

## A Single-Producer Multi-Retailer Integrated Inventory System with Scrap in Production

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**Abstract:** This study determines the replenishment-distribution policy for a single-producer multi-retailer integrated inventory system with scrap rate in production. The objective is to find the optimal lot-size and number of shipments that minimizes the total expected costs for such a specific system. It is assumed that an item is manufactured by a producer and then delivered to its  $n$  different retailers for sale. Each retailer has its own annual product demand and the demand will be satisfied by synchronized multiple shipments during each production cycle. Unlike the classic Finite Production Rate (FPR) model assuming perfect production and a continuous inventory issuing policies, the proposed system assumes that there is an inevitable random defective rate in production and all nonconforming items are scrap and delivery of product is under a practical multiple shipment policy. Mathematical modeling is employed and the renewal reward theorem is used to cope with the variable cycle length. Then the expected system cost function is derived and the convexity of this function is proved. Finally, a closed-form optimal replenishment-distribution policy for such a specific single-producer multi-retailer integrated system is obtained. A numerical example is provided to demonstrate the practical usage of the obtained results.

**Keywords:** FPR model, inventory, manufacturing, multiple retailers, multiple shipments, optimal batch size, scrap

### INTRODUCTION

The purpose of this study is to determine the optimal lot-size and number of shipments that minimizes the total expected cost for a single-producer multi-retailer integrated inventory system with scrap in production. The classic FPR model (also known as the Economic Production Quantity (EPQ) model) assumes a perfect production (Taft, 1918; Nahmias, 2009). However, in a real life manufacturing environment, due to different unpredictable factors it seems to be inevitable to have imperfect production (Hadley and Whitin, 1963; Schneider, 1979; Bielecki and Kumar, 1988; Chern and Yang, 1999; Boone *et al.*, 2000; Teunter and Flapper, 2003; Ojha *et al.*, 2007; Lin *et al.*, 2008; Chiu *et al.*, 2009a, 2012a; Lee *et al.*, 2011; Chen, 2011).

Another unrealistic assumption in the classic FPR model is the continuous inventory issuing policy to satisfy product demand. In the real world vendor-buyer integrated system, however, the multiple or periodic deliveries of finished products are often used. Schwarz (1973) first examined a one-warehouse  $N$ -retailer deterministic inventory system with the objective of determining the stocking policy that minimizes the long-run average system cost per unit time. The optimal solutions along with a few necessary properties are derived for such a one-retailer and  $N$  identical retailer

problems. Heuristic solutions for the general problem were also suggested. Goyal (1977) studied the integrated inventory model for the single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier and an example was provided to illustrate his proposed method. Schwarz *et al.* (1985) considered the system fill-rate of a one-warehouse  $N$ -identical retailer distribution system as a function of warehouse and retailer safety stock. They used an approximation model from a prior study to maximize system fill-rate subject to a constraint on system safety stock. As results, properties of fill-rate policy lines are suggested. They may be used to provide managerial insight into system optimization and as the basis for heuristics. Banerjee (1986) studied a joint economic lot-size model for purchaser and vendor, with the focus on minimizing the joint total relevant cost. He concluded that a jointly optimal ordering policy, together with an appropriate price adjustment, could be economically beneficial for both parties, but definitely not disadvantageous to either party. Studies have since been carried out to address various aspects of vendor-buyer supply chain optimization issues (Kim and Hwang, 1988; Banerjee and Burton, 1994; Parija and Sarker, 1999; Cetinkaya and Lee, 2000; Yao and Chiou, 2004; Hoque, 2008; Sarker and Diponegoro, 2009; Chiu *et al.*, 2011,

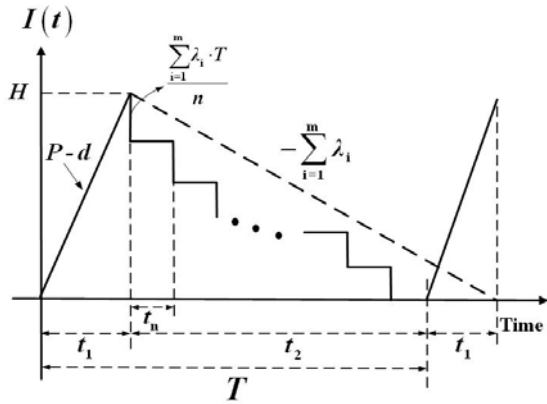


Fig. 1: On-hand inventory of perfect quality items in the producer side

2012b). Because little attention has been paid to the area of deriving the joint replenishment-distribution policy for a single-producer multi-retailer FPR model with scrap in production, this study intends to bridge the gap.

### MODELING AND ANALYSIS

This study considers a single-producer N-retailer integrated inventory system with scrap. It is assumed that the manufacturing process in producer's side may randomly produce an  $x$  portion of defective items at a production rate  $d$ . All items produced are screened and unit inspection cost is included in unit production cost  $C$ . All nonconforming items are assumed to be scrap; they are discarded at the end of production. Under the normal operation, to prevent shortages from occurring, the constant production rate  $P$  satisfies  $(P-d-\lambda) > 0$  or  $(1-x-\lambda/P) > 0$  and  $d$  can be expressed as  $d = Px$ . Unlike the classic FPR model assuming a continuous issuing policy for meeting demand, this research considers a multi-shipment policy. It is also assumed that the finished items can only be delivered to retailers if the whole lot is quality assured at the end of production.

Each retailer has its own annual demand rate  $\lambda_i$ . Fixed quantity  $n$  installments of the finished batch are delivered to multiple retailers synchronously at a fixed interval of time during the downtime  $t_2$  (Fig. 1). Cost parameters used in this study are as follows: the unit production cost  $C$ , unit disposal cost  $C_s$ , the production setup cost  $K$ , unit holding cost  $h$ , the fixed delivery cost  $K_{1i}$  per shipment delivered to retailer  $i$ , unit holding cost  $h_{2i}$  for item kept by retailer  $i$  and unit shipping cost  $C_i$  for item shipped to retailer  $i$ . Additional notation is listed as follows:

$Q$  = Production lot size per cycle, a decision variable (to be determined)

$n$  = Number of fixed quantity installments of the finished batch to be delivered to retailers for each cycle, a decision variable (to be determined)  
 $m$  = Number of retailers  
 $t_1$  = The production uptime for the proposed system  
 $H$  = Maximum level of on-hand inventory in units when regular production process ends  
 $t_2$  = Time required for delivering all quality assured finished products to retailers  
 $t_n$  = A fixed interval of time between each installment of finished products delivered during production downtime  $t_2$   
 $T$  = Production cycle length  
 $I(t)$  = On-hand inventory of perfect quality items at time  $t$

$I_c(t)$  = On-hand inventory at the retailers at time  $t$   
 $TC(Q, n)$  = Total production-inventory-delivery costs per cycle for the proposed system  
 $E[TCU(Q, n)]$  = Total expected production-inventory-delivery costs per unit time for the proposed system

From Fig. 1, the following equations can be directly obtained:

$$T = t_1 + t_2 \tag{1}$$

$$t_1 = \frac{Q}{P} = \frac{H}{P-d} \tag{2}$$

$$T = \frac{Q}{\lambda} (1-x) \tag{3}$$

$$t_2 = nt_n = Q \left( \frac{(1-x)}{\lambda} - \frac{1}{P} \right) \tag{4}$$

$$H = (P-d)t_1 = (P-d) \frac{Q}{P} = (1-x)Q \tag{5}$$

$$\lambda = \sum_{i=1}^m \lambda_i \tag{6}$$

The on-hand inventory of scrap items during production uptime  $t_1$  is:

$$dt_1 = Pxt_1 = xQ. \tag{7}$$

Cost for each delivery is:

$$\sum_{i=1}^m K_{1i} + \frac{1}{n} \left( \sum_{i=1}^m C_i \lambda_i T \right) \tag{8}$$

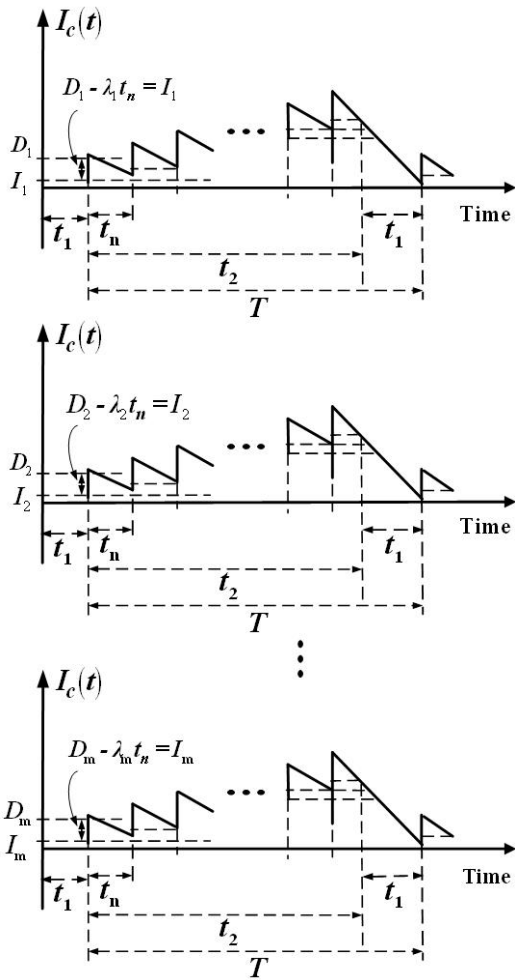


Fig. 2: On-hand inventory in  $m$  retailer sides

Total delivery costs for  $n$  shipments in a cycle are:

$$n \sum_{i=1}^m K_{li} + \sum_{i=1}^m C_i \lambda_i T \tag{9}$$

The variable holding costs for finished products kept by the manufacturer, during the delivery time  $t_2$  where  $n$  fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time, are as follows (Chiu *et al.*, 2009b):

$$h \left( \frac{n-1}{2n} \right) H t_2 \tag{10}$$

The variable holding costs for finished products kept by the customer during the delivery time  $t_2$ , are as follows (Fig. 2 and Appendix for details):

$$\frac{1}{2} \sum_{i=1}^m h_{2i} \lambda_i \left[ \frac{T t_2}{n} + T t_1 \right] \tag{11}$$

Total production-inventory-delivery cost per cycle  $TC(Q, n)$  consists of the setup cost, variable production cost, disposal cost, fixed and variable delivery cost, holding cost during production uptime  $t_1$  and holding cost for finished goods kept by both the manufacturer and the customer during the delivery time  $t_2$ . Therefore,  $TC(Q, n)$  is:

$$TC(Q, n) = CQ + K + C_s [xQ] + n \sum_{i=1}^m K_{li} + \sum_{i=1}^m C_i \lambda_i T + h \left[ \frac{H + dt_1}{2} (t_1) + \left( \frac{n-1}{2n} \right) H t_2 \right] + \frac{1}{2} \sum_{i=1}^m h_{2i} \lambda_i \left[ \frac{T t_2}{n} + T t_1 \right] \tag{12}$$

Because the scrap rate  $x$  during production is assumed to be a random variable with a known probability density function. In order to take the randomness of scrap rate into account, the expected values of  $x$  can be used in the cost analysis. Substituting all parameters from Eq. (1) to (11) in  $TC(Q, n)$  and with further derivations the expected cost  $E[TCU(Q, n)]$  can be obtained as follows:

$$E[TCU(Q, n)] = \frac{E[TC(Q, n)]}{E[T]} = \frac{C \sum_{i=1}^m \lambda_i}{(1-E[x])} + \frac{\left( K + n \sum_{i=1}^m K_{li} \right) \sum_{i=1}^m \lambda_i}{Q(1-E[x])} + \frac{C_s E[x] \sum_{i=1}^m \lambda_i}{(1-E[x])} + \sum_{i=1}^m C_i \lambda_i + \frac{hQ \sum_{i=1}^m \lambda_i}{2P(1-E[x])} + \left( \frac{n-1}{n} \right) \left( \frac{hQ \sum_{i=1}^m \lambda_i}{2} \right) \left[ \frac{(1-E[x])}{\sum_{i=1}^m \lambda_i} - \frac{1}{P} \right] + \left( \frac{n-1}{n} \right) \left( \frac{\sum_{i=1}^m h_{2i} \lambda_i Q}{2} \right) \left[ \frac{1}{P} \right] + \left( \frac{1}{n} \right) \frac{\sum_{i=1}^m h_{2i} \lambda_i Q (1-E[x])}{2 \sum_{i=1}^m \lambda_i} \tag{13}$$

### DERIVATION OF THE OPTIMAL SOLUTIONS

Proof of convexity: For the proof of convexity of  $E[TCU(Q, n)]$ , one can use the Hessian matrix equations (Rardin, 1998) and verify whether the following condition (Eq. (14)) holds:

$$\begin{bmatrix} Q & n \end{bmatrix} \begin{bmatrix} \frac{\partial^2 E[TCU(Q, n)]}{\partial Q^2} & \frac{\partial^2 E[TCU(Q, n)]}{\partial Q \partial n} \\ \frac{\partial^2 E[TCU(Q, n)]}{\partial Q \partial n} & \frac{\partial^2 E[TCU(Q, n)]}{\partial n^2} \end{bmatrix} \begin{bmatrix} Q \\ n \end{bmatrix} > 0 \tag{14}$$

From Eq. (13), one obtains:

$$\frac{\partial E[TCU(Q, n)]}{\partial Q} = - \frac{\left( K + n \sum_{i=1}^m K_{li} \right) \sum_{i=1}^m \lambda_i}{Q^2 (1-E[x])} + \frac{h \sum_{i=1}^m \lambda_i}{2P(1-E[x])} + \left( \frac{n-1}{n} \right) \left( \frac{h \sum_{i=1}^m \lambda_i}{2} \right) \left[ \frac{(1-E[x])}{\sum_{i=1}^m \lambda_i} - \frac{1}{P} \right] + \left( \frac{n-1}{n} \right) \left( \frac{\sum_{i=1}^m h_{2i} \lambda_i}{2} \right) \left[ \frac{1}{P} \right] + \left( \frac{1}{n} \right) \frac{\sum_{i=1}^m h_{2i} \lambda_i (1-E[x])}{2 \sum_{i=1}^m \lambda_i} \tag{15}$$

$$\frac{\partial^2 E [TCU(Q, n)]}{\partial Q^2} = \frac{2 \left( K + n \sum_{i=1}^m K_{1i} \right) \sum_{i=1}^m \lambda_i}{Q^3 (1-E[x])} \quad (16)$$

$$\frac{\partial E [TCU(Q, n)]}{\partial n} = \frac{\sum_{i=1}^m K_{1i} \cdot \sum_{i=1}^m \lambda_i}{Q(1-E[x])} - \left( \frac{Q}{2n^2} \right) \left( \sum_{i=1}^m h_{2i} \lambda_i - h \sum_{i=1}^m \lambda_i \right) \left[ \frac{(1-E[x])}{\sum_{i=1}^m \lambda_i} - \frac{1}{P} \right] \quad (17)$$

$$\frac{\partial^2 E [TCU(Q, n)]}{\partial n^2} = \left( \frac{Q}{n^3} \right) \left( \sum_{i=1}^m h_{2i} \lambda_i - h \sum_{i=1}^m \lambda_i \right) \left[ \frac{(1-E[x])}{\sum_{i=1}^m \lambda_i} - \frac{1}{P} \right] \quad (18)$$

$$\frac{\partial E [TCU(Q, n)]}{\partial Q \partial n} = - \frac{\sum_{i=1}^m K_{1i} \cdot \sum_{i=1}^m \lambda_i}{Q^2 (1-E[x])} - \left( \frac{1}{2n^2} \right) \left( \sum_{i=1}^m h_{2i} \lambda_i - h \sum_{i=1}^m \lambda_i \right) \left[ \frac{(1-E[x])}{\sum_{i=1}^m \lambda_i} - \frac{1}{P} \right] \quad (19)$$

Substituting Eq. (16), (18) and (19) in Eq. (14) one has:

$$[Q \ n] \cdot \begin{pmatrix} \frac{\partial^2 E [TCU(Q, n)]}{\partial Q^2} & \frac{\partial^2 E [TCU(Q, n)]}{\partial Q \partial n} \\ \frac{\partial^2 E [TCU(Q, n)]}{\partial Q \partial n} & \frac{\partial^2 E [TCU(Q, n)]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} = \frac{2K \sum_{i=1}^m \lambda_i}{Q(1-E[x])} > 0 \quad (20)$$

Equation (20) is resulting positive, because  $K, \lambda, (1-E[x])$  and  $Q$  are all positive. Hence,  $E [TCU(Q, n)]$  is a strictly convex function for all  $Q$  and  $n$  different from zero. Therefore, the convexity of  $E [TCU(Q, n)]$  is proved and there exists a minimum of  $E [TCU(Q, n)]$ .

**Derivation of the optimal policy:** To determine jointly the replenishment-distribution policy for the proposed single-producer multi-retailer integrated inventory system with scrap, one can solve the linear system of Eq. (15) and (17) by setting these partial derivatives equal to zero. Further derivations one obtains:

$$Q^* = \sqrt{\frac{2 \left( K + n \sum_{i=1}^m K_{1i} \right) \sum_{i=1}^m \lambda_i}{\frac{h \sum_{i=1}^m \lambda_i}{P} + \left( \frac{n-1}{n} \right) \left[ h(1-E[x])^2 + \left( \sum_{i=1}^m h_{2i} \lambda_i - h \sum_{i=1}^m \lambda_i \right) \left( \frac{1}{P} \right) (1-E[x]) \right]} + \frac{\sum_{i=1}^m h_{2i} \lambda_i (1-E[x])^2}{n \sum_{i=1}^m \lambda_i}} \quad (21)$$

and

$$n^* = \sqrt{\frac{K \left( \sum_{i=1}^m h_{2i} \lambda_i - h \sum_{i=1}^m \lambda_i \right) \left[ \frac{(1-E[x])^2}{\sum_{i=1}^m \lambda_i} - (1-E[x]) \left( \frac{1}{P} \right) \right]}{\sum_{i=1}^m K_{1i} \left\{ \frac{h \sum_{i=1}^m \lambda_i}{P} + \left( \sum_{i=1}^m h_{2i} \lambda_i - h \sum_{i=1}^m \lambda_i \right) \left( \frac{1}{P} \right) (1-E[x]) \right\}}} \quad (22)$$

It is noted that although in real-world situation the number of deliveries takes integer values only, Eq. (22) results in a real number. In order to find the integer value of  $n^*$  that minimizes the expected system cost, two adjacent integers to  $n$  must be examined respectively (Chiu *et al.*, 2009b). Let  $n^+$  denote the smallest integer greater than or equal to  $n$  (derived from Eq. (22)) and  $n^-$  denote the largest integer less than or equal to  $n$ . Substitute  $n^+$  and  $n^-$  respectively in Eq. (21), then applying the resulting  $(Q, n^+)$  and  $(Q, n^-)$  in Eq. (13) respectively. By selecting the one that gives the minimum long-run average cost as the optimal replenishment-distribution policy  $(Q^*, n^*)$ . A numerical example demonstrating their practical usages is provided in next section.

### NUMERICAL EXAMPLE

Assume a producer can manufacture a product at an annual rate of 60,000 units. Annual demands  $\lambda_i$  of this item from 5 different retailers are 400, 500, 600, 700 and 800 respectively (total demand is 3000/year). There is a random scrap rate during production uptime which follows a uniform distribution over the interval (0, 0.3). Values for additional parameters are:

- $h$  = Unit holding cost per item at the producer side, \$20 per item per year
- $K$  = \$35000 per production run
- $C$  = \$100 per item
- $C_S$  = \$20, disposal cost per scrap item
- $K_{1i}$  = The fixed delivery cost per shipment for retailer  $i$ , they are \$ 100, 200, 300, 400 and 500, respectively

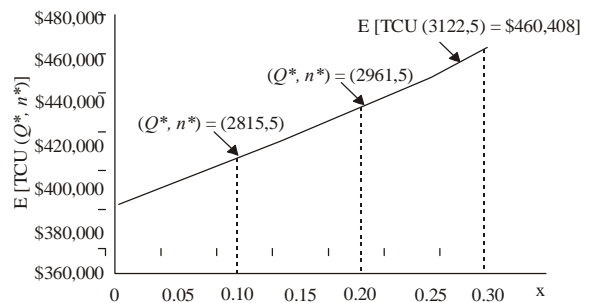


Fig. 3: Variation of random scrap rate effects on the  $(Q^*, n^*)$  policy and  $E [TCU(Q^*, n^*)]$

$h_{2i}$  = unit holding cost for item kept by retailer  $i$ , they are \$75, 70, 65, 60 and 55 per item, respectively  
 $C_i$  = unit transportation cost for item delivered to retailer  $i$ , they are \$0.5, 0.4, 0.3, 0.2 and 0.1, respectively

From Eq. (21), (22) and (13), one obtains the optimal number of delivery  $n^* = 5$ , the optimal replenishment lot size  $Q^* = 3122$  and total expected cost  $E [TCU(Q^*, n^*)] = \$ 460,408$ . It is noted that number of delivery should practically be an integer number while Eq. (22) results 5.39, by applying the aforementioned  $(Q, n^+)$  and  $(Q, n^-)$  in Eq. (13) respectively, one finds the optimal number of delivery  $n^* = 5$ . Variation of random scrap rate effects on the optimal  $(Q^*, n^*)$  policy and on the system cost  $E [TCU(Q^*, n^*)]$  are depicted in Fig. 3. It is noted that as the random scrap rate  $x$  increases, the value of the system cost  $E [TCU(Q^*, n^*)]$  increases significantly.

**CONCLUDING REMARKS**

In real world business environments, it is common to have a producer supplied a product to its multiple retailers. In such a specific internal supply chains, management would like to figure out a best replenishment-distribution policy in order to minimize total system costs. This study investigates the optimal replenishment-distribution policy for such a single-producer multi-retailer integrated inventory system with scrap rate in production. Mathematical modeling and analysis are used. The renewal reward theorem is employed to cope with the variable cycle length (due to the randomness of scrap rate in production). The long-run expected production-inventory-delivery cost per unit time for the proposed system model is derived and its convexity is proved. Then, a closed-form solution of the optimal replenishment-distribution policy for such a single-producer multi-retailer integrated system is obtained. Effects of various scrap rates on the optimal solution are also examined. One interesting topic for future research will be to investigate the effect of rework on the optimal replenishment-distribution policy in such a specific supply chains.

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**APPENDIX**

From Fig. 1,  $n$  installments (fixed quantity  $D$ ) of the finished lot are delivered to customer at a fixed interval of time  $t_n$ , one has:

$$D_i = \frac{H}{n} \tag{A1}$$

$$t_n = \frac{t_2}{n} \tag{A2}$$

From Fig. 2, the computations of retailers' holding cost during  $t_2$  (Eq. (11)) are as follows:

$$\begin{aligned} & h_{21} \left[ \frac{n(D_1 - I_1)t_n + \frac{n(n+1)}{2} I_1 \cdot t_n + \frac{nI_1(t_1)}{2} \right] \\ & + h_{22} \left[ \frac{n(D_2 - I_2)t_n + \frac{n(n+1)}{2} I_2 \cdot t_n + \frac{nI_2(t_1)}{2} \right] \\ & + \dots + h_{2m} \left[ \frac{n(D_m - I_m)t_n + \frac{n(n+1)}{2} I_m \cdot t_n + \frac{nI_m(t_1)}{2} \right] \\ & = \sum_{i=1}^m h_{2i} \left[ \frac{n(D_i - I_i)t_n + \frac{n(n+1)}{2} I_i \cdot t_n + \frac{nI_i(t_1)}{2} \right] \\ & = \sum_{i=1}^m h_{2i} \left[ n(D_i - I_i)t_n + n(n+1)I_i t_n + nI_i(t_1) \right] \end{aligned} \tag{A3}$$

Because  $n$  installments (fixed quantity  $D$ ) of the finished lot are delivered to customer at a fixed interval of time  $t_n$ , one has the following:

$$t_1 = \frac{nI_i}{\lambda_i} \tag{A4}$$

$$D_i = \lambda_i t_n + I_i \tag{A5}$$

where,  $I_i$  denotes number of left over items for each retailer after demand during each fixed interval of time  $t_n$  has been satisfied (Fig. 2). Eq. (A3), the retailers' holding cost during  $t_2$  becomes:

$$\begin{aligned} & \frac{1}{2} \sum_{i=1}^m h_{2i} \left[ (D_i - I_i)t_2 + (n+1)I_i t_2 + \lambda_i(t_1)^2 \right] \\ & = \frac{1}{2} \sum_{i=1}^m h_{2i} \left[ (\lambda_i t_n + I_i)t_2 + (\lambda_i t_1)t_2 + \lambda_i(t_1)^2 \right] \\ & = \frac{1}{2} \sum_{i=1}^m h_{2i} \left[ \frac{\lambda_i t_2}{n} + \lambda_i(t_1)T \right] \end{aligned} \tag{A6}$$

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