They attribute this unexpected result to the transfer of substantial uncertainty in the

inventory usage to the shop and to the heavy workload in the shop. The heavy workload prevents allocation of spare resources to newly-urgent jobs.

On the other hand, Maxwell [26], Maxwell and Mehra [27], Hausman and Scudder [20] and Baker and Bertrand [3] have shown that composite and dynamic rules perform better than simple and static rules. For example, Baker and Bertrand have developed a modified due-date rule which is the larger of the job's due-date or its early completion time. Results show that this composite and dynamic rule performed better than its static components. Findings such as this demonstrate the synergistic effect of some simple rules that produces a superior composite rule.

Researchers have also been concerned with the stability of the performance of various rules under different shop utilization levels. In Conway's experiments [7], a few composite rules, such as the SOPN outperformed the SPT rule in some of the runs. This erratic behavior shows that dispatching rules are sensitive to shifts in machine and shop utilization and that the SPT rule has a robust behavior. When machine utilization was balanced and shop utilization was a bit lower (88.8 percent), some compound rules showed better results than the SPT rule. However, with imbalance in the queues and slightly higher (91.9 percent) workload, the SPT rule gave better results.

The Conway study [7] raises several important points in job shop research. First, there is a disparity in the performance of different priority rules, and, some rules clearly outperform others. Specifically, formal rules outperform the "less formal" benchmark rules -- random selection and first-come first-served. This disparity

provides the rationale for developing and using formal rules to improve scheduling.

Second, among the better rules, it is extremely difficult (if not impossible) to establish a universally dominant rule. Rules are sensitive to changes in machine and shop conditions. And, since there are many variations in operating conditions, evaluating the performance of rules becomes an empirical matter.

Third, the SPT rule is an amazingly powerful rule in minimizing both mean flowtime and mean tardiness. Its attractiveness is enhanced by its simple information requirements and robust behavior under operational diversity. Given its simplicity and versatility, the SPT rule appears to be the rule to beat.

Fourth, caution must be exercised in interpreting results even for experiments with a large number of jobs. The Conway study demonstrates that merely changing the seed number for the random number generator, is sufficient to alter machine and shop load patterns to produce conflicting results between runs.

2.4.3. Dual-Constraint Shops

Dual-constraint shop problems refer to both machine-limited and labor-limited systems. Often, delays are caused by a lack of manpower to operate the available equipment.

Machine-limited systems are concerned with sequencing jobs on machines. In addition, labor-limited systems are concerned with effective labor assignment procedures, with dispatching rules which account for the interaction between labor assignment and job priorities, and with the effect of worker flexibility on shop performance.

Nelson [29, 30, 31] developed a single-echelon model which incorporates machine and labor limitations. The model is based on a job shop organization with multiple work centers in a single organizational unit. The model consists of a given configuration of several machine centers, each with multiple identical machines and a fixed labor force, with each worker having a relative efficiency on any machine. The performance of the shop is studied under different sets of labor assignment procedures, machine center selection procedures and dispatching rules for jobs. On one extreme, the labor assignment procedure consists of assigning the worker's next task each time the worker completes one operation in a machine center. On the other extreme, the procedure assigns a worker only after the worker completes all jobs in an assigned machine center. The machine center selection procedure determines which service center an available laborer is assigned to work at. For example, the worker may be assigned to a service center at which he is most efficient unless there is no work for him there and there is work at another service center.

Experiments on the model have shown that changes in machine center selection procedures have relatively little effect on shop performance. However, changes in dispatching rules have significant effect on performance. Likewise, performance increases when labor assignments are more flexible.

Fryer [13, 14] extended Nelson's model to a multi-echelon dual-constraint model. The model includes procedures to transfer workers between two organizational units. Fryer's results confirm Nelson's findings by showing that increasing labor flexibility, (measured by the ability to assign workers across organizational units),

improves shop performance.

2.4.4. Multiple Component (Assembly) Jobs

A single component job refers to a job which is not an assembled product of other jobs. All of the previously cited research has been concerned with single component jobs. This section will focus on research involving multiple component jobs, or, jobs which are assembled products of other jobs.

Multiple component jobs establish an interdependence between the set of jobs which must be finally assembled. Ideally, it is desirable to complete all the jobs in the set at the same time. But, due to the random events in the shop, this is very difficult to achieve for all sets of jobs in the shop. Thus, research on multiple component jobs has been concerned with developing dispatching rules which attempt to minimize the differences between the completion times of different jobs in a set. This is done by assigning a priority to a job dependent on the status of other jobs in its set.

Maxwell [26] appended an assembly shop model to the end of Conway's job shop to study dispatching rules for multiple component jobs. Job sets consisted of several individual jobs, similar to those of Conway, with a final assembly operation. New rules were developed that attempted to have jobs progress at the same rate or to have them completed at the same preset time. Maxwell found out that better overall performance could be achieved by combining the new rules with the SPT rule as a tie-breaker. In subsequent research, Maxwell and Mehra [27] again found that composite rules which incorporate the SPT rule with rules that account for the assembly structure of jobs perform

better than simple dispatching rules.

Hausman and Scudder [20], in their study of repairable inventory systems, extended the scope of Maxwell's work by providing for assembly jobs with interchangeable components and available spares. They have found that dynamic rules which use work-in-process inventory status information outperform both simple and dynamic rules which ignore inventory status.

2.5. General Observations

From the research just surveyed, we observe that:

- a. The SPT rule is a superior rule for simple component jobs.
- b. Composite rules which incorporate the strengths of the SPT rule with additional information, perform better for both single and multi-component jobs.
- c. Labor flexibility in dual-resource constrained systems, both for single- and multi-echelon shops, directly affects shop performance.

3. THE JOB SHOP SIMULATION MODEL

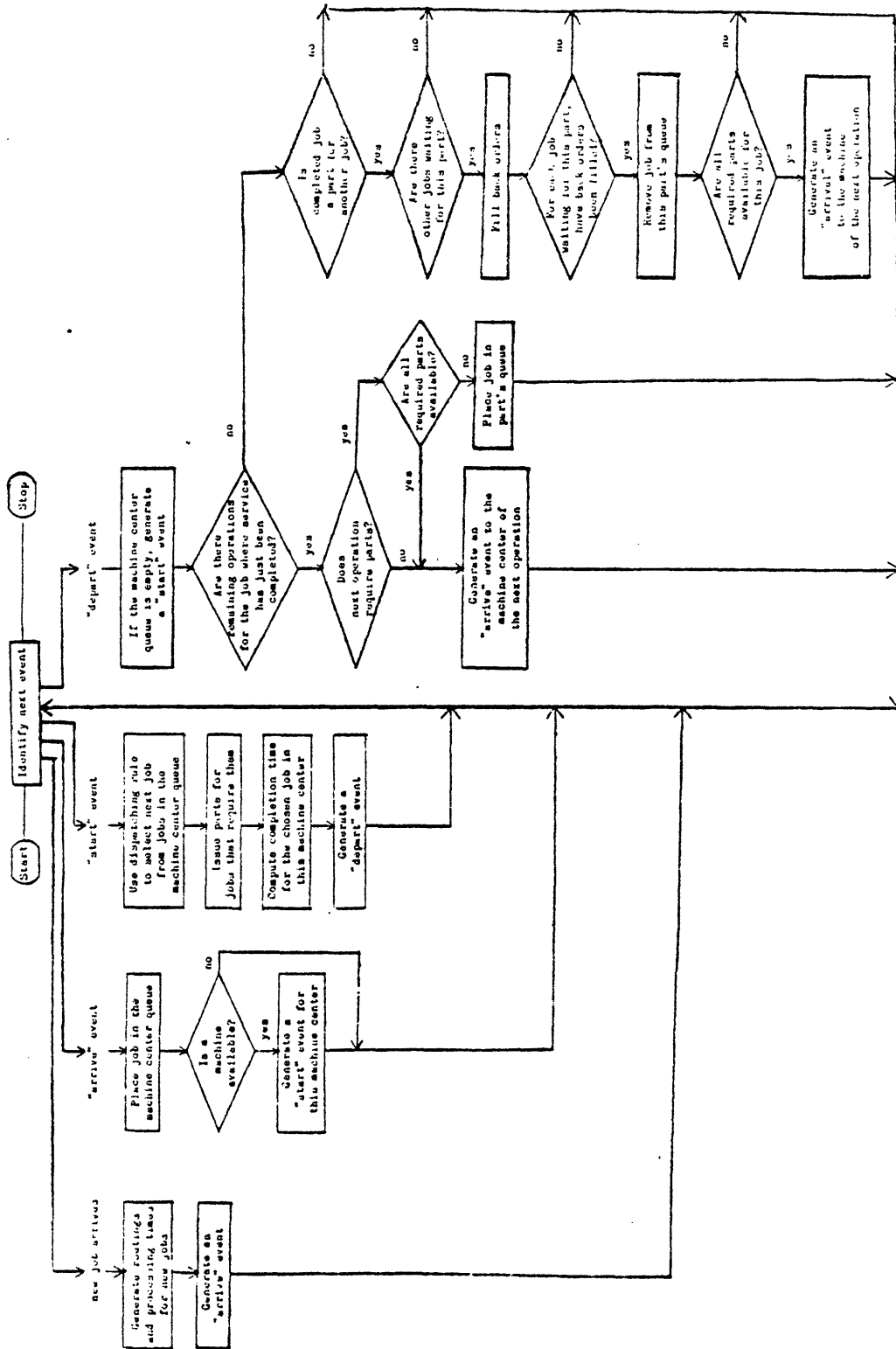
This section describes the features of the job shop simulation model. A general flow chart of the model is shown in Figure 3.1. The model is written in FORTRAN IV and uses list-processing techniques adapted from Gilsinn et al. [17].

3.1. General Features

The heart of a job shop simulation model is the mechanism that "drives" the shop through time. This mechanism can be either

FIGURE 3.1

A GENERAL FLOW CHART OF THE MODEL



time-driven or event-driven. A time-driven mechanism involves performing a standard procedure during each standard time unit. For example, if the standard time unit is defined as 1, then at each increment of 1, the model performs a standard procedure. The procedure involves checking each machine to: load and unload jobs, transfer jobs from one queue to another and gather statistics.

With an event-driven mechanism, the model performs the standard procedures pertaining to an event that is stored in a time-ordered list. The event-driven mechanism is preferred to the time-driven mechanism for the following reasons: a) it is a more realistic representation of a shop; and, b) it may run faster because the standard procedures for an event is a subset of the procedures for a time unit.

The general job shop simulation model developed for this paper is an event-driven, single resource constraint shop. It has three events -- "arrive", "start" and "depart". The events read as follows:

- "arrive" event: "At time X, job Y arrives at machine center Z."
- "start" event: "at time X, a machine at machine center Z starts working."
- "depart" event: "At time X, job Y departs from machine center Z."

The three events "bootstrap" on one another. For example, consider a simple job which does not require parts. The "arrive" event places the job in the queue of the machine center where its next operation will be performed. The "arrive" event also generates a "start" event if there is an available machine in this machine center. Then, the "start" event selects from among the jobs in the machine center queue, the next

job to process. It also generates a corresponding "depart" event for this job. Finally, the "depart" event directs the job to the machine center of the job's next operation by generating an "arrive" event. The "depart" event also generates a "start" event if the machine center which has just finished work on this job has other jobs waiting in queue.

The model maintains a precedence ordering of the events to increase the efficiency of their execution. Within the same time, events are performed in a "depart" - "arrive" - "start" precedence. Since transit time in the shop is assumed to be zero, a job which "departs" from one machine center can "start" in another machine center at the same time. The precedence convention assures that before a "start" event chooses the next job to process in the machine center, all jobs which "depart" from other machine centers and "arrive" at this machine center during the same time, are also considered in the selection process.

The model can handle problems with the following characteristics:

- a. single or several identical machines grouped by machine center with up to 20 machine centers.
- b. jobs with or without assembly operations with up to 10000 jobs.
- c. jobs with assembly operations, operations occurring at any stage and components made by the shop or supplied from outside the shop. Components may be common to many jobs.

3.2. Statistics

The model gathers statistics for a user-specified range of jobs. In simulation studies, the first and last few hundred jobs are discarded

from the observations to make sure that only "steady-state" conditions are examined [10]. Since the experiments usually begin with an empty job shop, the first few hundred jobs are required to "start-up" the shop and bring it to a "normal" operating level. Likewise, the last few hundred jobs are discarded to avoid observations related to a declining level of operations because no more jobs are arriving.

Researchers have been interested in rules which minimize work-in-process, meet due dates and maximize shop utilization. Job-related statistics provide a measure for the first two performance criteria, and shop-related statistics for the third. The model records these two types of statistics and miscellaneous statistics for diagnostic purposes.

3.2.1. Job Statistics

Job flowtime is a surrogate measure for work-in-process. The flowtime of a job is defined as follows:

Let C = completion time of a job

A = arrival time of a job

Then, flowtime, F is

$$F = C - A$$

The model records the flowtime for each job and computes the mean and variance of the flowtime for all jobs within the specified range.

Another measure for work-in-process is lateness. The lateness of a job is defined as follows:

Let D = due date of a job

Then, lateness, L is defined as

$$L = C - D$$

The model records the lateness for each job and computes the mean and variance of the lateness for all jobs within the specified range.

Tardiness is a measure for meeting due-dates. It is defined as follows:

Let T = tardiness of a job

Then,

$$T = \text{Maximum}(O, L)$$

The model records the tardiness for each job and computes: a) the mean and variance of the tardiness for all jobs within the specified range; b) the conditional tardiness (i.e. the mean and variance of the tardiness for tardy jobs only); and, c) the number and percentage of tardy jobs.

3.2.2. Shop Statistics

Shop utilization refers to the average percentage of the time that machines are busy. To measure shop utilization, the model gathers and computes statistics related to queues and utilization of machines each time the status of these measures changes. In computational terms, this means keeping a running average of the measures as follows:

Let T_0 = time when statistics are first gathered

T = current time

T_L = time when the status last changed

V = current value of the measure

RA_T = running average at time T

Then,

$$RA_T = \frac{(T_L - T_0) * RA_{T_L} + (T - T_L) * V}{T - T_0}$$

With regards to queues, the model computes the average number of jobs in each machine center queue, the maximum queue length per machine center, and the average job queue for the shop. For utilization, the model computes the average utilization for each machine center and the average utilization for the shop, shop utilization is computed as the weighted average of machine center utilization.

4. EXPERIMENTAL DESIGN AND ANALYSIS OF FINDINGS

The purpose of this section is to explain the experimental design of the verification runs and to discuss the findings of these runs. Each run is a combination of one "level" from each of the following factors:

- a. dispatching rule: 20 different rules
- b. job set: 4 different sets of 10000 jobs

The runs were primarily designed to verify the performance of the model and consequently, to evaluate the performance of some dispatching rules. A total of 80 runs were conducted involving all dispatching rules on all sets of jobs in a 9-machine shop.

For each set of jobs, three random numbers were used to initialize the random number generator. One number was used by the job generator program to generate the interarrival times. The second and third numbers were used by the route generator subroutine to generate the routings and processing times respectively.

4.1. Experimental Design

To evaluate the 20 dispatching rules, Conway's [7] experimental design was adopted. The salient features of the design are:

- a. Jobs: 4 sets of 10000 jobs, each job with a lot size of 1. A large set of jobs is intended for the shop to achieve "steady state" conditions. A lot size of 1 is a scaling technique for convenience.
- b. Processing time: Processing time at each machine is independently drawn from an exponential distribution with mean 1.
- c. Job arrival rate: exponentially distributed with mean time between arrivals set so that expected shop utilization is 90 %. Since expected processing time is 1 unit, this is equivalent to setting the mean time between arrivals as the reciprocal of .90, or 1.11 time units. The time unit can be any convenient unit such as weeks and days.
- d. Machine configuration: 9 different machines (In the context of our model, 9 machine centers with one machine per machine center.)
- e. Routing: randomly generated for each job so that the expected total number of operations on each machine for a run would be identical. Number of operations truncated at 39.
- f. range of jobs used for gathering statistics: 401st job to 9100th job.
- g. initial shop condition: empty shop with the first 50 jobs arriving simultaneously at the beginning of each run to hasten the achievement of "steady state".

The 20 dispatching rules selected for analysis are representative of the variety of rules studied in the literature. Appendix A contains

a rigorous mathematical description of each of these rules. Most of these rules were taken from Conway's study. The modified due-date (MDD) rule was adopted from Baker [3].

The analysis of this set of rules is intended to shed light on the following issues:

- a. Which rules are best for minimizing flowtime measures?
- b. Which rules are best for minimizing tardiness measures?
- c. What is the value of added information on the performance of dispatching rules?
- d. What is the trade-off between flowtime and tardiness?

The 20 dispatching rules consist of 8 simple rules and 12 composite rules which are derivatives of 5 of the simple rules. A dispatching rule is used to choose which job in a machine queue to process next. The rule assigns a value to each of the jobs in the queue. As a convention, the job with the lowest value is processed first. The values are assigned by each of the rules as follows:

4.1.1. Simple rules

1 FCFS, First-come-first-served. The value of the job is equal to its arrival time in the queue, and the job which arrived earliest is processed first.

2 RAND, Random. A job is chosen at random.

3 DDATE, Earliest due-date. The job with the earliest due-date is processed first. The due-date is equal to the arrival time plus a constant times the total processing time for the job.

4 FOPNR, Fewest number of operations remaining. The job with

the fewest number of operations remaining is processed first.

5 SPT, Shortest processing time. The job with the shortest processing time for the imminent operation is processed first.

6 LWRK, Least work remaining. The job with the least amount of work remaining (i.e. the sum of the processing times for the remaining operations including the imminent one) is processed first.

7 NINQ, Fewest jobs in next queue. The next queue of each job is identified. The job with the fewest number of jobs in its next queue is processed first.

8 WINQ, Least work in next queue. The next queue of each job is identified. The job whose next queue has the least amount of work (i.e. the sum of the processing time of the imminent operations) is processed first.

4.1.2. Composite rules

9 SLACK, Least slack. Slack is the difference between the due-date of the job and the earliest time that the job can be finished. It is defined as the due-date less the sum of current time and the amount of work remaining. The job with the least slack is processed first.

10 XWINQ, Least expected work in next queue. The expected work in the next queue is defined as the least work in the next queue (same as the WINQ rule) plus the remaining work of jobs being processed currently at other machines which will also join the next queue.

The job with the least expected work in the next queue is processed first.

11 OPNDD, Least operation due-date. The due-date for each job is divided by the job's number of operations to produce equally spaced due-dates for each operation. The job with the least operation due-date is processed first.

12 SOPN, Least slack per operation. The slack for each job is divided by the job's number of remaining operations. The job with the least slack per operation is processed first.

13 POPNR. The processing time for each job's imminent operation is divided by the job's number of remaining operations. The job with the smallest ratio is processed first.

14 PXWQ. The processing time for each job's imminent operation is added to the job's expected work in the next queue (same as the XWINQ rule). The job with the smallest sum is processed first.

15 PSP. The processing time for each job's next operation is subtracted from the processing time of the imminent operation. The job with the smallest difference is processed first.

16 PWRK. The processing time for each job's imminent operation is added to the job's amount of work remaining. The job with the smallest sum is processed first.

17 PWQP. The processing time for each job's imminent operation is added to the job's work in the next queue (same as the WINQ rule). This sum is divided by the processing time of the job's next operation. The job with the smallest ratio is

processed first.

18 PSOPN. The processing time for each job's imminent operation is added to the job's slack per operation (same as the SOPN rule). The job with the smallest sum is processed first.

19 MSOPN. The processing time for each job's imminent operation is multiplied by the job's slack (same as the SLACK rule). This product is divided by the job's amount of work remaining (same as the LWRK rule). The job with the smallest ratio is processed first.

20 MDD. The processing time of the job's imminent operation is multiplied by the difference between the job's due-date and the current time. This product is divided by the job's amount of work remaining (same as the LWRK rule). The ratio is compared with the processing time of the job's imminent operation, and the larger number is assigned as the job's value. The job which has the smallest value is processed first.

4.2. Experimental Results

For exposition purposes, the runs are categorized into trials as follows

trial 1: Job set 1, 9-machine shop

trial 2: Job set 2, 9-machine shop

trial 3: Job set 3, 9-machine shop

trial 4: Job set 4, 9-machine shop

Appendix B contains values obtained from all the runs. Appendix C contains the plots for these values. The results reveal the following:

a. Changing the random number seed which generates the set of jobs, alters the total work content of the set.

Table 4.1 shows the average and standard deviation of the total processing time per job for each trial and the average and standard deviation of the number of stages for each trial.

TABLE 4.1

RESULTS OF ROUTING GENERATION
PER JOB

LEGEND:

(X1) AVERAGE TOTAL PROCESSING TIME
 (X2) STANDARD DEVIATION OF
 TOTAL PROCESSING TIME
 (X3) AVERAGE NUMBER OF STAGES
 (X4) STANDARD DEVIATION OF
 THE NUMBER OF STAGES

TRIAL	X1	X2	X3	X4
1	9.308398	9.013644	9.323103	8.711934
2	8.902154	8.188431	8.818736	7.567665
3	9.314907	9.014271	9.329655	8.712777
4	8.891379	8.189765	8.813333	7.572082

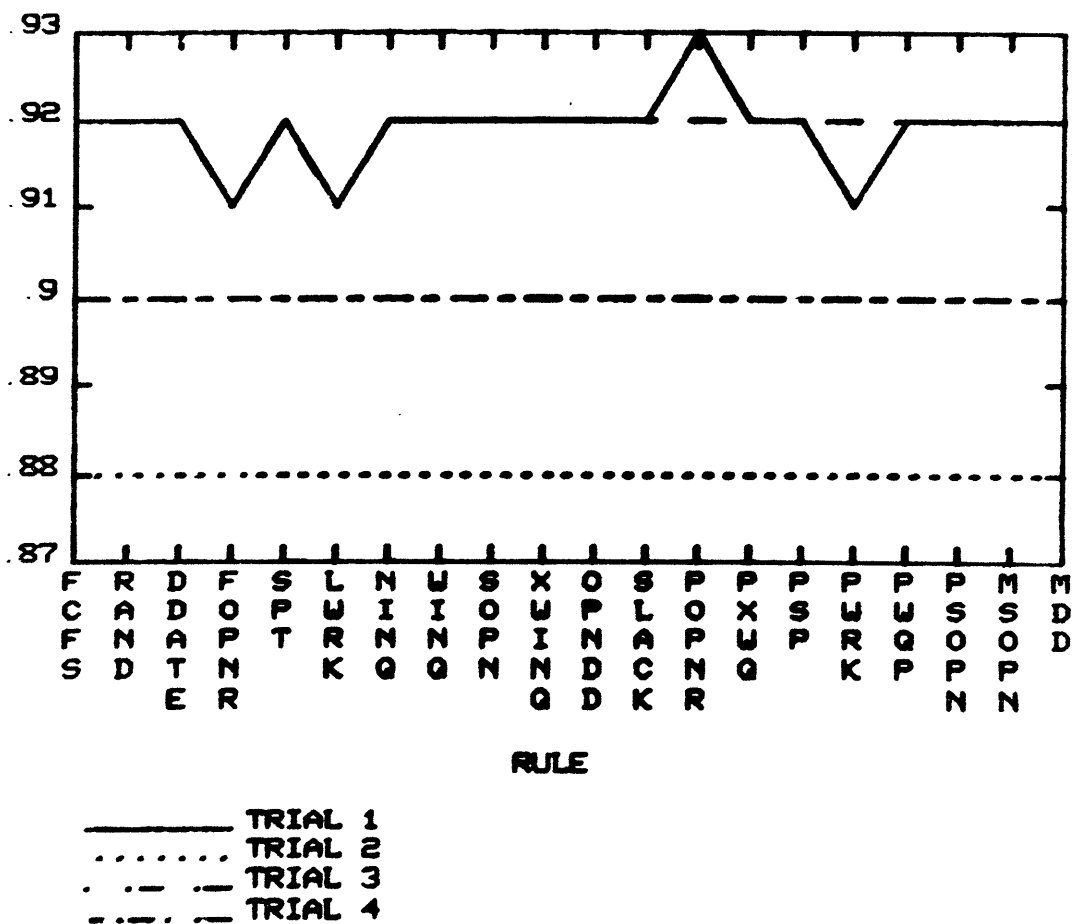
As a consequence of the different work content between trials, the shop utilization level varies correspondingly. However, the shop utilization level among rules in the same run are almost identical.

Figure 4.1 plots the shop utilization level for the four trials. The variation of the shop utilization level between trials may be due to the different arrival rates of the jobs or the work content of the jobs. If interarrival rates are tight, the shop utilization is expected to increase. And, as the work content increases, the shop utilization is likewise expected to increase.

The almost invariant utilization level for the same trial

FIGURE 4.1

SHOP UTILIZATION LEVEL BY TRIAL BY RULE



demonstrates that machine utilization is a function of the work content of the jobs rather than the dispatching rule.

b. The relative rankings of all statistics between rules across the trials are similar.

Each of the plots in Appendix C shows a similar pattern across all trials and no significant cross-over of lines which would indicate a change in relative rankings. For example, figure 4.2 plots the results for mean flowtime. The consistency suggests that the model has a tendency towards "steady state". Likewise, the rankings suggest that some rules perform much better than other rules.

c. Among all rules, the SOPN, PSOPN, MSOPN and MDD rules appear to produce the best results for the tardiness measures.

Figure 4.3 plots the results in decreasing order of performance (i.e. increasing values) for mean tardiness. The corresponding variances are plotted beside the means. The first four rules appear to perform better by an order of magnitude as a group than the next best rules. The strength of these rules lies primarily on the use of due-date information to regulate the pace of the jobs in the shop so that due dates are met. The SOPN rule, the simplest of these four rules, relies mainly on this strategy. The other three rules appear to show improved performance over the SOPN rule by adding processing time information.

d. Among all rules, the SPT and PSP rules appear to produce the best results for flowtime measures.

FIGURE 4.2

MEAN JOB FLOWTIME BY TRIAL BY RULE

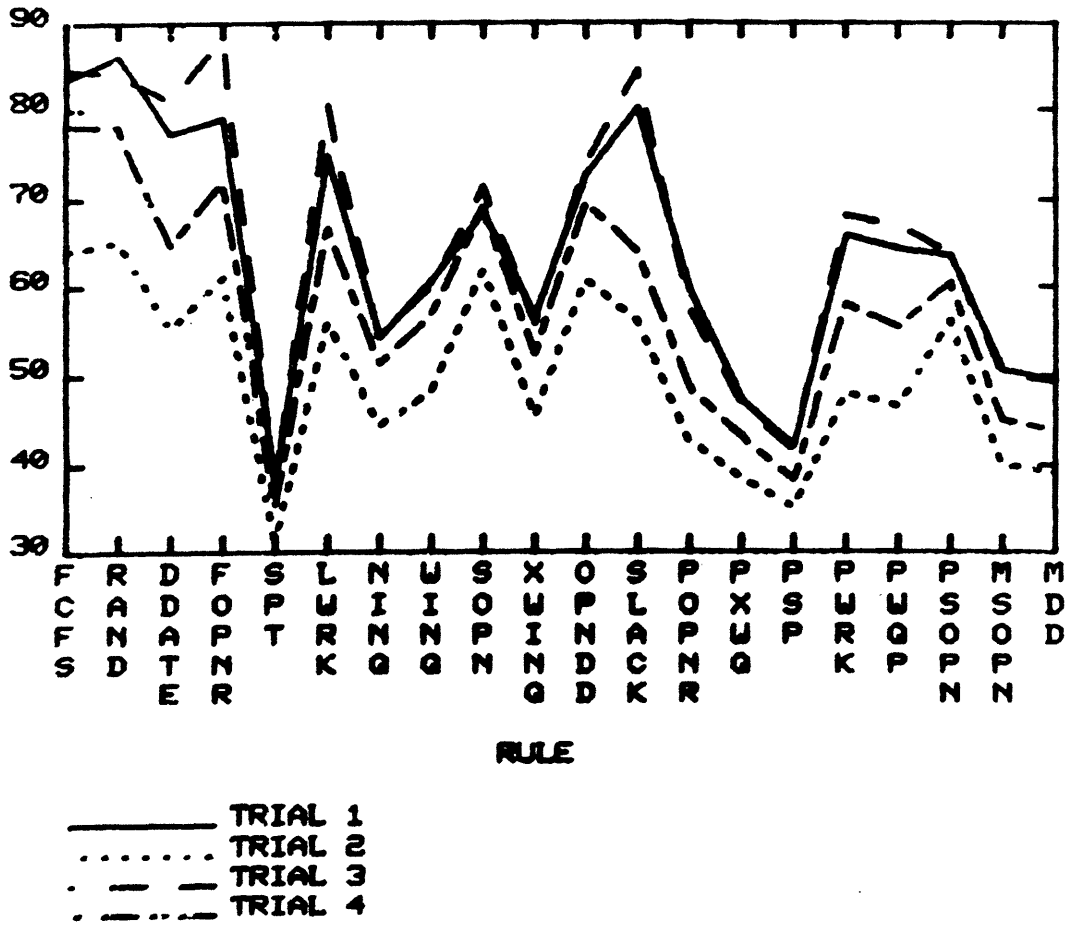


FIGURE 4.3
TARDINESS

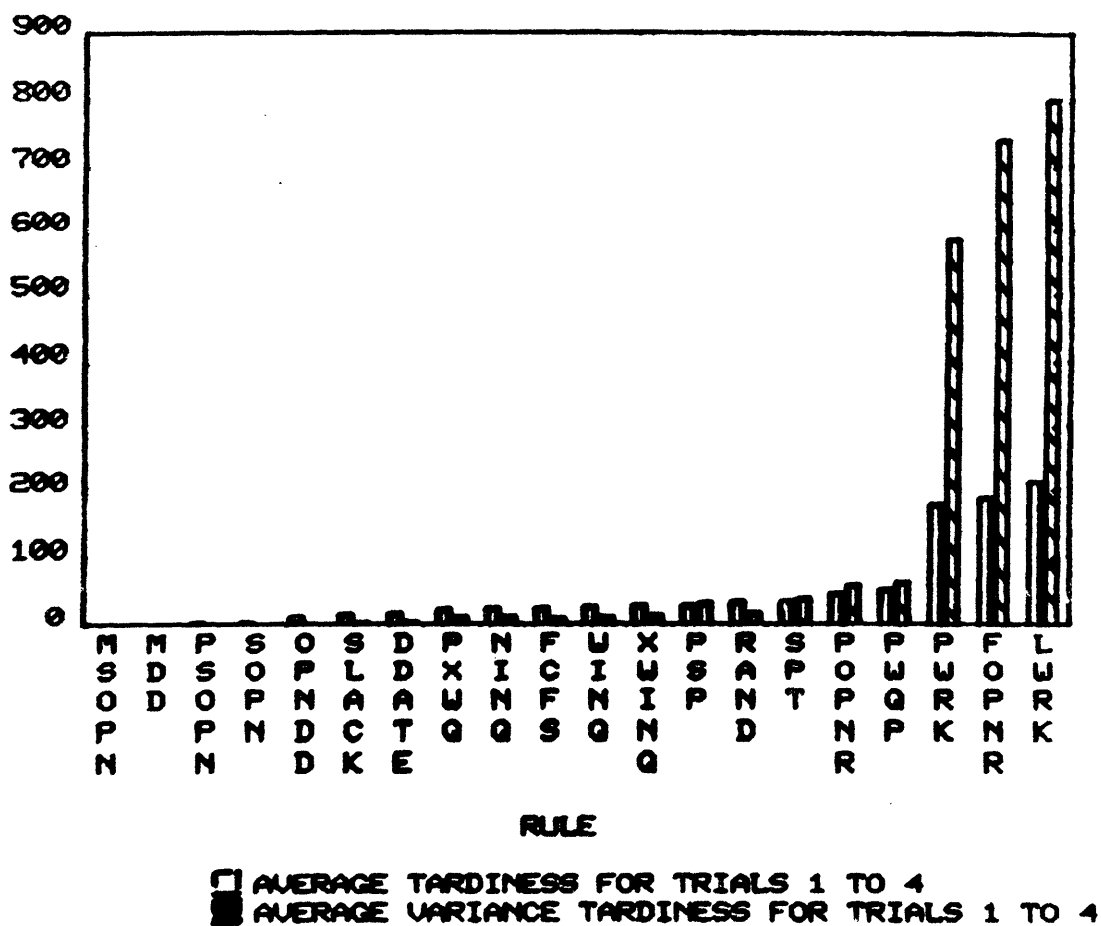


Figure 4.4 plots the results in decreasing order of performance for mean flowtime. The corresponding variances are also plotted beside the means. The figure shows that the SPT rule and its close derivative, the PSP rule, have the best flowtime measures. The other top rules in the list are also derivatives of the SPT rule.

e. Additional information can improve the performance of the rules.

Among the simple rules, the SPT rule shows the lowest flowtime mean and variance while the DDATE rule shows the best performance for the tardiness measures. Figures 4.5 and 4.6 illustrate these findings. Figures 4.3 and 4.4 illustrate the benefit of added information. For the tardiness measure, the six best rules are derivatives of the DDATE rule which ranks seventh. While the SPT rule shows the best performance on the flowtime measures, the next four best rules are derivatives of the SPT rule. The XWINQ rule, which is not a derivative of the SPT or the DDATE rule, also shows a better performance than its parent, the WINQ rule.

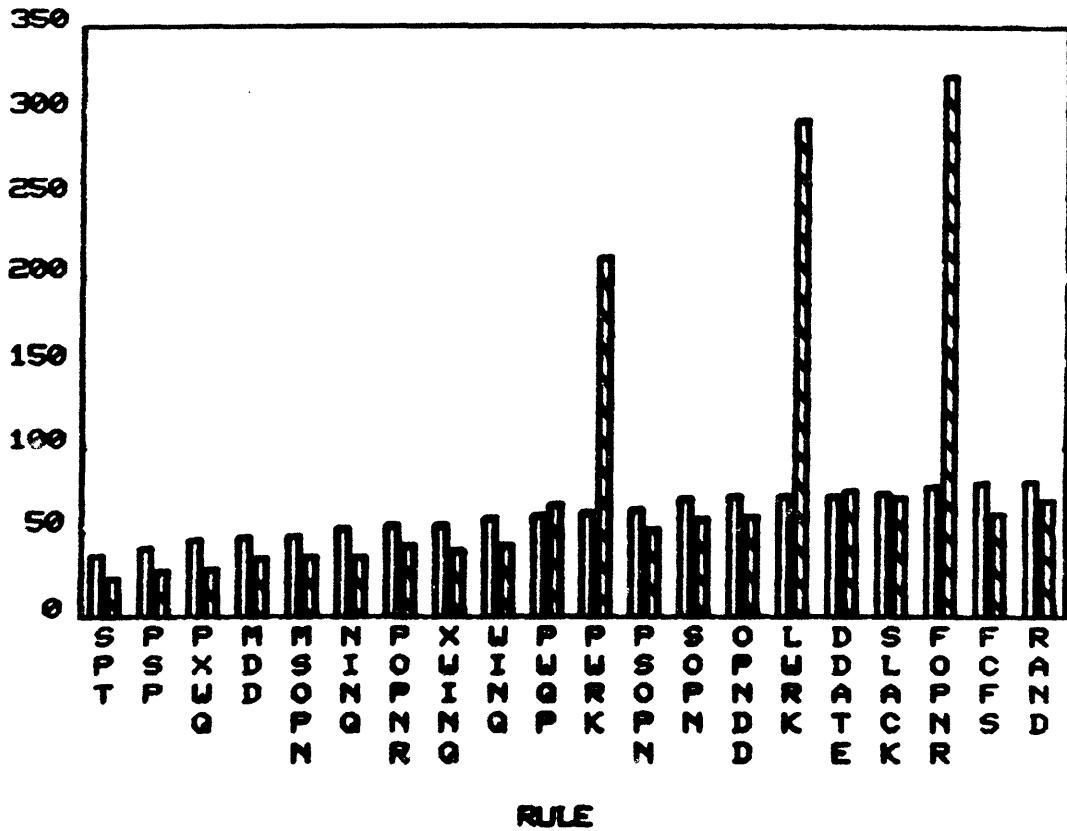
4.3. Dominance Analysis

The preceding analysis has shown that there is a difference among the rules which work best under the tardiness criteria and the flowtime criteria. The purpose of a dominance analysis is to attempt to capture trade-offs between different tardiness and flowtime criteria. The method used evaluates the performance of the dispatching rules based on several measures jointly.

A rule dominates another rule if all its relevant performance

FIGURE 4.4

FLOWTIME



AVERAGE FLOWTIME FOR TRIALS 1 TO 4
 AVERAGE VARIANCE FLOWTIME FOR TRIALS 1 TO 4

FIGURE 4.5
 FLOWTIME FOR SIMPLE RULES

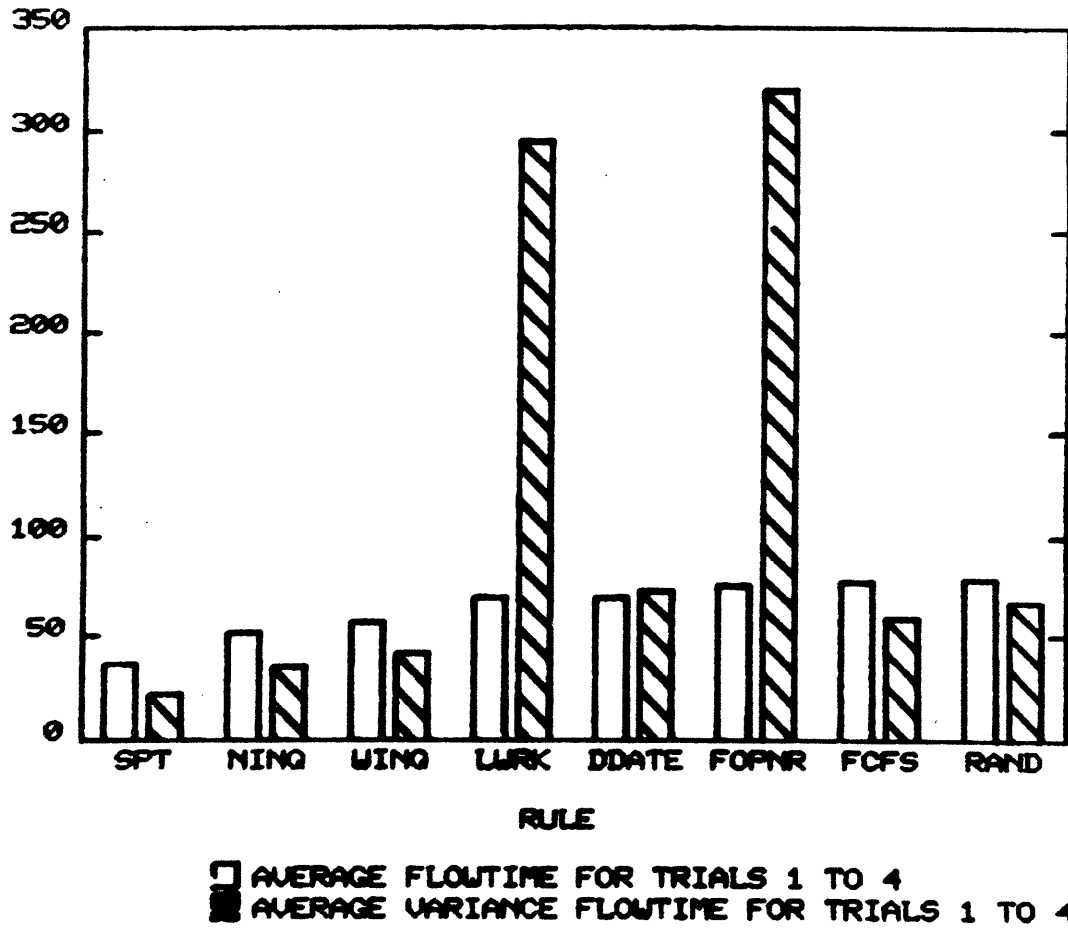
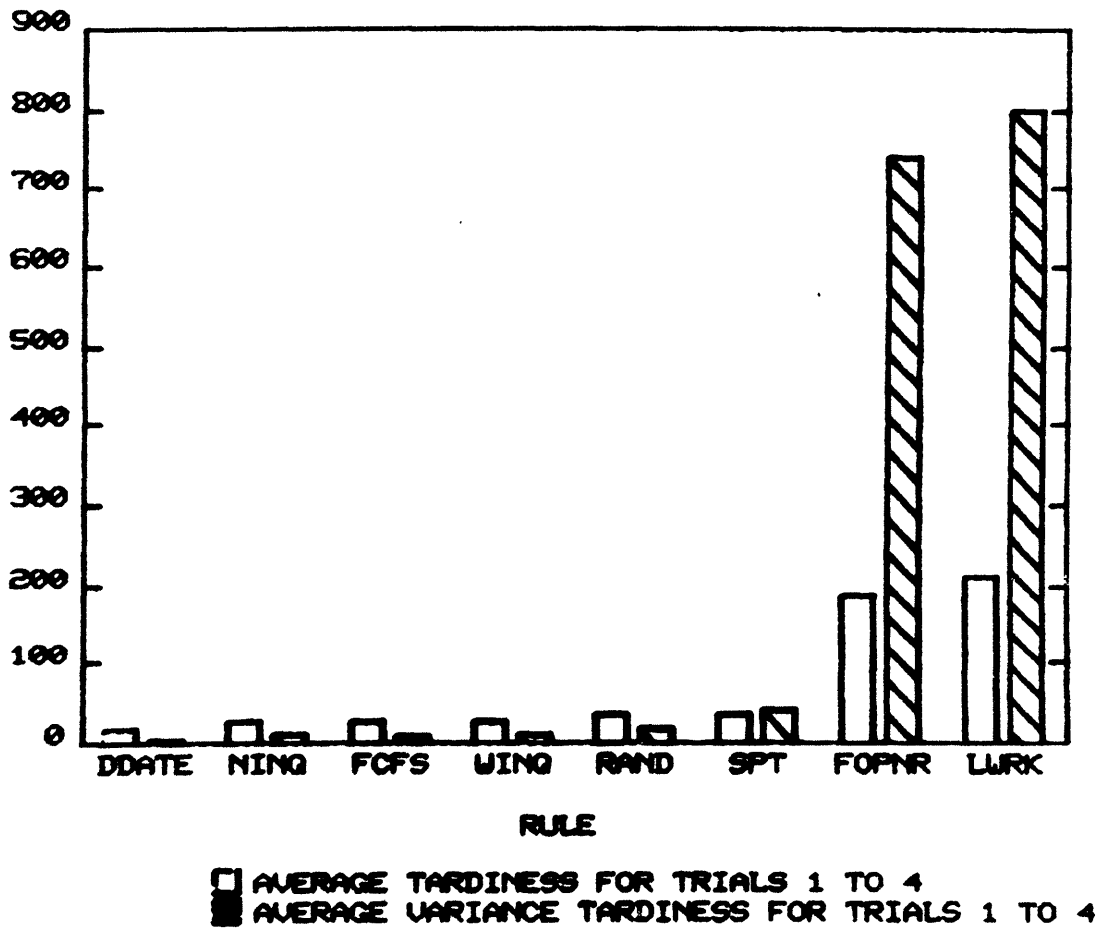


FIGURE 4.6
TARDINESS FOR SIMPLE RULES



measures are equal to or better than the other rule's measures. In mathematical terms, suppose we define a and b as the relevant measures and the criteria is to minimize these measures. Then, for two rules i and j , rule i dominates rule j if and only if $a_i \leq a_j$ and $b_i \leq b_j$, with at least one strict inequality.

Using mean flowtime and mean tardiness as the relevant measures reveals two dominant rules, SPT (rule 5) and MDD (rule 20). Using variance flowtime and variance tardiness reveals three dominant rules, SPT (rule 5), MDD (rule 20) and PXWQ (rule 14). A similar analysis using the four measures taken together, identifies the same three rules as being dominant.

SPT dominates processing time-related and status-related rules with superior performance in the flowtime measures and a reasonable performance in the tardiness measures. MDD dominates due-date-related rules with superior performance in the tardiness measures and reasonable performance in the flowtime measures.

Conway [7], [9] discusses the robustness of SPT. The dominance analysis reveals that PXWQ dominates SPT on the tardiness variance measure. In part, this is explained by recognizing that when using SPT, jobs with longer tasks are delayed in favor of those jobs with shorter tasks. Since the duration for each task was independently generated of the number of tasks, the longer jobs would have a greater probability of having longer tasks. Also, since the number of stages is geometrically distributed, there are fewer longer jobs than shorter ones. This minimizes the overall flowtime measure but causes the longer jobs to finish later and, consequently, increases their chances of being tardy. The higher variance of the tardiness of SPT indicates this delaying

effect.

On the two tardiness-related measures, SPT is dominated by MDD. This may be explained by the partitioning strategy used by MDD. Jobs are classified into two sets: those that are going to be tardy and those that will not be tardy. Jobs in the former set are scheduled using SPT and those in the latter, using DDATE. The power of this strategy is explained heuristically by recalling that for the static one-machine shop case, SPT minimizes mean tardiness if all jobs are tardy and DDATE minimizes mean tardiness when at most one job is tardy [2].

The PXWQ rule gains its strength from using the SPT rule to improve the flowtime measures and a look-ahead rule, XWINQ, to pace the jobs in the shop to improve the overall tardiness measure.

4.4. Lessons Learned

The analysis of the performance of dispatching rules reinforces the following results from other research work:

- a. The SPT rule is a superior rule in minimizing mean flowtime. Composite rules which improve flowtime mean and variance use the SPT rule as a component.
- b. The MDD rule is a superior rule in minimizing the tardiness mean and variance measures.
- c. The optimal rules derived for the one-machine case can be used to develop superior dispatching rules for the multiple machine case. Both the SPT and the DDATE rules have been analytically proven, in the one-machine case, to be optimal in minimizing mean flowtime and minimizing maximum tardiness respectively. The MDD rule produces

superior performance for tardiness measures because it is based on the SPT and the DDATE rules.

5. Conclusions.

In this paper, the performance of 20 dispatching rules in an open fabrication job shop was evaluated. Our findings and the current stage of research suggest the following issues for further study:

A. Extend the study of dispatching rules to open job shops with assembly. Issues to be resolved include developing methods of generating jobs and developing new dispatching rules that exploit the assembly structure.

B. A natural extension in the study of assembly job shops is the analysis of closed job shops. Examining the integration of inventory control and production sequencing is particularly relevant. A promising area of research appears to be the coordination of material requirements planning (MRP) and job shop scheduling.

C. Although many procedures have been suggested for assigning due-dates, the benchmark method is that suggested by Conway [7] and used in this study. Recently, Baker and Bertrand [3] have examined due-date assignment procedures for the one-machine shop. Their study needs to be extended to job shops. In particular, the impact of due-date assignment procedures on assembly shops remains to be studied.

D. Although most researchers report that dispatching rules that utilize more information are better than those that do not, there is no conclusive evidence to show that there has been a statistically significant improvement. This is in part due to the prohibitive cost of conducting a significant number of independent simulation runs.

E. Automated stopping rules to determine when a simulation has reached "steady-state" conditions would expedite the collection of independent observations. A promising area of research is developing such stopping rules.

F. Although simulation models provide insight into real life problems, the eventual justification of this avenue of research lies in demonstrating the effectiveness of applying these dispatching rules in actual manufacturing environments.

FORMULAE FOR DISPATCHING RULES

Definition of Symbols Used in Dispatching Rules

t	time at which a selection for machine assignment is to be made
i	index over the jobs to be processed by the shop
j	index over the sequence of operations of a job
J	specific value of j , the operation for which a job is in queue
M_i	the total number of operations on the i -th job $1 \leq j \leq M_i$
$T_{i,j}$	the time at which the i -th job becomes ready for its j -th operation (time at which the $j-1$ operation was finished) $T_{i,1}$ is the time at which the job arrived at the shop
D_i	the "due-date" (desired completion time) for the i -th job
$P_{i,j}$	the processing time for the j -th operation of the i -th job (including set-up and tear-down time, if any, assumed to be sequence independent)
$R_{i,j}$	a random variable, uniformly distributed between 0 and 1, assigned to the j -th operation of the i -th job
$N_{i,j}(t)$	the number of jobs, at time t , in the queue corresponding to the j -th operation of the i -th job
$W_{i,j}(t)$	the total work, at time t , in the queue corresponding to the j -th operation of the i -th job (total work is the sum of the imminent-operation processing times of the $N_{i,j}(t)$ jobs in the queue)
$X_{i,j}(t)$	the total work, at time t , which will "soon" arrive in the queue corresponding to the j -th operation of the i -th job (arrival of these jobs is imminent in the sense that, at time t , their preceding operation is being performed)

$Q_k(t)$ the set of jobs in the k-th queue at time t

$V_i(t)$ the priority value of the i-th job at time t

The queueing discipline specifies that when a selection for the machine assignment is to be made at time t from the k-th queue, a job I is chosen such that:

$$V_I(t) = \text{minimum } (V_i(t))$$
$$i \text{ in } Q_k(t)$$

In the event of a tie, the job with minimum value which arrived in the queue earliest is selected unless otherwise specified.

Source: Conway [7]

Dispatching Rules

a. Simple Rules

No. 1 FCFS First-come, first-served

$$V_i(t) = T_{i,j}$$

No. 2 RAND Random

$$V_i(t) = R_{i,j}$$

No. 3 DDATE Earliest due-date

$$V_i(t) = D_i$$

No. 4 FOPNR Fewest number of operations remaining

$$V_i(t) = M_i - J + 1$$

No. 5 SPT Shortest processing time

$$V_i(t) = P_{i,J}$$

No. 6 LWRK Least work remaining

$$V_i(t) = \sum_{j=J}^{M_i} P_{i,j}$$

No. 7 NINQ Fewest jobs in next queue
(ties resolved by SPT)

$$V_i(t) = N_{i,J+1}(t)$$

No. 8 WINQ Least work in next queue
(ties resolved by SPT)

$$V_i(t) = W_{i,J+1}(t)$$

b. Composite Rules

No. 9 SLACK Least slack

$$V_i(t) = D_i - t - \sum_{j=J}^{M_i} P_{i,j}$$

No. 10 XWINQ Least expected work in next queue
(ties resolved by SPT)

$$V_i(t) = W_{i,J+1}(t) + X_{i,J+1}(t)$$

No. 11 OPNDD Least operation due-date

$$V_i(t) = T_{i,1} + (D_i - T_{i,1})(J/M_i)$$

No. 12 SOPN Least slack per operation

$$V_i(t) = (D_i - t - \sum_{j=J}^{M_i} P_{i,j}) / (M_i - J + 1)$$

No. 13 POPNR

$$V_i(t) = P_{i,J} / (M_i - J + 1)$$

No. 14 PXWQ
(ties resolved by SPT)

$$V_i(t) = P_{i,J} + (W_{i,J+1}(t) + X_{i,J+1}(t))$$

No. 15 PSP

$$V_i(t) = \begin{cases} P_{i,J} - P_{i,J+1} & \text{if } J < M_i \\ P_{i,J} - 10.0 & \text{if } J = M_i \end{cases}$$

No. 16 PWRK

$$V_i(t) = P_{i,J} + \sum_{j=J}^{M_i} P_{i,j}$$

No. 17 PWQP

$$V_i(t) = \frac{P_{i,J} + W_{i,J+1}(t)}{P_{i,J+1}}$$

No. 18 PSOPN

$$V_i(t) = P_{i,J} + (D_i - t - \sum_{j=J}^{M_i} P_{i,j}) / (M_i - J + 1)$$

No. 19 MSOPN

$$V_i(t) = \frac{(D_i - t - \sum_{j=J}^{M_i} P_{i,j})(P_{i,J})}{\sum_{j=J}^{M_i} P_{i,j}}$$

No. 20 MDD Modified due date

$$V_i(t) = \frac{\text{maximum}(P_{i,J}, (D_i - t)(P_{i,J}))}{\sum_{j=J}^{M_i} P_{i,j}}$$

Source: Rules 1-18 Conway [7]
 Rule 20 Baker [3]

RESULTS OF THE EXPERIMENTS

A. Tardiness-related Results.

Legend:

MJTAJ.TR, Mean Tardiness For All Jobs
 VJTAJ.TR, Variance Tardiness For All Jobs
 MJTTJ.TR, Mean Tardiness For Tardy Jobs
 VJTTJ.TR, Variance Tardiness For Tardy Jobs
 NJT.TR, Number of Jobs Tardy
 PJT.TR, Percentage of Jobs Tardy

RULE	TRIAL	MJTAJ.TR	VJTAJ.TR	MJTTJ.TR	VJTTJ.TR	NJT.TR	PJT.TR
FCFS	1	12.96	648.92	26.62	969.06	4,237	48.7011
	2	6.16	192.55	17.3	347.76	3,100	35.6322
	3	13.6	603.94	27.11	837.58	4,364	50.1609
	4	14.18	773.33	29.65	1,158.66	4,160	47.8161
RAND	1	18.24	1,498.45	40.18	2,420.03	3,948	45.3793
	2	9.06	457.44	26.69	877.58	2,952	33.931
	3	16.33	1,052.51	36.37	1,615.15	3,907	44.908
	4	16.36	1,178.92	37.71	1,912.48	3,774	43.3793
DDATE	1	6.48	212.99	21.09	384.85	2,674	30.7356
	2	.38	7.73	7.52	100.2	437	5.02299
	3	8	238.43	21.17	351.94	3,289	37.8046
	4	3.22	91.26	16.67	248.38	1,680	19.3103
FOPNR	1	28.23	17,042	205.89	87,701.3	1,193	13.7126
	2	14.35	4,595.75	123.25	26,047.1	1,013	11.6437
	3	36.08	25,236	246.21	120,462	1,275	14.6552
	4	23.78	11,986.6	176.05	61,943	1,175	13.5057
SPT	1	2.03	364.35	38.61	5,523.9	457	5.25287
	2	.85	63.32	23.95	1,229.61	309	3.55172
	3	1.95	336.2	40.69	5,438.17	417	4.7931
	4	1.94	245.6	38.04	3,449.88	443	5.09195
LWRK	1	24.91	15,004.4	241.32	93,139.8	898	10.3218
	2	11.3	4,309.11	138.51	35,181.8	710	8.16092
	3	29.73	19,803.4	267.49	114,572	967	11.1149
	4	19.51	11,018.2	201.84	77,178.8	841	9.66667

Legend:

MJTAJ.TR, Mean Tardiness For All Jobs
 VJTAJ.TR, Variance Tardiness For All Jobs
 MJTTJ.TR, Mean Tardiness For Tardy Jobs
 VJTTJ.TR, Variance Tardiness For Tardy Jobs
 NJT.TR, Number of Jobs Tardy
 PJT.TR, Percentage of Jobs Tardy

RULE	TRIAL	MJTAJ.TR	VJTAJ.TR	MJTTJ.TR	VJTTJ.TR	NJT.TR	PJT.TR
NINQ	1	4.25	268.65	25.52	1,071.28	1,448	16.6437
	2	2.59	134.21	20.49	695.51	1,099	12.6322
	3	4.08	234.15	24.31	904.4	1,459	16.7701
	4	5.3	394.48	30.3	1,497.61	1,522	17.4943
WINQ	1	5.77	387.09	27.01	1,237.14	1,860	21.3793
	2	3.22	163.92	20.87	695.2	1,341	15.4138
	3	5.69	346.45	27.29	1,072.95	1,813	20.8391
	4	6.67	479.2	30.62	1,467.57	1,894	21.7701
SOPN	1	.07	1.11	2.72	34.57	231	2.65517
	2	.02	.04	1.08	1.29	157	1.8046
	3	.03	.08	1.32	2.02	195	2.24138
	4	.71	11.91	6.55	71.38	945	10.8621
XWINQ	1	5.32	362.28	29	1,288.04	1,596	18.3448
	2	3.45	234.34	24.81	1,157.56	1,208	13.8851
	3	5.17	319.51	27.77	1,089.43	1,619	18.6092
	4	5.8	450.12	31.49	1,635.56	1,602	18.4138
OPNDD	1	2.51	37.24	10.41	72.36	2,096	24.092
	2	.71	7.74	6.44	33.44	958	11.0115
	3	2.77	33.83	9.21	53.24	2,613	30.0345
	4	4.18	78.06	14.97	117.99	2,431	27.9425
SLACK	1	7.43	202.96	20.4	292.49	3,170	36.4368
	2	.27	3.59	5.44	45.04	427	4.90805
	3	9.53	230.46	20.62	270.03	4,021	46.2184
	4	2.15	46.59	12.84	141	1,457	16.7471

Legend:

MJTAJ.TR, Mean Tardiness For All Jobs
 VJTAJ.TR, Variance Tardiness For All Jobs
 MJTTJ.TR, Mean Tardiness For Tardy Jobs
 VJTTJ.TR, Variance Tardiness For Tardy Jobs
 NJT.TR, Number of Jobs Tardy
 PJT.TR, Percentage of Jobs Tardy

RULE	TRIAL	MJTAJ.TR	VJTAJ.TR	MJTTJ.TR	VJTTJ.TR	NJT.TR	PJT.TR
POPNR	1	16.18	3,284.89	58.44	9,397.87	2,408	27.6782
	2	6.89	736.17	35.32	2,768.08	1,698	19.5172
	3	14.7	2,539.62	54.82	7,271.15	2,333	26.8161
	4	10.31	1,539.61	44.75	5,139.41	2,005	23.046
PXWQ	1	2.52	177.67	21.23	1,100.41	1,032	11.8621
	2	1.78	132.54	22.23	1,197.51	698	8.02299
	3	2.39	145.02	21.15	885.44	984	11.3103
	4	2.78	192.46	25.24	1,181.1	958	11.0115
PSP	1	2.36	452.1	34.4	5,475.63	598	6.87356
	2	1.13	95.9	21.52	1,382.96	458	5.26437
	3	2.33	344.99	31.83	3,779.85	636	7.31034
	4	2.07	241.57	33.08	2,837.69	544	6.25287
PWRK	1	18.26	9,888.96	204.24	72,599.9	778	8.94253
	2	6.87	2,264.81	107.91	24,663.3	554	6.36782
	3	20.02	11,091.1	217.18	77,495	802	9.21839
	4	14.06	6,948.02	177.77	58,757.9	688	7.90805
PWQP	1	13.18	2,368.16	62.37	8,135.56	1,839	21.1379
	2	5.43	508.91	35.56	2,260.29	1,329	15.2759
	3	15.18	3,147.13	69.45	10,633.6	1,901	21.8506
	4	9.95	1,408.78	49.93	5,076.14	1,733	19.9195
PSOPN	1	.02	.04	1.02	1.12	145	1.66667
	2	.02	.04	1.13	1.64	130	1.49425
	3	.02	.04	1.05	1.08	153	1.75862
	4	.02	.05	1.2	1.76	124	1.42529
MSOPN	1	.01	.03	.89	.84	139	1.5977
	2	.01	.03	1.06	1.07	108	1.24138
	3	.02	.03	1	.96	137	1.57471
	4	.01	.02	.95	.83	112	1.28736
MDD	1	.01	.02	.9	.67	128	1.47126
	2	.01	.03	1.04	1.06	108	1.24138
	3	.01	.03	.98	.75	129	1.48276
	4	.01	.03	1.08	1.06	114	1.31034

B. Flowtime-related Results.

Legend:

MJF.TR, Mean Flowtime
 VJF.TR, Variance Flowtime
 MJL.TR, Mean Lateness
 VJL.TR, Variance Lateness
 NJBAR.TR, Average Number of Jobs in the Shop

RULE	TRIAL	MJF.TR	VJF.TR	MJL.TR	VJL.TR	NJBAR.TR
FCFS	1	83.43	7,117.89	-.31	1,539.72	79.2
	2	64.13	3,496.53	-15.97	1,454.76	57.87
	3	84.51	7,173.65	.71	1,501.95	75.25
	4	78.26	5,634.98	-1.73	1,940.83	71.1
RAND	1	86.07	8,621.6	2.33	2,794.07	82.02
	2	65.28	4,119.35	-14.82	1,978.52	59.17
	3	83.95	7,849.82	.15	2,309.41	75.08
	4	78.23	6,156.14	-1.76	2,601.98	71.06
DDATE	1	77.44	8,629.29	-6.31	593.87	73.8
	2	55.17	5,259	-24.94	346.05	49.63
	3	81.04	8,856.2	-2.76	610.62	72.28
	4	64.71	6,237.16	-15.29	500.46	58.86
FOPNR	1	79	36,326.3	-3.92	19,799.9	74.14
	2	61.03	15,678.2	-19.07	6,513.92	54.6
	3	88.07	48,658.1	4.38	28,455.6	77.77
	4	72.16	27,417.6	-7.75	14,366.8	65.23
SPT	1	38.38	2,701.4	-45.36	2,927	36.27
	2	31.45	1,456.97	-48.68	2,466.35	28.45
	3	37.65	2,533.73	-46.19	2,982.81	33.43
	4	35.22	2,235.05	-44.81	2,562.12	32.25
LWRK	1	74.89	34,444.5	-8.22	17,417.4	69.08
	2	56.05	15,143	-24.06	5,895.12	50.14
	3	80.49	42,005.1	-2.98	22,495.4	71.44
	4	66.6	26,102.5	-13.4	13,010.7	59.95

Legend:

MJF.TR, Mean Flowtime
 VJF.TR, Variance Flowtime
 MJL.TR, Mean Lateness
 VJL.TR, Variance Lateness
 NJBAR.TR, Average Number of Jobs in the Shop

RULE	TRIAL	MJF.TR	VJF.TR	MJL.TR	VJL.TR	NJBAR.TR
NINQ	1	54.14	4,037.13	-29.63	2,153.83	51.23
	2	44.61	2,409.93	-35.51	2,088.39	40.29
	3	54.36	3,999.34	-29.46	2,106.34	48.65
	4	51.32	3,506.49	-28.69	2,329.44	46.85
WINQ	1	60.55	4,951.03	-23.21	2,045	57.19
	2	48.26	2,705.11	-31.85	1,943.93	43.75
	3	60.04	4,859.79	-23.78	2,051.24	53.14
	4	56.71	4,079.9	-23.29	2,244.85	51.58
SOPN	1	68.49	6,075.61	-15.27	281.64	64.01
	2	61.77	4,660.83	-18.33	364.15	55.7
	3	71.43	6,205.06	-12.38	224.87	63.83
	4	69.09	5,278.76	-10.92	179.3	62.95
XWINQ	1	56.5	4,486.58	-27.27	2,277.29	53.28
	2	45.57	2,631.79	-34.54	2,273.79	41.17
	3	55.86	4,359.76	-27.96	2,298.35	49.77
	4	52.34	3,665.73	-27.67	2,382.31	47.57
OPNDD	1	72.79	6,472.06	-10.97	314.69	69.42
	2	60.81	4,787.83	-19.3	355.3	54.9
	3	74.03	6,474.9	-9.77	338.16	66.28
	4	69.73	5,307.63	-10.26	449.22	63.47
SLACK	1	80.16	8,170.28	-3.59	552.29	76.16
	2	56	5,031.66	-24.11	320.86	50.49
	3	84.34	8,409.19	.54	579.06	75.14
	4	63.95	5,739.34	-16.05	387.41	58.38

Legend:

MJF.TR, Mean Flowtime
 VJF.TR, Variance Flowtime
 MJL.TR, Mean Lateness
 VJL.TR, Variance Lateness
 NJBAR.TR, Average Number of Jobs in the Shop

RULE	TRIAL	MJF.TR	VJF.TR	MJL.TR	VJL.TR	NJBAR.TR
POPNR	1	59.59	5,977.14	-24.05	7,631.01	57.12
	2	42.94	2,188.94	-37.18	4,252.39	39.15
	3	57.52	4,967.48	-26.3	6,833.51	50.65
	4	48.79	3,371.44	-31.22	5,223.94	45.13
PXWQ	1	47.36	3,283.78	-36.41	2,271.53	44.54
	2	38.56	1,933.14	-41.56	2,289.53	34.85
	3	47.04	3,183.76	-36.79	2,245.62	41.94
	4	43.32	2,613.02	-36.7	2,228.08	39.71
PSP	1	42.19	3,283.58	-41.57	2,759.49	39.53
	2	35	1,745.14	-45.13	2,308.98	31.52
	3	41.88	3,059.26	-41.96	2,704.19	37.35
	4	38.37	2,460.9	-41.65	2,355.58	35.09
PWRK	1	65.8	25,897.7	-17.43	12,036.4	60.96
	2	48.34	10,472.4	-31.73	3,688.94	43.58
	3	68.08	28,391.1	-15.64	13,351.3	60.23
	4	57.99	19,393.7	-21.98	8,752.53	52.44
PWQP	1	64.31	8,106.3	-19.17	4,815.72	62.4
	2	46.83	3,141.08	-33.29	2,811.66	42.32
	3	67.07	9,379.51	-16.74	5,686.67	59.64
	4	55.39	5,239.9	-24.62	3,789.98	50.5
PSOPN	1	63.52	5,655.29	-20.24	414.61	58.29
	2	56.09	4,145.21	-24.02	569.19	50.61
	3	63.86	5,690.4	-19.96	387	57.01
	4	60.41	4,551.8	-19.6	413.95	55.5
MSOPN	1	50.62	4,303.22	-32.89	932.64	47.04
	2	40.1	2,317.19	-39.59	1,361.55	36.25
	3	50.15	4,194.97	-33.41	983.48	44.49
	4	45.21	2,985.03	-34.38	1,109.37	41.57
MDD	1	49.6	4,148.54	-33.9	984.73	46.11
	2	39.26	2,236.44	-40.44	1,375.41	35.48
	3	49.41	4,144.25	-34.15	995.43	43.77
	4	44.1	2,877.43	-35.49	1,131.51	40.5

Legend:

NBAR.TR, Mean Machine Queue Length
 LBAR.TR, Shop Utilization Level

RULE	TRIAL	NBAR.TR	LBAR.TR
FCFS	1	7.88	.92
	2	5.55	.88
	3	7.44	.92
	4	7	.9
RAND	1	8.19	.92
	2	5.69	.88
	3	7.42	.92
	4	7	.9
DDATE	1	7.28	.92
	2	4.63	.88
	3	7.12	.92
	4	5.64	.9
FOPNR	1	7.32	.91
	2	5.19	.88
	3	7.73	.91
	4	6.35	.9
SPT	1	3.1	.92
	2	2.28	.88
	3	2.79	.92
	4	2.68	.9
LWRK	1	6.76	.91
	2	4.69	.88
	3	7.03	.91
	4	5.76	.9

Legend:

NBAR.TR, Mean Machine Queue Length
 LBAR.TR, Shop Utilization Level

RULE	TRIAL	NBAR.TR	LBAR.TR
NINQ	1	4.77	.92
	2	3.6	.88
	3	4.48	.92
	4	4.3	.9
WINQ	1	5.43	.92
	2	3.98	.88
	3	4.98	.92
	4	4.83	.9
SOPN	1	6.19	.92
	2	5.31	.88
	3	6.17	.92
	4	6.09	.9
XWINQ	1	4.99	.92
	2	3.69	.88
	3	4.61	.92
	4	4.38	.9
OPNDD	1	6.79	.92
	2	5.22	.88
	3	6.44	.92
	4	6.15	.9
SLACK	1	7.54	.92
	2	4.73	.88
	3	7.43	.92
	4	5.59	.9

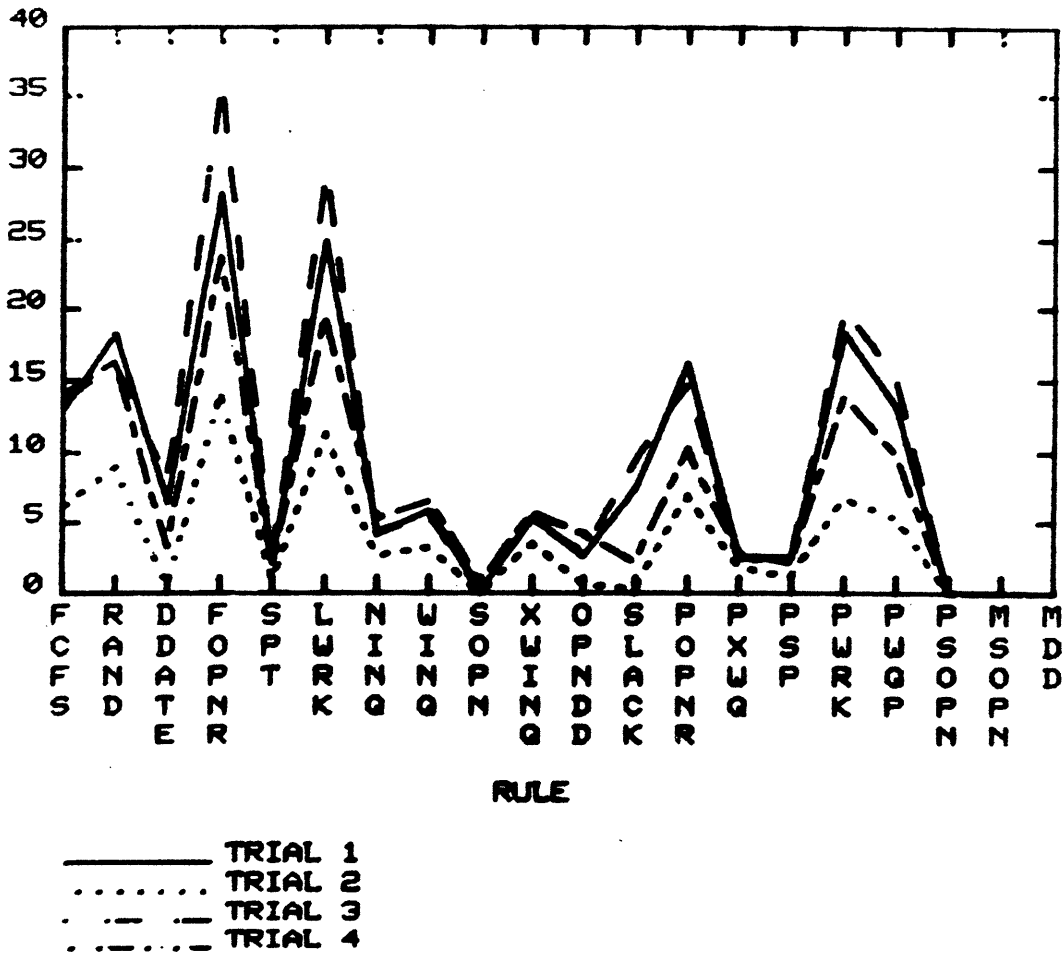
Legend:

NBAR.TR, Mean Machine Queue Length
 LBAR.TR, Shop Utilization Level

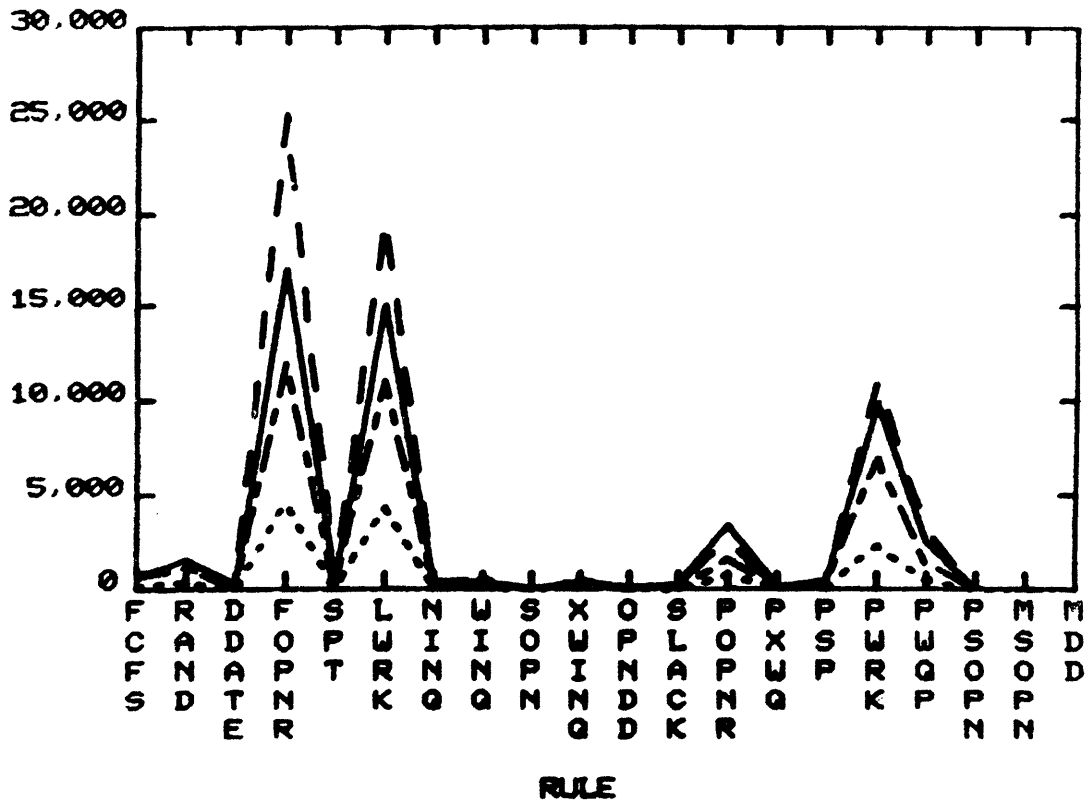
RULE	TRIAL	NBAR.TR	LBAR.TR
POPNR	1	5.42	.93
	2	3.47	.88
	3	4.7	.92
	4	4.11	.9
PXWQ	1	4.02	.92
	2	2.99	.88
	3	3.74	.92
	4	3.51	.9
PSP	1	3.47	.92
	2	2.62	.88
	3	3.23	.92
	4	3	.9
PWRK	1	5.86	.91
	2	3.96	.88
	3	5.77	.92
	4	4.93	.9
PWQP	1	6.01	.92
	2	3.82	.88
	3	5.7	.92
	4	4.71	.9
PSOPN	1	5.55	.92
	2	4.74	.88
	3	5.41	.92
	4	5.27	.9
MSOPN	1	4.3	.92
	2	3.15	.88
	3	4.02	.92
	4	3.72	.9
MDD	1	4.2	.92
	2	3.06	.88
	3	3.94	.92
	4	3.6	.9

PLOTS OF EXPERIMENTAL RESULTS

MJTAJ. TR. MEAN TARDINESS FOR ALL JOBS
BY TRIAL BY RULE

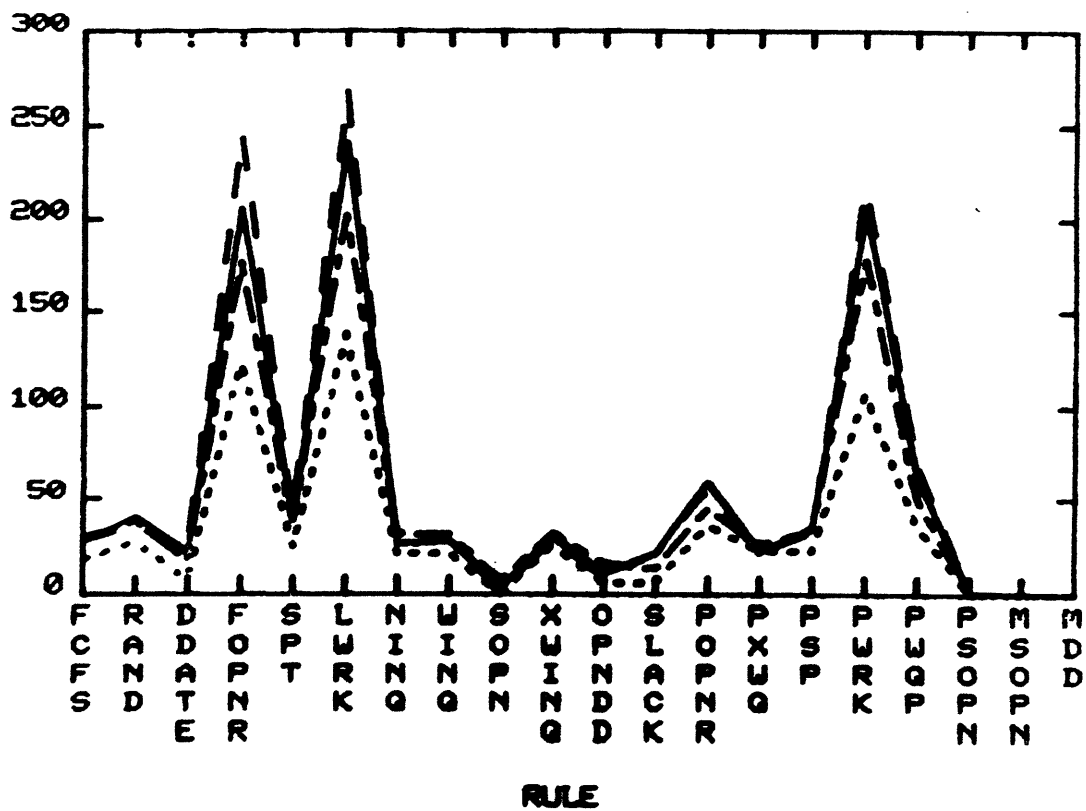


UJTAJ.TR, TARDINESS VARIANCE FOR ALL JOBS
BY TRIAL BY RULE



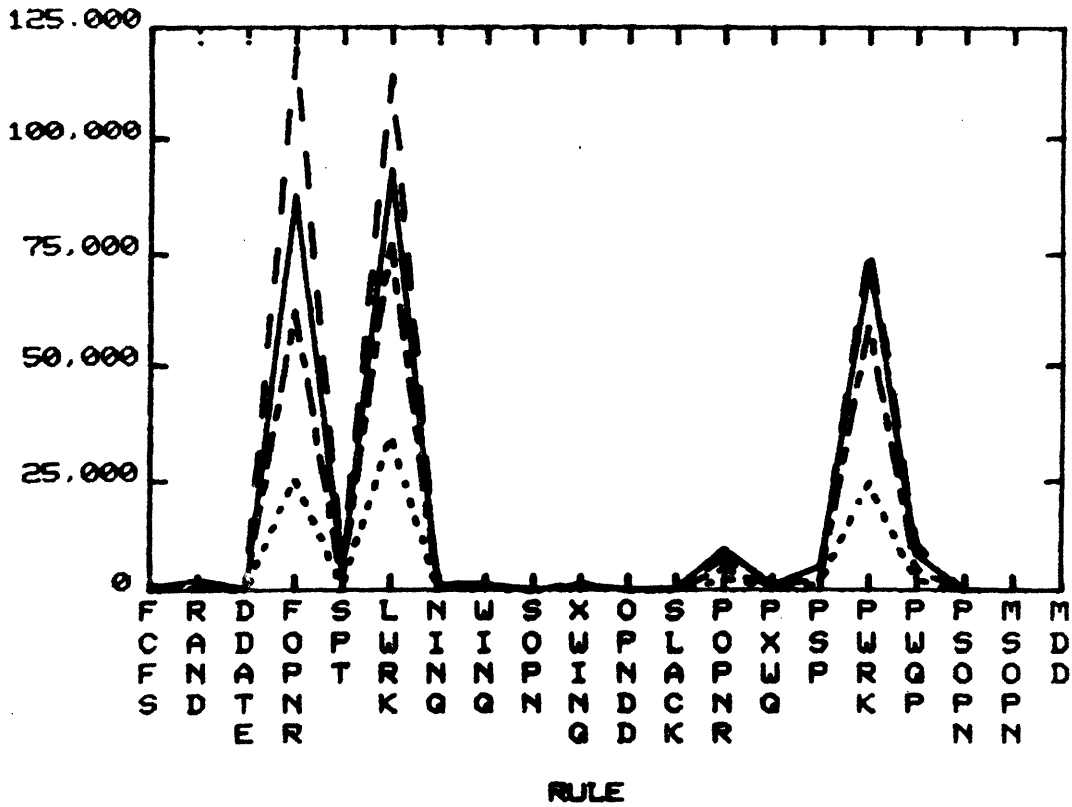
_____ TRIAL 1
 TRIAL 2
 - - - - TRIAL 3
 TRIAL 4

MJTTJ.TR. MEAN TARDINESS FOR TARDY JOBS
BY TRIAL BY RULE



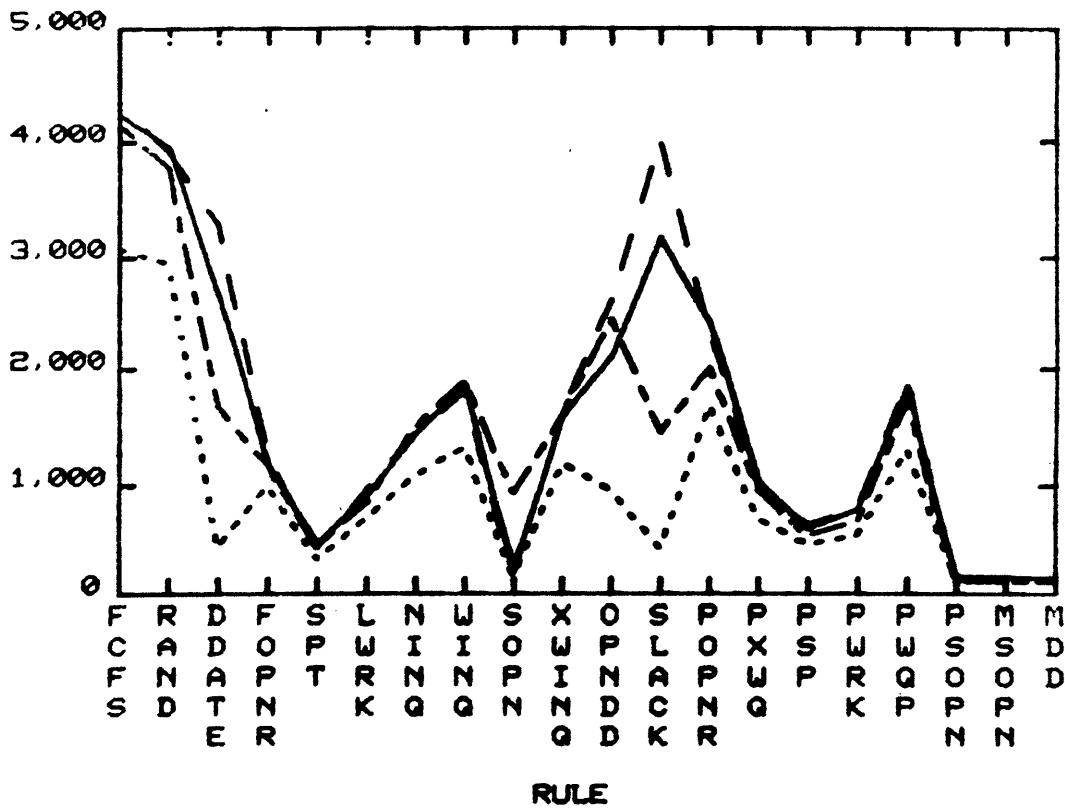
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 TRIAL 2
 - - - TRIAL 3
 - . - . TRIAL 4

UJTJ TR. TARDINESS VARIANCE FOR TARDY JOBS
BY TRIAL BY RULE



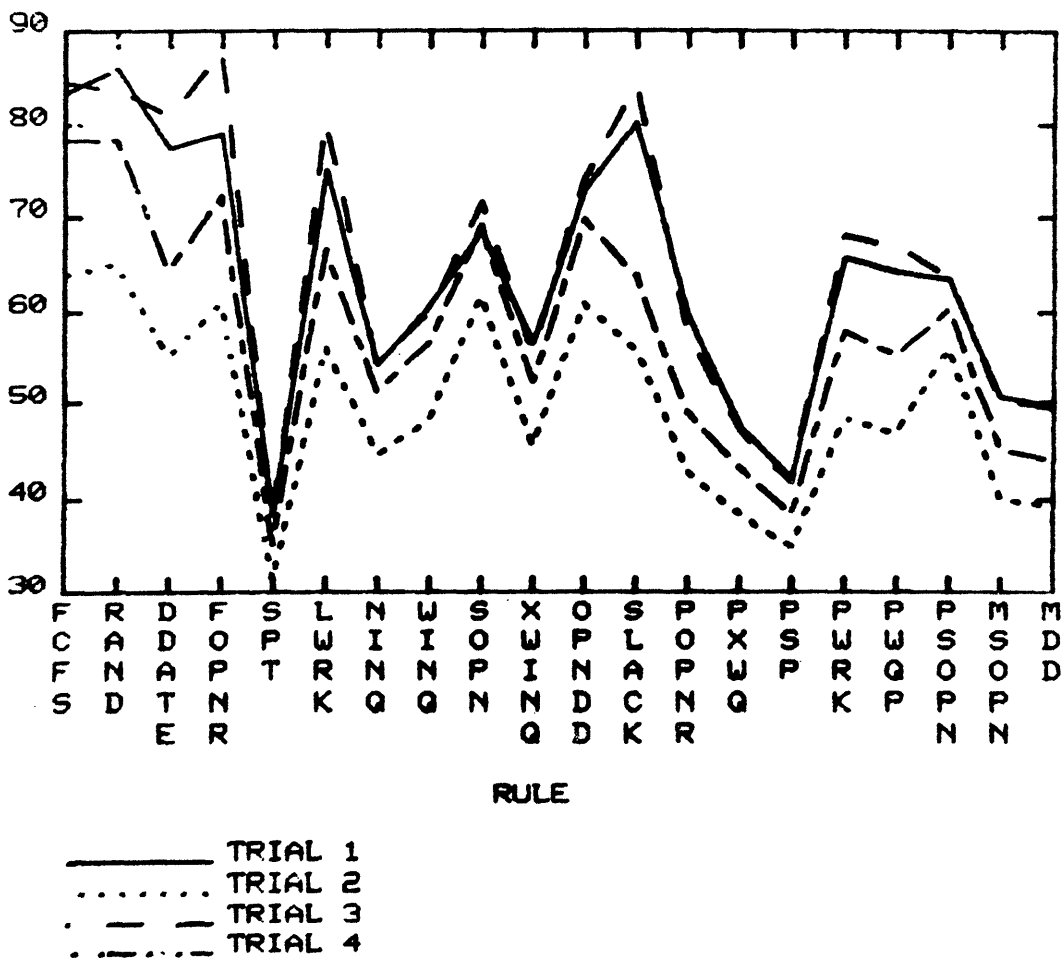
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 TRIAL 2
 - - - - TRIAL 3
 - . . - TRIAL 4

NJT TR. NUMBER OF JOBS TARDY
BY TRIAL BY RULE

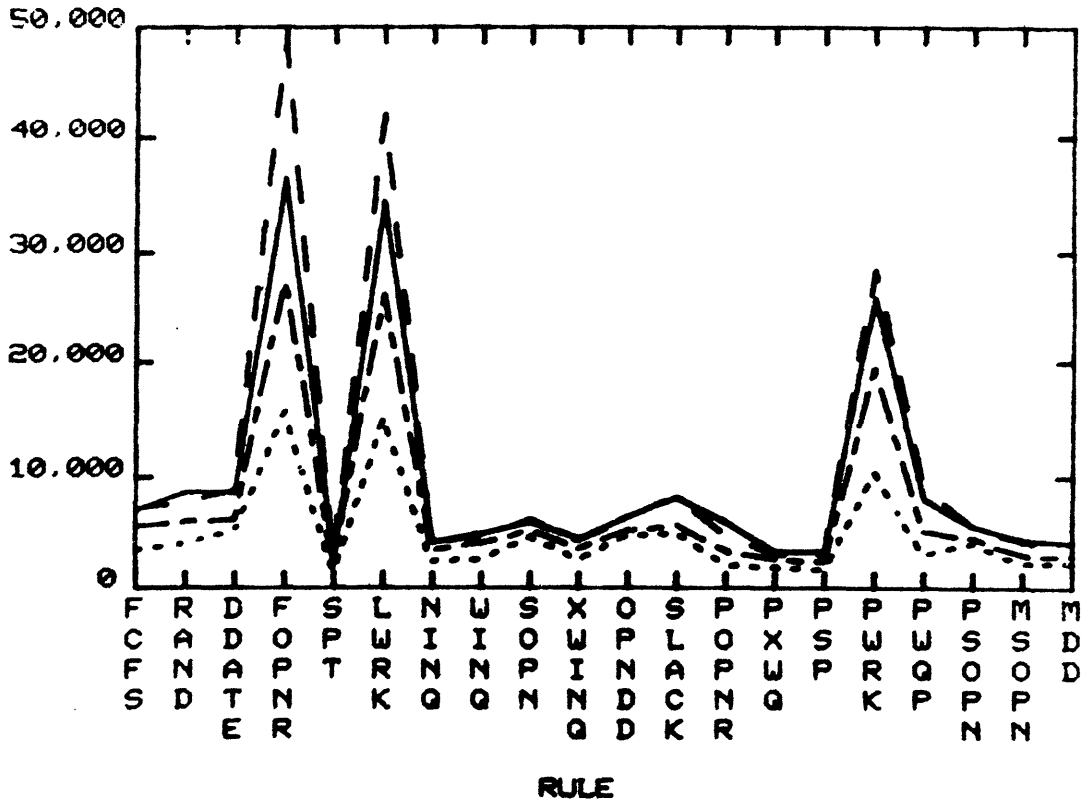


_____ TRIAL 1
 TRIAL 2
 - - - - TRIAL 3
 - . - . TRIAL 4

MJF TR. MEAN FLOWTIME
BY TRIAL BY RULE

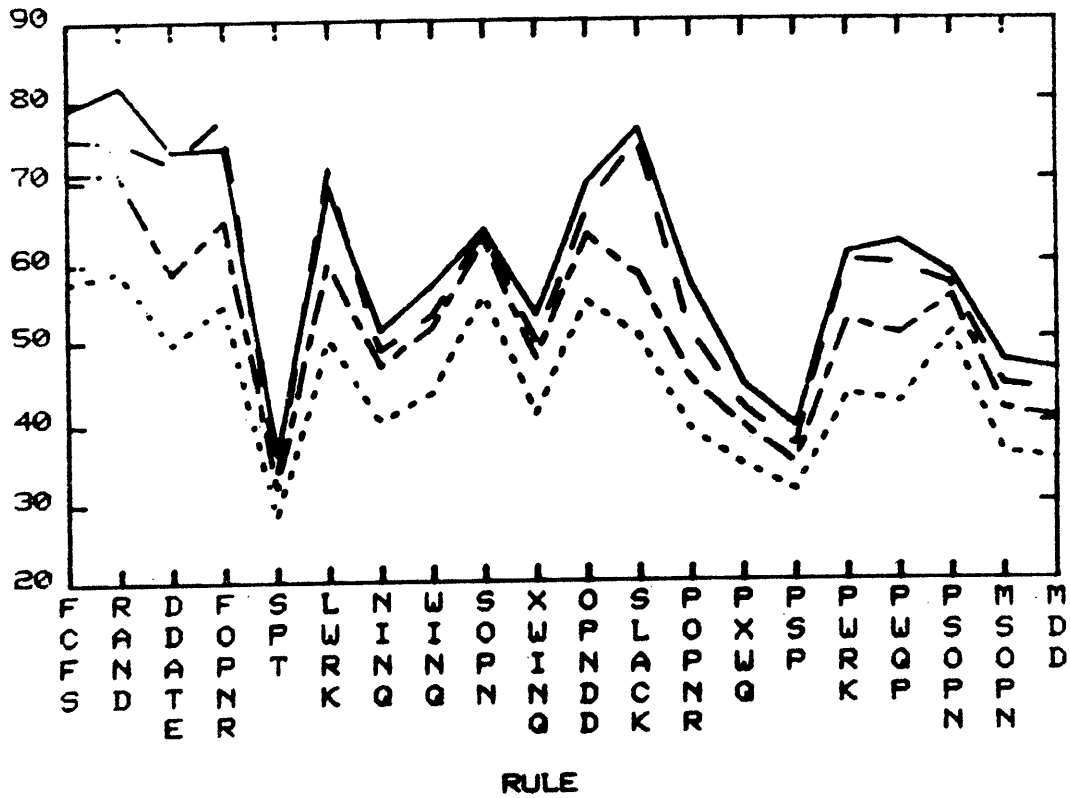


UJF TR. FLOWTIME VARIANCE
BY TRIAL BY RULE



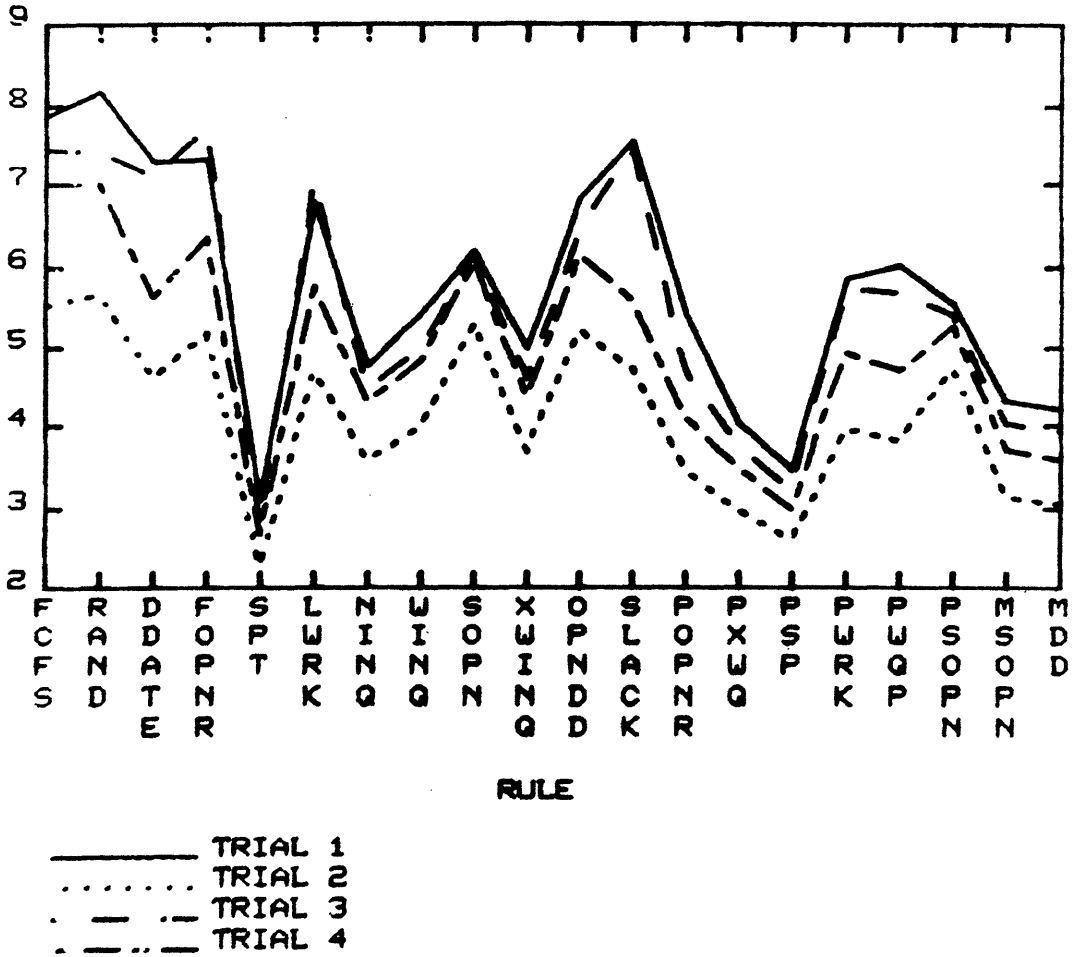
_____ TRIAL 1
 TRIAL 2
 - . - . TRIAL 3
 - - - - TRIAL 4

NJBAR TR. AVERAGE NUMBER OF JOBS IN THE SHOP
BY TRIAL BY RULE

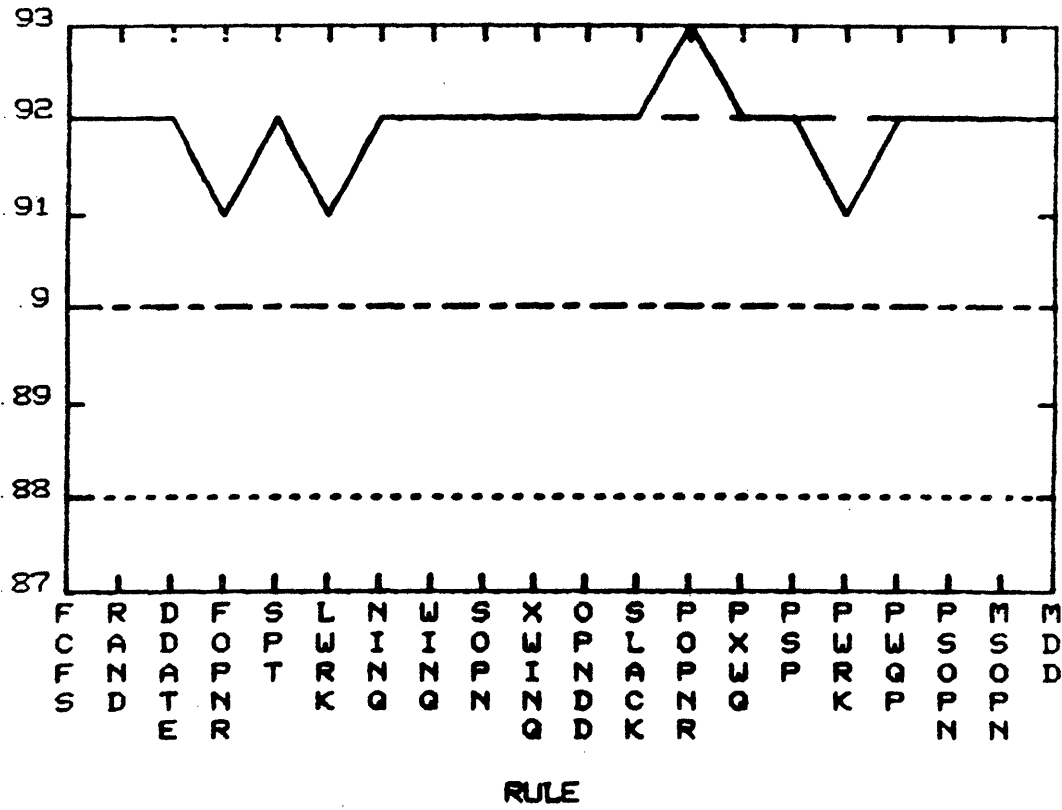


_____ TRIAL 1
 TRIAL 2
 - - - - TRIAL 3
 - . - . TRIAL 4

NBAR TR. MEAN MACHINE QUEUE LENGTH
BY TRIAL BY RULE



LBAR TR. SHOP UTILIZATION LEVEL
BY TRIAL BY RULE



_____ TRIAL 1
 TRIAL 2
 - - - - TRIAL 3
 - . - . TRIAL 4

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