International Journal of Engineering & Technology, 7 (4.10) (2018) 476-479

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

A Single Server Queue with Immediate Feedback, Working Vacation and Server Breakdown

Varalakshmi M, Chandrasekaran V M*, Saravanarajan M C

Department of Mathematics, School of Advanced Sciences,
VIT University, Vellore, Tamilnadu, India.
**Corresponding author E-mail: vmcsn@vit.ac.in*

Abstract

This paper deals with analyze of a single server queueing system with immediate feedbacks and working vacation. Upon arrival if the customer sees the server to be busy then it joins the tail end of queue. Otherwise if server is idle, the customer gets into service. After completion of service, the customer is allowed to make an immediate feedback in finite number. Busy server may fail for a short interval of time. Using supplementary variable technique the steady state results are deduced. Some system performance measures are discussed

Keywords: *Immediate feedback, working vacation and breakdown.*

1. Introduction

A considerable amount of work has been done in the past in queueing system models with in working vacation period, the server gives service to the customer at lower service rate. This queueing system has subsequent application in the field of mailing service, file transfer and network service. M/M/1 working vacation queue has been introduced by Servi and Finn [6].
In the history of queue, Kalidass and Kasturi [2] contributed different way to the theory of feedback called immediate feedback. That is if arrival wants service one more time, it will immediately get into service once again without joining the queue after receiving first service. For further details on queues with feedback and server down refer Takacs [7], Krishnakumar et al. [3], Choudhury and Paul [1], Rajadurai et al. [[4], [5]], Varalakshmi et al. [[9], [10]].
This model has application in Computer Processing System, LAN, Simple Mail Transfer Protocol, etc. Whatever remains of the sections are given in short. The model portrayed numerically is given in section 2. The steady-state results and performance measures are talked about in section 3. Unique cases are discussed in section 4.The conclusion and the uses of the model considered are conveyed in section 5.

2. Description of the Model

In this section, "we develop a model for an unreliable M/G/1 queue with immediate feedback and working vacation. The detailed description of the model is given as follows:

- The new customers arrive at the system according to a Poisson process of rate λ.
- We assume that there is a waiting space and therefore if an arriving customer finds the server being busy or on repair, the customer oblique to join the tail end of the queue.
- The service discipline for new arriving customers is first come first served (FCFS). The service time is denoted by the random variable S with distribution function $S(t)$ and LST $S^*(\vartheta)$.
- When the queue is empty at the service completion epoch, the server goes for working vacation and the vacation time follows exponential distribution with the parameter θ. During vacation period, the server provides service at lower rate if any customers arrive. If any customers are present at a service completion instant in the vacation period, the server will stop the vacation and comes back to the normal busy period which is referred as vacation interruption. Otherwise, the server continues the vacation.
- At the completion epoch of the vacation, if any customer is found in the orbit, the server switches to regular service else, the server goes for another vacation. The service time during working vacation follows general random variable S_v with distribution function $S_v(t)$; LST $S_v^*(x)$ and the first moment is given by $S_v^*(\theta)=\int_0^\infty xe^{-\theta x}dS_v(x)$. During working vacation, no feedback is allowed.
- While the server is of servicing, it may breakdown at any time and the service channel will fail for a short interval of time i.e. server is down for a short interval of time. The breakdown rate is β.
- As soon as breakdown occurs the server is sent for repair, during that time the server stops providing service. The customer who was just being served before server breakdown waits for the remaining service to complete. The repair time (denoted by H) distribution of the server is assumed to be arbitrarily distributed with distribution function $G(t)$ and LST $G^*(\vartheta)$.
- Various stochastic processes involved in the system are assumed to be independent of each other.

In the steady state, we assume that $S_b(0) = 0$, $S_b(\infty) = 1$, $S_v(0) = 0$, $S_v(\infty) = 1$ are continuous at $x = 0$ and $H(0) = 0$, $H_i(\infty) = 1$ are continuous at $y = 0$.
So that the function $\mu_b(x)$, $\mu_v(x)$ and $\xi(y)$ are the conditional completion rates for service on regular busy, working vacation and repair time respectively.

$$\mu_b(x)dx=\frac{dS_b(x)}{1-S_b(x)}\ \ \mu_v(x)dx=\frac{dS_v(x)}{1-S_v(x)}\ \ \xi(y)dy\ =\frac{dH(y)}{1-H(y)}.$$

Also let $S_b^0(t)$ = the elapsed regular service time, $S_v^0(t)$ = the elapsed working vacation time and $H^0(t)$ = the elapsed repair time. The state of the system at time *t* can be described by the bivariate Markov process $\{C(t),\ N(t);\ t\geq 0\}$ where $C(t)$ denotes the server state (0,1,2,3,4,) depending if server is idle on normal busy and working vacation period, busy on regular service and working vacation period and under repair respectively. $N(t)$ denotes the number of customers in the system at time *t*.

If $C(t)=1$, and $N(t)\geq 0$ then $S^0(t)$ corresponds to the elapsed time of the customer being served on FPS. If $C(t)=2$ and $N(t)\geq 0$ then $S_b^0(t)$ corresponds to the elapsed service time of the customer being served during regular busy period. If $C(t)=3$, and $N(t)\geq 0$, then $S_v^0(t)$ represents the elapsed time of the customer being served in lower rate service period. If $C(t)=4$, and $N(t)\geq 0$, then $G^0(t)$ represents elapsed time of the server being repaired.

Let $\{t_n;\ n = 1,2,\dots\}$ be the sequence of epochs at which either a service period completion occurs or a vacation time ends. The sequence of random vectors $Z_n=\{C(t_n+),\ N(t_n+)\}$ forms a Markov chain which is embedded in the retrial queueing system.

The embedded Markov chain $\{Z_n;\ n\in N\}$ is ergodic if and only if $\rho< 1$ for our system to be stable, where

$$\rho=(1+\alpha_1+\alpha_1\alpha_2+.....+\alpha_1\alpha_2....\alpha_{m-1})\lambda E(S_b)(1+\alpha E(H))$$

3. Analysis of Steady State Probability Distribution

In this section, we first develop the steady state difference-differential equations for our queueing system by treating the elapsed service times, the elapsed vacation times and the elapsed repair times as supplementary variables. Then we derive the probability generating functions for the server state and the number of customers in the system and in the queue.

For the process $\{N(t),\ t\geq 0\}$, we define the probabilities $P_0(t)=P\{C(t)=0,\ N(t)=0\}$ and the probability densities (for $0\leq j\leq m$-1)

$$Q_{j_b,n}(x,t)dx=P\left\{C(t)=2,N(t)=n,\ x\leq S_b^0(t)<x+dx\right\},\ \text{for } t\geq 0,\ x\geq 0 \text{ and } n\geq 1,$$

$$\Omega_{v,n}(x,t)dx\ =P\left\{C(t)=3,\ N(t)=n,\ x\leq (t)S_v^0<x+dx\right\},\ \text{for } t\geq 0,\ x\geq 0 \text{ and } n\geq 0,$$

$$R_{j,n}(x,y,t)dy=P\left\{C(t)=4,\ N(t)=n,\ y\leq G^0(t)<y+dy/S_b{}^0(t)=x\right\},$$

$$\text{for } t\geq 0,\ (x,y)\geq 0 \text{ and } n\geq 0,$$

We assume that the stability condition is fulfilled in the sequel and so that we can set $t\geq 0, x\geq 0, y\geq 0.$ $(0\ \leq j\leq m-1)$

$$P_0=\lim_{t\to\infty}P_0(t),\ Q_{j_b,n}(x)=\lim_{t\to\infty}Q_{j_b,n}(x,t),\ \Omega_{v,n}(x)=\lim_{t\to\infty}\Omega_{v,n}(x,t),$$

$$R_{j,n}(x,y)=\lim_{t\to\infty}R_{j,n}(x,y,t),$$

By the method of supplementary variable technique, we obtain the following system of equations for the values $0\leq j\leq m$-1,

$$(\lambda+\theta)P_0\ =\left[\sum_{l=0}^{m-2}\bar{\alpha}_{l+1}\int_0^\infty Q_{l_b,0}(x)\mu_b(x)dx+\int_0^\infty Q_{m-1_b,0}(x)\mu_b(x)dx\right]+\int_0^\infty\Omega_{v,0}(x)\mu_v(x)dx+\theta P_0 \tag{1}$$

$$\frac{dQ_{j_b,n}(x)}{dx}+\left(\lambda+\mu_b(x)+\beta\right)Q_{j_b,n}(x)\ =\ \lambda Q_{j_b,n-1}(x)+\int_0^\infty R_n(x.y)dy \tag{2}$$

$$\frac{d\Omega_{v,n}(x)}{dx}+\left(\lambda+\mu_v(x)+\theta\right)\Omega_{v,n}(x)\ =\ \lambda\Omega_{v,n-1}(x) \tag{3}$$

$$\frac{dRj,_n(x,y)}{dy}+\left(\lambda+\xi(y)\right)R_{j,n}(x,y)\ =\ \lambda R_{j,n-1}(x,u)\ n\geq 1 \tag{4}$$

The steady state boundary conditions at x=0 are

$$Q_{j_b,n}(0)\ =\left[\sum_{l=0}^{m-2}\bar{\alpha}_{l+1}\int_0^\infty Q_{l_b,n+1}+(x)\mu_b(x)dx+\int_0^\infty Q_{m-1_b,n+1}(x)\mu_b(x)dx\right]+\int_0^\infty\Omega_{v,n}(x)\mu_v(x)dx+\theta\int_0^\infty\Omega_{v,n-1}(x)dx, \tag{5}$$

$$Q_{j_b,n}(0)\ =\alpha_j\int_0^\infty Q_{j-1_b,n}(x)\mu_b(x)dx,\ \text{j}=1,2,3,...,m-1 \tag{6}$$

$$\Omega_{v,n}(0)\ =\begin{Bmatrix}\lambda P_0, n=0\\ 0, n\geq 1\end{Bmatrix} \tag{7}$$

$$R_{j,n}(x,0)\ =\beta Q_{j_b,n}(x)dx, n\geq 0,\ j=0,1,2,...,m-1. \tag{8}$$

The normalizing condition is

$$P_0+\sum_{n=0}^\infty\left(\sum_{j=0}^{m-1}\left(\int_0^\infty Q_{j_b,n}(x)dx+0\int_0^\infty\int R_{j,n}(x,y)dxdy\right)+\int_0^\infty\Omega_{v,n}(x)dx\right)=1 \tag{9}$$

3.1. The Steady State Solutions

The probability generating function technique has been employed to obtain the steady state solution for the queueing model considered. In order to solve the system equations, let us define the following generating functions for $|z|\leq 1$, for $0\leq j\leq m-1$.

$$Q_{j_b}(x,z)=\sum_{n=1}^\infty Q_{j_b,n}(x)z^n;\ Q_{j_b}(0,z)=\sum_{n=1}^\infty Q_{j_b,n}(0)z^n;\ \Omega_v(x,z)=\sum_{n=0}^\infty\Omega_{v,n}(x)z^n\ ;$$

$$\Omega_v(0,z)=\sum_{n=0}^\infty\Omega_{v,n}(0)z^n\ ;R_j(x,y,z)=\sum_{n=1}^\infty R_{j,n}(x,y)z^n\ ;\ R_j(x,0,z)=\sum_{n=1}^\infty R_{j,n}(x,0)z^n;$$

Now multiplying the steady state equations and steady state boundary conditions (2) to (8) by z^n and summing over *n*, $(n=\ 1,2,...,\infty)$ and solving the partial generating functions, we get the limiting PDFs $Q_{j_b}(x,z),\Omega_v(x,z)$ and $R_j(x,y,z)$. Next we are interested in investigating the marginal queue size distribution due to system state of the server.

Theorem 1

Under the stability condition ρ<1, the marginal probability distributions of the number of customers in the system when the server is idle, busy on regular service, busy on working vacation and under repair are given by

$$Q_{0_b}(z) = \frac{\lambda P_0\left(1 - S_b^*\left(A(z)\right)\right)}{A(z)}\left\{\frac{S_v^*\left(A_v(z)\right) + z\theta V(z) - 1}{z - \Phi(z)}\right\} \quad (10)$$

$$Q_{j_b}(z) = \left(\prod_{j=1}^{m-1}\alpha_j\left(S^*\left(A(z)\right)\right)^j\right)\frac{\lambda P_0\left(1 - S_b^*\left(A(z)\right)\right)}{A(z)}\left\{\frac{S_v^*\left(A_v(z)\right) + z\theta V(z) - 1}{z - \Phi(z)}\right\} \quad (11)$$

$$\Omega(z) = \lambda P_0 V(z) \quad (12)$$

$$R_j(z) = \left(\prod_{j=0}^{m-1}\alpha_j\left(S^*\left(A(z)\right)\right)^j\right)\frac{\lambda\beta P_0\left(1 - S_b^*\left(A(z)\right)\right)\left(1 - H^*\left(b(z)\right)\right)}{A(z)b(z)} \times\left\{\frac{S_v^*\left(A_v(z)\right) + z\theta V(z) - 1}{z - \Phi(z)}\right\} \quad (13)$$

Where

$$P_0 = \frac{\left(1 - \Phi'(1)\right)}{\left(1 - \Phi'(1)\right) - \lambda\left(\left(S_v^*(\theta)/\theta\right) + 2\Phi(1)E(S)\left(1 + \beta E(G)\right)\right)} \quad (14)$$

$$A(z) = b(z) + \beta[1 - H^*(b(z))],\ A_v(z) = \theta + u(z)$$

$$b(z) = \lambda(1 - z);$$

$$\rho = \Phi'(1) = \left\{1 + \alpha_1 + \alpha_1\alpha_2 + ... + \alpha_1\alpha_2...\alpha_{m-1}\right\}\lambda E(S)\left(1 + \beta E(G)\right)$$

$$\Phi(z) = \left\{(1 - \alpha_1)S^*(A(z)) + \sum_{l=1}^{m-1}(1 - \alpha_{l+1})\alpha_1....\alpha_l\left(S^*(A(z))\right)^{l+1}\right\},\ \alpha_m = 0$$

$$V(z) = \left(1 - S_v^*\left(A_v(z)\right)\right)\Big/A_v(z)$$

Proof. Integrating the above partial generating functions w.r.to x and y, then the partial PGFs

$$Q_{j_b}(z) = \int_0^\infty Q_{j_b}(x,z)dx;\ \Omega_v(z) = \int_0^\infty \Omega_v(x,z)dx;$$

$$R_j(x,z) = \int_0^\infty R_j(x,y,z)dy;\ R_j(z) = \int_0^\infty R_j(x,z)dx$$

can be obtained. By using the normalized condition the probability of server being idle P0 can be determined by setting z=1 in the-above equations $\left(P_0 + \Omega_v(1) + \sum_{j=0}^{m-1}\left(Q_j(1) + R_j(1)\right) = 1\right)$ and applying L'Hôpital's rule whenever necessary."

Corollary 1

The probability generating function of number of customers in the system is

$$K_s(z) = P_0 + \Omega_v(z) + \sum_{j=0}^{m-1}\left(Q_{j_b}(z) + R_j(z)\right)$$

$$= \frac{P_0}{\left(1 - z\right)z - \Phi(z)}\left[\begin{array}{l}\left[(1 - z)\left(z - \Phi(z)\right) + V(z)\right] \\ + \left\{\begin{array}{l}\sum_{j=0}^{m-1}\left(\prod_{j=0}^{m-1}\alpha_j\left(S^*\left(A(z)\right)\right)^j\right) \\ \left(1 - S_b^*\left(A(z)\right)\right)\left(S_v^*\left(A_v(z)\right) + z\theta V(z) - 1\right)\end{array}\right\}\end{array}\right]$$

Corollary 2

The mean number of customers in the sysem (L_s) and the mean waiting time of a customer (W_s) in the system are given by

$$L_s = \left[\frac{dK(z)}{dz}\right]_{z=1}$$

$$L_q = L_s - \rho$$

$$W_s = \frac{L_s}{\lambda}$$

4. Special Cases

Some special cases of the considered model are discussed in this section, which are consistent with the existing literature

Case (i):

No immediate feedback, No vacation and No breakdown

Let $\beta = 0$, $\theta = 0$ and $\alpha_j = 0$ our model can be reduced to M/G/1 queue.

$$K(z) = \frac{\left[1 - \lambda E(S)\right]S^*\left[\lambda - \lambda z\right]\left[z - 1\right]}{z - \left\{S^*\left[\lambda - \lambda z\right]\right\}}$$

The above form and result agree with Takagi [8].

Case (ii):

Working vacation, No retrial, No immediate feedback and No breakdown

Let $\alpha_j = \theta = 0$; our model can be reduced to M/G/1 queue with working vacation policy. The result coincides with Zhang and Hou [11].

$$K(z) = \frac{\left(1 - \lambda E(S)\right)}{\left(1 - \lambda E(S)S_v^*(\theta) + \frac{\lambda}{\theta}(1 - S_v^*(\theta)\right)} \times \frac{\left\{\begin{array}{l}(1 - z)\left\{\left(z - \left(S_b^*(A(z))\right)\right)(1 + \lambda V(z))\right\} \\ + z\left[(1 - S_b^*(A(z)))\left[z\theta V(z) + \left(S_v^*\left(A_v(z)\right) - 1\right)\right]\right]\end{array}\right\}}{(1 - z)\left(z - S_b^*(A(z))\right)}$$

5. Conclusion

In this paper, we analyzed an unreliable queueing system with immediate feedbacks under working vacation. The steady state results are found by using supplementary variable technique. Some performance measures were deduced. The results discover applications in mailing system, ATMs, software design, production lines and satellite communication.

References

[1] Choudhury G & Paul, M (2005), A two phase queueing system with Bernoulli feedback. Journal of Information and Management Sciences, 16(1), 35-52.

[2] Kalidass, K & Kasturi R (2014), A two phase service M/G/1queue with a finite number of immediate Bernoulli feedbacks. OPSEARCH, 51(2), 201-218.

[3] Krishna Kumar B, Vijayakumar A & Arivudainambi D (2002), The M/G/1 retrial queue with Bernoulli schedules and general retrial times. Computers and Mathematics with Applications, 43, 15-30.

[4] Rajadurai P, Saravanarajan MC & Chandrasekaran VM (2014), Analysis of an $M^{[X]}$/(G1,G2)/1 retrial queueing system with balking, optional re-service under modified vacation policy and service interruption. Ains Shams Engineering Journal, 5, 935-950.

[5] Rajadurai P, Varalakshmi M, Saravanarajan MC & Chandrasekaran VM (2015), Analysis of $M^{[X]}$/G/1 retrial queue with two phase service under Bernoulli vacation schedule and random breakdown. International Journal of Math. Oper. Res., 7(1), 19-41.

[6] Servi LD & Finn SG (2002), M/M/1 queues with working vacations (M/M/1/WV). Performance Evaluation, 50, 41–52.

[7] Takacs L (1963), A Single Server Queue with Feedback. Bell Syst. Tech. journal, 42, 505-519.

[8] Takagi H (1991), Queueing Analysis, Vol. 1, North Holland, Amsterdam, New York.

[9] Varalakshmi M, Rajadurai P, Saravanarajan MC & Chandrasekaran VM (2016), Analysis of an $M^{[x]}$/(G1,G2)/1 Retrial Queue with

Feedback, K-Optional Vacation and Server Breakdowns. Global J. Pure and Appl. Math., 12(1), 436-441.
[10] Varalakshmi M, Saravanarajan MC & Chandrasekaran VM (2017), A study on M/G/1 retrial G-queue with two phase of service, immediate feedback and working vacation. IOP Conference Series: Materials Science and Engineering, 263, 042156.
[11] Zhang M & Hou Z (2012), M/G/1 queue with single working vacation. *Journal* of *Applied Mathematics* and *Computing*, 39(1-2), 221–234.