A Small-Sample Correction for Testing for gth-Order Serial Correlation with Artificial Regressions

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Monte Carlo experiments establish that the usual "t-statistic" used for testing for first-order serial correlation with artificial regressions is far from being distributed as a Student's t in small samples. Rather, it is badly biased in both mean and variance and results in grossly misleading tests of hypotheses when treated as a Student's t. Simply computed corrections for the mean and variance are derived, however, which are shown to lead to a transformed statistic producing acceptable tests. The test procedure is detailed and exemplar code provided.

1. Introduction

Artificial regression has been proposed [Durbin (1970), Breusch (1978), Godfrey (1978a and b), MacKinnon (1992), Davidson and MacKinnon (1993)] as a simple means for testing for the presence of serial correlation in the error structure of the standard regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta}+\boldsymbol{\varepsilon}$. To test, for example, for simple gth-order serial correlation, one merely regresses $\mathbf{y}$ on $\mathbf{X}$, lags the residuals $\mathbf{e}$ from this regression g times to get $\mathbf{e}_{-g}$, and then regresses $\mathbf{y}$ on $\mathbf{X}$ and $\mathbf{e}_{-g}$. The t statistic t_c of the estimated coefficient c of $\mathbf{e}_{-g}$ provides the desired test.

While t_c is known asymptotically to be distributed as $N(0, 1)$ under the null of no serial correlation [Durbin (1970)], its small-sample distribution is little known. General practice, however, clearly presumes it to be distributed as a Student's t with appropriate degrees of freedom. In this paper, we first use Monte Carlo experiments to demonstrate this presumption to be false. Relative to a Student's t distribution, t_c tends to be biased downwards, have lower-than-expected variance, be skewed, and have varying degrees of kurtosis. All of this results in actual test sizes, based on the assumed relevance of the standard t-test, that are asymmetric and badly misleading. For example, relative to the 5% expected in each tail when using the 10% two-tailed critical values for a Student's t, a typical outcome when testing for first-order serial correlation might show actual test sizes of 3.5% in the lower tail and 0.3% in the upper.

There are three ways to proceed in such a situation. One could try to derive the actual small-sample distribution of t_c, one could use Monte Carlo techniques to establish appropriate critical values, or one could attempt to transform t_c to behave more nearly like a Student's t. The first option is available less often than we would like, but the second and third are always possibilities. The second is adopted, for example, by Dickey and Fuller (1981) in their test for random walks, but, to be useful, this option requires either stability across cases or many runs (summarized in many tables) under differing situations. Here we adopt the third option, for we are able to find simply computed estimates of the small-sample biases both for the mean and variance. Once corrected for these, t_c becomes a quite well-behaved statistic, still showing some asymmetry, but having acceptable sizes in both tails. The use of a 10% two-tailed critical value in testing for first-order serial correlation now results in actual sizes, say in a typical example, of 5.5% in the lower tail and 4.5% in the upper. Furthermore, this correction, with simple modifications for extreme cases, appears remarkably effective for all of the widely different test models, orders of serial correlation, and degrees of freedom (with some minor qualifications). Monte Carlo experiments are used to show both the effectiveness of the small-sample corrections and the suitability of the corrected t_c for small-sample tests.

The next section derives the artificial-regression test procedure for simple gth order serial correlation. Section 3 presents Monte Carlo results showing the inappropriateness of small-sample tests based on the assumption that t_c is distributed as a Student's t. Section 4 introduces a simply effected correction for t_c that alleviates this problem and presents Monte Carlo results demonstrating its value. Section 5 provides simple modifications for certain extreme cases. Section 6 briefly examines an alternative, asymptotically equivalent test. Section 7 presents step-by-step instructions for carrying out the procedure. Appendices contain formal derivations of the relevant statistics and correction factors, discuss computational considerations, and provide appropriate *Mathematica* code.

2. The Test Procedure

Assume a standard linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ with a disturbance term $\boldsymbol{\varepsilon}$ that has simple gth-order serial correlation, i.e.,

$$y_n = \mathbf{x}_n^T\boldsymbol{\beta} + \varepsilon_n, \qquad n = 1, \ldots, N, \tag{2.1}$$

$$\varepsilon_n = \rho\varepsilon_{n-g} + u_n, \tag{2.2}$$

where the y_n are the N elements of $\mathbf{y}$, the $\mathbf{x}_n^T$ are the rows of the $N \times K$ data matrix $\mathbf{X}$, u_n is an iid disturbance term with mean zero and variance σ_u^2, and $|\rho| < 1$. Substituting (2.2) into (2.1) and using (2.1) lagged g times gives

$$y_n = \mathbf{x}_n^T\boldsymbol{\beta} + \rho(y_{n-g} - \mathbf{x}_{n-g}^T\boldsymbol{\beta}) + u_n, \tag{2.3}$$

an equation with a well-behaved disturbance term, but nonlinear in the parameters $\boldsymbol{\beta}$ and ρ. A first-order expansion of (2.3) about arbitrary $(\boldsymbol{\beta}_0, \rho_0)$ gives the Gauss-Newton Regression (GNR)

$$y_n - \mathbf{x}_n^T\boldsymbol{\beta}_0 - \rho_0(y_{n-g} - \mathbf{x}_{n-g}^T\boldsymbol{\beta}_0)$$

$$= (\mathbf{x}_n^T - \rho_0 \mathbf{x}_{n-g}^T)(\boldsymbol{\beta} - \boldsymbol{\beta}_0) + (y_{n-g} - \mathbf{x}_{n-g}^T \boldsymbol{\beta}_0)(\rho - \rho_0) + v_n, \tag{2.4}$$

where v_n includes the higher-order expansion terms along with u_n.

Under the null hypothesis $H_0{:}\, \rho = 0$, $\boldsymbol{\beta}$ is consistently and efficiently estimated with the ordinary least-squares estimator $\mathbf{b}_{OLS} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$ from (2.1) alone. Evaluating (2.4) at $(\boldsymbol{\beta}_0, \rho_0) = (\mathbf{b}_{OLS}, 0)$ gives the artificial regression

$$e_n = \mathbf{x}_n^T \boldsymbol{\delta} + \gamma e_{n-g} + v_n, \qquad n = g+1, \ldots, N, \tag{2.5}$$

where $\boldsymbol{\delta} \equiv \boldsymbol{\beta} - \mathbf{b}_{OLS}$, $\gamma \equiv \rho$, and the $e_n \equiv y_n - \mathbf{x}_n^T \mathbf{b}_{OLS}$ are simply the OLS residuals from the $\mathbf{y}$'s regressed on the $\mathbf{x}$'s. It is proposed [MacKinnon 1992] that we test $H_0{:}\, \rho = 0$ through a standard test of the hypothesis that $\gamma = 0$; that is, we simply (a) regress the y's on the $\mathbf{x}$'s, (b) take the residuals e_n from this regression and regress them in turn on the same set of $\mathbf{x}$'s along with their own lagged values e_{n-g}, and then (c) test whether the OLS estimate c of the coefficient γ of the lagged residuals in this regression is significantly different from zero. Asymptotically, a t-test based on a Student's t distribution with appropriate degrees of freedom is relevant for this purpose, but, of course, interest centers here on whether this remains so for truly small samples.

As a practical matter, it makes no difference whether e_n or y_n appears on the left-hand side of this artificial regression. Since $e_n \equiv y_n - \mathbf{x}_n^T \mathbf{b}_{OLS}$, substituting y_n for e_n would result in different estimates for $\boldsymbol{\delta}$, but would have no effect on c or its variance, and hence on the value of t_c, the standard t-statistic for c. This latter statistic is derived formally in Appendix C, but in essence it is determined as follows: Let $\mathbf{X}_{[g]}$ be the $(N-g) \times K$ matrix that results from deleting the first g rows from $\mathbf{X}$, and let $\mathbf{e}_{-g}$ be the $(N-g)$-vector obtained by lagging the residual vector $\mathbf{e} \equiv y - \mathbf{X}\mathbf{b}_{OLS}$ g times (i.e., removing its last g elements). Define the $(N-g) \times (K+1)$ matrix $\mathbf{Z} \equiv [\mathbf{X}_{[g]}\ \mathbf{e}_{-g}]$, and let s_c^2 denote the $(K+1)$st diagonal element of $s^2(\mathbf{Z}^T\mathbf{Z})^{-1}$, where $s^2 \equiv \hat{\mathbf{v}}^T\hat{\mathbf{v}}/(N-K-g-1)$, $\hat{\mathbf{v}}$ being the residuals from the artificial regression of $\mathbf{y}_{[g]}$ ($\mathbf{y}$ with its first g rows deleted) on $\mathbf{Z}$, i.e., $\mathbf{y}_{[g]} \equiv \mathbf{X}_{[g]}\mathbf{d} + c\mathbf{e}_{-g} + \hat{\mathbf{v}}$. Then

$$t_c = \frac{c}{s_c}. \tag{2.6}$$

3. The non-Studentness of t_c in small samples

Monte Carlo experiments are used to demonstrate the divergence of the small-sample distribution of t_c from a Student's t. A Student's t with r degrees of freedom, we recall, is a symmetric distribution with mean zero and variance $r/(r-2)$. Relative to this, we find for small samples that t_c is biased (often severely), has depressed variance, and is asymmetric. Monte Carlo experiments for this use are quite straightforward. Twenty-two models, differing in the structure of the data comprising $\mathbf{X}$, are examined for $N = 20$, a very small sample (leading to even smaller degrees of freedom), and $N = 60$, a moderately small sample. 10,000 replications are used with the smaller sample size and 5,000 with the larger. For each replication one must essentially

(a) generate the N elements of $\boldsymbol{\varepsilon}$ as iid normals with mean zero and variance σ^2,

(b) derive $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$,

(c) determine $\mathbf{e} \equiv \mathbf{y} - \mathbf{X}\mathbf{b}_{OLS} = \mathbf{M}\mathbf{y} \equiv \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right)\mathbf{y}$,

(d) calculate t_c as explained at the end of the previous section.

The value of t_c is readily shown to be invariant to the choice of $\boldsymbol{\beta}$ or σ^2, so the experiments need not explore these dimensions. The results, however, could depend upon the order g of serial correlation being tested for and upon the character and numbers of the $\mathbf{X}$ data. Experiments are conducted for five values of g: 1, 2, 4, 8, and 12, and, for each of these, twenty-two models are examined containing from 1 to 6 variates, built up from 8 different data types:

1. Constant (C)—a vector of 1's.
2. Random (U and N)—U is $U(0, 1)$ and N is $N(0, 16)$.
3. Trend (T)—values $1, \ldots, N$.
4. Sinusoidal (S)—20 values of $\mathrm{Sin}(i)$ for $i = .15$ to π in steps of .15, repeated three times for $N = 60$.
5. Oscillating (O)—alternating 1 and -1.
6. Basic economic data (X)—20 annual consumption figures, repeated three times for $N = 60$.
7. First differences (D)—first differences of previous series, repeated three times for $N = 60$.
8. Second differences (D*)—first differences of D, repeated three times for $N = 60$.

The models are named according to the variates included. Thus, CUX contains an intercept C, the uniform random variate U, and the basic economic data series X. Models with names not including C have no intercept. All the artificial test regressions, of course, also include the appropriately lagged residual term. Table 1 lists the models examined.

TABLE 1 HERE

The random disturbances $\boldsymbol{\varepsilon}$ in step (a) are generated as iid Normals using the random number generator resident in *Mathematica*. This software environment has been used for all the calculations in this study, variously on a Macintosh IIfx and a PowerMac 8100/110. *Mathematica*'s random number generator has excellent properties detailed in Belsley (1995). For $N = 20$, then, a run for a particular g consists of a given model being replicated 10,000 times, resulting in that many values for t_c. From these it is straightforward to calculate the sample mean, variance, skewness, and kurtosis of t_c, as well as the actual proportion of t_c's falling in each tail region as determined by a given two-tailed critical value for a Student's t with appropriate degrees of freedom,

$$r \equiv (N-g) - (K+1) = (N-K-g-1). \tag{3.1}$$

Here K is the number of variates included in the basic model—$K = 3$, for example, for the model CTD. In addition to the usual $N - K$ degrees of freedom, the above figure reflects the loss of g degrees of freedom to lagging and one to the additional lagged-residual regressor present in the artificial regression. The actual proportions are calculated for two different test regions: the first determined by the critical values for the 10% two-tailed region for a Student's t—so that the actual proportions should be roughly 5% in each tail if t_c followed a Student's t—and the second determined by the critical values for the 5% two-tailed region—so that roughly 2.5% should fall in each tail. For $N = 60$, similar results are obtained, but using only 5,000 replications.

The basic, pre-correction results for $N = 20$ and $g = 1$, 2, 4, 8, and 12 are summarized in Tables 2*a*–*e*, respectively, and those for $N = 60$, $g = 1$, 2, 4, 8, and 12 in Tables 3*a*-*e*.

TABLES 2*a*-*e* AND 3*a*-*e* HERE

We first examine in some detail the results for testing for first-order serial correlation ($g = 1$) and $N = 20$ given in Table 2*a*, since this is arguably the most frequently encountered case in practice. We can then treat the remaining cases more briefly. The departure of the small-sample distribution of t_c from a Student's t is immediately evident here. Almost all the models show biased means, mostly downward. Only the oscillating model without intercept (O) has positive bias. There is also a definite tendency for models involving trended or highly autocorrelated variates—T, S, and X—to have larger biases, and, indeed, only the completely nontrended model, CO, shows an absence of bias. The variances too are biased downwards from a Student's t, whose variance for r degrees of freedom is $r/(r-2) > 1$. This bias tends to be larger for those models having more variates.

The small-sample distribution of t_c also appears to be negatively skewed, a phenomenon that tends to be stronger for models with autocorrelated variates. The oscillating model O again shows a contrary pattern, as does the model CO, which appears not only to have small bias but also little skewness. Finally, no patterns or tendencies appear relative to kurtosis. A Student's t with 20 degrees of freedom has a kurtosis of roughly 3.5, relative to which we see both lepto- and platykurtic behavior. In any event, it appears that the variety of models chosen for these experiments provides a wide spectrum of possible distributions for t_c, none of which is a Student's t.

The net effect of the above distortions from a Student's t is seen in the actual percentages of t_c appearing in the tail regions relevant to a Student's t. For the 10% two-tailed test, we would expect the actual percentages to be evenly distributed between the upper and lower tails with roughly 5% in each if t_c were distributed as a Student's t. Instead we see gross asymmetry, often with totals not even near 10%. The lower tail typically contains a much higher proportion than the upper, particularly with models containing the trended or autocorrelated variates T, S, and X, clearly the result of the negative mean bias. Model O, with its positive mean bias, shows contrary asymmetry, but asymmetry nevertheless. The model CO shows the best overall behavior, but its proportions, while more nearly symmetric, fall quite short of the requisite 5% in each tail.

Similar patterns occur with the 5% two-tailed tests, which should produce actual percentages near 2.5% in each tail. But even further evidence of distortion is seen here since, in many cases, these proportions are far from being one-half the corresponding value of the 10% two-tailed test, as would be expected if t_c were distributed as a Student's t.

Table 3a shows the sample statistics for t_c when testing for first-order serial correlation with sample size $N = 60$, and we see a notable improvement: biases have been reduced by roughly one-half; variances, while still low, are closer to the 1.03 figure expected of an appropriate Student's t; skewness is now typically positive but smaller in absolute value; and the kurtosis figures bracket the value of approximately 3.1 relevant to a Student's t with 60 degrees of freedom. The totals in the tails are now somewhat closer to nominal, but there remains substantial asymmetry in the actual proportions falling into the tail regions that would be relevant to a Student's t. Thus, standard tests that assume t_c to follow a Student's t distribution are quite misleading.

Substantial distortions continue to appear for t_c when testing for higher orders of serial correlation—but are slightly different in character. For 2nd- and 4th-order serial correlation with $N = 20$, we see from Tables 2b and 2c that biases continue to be downward, both for mean and variance, but there are more cases of positive skewness. The asymmetry of the actual proportions in the tails continues very strongly, particularly for those models containing more variates and having the lowest degrees of freedom. The results in Tables 2d and 2e for 8th- and 12th-order serial correlation, respectively, are notably different. The mean bias tends to be less and the actual test sizes tend to be closer to expectation, giving the rather interesting hint that the uncorrected testing procedure may give more reasonable results when testing for high orders of serial correlation with low degrees of freedom. However, in those 8th-order cases having the highest mean biases, such as models CN, CO, or CDO in Table 2d, there is still substantial asymmetry in the actual proportions in each tail. The 12th-order results for the last three models in Table 2e deserve mention: these models have only one or two degrees of freedom, resulting, as is to be expected, in volatile summary statistics. The test sizes, however, are excellent. Tables 3b-e show that results for the $N = 60$ cases for all orders of serial correlation beyond the first, including 8 and 12, are much the same as those for described above for the 2nd and 4th orders for $N = 20$, except that the distortions tend to be tempered somewhat and the skewnesses are now virtually all positive.

In general, then, we can conclude that t_c cannot be assumed distributed as a Student's t, and tests based on that assumption are highly misleading.

4. Transforming t_c to be more nearly a Student's t.

Computational solutions for making t_c a useful test statistic could take several directions. One could, for example, use Monte Carlo experiments to determine small-sample critical values for t_c. The previous analysis, however, shows that such experiments must needs be extensive; separate positive and negative critical values would be required for many different degrees of freedom, orders of serial correlation, and model types—and even then the ultimate usefulness of the results would be problematic. Further, many more replications would be required of each experiment to obtain the desired degree of refinement. It would clearly be simpler, if possible, to transform t_c into a more nearly Student-like statistic. This is the approach adopted here, where, for any given model (i.e., any given $T \times K$ data matrix $\mathbf{X}$) and any order of serial correlation g desired to be tested for, we determine a computationally simple but effective means for approximating the population mean μ_{t_c} and variance $\sigma^2_{t_c}$ of t_c. Using these values, along with the degrees of freedom r as defined in (3.1), t_c can be transformed into

$$t_c^* \equiv \sqrt{\frac{r}{r-2}}\,\frac{t_c - \mu_{t_c}}{\sigma_{t_c}}. \tag{4.1}$$

While not perfectly distributed as a Student's t with r degrees of freedom, t_c^* is approximately correct through the first two moments and, as a practical matter, is seen through Monte Carlo experiments to be close enough to a Student's t to produce acceptable test results for all models, orders of serial correlation, and degrees of freedom (with some qualifications) when treated as if it were.

For a given model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta}+\boldsymbol{\varepsilon}$ and order of serial correlation g, the test statistic t_c is derived in Appendix A, and the small-sample approximations to its mean μ_{t_c} and variance $\sigma^2_{t_c}$ are derived in Appendix B. While these small-sample approximations are based on asymptotic considerations, we shall see momentarily that they are quite effective. Appendix C provides computationally efficient means for carrying out the needed calculations along with exemplar *Mathematica* code.

TABLES $4a$–e AND $5a$–e HERE

We first examine the corrected results for testing for first-order serial correlation with $N = 20$ given in Table $4a$. The efficacy of the asymptotic approximations to the means μ_{t_c} and variances $\sigma^2_{t_c}$ of the t_c is readily apparent. These values, as calculated from (B.2) and (B.11), are given in columns (4) and (5), respectively. For comparison, the Monte Carlo sample estimates for these means and variances are given in columns (2) and (3), repeated here from Table $2a$ for convenience. It is clear that both of these approximations are very good indeed, doing a remarkable job for each of the very different model conditions. Turning now to the empirical test sizes, we see there has been a significant improvement. The "10%" tests—columns (7) and (8)—now show actual percentages quite closely in line with what one would expect from a Student's t: the total in both tails is roughly 10%, distributed approximately equally. Some asymmetry clearly remains, but it is greatly reduced. The greatest distortions continue to be associated with models containing the trended and highly autocorrelated variates T, X, and S—but even here the improvement is striking and satisfying. Similar conclusions apply to the "5%" tests, which now also demonstrate proportions much closer to one-half those of the corresponding 10% test.

Table $5a$ shows the results of the correction when testing for first-order serial correlation when $N = 60$. Again, the asymptotic approximations to the mean and variance are very good (although we would expect this for higher degrees of freedom), and similar conclusions can be drawn for the improvement in the size tests. All in all, the transformed statistic t_c^* appears to provide quite acceptable small-sample tests for first-order serial correlation over the full range of models and degrees of freedom.

The corrected results for higher orders of serial correlation (2, 4, 8, and 12) are shown in Tables $4b$–e and $5b$–e for $N = 20$ and 60, respectively. Let us begin with the results in Table $4b$ for the test for second-order serial correlation with $N = 20$. Comparing columns (4) and (5) with (2) and (3), we again see that the asymptotic approximations to the mean and variance are quite good, although this is less so for the last three cases where the degrees of freedom get as low as 12. And, except for these cases, the corrections result in tail percentages that are much improved over those in Table $2b$, again having approximately correct totals (tending to be somewhat smaller than 10%) and distributions. It is clear that the inaccuracy of the asymptotic approximations for very low degrees of freedom (≤ 12) has not allowed fully appropriate corrections to be made here, but otherwise the correction is working well. Similar results are seen from Table $4c$ for the corrections for the test for 4th-order serial correlation. Again, for degrees of freedom above 12, the asymptotic approximations are working very well and the test sizes are quite acceptable. For degrees of freedom twelve and below, there is a tendency for the mean estimate to be somewhat less reliable and for the variance estimate to be understated. Even here, however, it is clear that the correction for mean has substantially improved the distribution of the outcomes between the tails.

Much of the preceding description remains true for the corrected tests for 8th- and 12th-order serial correlation shown in Tables 4*d* and 4*e*—where all of the cases involve degrees of freedom below 12—, but a notable difference arises. In these cases, g is either large relative to $N/2$ or exceeds this value, causing the various truncations involved in producing the matrices in (B.2) and (B.11) that determine the asymptotic mean and variance to produce values near (as is the case for the 8th-order tests) or at (as is the case for the 12th-order tests) their asymptotic values of 0 and 1, respectively. For the most part, the asymptotic mean remains a good approximation to the sample mean, but the asymptotic variance substantially underestimates the sample variance. This results in proportions for the corrected test statistic that are distributed well between the tails but are too large overall, now in the range of 12% for the 8th-order cases, and 18% for the 12th-order cases. We discuss a simple modification for cases like this in the next section.

For $N = 60$—and the higher degrees of freedom thereby implied—, we see from Tables 5*b*–*e*, that the corrected test statistic is doing a very reasonable job for all orders of serial correlation. Some instances are better or worse than others, but overall the resulting test sizes appear quite acceptable.

5. Modifications for higher and lower degrees of freedom

We have seen from above that the test statistic t_c^* behaves quite well under the test conditions when the degrees of freedom are in excess of 12 and g is not too large relative to $N/2$. Under these circumstances t_c^* provides a useful small-sample correction for testing for gth-order serial correlation. Here we extend the analysis to help answer several ancillary questions: are there any modifications that help make the test more suitable for degrees of freedom below 12? or when g is large relative to $N/2$? and how large must a sample be before it is no longer "small", i.e., before correction is no longer needed? To help answer the first and last of these questions, additional Monte Carlo experiments of 5000 replications are conducted with two models, CT and CD, exploring very low and much higher degrees of freedom when testing for first-order serial correlation. The results of these runs are given in Tables 6*a* and *b*.

TABLES 6*a* and 6*b* HERE

The first few rows of Table 6*a* show the results for model CT with very small degrees of freedom: 7-16 (i.e., N's of 11-20). We see a tendency for the asymptotic mean in column (4) increasingly to overestimate the sample mean in column (2) and for the asymptotic variance in column (5) increasingly to underestimate the sample variance in column (3) as the degrees of freedom get smaller, particularly for twelve and below.

Thus, while the corrected 10% test sizes shown in columns (8) and (9) are far better than the uncorrected sizes given in columns (6) and (7), the net effect of the above tendencies is to result in corrected test sizes that become more asymmetric and overly large with decreasing degrees of freedom. Similar results obtain in Table 6*b* for model CT.

The tendencies for the asymptotic approximations to become worse with lower degrees of freedom is due to the absence of higher order corrective terms in the asymptotic approximations, terms that become more relevant for these conditions. Short of including these higher-order corrections—clearly fodder for future research—, there seems little one can do to alleviate this problem for t_c^*. We do see in the next section, however, that an asymptotically equivalent alternative to t_c^* can offer some relief here, but only for the case of first-order serial correlation. For the moment, we simply note that, even with the problems that occur when the degrees of freedom are twelve and below, the corrected test t_c^* is not unreasonable and is certainly a vast improvement over no correction at all.

The last few lines of Tables 6*a* and 6*b* show the results for higher degrees of freedom, coming from sample sizes of 120 and 240. Even at these larger sample sizes, there is clearly substantial downward bias in the actual mean of the standard t-statistic as seen from column (2), resulting in very asymmetric size distributions in the uncorrected tails, as seen from columns (6) and (7). The corrections, however, are doing an excellent job: both asymptotic mean and variance, shown in columns (4) and (5), provide good estimates of the sample means and variances, and the corrected test sizes, given in columns (8) and (9), are quite good. Thus, even for these larger sample sizes, it is clear we are not warranted in assuming t_c to be distributed as a Student's t; some correction continues to be needed.

But, up to what sample size? Fortunately this potentially thorny question need not be answered exactly, because a simple testing strategy is apparent from Tables 6. Note in both Table 6*a* and 6*b* that, as the degrees of freedom increase, the sample mean tends downward, ultimately to its asymptotic value of zero, and the sample variance, with some fluctuation, tends towards its asymptotic limit of unity. As is to be expected, these tendencies are even more apparent for the asymptotic approximations. Since both the asymptotic approximation to the variance $\sigma_{t_c}^2$ and the ratio $r/(r-2)$ go to unity with increasing r, the scale correction in (4.1)—multiplying by $\sqrt{r/(r-2)}$ and dividing by σ_{t_c}—becomes less and less important for higher degrees of freedom. The mean correction, however, can remain important—as is seen in Table 6. The asymmetry in the uncorrected test sizes is clearly due almost entirely to this mean bias. To see this, I have included in columns (10) and (11)—for these higher degrees of freedom only—the empirical test sizes that result from mean correction only, ignoring any scale correction in (4.1), and it is clear that there is very little difference between these sizes and the fully corrected sizes in columns (8) and (9). Thus, for high degrees of freedom, one need only check the asymptotic approximation to the mean. And if it is not inconsequential—say, greater than .03 or .04 in absolute value—, then simply correct t_c for the mean, and use the result as the test statistic. Further correction for scale is unnecessary.

We can now examine the remaining ancillary question dealing with modifications when g becomes large relative to $N/2$. Recall from section 3 that this situation arises for the tests for 8th- and 12th-order serial correlation with $N = 20$. We note from Tables $2d$ and $2e$ that the uncorrected t_c for these cases appears not badly behaved, often having low mean bias, fairly good overall size, and decent distribution between the tails. In those cases, however, where the mean bias is still high, such as models T, O, CN, CO, and CDO in Table $2d$, the distribution between the tails is poor. The standard correction, on the other hand, leads to mixed results, as we see from Tables $4d$ and $4e$. The mean correction shapes up the distribution between the tails for all cases, but the scale correction leads to overall sizes that are too large, the more so as the degrees of freedom get lower.

TABLES $7a$ and $7b$ HERE

All this suggests correcting these cases of high-order serial correlation and low degrees of freedom for mean only, ignoring scaling. Tables $7a$ and $7b$—which are otherwise just like Tables $4d$ and $4e$—show the sizes for the test for 8th- and 12th-order serial correlation, respectively, with mean correction only. It is clear that this simple expedient produces very reasonable tests, having overall sizes that are only somewhat understated (ranging around 9-9.5%) and displaying excellent tail symmetry. The strategy for these high-order, low-degree-of-freedom cases, then, is similar to that given above: use t_c corrected by the asymptotic mean, but ignore any further scale correction. The correction for mean is particularly important if the asymptotic mean exceeds .04 in absolute value, but it clearly cannot hurt to make this correction quite generally. Scale correction leads only to mild size inflation in the case of the tests for 8th-order serial correlation, but the size inflation is substantial for the 12th-order tests.

6. An Alternative Test Statistic

An asymptotically equivalent alternative to the transformation (4.1) would be

$$\tilde{t}_c^* \equiv \sqrt{\frac{r}{r-2}}\frac{1}{\sigma_{t_c}}\frac{c-\mu_c}{s_c} = \sqrt{\frac{r}{r-2}}\frac{\tau_c}{\sigma_{t_c}}, \tag{6.1}$$

where c is the least-squares estimate of the lagged residual term in the artificial regression and s_c is its estimated standard error, both as defined in (2.6), μ_c is an asymptotic approximation to the mean of c, and $\tau_c \equiv s_c^{-1}(c - \mu_c)$. Here, rather than removing the mean bias μ_{t_c} from t_c and scaling, as in (4.1), we first remove the coefficient mean bias μ_c from c, then form the corresponding asymptotically unbiased t statistic τ_c, and scale as before. While (6.1) is clearly asymptotically equivalent to (4.1), it could well have different small-sample properties and so warrants examination.

We see from Appendix B that the elements needed to produce (6.1) are minor variants of those needed for (4.1). However, (6.1) is somewhat more awkward to use in practice. The transformation (4.1) acts directly on the standard t-statistic routinely reported in a regression package. The transformation (6.1), however, requires that a new, corrected t-statistic be computed after the estimate c is adjusted by μ_c. Then the same scaling is applied.

TABLES 8*a*–*d* HERE

Tables 8*a*–*d* show the results for $\tilde{t}_c^*$ when $N = 20$ and $g = 1$, 2, 4, and 8, respectively. No results are given for $g = 12$, for when $g > N/2$, $\mu_c = \mu_{t_c} \equiv 0$ and $\sigma_{t_c} \equiv 1$, and hence (6.1) and (4.1) are identical. For these experiments, 10,000 replications are done for each model, resulting in that many values for τ_c to be corrected as in (6.1). The same seeds for the random number generator are used for these experiments as were used in their t_c^* counterparts given in Tables 4*a*–*e*. This allows direct comparison of the relative behavior of the two statistics when confronting identical underlying situations.

Let us first examine the results for the tests for first-order serial correlation given in Table 8*a*. Column (2) shows the sample mean of τ_c, which is only slightly warped downwards from its ideal value of 0. Columns (5) and (3) show the closeness of the sample variance of τ_c to its asymptotic approximation—which approximation, it is to be noted, is the same as that of t_c. Now, comparing the test sizes for $\tilde{t}_c^*$ given in columns (7)-(10) of Table 8*a* with their t_c^* counterparts of Table 4*a*, we see a notable improvement. The overall test sizes are more in line with their nominal values of 10% and 5%, and the distribution between the tails is more even. Furthermore, if we look at Tables 10*a* and *b*, which correspond to Tables 6*a* and *b*, we see there is less size inflation and better tail symmetry even for very low degrees of freedom.

TABLES 10*a* and 10*b* HERE

Unfortunately, the virtues of (6.1) over (4.1) seem to end here. Examining the results for the tests for higher orders of serial correlation in Tables 8*b*–*d*, and comparing them with their t_c^* counterparts in Tables 4*b*–*d*, we see that $\tilde{t}_c^*$ is rarely better than t_c^* and, indeed, tends to exacerbate any problems in overall size and tail symmetry that t_c^* may have. No results are given for $\tilde{t}_c^*$ for $N = 60$ or higher because, at these degrees of freedom, the test sizes for this statistic are virtually identical to those for t_c^*, often being exactly the same and rarely deviating in value by more than ± 0.002.

The statistic $\tilde{t}_c^*$, then, seems to be preferable to t_c^* only for testing for first-order serial correlation when there are few degrees of freedom. This, of course, is an important case, and therefore the transformation (6.1) is not to be ignored. However, t_c^* is not bad in dealing with this situation, and, as an overall test statistic, appears more generally applicable.

7. Practical Considerations

The test procedure described above is extremely simple to employ. We assume an N-vector of observations on the dependent variate $\mathbf{y}$ and on each of K independent variates, collected together in the $N \times K$ matrix $\mathbf{X}$, which includes a constant column of ones if an intercept term is included in the model. We begin the test for gth-order serial correlation exactly as stipulated in, say, MacKinnon (1992), namely,

(a) regress $\mathbf{y}$ on $\mathbf{X}$, keeping the regression residuals $\mathbf{e} \equiv \mathbf{y} - \mathbf{X}\mathbf{b}_{OLS}$;

(b) regress $\mathbf{y}_{[g]}$ on $\mathbf{Z} \equiv [\mathbf{X}_{[g]}\ \mathbf{e}_{-g}]$, where $\mathbf{y}_{[g]}$ is the $(N-g)$-vector formed by deleting the first g elements of $\mathbf{y}$, $\mathbf{X}_{[g]}$ is the $(N-g) \times K$ matrix formed by deleting the first g rows of $\mathbf{X}$, and $\mathbf{e}_{-g}$ is the $(N-g)$-vector of residuals $\mathbf{e}$ lagged g times. t_c is the standard "t" statistic for the coefficient c of $\mathbf{e}_{-g}$ in this artificial regression.

Then

(c) calculate the asymptotic approximations to the mean μ_{t_c} from (B.2) and variance $\sigma_{t_c}^2$ from (B.11) and determine t_c^* as in (4.1), where $r = N - K - g - 1$ (see comment (1) below), and

(d) conduct a test with t_c^* just as you would any Student's t with r degrees of freedom. Thus, if $r = 27$, and you wish to do a 5% two-tailed test, compare t_c^* to the values $\pm t_{27}(.025)$, the .025 critical value for a Student's t with 27 degrees of freedom gotten from your favorite Student's t tables. If you wish to do, say, a 1% one-tailed test on the down side, use $-t_{27}(.01)$, etc. (See comment (2) below.)

Caveats

(e) this test appears less reliable when $r \leq 12$. The test sizes are somewhat larger than the theoretical sizes and the distribution between tails can be poor. Even here, however, the corrected test statistic is superior to no correction at all, but caution is warranted. (See comment (4) below.)

(f) for high degrees of freedom, say $r > 250$, scale correction in (4.1) is no longer necessary, but mean correction is still required if the asymptotic approximation to μ_{t_c} is greater than .04 in absolute value. (See comment (3) below.)

(g) for high orders of serial correlation and low degrees of freedom (when g is large relative to $N/2$ and particularly when $g > N/2$), correct for the mean, but consider no scale corrections unless it is desirable to have oversized tests. (See comment (4) below.)

Comments:

(1) The considerations given in Appendix C can be used to make the calculations in step (c) as efficient as possible.

(2) In some instances it may be preferable in step (d) to transform the critical values rather than t_c. Let $t_r(\alpha)$ be the α-critical value for a Student's t with r degrees of freedom. Then it is quite the same to compare t_c (untransformed) to $t_r(\alpha)$ transformed according to

$$\sqrt{\frac{r-2}{r}}\sigma_{t_c} t_r(\alpha) + \mu_{t_c} \tag{7.1}$$

as to compare t_c^* to $t_r(\alpha)$.

(3) For those who insist on making scale corrections for high degrees of freedom, the following consideration can significantly reduce computational cost, for it is unnecessary to calculate the asymptotic approximation to the variance, which is quite expensive with large r. Specifically, both $(r/(r-2))^{1/2}$ and $\sigma_{t_c}^{-1}$ tend toward their limit of unity at roughly equal rates and hence scaling by $r/(r-2)$ is a good alternative. For $r > 250$, however, the practical size differences that result from this scaling, full scaling, and no scaling are very small.

(4) In the situations of (e) and (g), somewhat better results can be obtained when testing for first-order serial correlation (but this case only) by using the alternative test statistic $\tilde{t}_c^*$ defined in (6.1).

8. Appendices

Appendix A: Some notation and the OLS estimates of c and t_c

Begin with the basic model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{A.1}$$

where $\mathbf{y}$ is an N-vector, $\mathbf{X}$ is an $N \times K$ data matrix, and $\varepsilon \sim N(0,\ \sigma^2\mathbf{I})$. Estimation of (A.1) with ordinary least squares results in the OLS residuals

$$\mathbf{e} = \mathbf{M_X y} = \mathbf{M_X}\boldsymbol{\varepsilon}, \tag{A.2}$$

where, for the $m \times n$ matrix $\mathbf{Z}$, the notation $\mathbf{M_Z}$ will be used throughout to mean the $m \times m$ projection matrix

$$\mathbf{M_Z} \equiv \mathbf{I} - \mathbf{Z}(\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T. \tag{A.3}$$

Now define the $(N-g) \times N$ "drop matrix"

$$\mathbf{D}_g \equiv [\mathbf{0} \;\; \mathbf{I}_{N-g}] \tag{A.4}$$

and the $(N-g) \times N$ "lag matrix"

$$\mathbf{L}_g \equiv [\mathbf{I}_{N-g} \;\; \mathbf{0}], \tag{A.5}$$

where $\mathbf{I}_{N-g}$ is the $(N-g)$ identity matrix.

In this notation, the artificial regression (2.5) becomes

$$\begin{aligned}\mathbf{D}_g\mathbf{e} &= \mathbf{D}_g\mathbf{X}\boldsymbol{\delta} + \gamma\mathbf{L}_g\mathbf{e} + \mathbf{v} \\ &= [\mathbf{D}_g\mathbf{X} \;\; \mathbf{L}_g\mathbf{e}]\begin{bmatrix}\boldsymbol{\delta} \\ \gamma\end{bmatrix} + \mathbf{v}.\end{aligned} \tag{A.6}$$

The OLS estimates $\begin{bmatrix}\mathbf{d} \\ c\end{bmatrix}$ of $\begin{bmatrix}\boldsymbol{\delta} \\ \gamma\end{bmatrix}$ are

$$\begin{bmatrix}\mathbf{d} \\ c\end{bmatrix} = \left\{[\mathbf{D}_g\mathbf{X} \;\; \mathbf{L}_g\mathbf{e}]^T[\mathbf{D}_g\mathbf{X} \;\; \mathbf{L}_g\mathbf{e}]\right\}^{-1}[\mathbf{D}_g\mathbf{X} \;\; \mathbf{L}_g\mathbf{e}]^T\mathbf{D}_g\mathbf{e}. \tag{A.7}$$

Solving for c gives

$$\begin{aligned}c &= (\mathbf{e}^T\mathbf{L}_g^T\mathbf{M}_{\mathbf{D}_g\mathbf{X}}\mathbf{L}_g\mathbf{e})^{-1}(\mathbf{e}^T\mathbf{L}_g^T\mathbf{M}_{\mathbf{D}_g\mathbf{X}}\mathbf{D}_g\mathbf{e}) \\ &= (\boldsymbol{\varepsilon}^T\mathbf{M}_\mathbf{X}\mathbf{L}_g^T\mathbf{M}_{\mathbf{D}_g\mathbf{X}}\mathbf{L}_g\mathbf{M}_\mathbf{X}\boldsymbol{\varepsilon})^{-1}(\boldsymbol{\varepsilon}^T\mathbf{M}_\mathbf{X}\mathbf{L}_g^T\mathbf{M}_{\mathbf{D}_g\mathbf{X}}\mathbf{D}_g\mathbf{M}_\mathbf{X}\boldsymbol{\varepsilon}) \\ &\equiv \frac{\boldsymbol{\varepsilon}^T\mathbf{A}_g\boldsymbol{\varepsilon}}{\boldsymbol{\varepsilon}^T\mathbf{B}_g\boldsymbol{\varepsilon}},\end{aligned} \tag{A.8}$$

where

$$\begin{aligned}\mathbf{A}_g &\equiv \mathbf{M}_\mathbf{X}\mathbf{L}_g^T\mathbf{M}_{\mathbf{D}_g\mathbf{X}}\mathbf{D}_g\mathbf{M}_\mathbf{X} \quad \text{and} \\ \mathbf{B}_g &\equiv \mathbf{M}_\mathbf{X}\mathbf{L}_g^T\mathbf{M}_{\mathbf{D}_g\mathbf{X}}\mathbf{L}_g\mathbf{M}_\mathbf{X}.\end{aligned} \tag{A.9}$$

The estimated variance of c is

$$\text{var}(c) = s_v^2(\boldsymbol{\varepsilon}^T\mathbf{B}_g\boldsymbol{\varepsilon})^{-1}, \tag{A.10}$$

where $s_v^2 \equiv \hat{\mathbf{v}}^T\hat{\mathbf{v}}/(N-K-g-1)$, $\hat{\mathbf{v}}$ being the OLS residuals from the artificial regression. Letting $\tilde{\mathbf{X}}_g \equiv [\mathbf{D}_g\mathbf{X} \;\; \mathbf{L}_g\mathbf{e}]$ be the $(N-g) \times (K+1)$ data matrix for the artificial regression (A.6), we have

$$\begin{aligned}\hat{\mathbf{v}}^T\hat{\mathbf{v}} &= \mathbf{e}^T\mathbf{D}_g^T\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\mathbf{e} \\ &= \boldsymbol{\varepsilon}^T\mathbf{M}_\mathbf{X}\mathbf{D}_g^T\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\mathbf{M}_\mathbf{X}\boldsymbol{\varepsilon} \\ &= \boldsymbol{\varepsilon}^T\mathbf{D}_g^T\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\boldsymbol{\varepsilon},\end{aligned} \tag{A.11}$$

this last equality holding since $\mathbf{D}_g\mathbf{X}$ is in the column space of $\tilde{\mathbf{X}}_g$, so $\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\mathbf{X} = \mathbf{0}$, and hence, $\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\mathbf{M}_{\mathbf{X}}$ $= \mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g$. Thus,

$$s_v^2 = \frac{\boldsymbol{\varepsilon}^T\mathbf{D}_g^T\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\boldsymbol{\varepsilon}}{N-K-g-1}. \tag{A.12}$$

Because $\mathbf{M}_{\tilde{\mathbf{X}}_g}$ is idempotent with rank $N-K-g-1$ and $\mathbf{D}_g\mathbf{D}_g^T = \mathbf{I}_{N-g}$, $Es_v^2 = \sigma^2$.

Now, putting (A.12) into (A.10), we have

$$\mathrm{var}(c) = \frac{1}{N-K-g-1}\frac{\boldsymbol{\varepsilon}^T\mathbf{D}_g^T\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\boldsymbol{\varepsilon}}{\boldsymbol{\varepsilon}^T\mathbf{B}_g\boldsymbol{\varepsilon}}, \tag{A.13}$$

and hence

$$t_c \equiv \frac{c}{\sqrt{\mathrm{var}(c)}} = \frac{\sqrt{N-K-g-1}}{\sqrt{\boldsymbol{\varepsilon}^T\mathbf{D}_g^T\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\boldsymbol{\varepsilon}}}\frac{\boldsymbol{\varepsilon}^T\mathbf{A}_g\boldsymbol{\varepsilon}}{\sqrt{\boldsymbol{\varepsilon}^T\mathbf{B}_g\boldsymbol{\varepsilon}}}. \tag{A.14}$$

Appendix B: Asymptotic approximations to the mean and variance of t_c

We note the following probability limits and asymptotic approximations to the various components of t_c in (A.14):

$$\mathrm{plim}\,\frac{1}{N}\boldsymbol{\varepsilon}^T\mathbf{A}_g\boldsymbol{\varepsilon} = \lim\frac{1}{N}E\boldsymbol{\varepsilon}^T\mathbf{A}_g\boldsymbol{\varepsilon} = \sigma^2\frac{1}{N}\,\mathrm{tr}(\mathbf{A}_g) \tag{B.1a}$$

$$\mathrm{plim}\,\frac{1}{N}\boldsymbol{\varepsilon}^T\mathbf{B}_g\boldsymbol{\varepsilon} = \lim\frac{1}{N}E\boldsymbol{\varepsilon}^T\mathbf{B}_g\boldsymbol{\varepsilon} = \sigma^2\frac{1}{N}\,\mathrm{tr}(\mathbf{B}_g) \tag{B.1b}$$

$$\begin{aligned}\mathrm{plim}\,\frac{1}{N}\boldsymbol{\varepsilon}^T\mathbf{D}_g^T\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\boldsymbol{\varepsilon} &= \lim\frac{1}{N}E\boldsymbol{\varepsilon}^T\mathbf{D}_g^T\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\boldsymbol{\varepsilon}\\ &= \sigma^2\lim\frac{1}{N}\,\mathrm{tr}(\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\mathbf{D}_g^T) = \sigma^2\lim\frac{1}{N}\,\mathrm{tr}(\mathbf{M}_{\tilde{\mathbf{X}}_g})\\ &= \sigma^2\lim\frac{1}{N}(N-K-g-1) = \sigma^2.\end{aligned} \tag{B.1c}$$

Thus,

$$\begin{aligned}\mathrm{plim}\,\frac{t_c}{\sqrt{N}} &= \frac{\mathrm{plim}\,\frac{1}{N}\boldsymbol{\varepsilon}^T\mathbf{A}_g\boldsymbol{\varepsilon}}{\sqrt{\mathrm{plim}\,\frac{1}{N}\boldsymbol{\varepsilon}^T\mathbf{D}_g^T\mathbf{M}_{\tilde{\mathbf{X}}_g}\mathbf{D}_g\boldsymbol{\varepsilon}}\sqrt{\mathrm{plim}\,\frac{1}{N}\boldsymbol{\varepsilon}^T\mathbf{B}_g\boldsymbol{\varepsilon}}}\\ &\overset{A}{\approx} \frac{\sigma^2\frac{1}{N}\,\mathrm{tr}(\mathbf{A}_g)}{\sqrt{\sigma^2}\sqrt{\sigma^2\frac{1}{N}\,\mathrm{tr}(\mathbf{B}_g)}} = \frac{1}{\sqrt{N}}\frac{\mathrm{tr}(\mathbf{A}_g)}{\sqrt{\mathrm{tr}(\mathbf{B}_g)}}\end{aligned}$$

and hence,

$$Et_c \overset{A}{\approx} \frac{\mathrm{tr}(\mathbf{A}_g)}{\sqrt{\mathrm{tr}(\mathbf{B}_g)}}, \tag{B.2}$$

which is the value used to approximate μ_{t_c} in (4.1). And using (B.1a) and (B.1b) with (A.8), we see

$$Ec \overset{A}{\approx} \frac{\mathrm{tr}(\mathbf{A}_g)}{\mathrm{tr}(\mathbf{B}_g)}, \tag{B.3}$$

which is the value used to approximate μ_c in (6.1).

We can derive an asymptotic approximation for $\sigma_{t_c}^2$, the variance of t_c, as follows: From above, we know asymptotically that $\varepsilon^T \mathbf{D}_g^T \mathbf{M}_{\tilde{\mathbf{X}}_g} \mathbf{D}_g \varepsilon/(N-K-g-1) \to \sigma^2$ and $\varepsilon^T \mathbf{B}_g \varepsilon \to \sigma^2 \operatorname{tr}(\mathbf{B}_g)$. Hence, from (A.14),

$$\operatorname{var}(t_c) \overset{A}{\approx} \frac{\operatorname{var}(\varepsilon^T \mathbf{A}_g \varepsilon)}{\sigma^2 \cdot \sigma^2 \operatorname{tr}(\mathbf{B}_g)} = \frac{\operatorname{var}(\varepsilon^T \mathbf{A}_g \varepsilon)}{\sigma^4 \operatorname{tr}(\mathbf{B}_g)}. \tag{B.4}$$

To derive $\operatorname{var}(\varepsilon^T \mathbf{A}_g \varepsilon)$, we first note that

$$\operatorname{var}(\varepsilon^T \mathbf{A}_g \varepsilon) = E(\varepsilon^T \mathbf{A}_g \varepsilon)^2 - (E\varepsilon^T \mathbf{A}_g \varepsilon)^2, \tag{B.5}$$

the first term of which is

$$\begin{aligned} E(\varepsilon^T \mathbf{A}_g \varepsilon)^2 &= E(\varepsilon^T \mathbf{A}_g \varepsilon)(\varepsilon^T \mathbf{A}_g \varepsilon) \\ &= E \sum_i \sum_j \sum_k \sum_l a_{ij} a_{kl} \epsilon_i \epsilon_j \epsilon_k \epsilon_l \\ &= \sigma^4 (3 \sum_i a_{ii}^2 + \sum_{i \neq j} a_{ii} a_{jj} + \sum_{i \neq j} a_{ij} a_{ji} + \sum_{i \neq j} a_{ij}^2) \\ &\equiv \sigma^4 (3\alpha + \beta + \gamma + \delta), \end{aligned} \tag{B.6}$$

where α, β, γ, and δ are defined by context, and in the first term of the third equality we have used the fact that the fourth moment $\sigma_4 = 3\sigma^4$ for the Normal distribution. The second term of (B.5) is

$$\begin{aligned} (E\varepsilon^T \mathbf{A}_g \varepsilon)^2 = (\sigma^2 \operatorname{tr}(\mathbf{A}_g))^2 = \sigma^4 (\sum_i a_{ii})^2 &= \sigma^4 \sum_{i,j} a_{ii} a_{jj} \\ = \sigma^4 (\sum_i a_{ii}^2 + \sum_{i \neq j} a_{ii} a_{jj}) \\ = \sigma^4 (\alpha + \beta). \end{aligned} \tag{B.7}$$

Thus, putting (B.7) and (B.6) into (B.5)

$$\begin{aligned} \operatorname{var}(\varepsilon^T \mathbf{A}_g \varepsilon) &= \sigma^4 (3\alpha + \beta + \gamma + \delta) - \sigma^4 (\alpha + \beta) \\ &= \sigma^4 (2\alpha + \gamma + \delta). \end{aligned} \tag{B.8}$$

Now we note that

$$\boldsymbol{\iota}^T (\mathbf{A}_g * \mathbf{A}_g) \boldsymbol{\iota} = \sum_{i,j} a_{ij}^2 = \sum_i a_{ii}^2 + \sum_{i \neq j} a_{ij}^2 = \alpha + \delta, \quad \text{and} \tag{B.9a}$$

$$\boldsymbol{\iota}^T (\mathbf{A}_g * \mathbf{A}_g^T) \boldsymbol{\iota} = \sum_{i \neq j} a_{ij} a_{ji} = \sum_i a_{ii}^2 + \sum_{i \neq j} a_{ij} a_{ji} = \alpha + \gamma, \tag{B.9b}$$

where $\boldsymbol{\iota}$ is a vector of ones and $*$ denotes parallel matrix multiplication, i.e., if $\mathbf{S} = (s_{ij})$ and $\mathbf{R} = (r_{ij})$ are two $n \times m$ matrices, $\mathbf{S} * \mathbf{R} = (s_{ij} r_{ij})$. Using (B.9a and b) in (B.8) gives

$$\operatorname{var}(\varepsilon^T \mathbf{A}_g \varepsilon) = \sigma^4 \left[\boldsymbol{\iota}^T (\mathbf{A}_g * \mathbf{A}_g) \boldsymbol{\iota} + \boldsymbol{\iota}^T (\mathbf{A}_g * \mathbf{A}_g^T) \boldsymbol{\iota} \right] = \sigma^4 \boldsymbol{\iota}^T \left[\mathbf{A}_g * (\mathbf{A}_g + \mathbf{A}_g^T) \right] \boldsymbol{\iota}. \tag{B.10}$$

From (B.4) we get

$$\operatorname{var}(t_c) \overset{A}{\approx} \frac{\boldsymbol{\iota}^T \left[\mathbf{A}_g * (\mathbf{A}_g + \mathbf{A}_g^T)\right] \boldsymbol{\iota}}{\operatorname{tr}(\mathbf{B}_g)}, \tag{B.11}$$

which is the value used to approximate $\sigma_{t_c}^2$ in (4.1) and (6.1).

Appendix C: Computational formulae and code

Here we consider shortcuts for computing $\operatorname{tr}(\mathbf{A}_g)$, $\operatorname{tr}(\mathbf{B}_g)$, and the matrix $\mathbf{A}_g$, which are used in approximating μ_{t_c}, μ_c, and $\sigma_{t_c}^2$. We also provide *Mathematica* code to accomplish these tasks. Beginning from (A.9), we have

$$\begin{aligned}
\operatorname{tr}(\mathbf{A}_g) &= \operatorname{tr}(\mathbf{M}_\mathbf{X} \mathbf{L}_g^T \mathbf{M}_{\mathbf{D}_g\mathbf{X}} \mathbf{D}_g \mathbf{M}_\mathbf{X}) \\
&= \operatorname{tr}(\mathbf{L}_g^T \mathbf{M}_{\mathbf{D}_g\mathbf{X}} \mathbf{D}_g \mathbf{M}_\mathbf{X}) \\
&= \operatorname{tr}(\mathbf{L}_g^T \mathbf{M}_{\mathbf{D}_g\mathbf{X}} \mathbf{D}_g) \\
&= \operatorname{tr}(\mathbf{L}_g^T [\mathbf{0} \quad \mathbf{M}_{\mathbf{D}_g\mathbf{X}}]) \\
&= \operatorname{tr} \begin{bmatrix} \mathbf{0} & \mathbf{M}_{\mathbf{D}_g\mathbf{X}} \\ \mathbf{0} & \mathbf{0}^T \end{bmatrix} \\
&= \operatorname{subTr}_g(\mathbf{M}_{\mathbf{D}_g\mathbf{X}}).
\end{aligned} \tag{C.1}$$

The second equality holds because of the idempotency of $\mathbf{M}_\mathbf{X}$, the third because $\mathbf{D}_g\mathbf{X}$ is in the null space of $\mathbf{M}_{\mathbf{D}_g\mathbf{X}}$ so that $\mathbf{M}_{\mathbf{D}_g\mathbf{X}}\mathbf{D}_g\mathbf{X} = \mathbf{0}$, and the fourth and fifth simply by the definitions (A.3) and (A.4) of $\mathbf{D}_g$ and $\mathbf{L}_g$, respectively. SubTr_g is the operator that sums the elements of a square matrix along its gth sub diagonal.

Let $\mathbf{X}_{[g]}$ and $\mathbf{X}_{\{g\}}$ denote the $(N-g)\times K$ matrices formed by deleting the first g and last g rows of $\mathbf{X}$, respectively. Then $\mathbf{D}_g\mathbf{X} = \mathbf{X}_{[g]}$ and $\mathbf{L}_g\mathbf{X} = \mathbf{X}_{\{g\}}$. First note that

$$\mathbf{M}_{\mathbf{D}_g\mathbf{X}} = \mathbf{I} - \mathbf{X}_{[g]}(\mathbf{X}_{[g]}^T\mathbf{X}_{[g]})^{-1}\mathbf{X}_{[g]}^T. \tag{C.2}$$

Now beginning again from (A.9), we have

$$\begin{aligned}
\operatorname{tr}(\mathbf{B}_g) &= \operatorname{tr}(\mathbf{M}_\mathbf{X} \mathbf{L}_g^T \mathbf{M}_{\mathbf{D}_g\mathbf{X}} \mathbf{L}_g \mathbf{M}_\mathbf{X}) \\
&= \operatorname{tr}(\mathbf{M}_{\mathbf{D}_g\mathbf{X}} \mathbf{L}_g \mathbf{M}_\mathbf{X} \mathbf{L}_g^T) \\
&= \operatorname{tr}(\mathbf{M}_{\mathbf{D}_g\mathbf{X}} \mathbf{L}_g \mathbf{L}_g^T + \mathbf{M}_{\mathbf{D}_g\mathbf{X}} \mathbf{L}_g \mathbf{X} (\mathbf{X}^T\mathbf{X})^{-1} \mathbf{X}^T \mathbf{L}_g^T) \\
&= \operatorname{tr}(\mathbf{M}_{\mathbf{D}_g\mathbf{X}}) + \operatorname{tr}(\mathbf{M}_{\mathbf{D}_g\mathbf{X}} \mathbf{X}_{\{g\}} (\mathbf{X}^T\mathbf{X})^{-1} \mathbf{X}_{\{g\}}^T) \\
&= (N-K-g) + \operatorname{tr}((\mathbf{X}^T\mathbf{X})^{-1} \mathbf{X}_{\{g\}}^T \mathbf{M}_{\mathbf{D}_g\mathbf{X}} \mathbf{X}_{\{g\}}).
\end{aligned} \tag{C.3}$$

The second equality holds because of the idempotency of $\mathbf{M_X}$ and the well-known properties of trace, the third by the definition of $\mathbf{M_X} = \mathbf{I}_N - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$, the fourth by the definition of $\mathbf{L}_g$, which implies $\mathbf{L}_g\mathbf{L}_g^T = \mathbf{I}_{N-g}$ and $\mathbf{L}_g\mathbf{X} = \mathbf{X}_{\{g\}}$, and the last by the fact that $\mathbf{M}_{\mathbf{D}_g\mathbf{X}}$ is idempotent with rank $N-K-g$. The commutation in the last term of the last line allows working with $K\times K$ matrices rather than $(N-g)\times(N-g)$ matrices.

We also need an efficient way to calculate the matrix $\mathbf{A}$ for use in the numerator of (B.11). Thus, from (A.9)

$$\begin{aligned}\mathbf{A} &= \mathbf{M_X}\mathbf{L}_g^T\mathbf{M}_{\mathbf{D}_g\mathbf{X}}\mathbf{D}_g\mathbf{M_X} \\ &= \mathbf{M_X}\mathbf{L}_g^T\mathbf{M}_{\mathbf{D}_g\mathbf{X}}\mathbf{D}_g \\ &= \mathbf{M_X}\mathbf{L}_g^T[\mathbf{0} \quad \mathbf{M}_{\mathbf{D}_g\mathbf{X}}] \\ &= \mathbf{M_X}\begin{bmatrix}\mathbf{0} & \mathbf{M}_{\mathbf{D}_g\mathbf{X}} \\ \mathbf{0} & \mathbf{0}^T\end{bmatrix}.\end{aligned} \tag{C.4}$$

The reasons for these steps can be found above following (C.1).

Everything is now in place to compute the t_c^* test described in Section 4 and the $\tilde{t}_c^*$ test described in Section 6. Here is a series of *Mathematica* functions that culminates in routines for calculating the asymptotic approximations to the mean μ_{t_c} and variance $\sigma_{t_c}^2$ of t_c in (B.2) and (B.11), respectively, and to the mean μ_c of c in (B.3), and for computing t_c^* from t_c and $\tilde{t}_c^*$ from c and s_c using the preceding computational simplifications.

Preliminary utility routines

To produce a vector of n ones:

```
iota[n_] := Table[1., {n}]
```

Test to return `TRUE` if `mat` is a square matrix, `FALSE` otherwise:

```
SquareQ[mat_] := TrueQ[MatrixQ[mat] &&
                  Apply[Equal,Dimensions[mat]]]
```

To orient data matrix correctly for following routines. Input can be either $N\times K$ or $K\times N$: Output is $N\times K$:

```
alignDataMatrix[X_] :=
Block[{T,K},
   {T,K} = Dimensions[X];
   If[T<K,
      {K,T,Transpose[X//N]},
      {T,K,X//N}
   ]
]
```

To produce the matrix $\mathbf{M}_{\mathbf{D}_g\mathbf{X}}$:

Note: $\mathbf{X}$ should include a vector of ones if there is an intercept term in the basic model (A.1). Omitting the second parameter results in a default of $g = 0$, which is the basic projection matrix $\mathbf{M_X} = \mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$. This routine uses the QR decomposition to produce the matrix $\mathbf{X}_{[g]}(\mathbf{X}_{[g]}^T)\mathbf{X}_{[g]})^{-1}\mathbf{X}_{[g]}^T$.

```
Mx[X_, g_:0] :=
Block[{q,T,K,i},
   {T,K,q} = alignDataMatrix[X];
   q = Drop[q,g];
   q = QRDecomposition[q][[1]];
   q = -Transpose[q].q;
   Do[q[[i,i]] = 1.+ q[[i,i]],{i,T-g}];
   q
]
```

To calculate the gth subtrace of a matrix, as needed in (C.1):

Note: omitting the second argument results in the default $g = 0$, which is the trace of the matrix.

```
tr[mat_?SquareQ, g_:0] :=
   Apply[Plus,Table[mat[[i+g,i]],{i,Dimensions[mat][[1]]-g}]]
```

To preborder a matrix $\mathbf{Z}$ with g leading columns and g bottom rows of zeroes, as needed, for example, in (C.1) or (C.4):

Note: omitting the second argument results in the default $g = 1$, which is relevant to first-order serial correlation.

```
zeroBorder[Z_, g_:1] :=
   Transpose[Join[Table[0.,{g},{Dimensions[Z][[1]]+g}],
      Transpose[Join[Z,Table[0.,{g},{Dimensions[Z][[2]]}]]]]]
```

To calculate the numerator of the asymptotic variance (B.11).

```
varQuadraticForm[A_?SquareQ] :=
   iota[Dimensions[A][[2]]].(A*(A+Transpose[A])).
   iota[Dimensions[A][[2]]]
```

Asymptotic mean and variance routines

Note: in the following routines, $\mathbf{X}$ should include a vector of ones if there is an intercept term in the basic model (A.1).

To calculate the asymptotic approximation to the mean μ_{t_c} (B.2) for gth-order serial correlation:

Note: omitting second argument results in default of $g = 1$, which is relevant to first-order serial correlation.

```
tcAsymptoticMean[X_,g_:1] :=
Block[{T,K,x,Mdx},
   {T,K,x} = alignDataMatrix[X];
   Mdx = Mx[x,g];
   tr[Mdx,g]/Sqrt[(T-K-g)-tr[Inverse[Transpose[x].x].
      Transpose[Drop[x,-g]].Mdx.Drop[x,-g]]]
]
```

To calculate the asymptotic approximation to the variance $\sigma^2_{t_c}$ (B.11) for gth-order serial correlation:

Note: omitting second argument results in default of $g = 1$, which is relevant to first-order serial correlation.

```
asymptoticVariance[X_, g_:1] :=
Block[{T,K,x,Mdx},
   {T,K,x} = alignDataMatrix[X];
   Mdx = Mx[x,g];
   varQuadraticForm[Mx[x].zeroBorder[Mdx,g]]/((T-K-g)
      - tr[Inverse[Transpose[x].x].Transpose[Drop[x,-g]].
        Mdx.Drop[x,-g]])
]
```

Corrected t routine

Use this routine to calculate t_c^* as in (4.1) from t_c gotten from an artificial regression like (A.6) using basic data matrix $\mathbf{X}$. This integrates previous routines.

Note: in the following routine, $\mathbf{X}$ should include a vector of ones if there is an intercept term in the basic model (A.1). Omitting the last argument results in the default $g = 1$, which is relevant to first-order serial correlation.

```
tcCorrected[X_,tc_,g_:1]:=
Block[{T,K,r,x,Mdx,trB,asyMean,asyVar},
   {T,K,x} = alignDataMatrix[X];
   r = T-K-g-1;
   Mdx = Mx[x,g];
   trB = (T-K-g) - tr[Inverse[Transpose[x].x].
      Transpose[Drop[x,-g]].Mdx.Drop[x,-g]];
   asyMean = tr[Mdx,g]/Sqrt[trB];
   asyVar=varQuadraticForm[Mx[x].zeroBorder[Mdx,g]]/trB;
   Sqrt[r/(r-2.)](tc-asyMean)/Sqrt[asyVar]
]
```

Use this routine to calculate $\tilde{t}_c^*$ as in (6.1) from c and s_c gotten from an artificial regression like (A.6) using basic data matrix $\mathbf{X}$. This integrates previous routines.

Note: in the following routine, $\mathbf{X}$ should include a vector of ones if there is an intercept term in the basic model (A.1). Omitting the last argument results in the default $g = 1$, which is relevant to first-order serial correlation.

```
ccCorrected[X_,c_,sc_,g_:1]:=
Block[{T,K,r,x,Mdx,trB,asyMeanC,asyVar},
   {T,K,x} = alignDataMatrix[X];
   r = T-K-g-1;
   Mdx = Mx[x,g];
   trB = (T-K-g) - tr[Inverse[Transpose[x].x].
      Transpose[Drop[x,-g]].Mdx.Drop[x,-g]];
   asyMeanC = tr[Mdx,g]/trB;
   asyVar=varQuadraticForm[Mx[x].zeroBorder[Mdx,g]]/trB;
   Sqrt[r/(r-2.)](c-asyMeanC)/(sc*Sqrt[asyVar])
]
```

7. Bibliography

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Table 1
Variates included in each model

Model	C	U	N	T	X	D	S	O	D*
C	x								
CU	x	x							
CN	x		x						
CT	x			x					
CX	x				x				
CD	x					x			
CS	x						x		
CO	x							x	
T				x					
X					x				
D						x			
S							x		
O								x	
CUX	x	x			x				
CTX	x			x	x				
CTD	x			x		x			
CTS	x			x			x		
CDS	x					x	x		
CDO	x					x		x	
CUTXDO	x	x		x	x	x		x	
UTXDO		x		x	x	x		x	
CUNDD*O	x	x	x			x		x	x

Table 2a
Monte Carlo Summary Statistics for t_c
Tests for 1st Order Serial Correlation
Sample size = 20; Replications = 10,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	−0.2325	0.9642	−0.0197	3.4397	5.84	2.29	2.69	1.02
T	−0.2282	0.9603	−0.0188	3.2903	5.77	2.19	2.76	1.02
X	−0.2299	0.9832	0.0362	3.3898	6.19	2.58	2.94	1.12
D	−0.2110	0.9495	−0.0258	3.3440	5.80	2.38	2.58	1.05
S	−0.2482	0.9815	0.0555	3.5450	6.14	2.52	2.90	1.14
O	0.2510	0.9591	0.2258	3.5518	1.74	6.47	0.67	3.39
CU	−0.2617	1.0139	−0.0100	3.5007	6.68	2.36	3.16	1.08
CN	−0.2517	0.9916	0.0151	3.3721	6.31	2.23	2.97	1.10
CT	−0.4595	0.9461	−0.1344	3.4855	9.07	1.18	4.13	0.37
CX	−0.4592	0.9468	−0.0866	3.4162	8.85	1.24	4.50	0.45
CD	−0.3124	0.9119	−0.0523	3.4527	6.66	1.54	3.06	0.69
CS	−0.4503	0.9422	−0.0591	3.4812	8.79	1.18	4.40	0.49
CO	−0.0001	0.8804	0.0552	3.3430	2.91	3.29	1.35	1.51
CUX	−0.4399	0.9061	−0.1752	3.4969	8.31	1.04	4.00	0.37
CTX	−0.6370	0.8949	−0.1121	3.4283	11.39	0.54	5.59	0.17
CTD	−0.4747	0.8741	−0.2190	3.4926	8.38	0.74	4.27	0.24
CTS	−0.6579	0.8974	−0.2008	3.6215	11.80	0.53	5.99	0.17
CDS	−0.4823	0.9175	−0.1362	3.4674	8.76	0.81	4.56	0.40
CDO	−0.0902	0.8157	0.0175	3.3907	3.19	2.26	1.44	0.90
UTXDO	−0.1953	0.7299	−0.0912	3.4742	3.31	1.01	1.43	0.42
CUTXDO	−0.2823	0.7402	−0.1411	3.3997	4.31	0.74	1.70	0.25
CUNDD*O	−0.0101	0.8142	−0.0622	3.6011	2.73	2.45	0.95	0.84

Table 2b

Monte Carlo Summary Statistics for t_c

Tests for 2nd Order Serial Correlation

Sample size = 20; Replications = 10,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	−0.2177	0.9690	0.1184	3.4372	5.52	2.68	2.55	1.25
T	−0.2054	0.9305	0.1792	3.5527	4.87	2.77	2.13	1.33
X	−0.2154	0.9304	0.1486	3.5763	4.93	2.58	2.35	1.22
D	−0.1625	0.9487	0.1717	3.4888	4.80	3.09	2.12	1.37
S	−0.2407	0.9547	0.1622	3.4610	5.58	2.56	2.31	1.23
O	−0.2022	0.9391	0.1096	3.3187	5.09	2.72	2.24	1.16
CU	−0.2097	0.9534	0.1164	3.5644	4.98	2.56	2.27	1.26
CN	−0.1454	0.9576	0.1028	3.5833	4.54	3.06	2.13	1.43
CT	−0.4065	0.9350	−0.0629	3.4666	7.87	1.34	3.90	0.59
CX	−0.4114	0.8969	−0.0731	3.4291	7.38	1.13	3.59	0.48
CD	−0.3220	0.9612	0.0609	3.4921	6.56	2.03	3.17	0.96
CS	−0.3665	0.9515	0.1139	3.7138	7.11	1.98	3.40	0.93
CO	−0.4691	0.8826	0.1114	3.6040	7.81	1.37	3.24	0.63
CUX	−0.4438	0.9162	0.0527	3.5840	7.81	1.60	3.57	0.72
CTX	−0.5220	0.9577	−0.0686	3.4771	9.77	1.06	4.80	0.39
CTD	−0.4619	0.9437	−0.0799	3.4429	9.02	1.19	4.31	0.40
CTS	−0.5376	0.9346	−0.0195	3.6381	9.59	1.07	4.75	0.48
CDS	−0.4174	0.9591	0.0639	3.4840	7.76	1.63	3.70	0.76
CDO	−0.5822	0.9106	−0.0394	3.6312	9.78	0.98	4.87	0.43
UTXDO	−0.8383	0.8802	−0.2192	3.8285	14.69	0.36	7.63	0.13
CUTXDO	−0.8955	0.8896	−0.1925	3.9324	15.47	0.29	8.21	0.11
CUNDD*O	−0.7277	0.9273	−0.1217	3.8535	12.38	0.58	6.08	0.28

Table 2c

Monte Carlo Summary Statistics for t_c

Tests for 4th Order Serial Correlation

Sample size = 20; Replications = 10,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	−0.2022	0.9991	0.0020	3.5616	5.30	2.68	2.50	1.24
T	−0.1802	1.0048	0.0268	3.5200	5.34	2.75	2.65	1.26
X	−0.1852	1.0130	0.0736	3.4430	5.52	2.88	2.51	1.33
D	−0.1631	1.0100	0.0255	3.6781	5.35	2.68	2.63	1.20
S	−0.2049	1.0143	0.0458	3.5688	5.51	2.87	2.59	1.37
O	−0.1981	1.0065	0.0362	3.5739	5.77	2.95	2.77	1.15
CU	−0.2160	1.0561	0.1018	3.7249	5.99	2.98	2.86	1.47
CN	−0.2639	0.9846	−0.0062	3.6277	6.30	2.13	2.92	1.01
CT	−0.2758	1.0246	0.0608	3.5403	6.45	2.48	3.24	1.10
CX	−0.2630	1.0051	0.0183	3.5292	6.48	2.14	2.77	0.97
CD	−0.2052	1.0226	−0.0061	3.6168	5.98	2.58	2.90	1.19
CS	−0.2317	1.0838	0.0356	3.7305	6.29	3.00	3.08	1.43
CO	−0.4140	0.9932	−0.1366	3.6585	7.78	1.36	3.84	0.57
CUX	−0.3254	1.0369	−0.0343	3.6640	7.03	2.11	3.62	1.08
CTX	−0.2380	1.0704	0.0184	3.8686	6.04	2.82	2.85	1.32
CTD	−0.2753	1.0926	−0.0541	3.7545	6.79	2.50	3.44	1.19
CTS	−0.2678	1.0553	−0.0809	3.7025	6.62	2.35	3.45	1.12
CDS	−0.1703	1.0911	0.0096	3.7715	5.78	3.12	2.78	1.39
CDO	−0.4372	0.9861	−0.0781	3.9460	8.09	1.45	3.95	0.68
UTXDO	−0.5873	1.1247	−0.2898	4.1027	11.27	1.14	6.04	0.45
CUTXDO	−0.5183	1.1364	−0.2766	4.1897	10.13	1.25	5.28	0.54
CUNDD*O	−0.4339	1.1970	−0.0973	3.8459	9.25	1.93	4.94	0.79

Table 2d

Monte Carlo Summary Statistics for t_c

Tests for 8th Order Serial Correlation

Sample size = 20; Replications = 10,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	−0.0939	1.1216	−0.0550	3.8514	5.25	3.56	2.55	1.65
T	−0.1135	1.1503	−0.0458	3.6702	5.55	3.78	2.71	1.62
X	−0.1055	1.1824	−0.0173	3.8432	5.70	3.92	2.93	1.85
D	−0.0833	1.1460	−0.0730	3.8931	5.25	3.77	2.51	1.68
S	−0.0780	1.1336	−0.0449	4.0783	4.87	3.76	2.41	1.69
O	−0.1066	1.1570	−0.0362	4.0066	5.43	3.69	2.60	1.92
CU	−0.0547	1.2231	0.0311	4.7718	5.11	4.40	2.40	2.13
CN	−0.1477	1.1926	−0.1135	4.2448	5.79	3.38	3.10	1.58
CT	0.0144	1.1807	−0.0616	4.2447	4.17	4.66	2.20	2.31
CX	0.0146	1.2248	−0.0323	4.3764	4.65	4.60	2.20	2.21
CD	0.0048	1.2196	−0.0889	4.5408	4.65	4.51	2.28	2.18
CS	0.0150	1.2376	0.0126	4.1773	4.67	4.78	2.25	2.32
CO	−0.2202	1.1753	−0.1158	3.7910	7.06	2.72	3.41	1.32
CUX	0.0855	1.2342	−0.0219	4.4024	4.02	5.09	2.01	2.54
CTX	0.0781	1.2657	0.0605	4.6655	3.77	5.15	1.81	2.50
CTD	0.0625	1.2545	−0.0543	4.1847	4.24	5.05	2.11	2.32
CTS	0.0743	1.2850	0.0190	4.5629	4.14	5.49	2.11	2.68
CDS	0.0087	1.2484	−0.0195	4.3878	4.36	4.54	2.09	2.16
CDO	−0.1270	1.2604	−0.1695	4.4129	5.88	3.70	2.85	1.69
UTXDO	0.0607	1.4572	−0.0958	4.6598	4.86	5.27	2.66	2.43
CUTXDO	0.0204	1.6465	0.0481	6.6785	4.96	4.76	2.26	2.46
CUNDD*O	0.0361	1.5355	−0.2261	7.5214	4.69	4.69	2.34	2.21

Table 2e

Monte Carlo Summary Statistics for t_c

Tests for 12th Order Serial Correlation

Sample size = 20; Replications = 10,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	0.0169	1.4524	0.0412	5.3031	4.59	4.87	2.11	2.35
T	0.0109	1.3904	0.0795	5.6845	4.45	4.85	2.07	2.30
X	0.0022	1.4620	0.1292	5.9951	4.78	4.85	2.22	2.49
D	−0.0105	1.3923	−0.0980	4.9190	4.77	4.63	2.28	2.14
S	−0.0300	1.3938	0.0002	5.2748	4.84	4.41	2.32	2.10
O	0.0033	1.3944	0.0080	5.2658	4.40	4.62	2.31	2.30
CU	0.0074	1.5982	0.0338	5.7932	4.75	4.94	2.48	2.41
CN	−0.0074	1.6794	−0.0109	6.9916	5.25	4.87	2.63	2.43
CT	0.0152	1.5204	0.0164	5.7445	4.41	4.83	2.12	2.27
CX	−0.0093	1.6346	0.2741	10.1174	4.62	4.52	2.23	2.30
CD	0.0161	1.4774	0.0278	6.0681	4.51	4.46	2.16	2.15
CS	−0.0101	1.6573	−0.1157	6.6673	5.20	4.74	2.62	2.37
CO	0.0056	1.4309	0.0444	6.1776	3.90	4.19	1.85	1.99
CUX	0.0173	1.8059	0.2507	9.4117	4.45	4.79	2.03	2.30
CTX	−0.0010	1.7781	0.1146	8.5693	4.42	4.26	2.24	2.07
CTD	−0.0188	1.9061	0.4211	13.9171	4.83	4.42	2.59	2.10
CTS	0.0006	1.8016	−0.0625	8.2479	4.87	4.50	2.32	2.20
CDS	−0.0037	1.8322	0.0763	8.8564	4.56	4.53	2.20	2.17
CDO	0.0215	1.8124	0.4981	13.7751	4.38	4.49	2.08	2.12
UTXDO	0.0182	7.3619	−7.8383	3.4E2	4.33	4.95	1.99	2.28
CUTXDO	1.6809	1.6E4	8.6E1	8.1E3	4.95	4.86	2.54	2.44
CUNDD*O	−0.0345	1.3E3	−1.7E1	1.4E3	5.08	5.08	2.57	2.60

Table 3a
Monte Carlo Summary Statistics for t_c
Tests for 1st Order Serial Correlation
Sample size = 60; Replications = 5,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	−0.1438	1.0027	0.0683	3.0965	5.78	3.64	2.74	1.76
T	−0.1355	0.9631	0.1607	3.2732	5.50	3.78	2.62	2.06
X	−0.1230	0.9862	0.0239	3.1344	5.94	3.56	3.12	1.68
D	−0.1180	0.9771	0.1066	3.0159	5.32	3.66	2.40	1.92
S	−0.1398	0.9838	0.1094	3.1972	5.88	3.70	2.60	1.90
O	0.1020	0.9993	0.2160	3.3439	3.64	5.82	1.28	3.16
CU	−0.1463	1.0006	0.1124	3.3081	6.06	3.94	3.04	1.82
CN	−0.1326	0.9977	0.1461	3.2626	5.70	3.82	2.86	1.76
CT	−0.2444	0.9858	0.0325	3.2637	7.28	2.54	3.62	1.42
CX	−0.2176	0.9766	0.0910	3.1728	6.52	2.90	3.18	1.48
CD	−0.1869	0.9956	0.0892	3.1861	6.42	3.46	3.00	1.86
CS	−0.2761	0.9924	0.1721	3.1946	7.94	3.00	3.96	1.40
CO	0.0033	0.9960	0.1551	3.1871	3.98	4.82	1.82	2.62
CUX	−0.2610	0.9685	0.0740	3.2458	7.42	2.48	3.62	1.30
CTX	−0.3634	0.9913	0.0568	3.1242	9.36	2.24	4.94	1.22
CTD	−0.2974	0.9974	0.0467	3.2659	7.96	2.70	4.32	1.12
CTS	−0.4014	0.9609	0.0897	3.1566	9.22	1.98	4.72	0.88
CDS	−0.2776	0.9302	0.0428	3.0229	7.28	2.28	3.62	0.96
CDO	−0.0295	0.9736	0.1157	3.1049	4.36	4.58	1.94	2.36
UTXDO	−0.1756	0.9377	0.0775	3.2190	5.50	3.02	2.56	1.58
CUTXDO	−0.2030	0.9548	0.0797	3.0266	6.06	2.88	2.86	1.46
CUNDD*O	−0.0190	0.9570	0.0738	3.0525	4.54	4.20	1.96	2.16

Table 3b

Monte Carlo Summary Statistics for t_c

Tests for 2nd Order Serial Correlation

Sample size = 60; Replications = 5,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	−0.1238	0.9490	0.1210	3.1370	4.98	3.54	2.48	1.80
T	−0.1197	0.9492	0.2654	3.0396	4.46	3.84	1.88	2.04
X	−0.1246	0.9537	0.2239	3.1765	4.94	3.98	2.18	2.22
D	−0.1122	0.9673	0.1942	3.2645	4.96	3.98	2.42	1.98
S	−0.1335	0.9887	0.2075	3.1891	5.28	4.02	2.18	2.08
O	−0.1187	0.9486	0.2214	3.1114	4.84	3.96	2.08	2.10
CU	−0.1225	0.9497	0.1230	3.1300	5.48	3.66	2.48	1.74
CN	−0.1102	0.9630	0.2364	3.2645	4.88	4.08	2.20	1.90
CT	−0.2429	0.9253	0.2060	3.4508	6.26	2.86	2.88	1.40
CX	−0.1854	0.9925	0.1517	3.0735	6.20	3.74	2.98	1.68
CD	−0.1593	0.9915	0.2385	3.2305	5.76	3.86	2.58	2.06
CS	−0.2366	0.9368	0.1821	3.1476	6.36	3.18	2.90	1.60
CO	−0.2647	0.9055	0.1535	3.0768	6.46	2.58	2.74	1.34
CUX	−0.1991	0.9151	0.1613	3.1041	5.46	3.34	2.54	1.52
CTX	−0.3211	0.8940	0.1662	3.1472	7.28	2.12	3.56	0.98
CTD	−0.3248	0.9436	0.1541	3.2759	7.60	2.60	3.76	1.26
CTS	−0.3724	0.9428	0.1456	3.1613	8.68	2.32	3.80	1.00
CDS	−0.2494	0.9630	0.2035	3.3778	6.36	2.98	2.98	1.64
CDO	−0.3058	0.8926	0.1704	3.1642	7.20	2.00	3.32	0.98
UTXDO	−0.4163	0.8515	0.0965	2.9588	8.42	1.26	4.06	0.40
CUTXDO	−0.4697	0.9176	0.1901	3.3298	9.76	1.86	5.04	0.92
CUNDD*O	−0.3376	0.8934	0.0696	3.1831	7.38	1.80	3.52	0.90

Table 3c
Monte Carlo Summary Statistics for t_c
Tests for 4th Order Serial Correlation
Sample size = 60; Replications = 5,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	−0.0974	0.9768	0.0864	3.1097	5.40	3.90	2.46	1.92
T	−0.1352	1.0112	0.1061	3.2257	5.74	3.76	2.96	1.98
X	−0.1256	1.0067	0.1196	3.0688	5.66	3.94	2.74	1.92
D	−0.0689	0.9577	0.0633	3.1549	4.90	3.96	2.30	1.84
S	−0.0885	0.9530	0.1077	3.0549	5.20	3.86	2.14	1.78
O	−0.1160	0.9531	0.0598	2.9694	5.38	3.60	2.52	1.40
CU	−0.1026	0.9877	0.0727	3.0337	5.60	3.88	2.78	1.94
CN	−0.1366	1.0134	0.1257	3.1129	5.76	3.98	2.66	2.06
CT	−0.2456	0.9742	0.0474	2.9324	7.24	2.92	3.52	1.20
CX	−0.1478	1.0131	0.1611	3.2316	5.88	3.78	2.72	2.00
CD	−0.0965	0.9694	0.1472	3.2158	5.34	3.82	2.36	1.92
CS	−0.1792	0.9943	0.1359	3.1517	6.52	3.46	3.06	1.76
CO	−0.2689	0.9607	0.0663	3.0545	7.20	2.58	3.66	1.02
CUX	−0.1684	0.9940	0.1825	3.1084	6.04	3.58	2.82	1.80
CTX	−0.2564	0.9714	0.1153	3.1494	7.18	2.54	3.68	1.36
CTD	−0.2229	0.9566	0.0478	3.1639	6.22	2.76	3.38	1.24
CTS	−0.2778	0.9910	0.0923	3.0409	7.88	2.92	3.98	1.24
CDS	−0.1420	0.9786	0.1896	3.3834	5.68	3.74	2.38	2.02
CDO	−0.2665	0.9918	0.1536	3.2102	7.22	2.94	3.58	1.60
UTXDO	−0.3823	0.9285	0.0131	2.9548	8.92	1.06	4.48	0.62
CUTXDO	−0.3788	0.9464	0.1007	3.0605	8.48	2.18	4.18	0.82
CUNDD*O	−0.2033	0.9988	0.0722	3.1253	6.16	3.14	3.16	1.60

Table 3d

Monte Carlo Summary Statistics for t_c

Tests for 8th Order Serial Correlation

Sample size = 60; Replications = 5,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	−0.1290	0.9310	0.0941	3.3280	4.98	3.18	2.46	1.38
T	−0.0965	0.9677	0.1497	3.1905	4.80	4.08	2.18	2.38
X	−0.1205	0.9877	0.1521	3.1377	5.10	4.16	2.48	2.12
D	−0.0732	0.9839	0.1016	3.3306	4.62	4.34	2.32	2.14
S	−0.0689	0.9563	0.1951	3.2260	4.46	4.16	2.02	2.06
O	−0.1280	0.9606	0.1576	3.3509	5.26	3.64	2.50	1.98
CU	−0.1288	0.9292	0.1499	3.3656	4.88	3.36	2.16	1.72
CN	−0.0835	0.9754	0.1413	3.2018	4.76	3.98	1.96	2.26
CT	−0.1895	0.9372	0.1384	3.2914	5.70	3.14	2.60	1.50
CX	−0.0733	0.9449	0.0900	3.1133	4.56	3.74	2.08	1.70
CD	−0.0579	0.9908	0.1225	3.0914	4.54	4.14	2.20	2.28
CS	−0.0254	0.9763	0.0938	3.2117	4.12	4.74	2.06	2.40
CO	−0.2635	0.9866	0.0324	3.1906	7.60	2.90	3.78	1.40
CUX	−0.1029	0.9638	0.0837	3.3972	5.02	3.92	2.30	1.96
CTX	−0.1558	0.9591	0.0855	3.1883	6.16	3.26	2.70	1.52
CTD	−0.1453	0.9791	0.1478	3.2778	5.74	3.66	2.66	1.76
CTS	−0.1117	0.9951	0.0526	3.2136	5.32	3.90	2.74	1.92
CDS	0.0177	0.9976	0.1116	3.2462	3.94	4.98	1.90	2.84
CDO	−0.2060	0.9319	0.0962	3.3350	5.78	3.08	2.82	1.56
UTXDO	−0.2523	0.9393	0.0887	3.4039	6.54	2.36	3.28	1.18
CUTXDO	−0.2528	0.9338	−0.0386	3.1494	7.08	2.14	3.52	0.94
CUNDD*O	−0.1433	0.9669	0.0460	3.1474	5.56	3.22	2.80	1.72

Table 3e

Monte Carlo Summary Statistics for t_c

Tests for 12th Order Serial Correlation

Sample size = 60; Replications = 5,000

					10% two-tailed test		5% two-tailed test	
					actual percentage in		actual percentage in	
model	mean	variance	skewness	kurtosis	lower tail	upper tail	lower tail	upper tail
C	−0.1069	0.9413	0.1748	3.1689	4.56	3.66	1.96	1.84
T	−0.0759	0.9507	0.2514	3.2718	4.38	4.16	1.74	2.10
X	−0.1029	0.9453	0.1933	3.2161	4.82	3.68	1.90	2.12
D	−0.0545	0.9681	0.2956	3.2617	3.82	4.94	1.54	2.72
S	−0.0850	0.9424	0.1541	3.1897	4.36	3.68	1.96	2.10
O	−0.1036	0.9766	0.4508	4.1558	4.32	4.10	1.60	2.48
CU	−0.0756	0.9383	0.2088	3.1684	4.50	3.96	1.82	2.02
CN	−0.1348	0.9578	0.1569	3.1858	5.36	3.64	2.48	1.90
CT	−0.1474	0.9734	0.2744	3.4570	5.22	3.82	2.50	2.28
CX	−0.0662	0.9944	0.1492	3.1521	4.92	4.70	2.18	2.30
CD	−0.0583	1.0116	0.2404	3.2942	4.92	4.46	1.98	2.50
CS	−0.0409	0.9894	0.2121	3.2652	4.24	4.62	1.86	2.52
CO	−0.2210	0.9124	0.1599	3.3040	6.10	2.54	2.98	1.44
CUX	−0.0733	0.9667	0.2828	3.1643	4.18	4.36	1.66	2.14
CTX	−0.1292	0.9754	0.1784	3.1566	5.04	3.60	2.26	1.62
CTD	−0.1071	0.9904	0.2216	3.2730	5.06	4.00	2.20	2.30
CTS	−0.0783	0.9996	0.1952	3.1925	4.68	4.42	1.88	2.44
CDS	0.0102	0.9850	0.2296	3.3606	3.80	5.28	1.72	2.66
CDO	−0.1847	0.9361	0.2623	3.2883	5.30	3.38	2.44	1.90
UTXDO	−0.2010	0.9475	0.1618	3.2628	5.72	2.86	2.62	1.54
CUTXDO	−0.1700	0.9359	0.1571	3.2626	5.00	2.98	2.48	1.50
CUNDD*O	−0.1463	0.9600	0.0964	3.2770	5.66	3.40	2.80	1.66

Table 4a
Monte Carlo Summary Statistics for t_c^* (corrected t_c)
Tests for 1st Order Serial Correlation
Sample size = 20; Replications = 10,000

model	sample mean	sample variance	asymptotic approximation to mean	asymptotic approximation to variance	degrees of freedom	10% two-tailed test actual percentage in lower tail	10% two-tailed test actual percentage in upper tail	5% two-tailed test actual percentage in lower tail	5% two-tailed test actual percentage in upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.2325	0.9642	−0.2233	0.9504	17	5.08	4.89	2.45	2.46
T	−0.2282	0.9603	−0.2184	0.9529	17	5.04	4.97	2.51	2.33
X	−0.2299	0.9832	−0.2202	0.9516	17	5.49	5.23	2.71	2.67
D	−0.2110	0.9495	−0.1764	0.9526	17	5.46	4.39	2.57	2.33
S	−0.2482	0.9815	−0.2327	0.9481	17	5.13	5.20	2.60	2.75
O	0.2510	0.9591	0.2233	0.9504	17	4.37	5.70	1.88	3.15
CU	−0.2617	1.0139	−0.2291	0.9647	16	5.81	4.99	2.84	2.49
CN	−0.2517	0.9916	−0.2527	0.9693	16	4.96	5.01	2.43	2.39
CT	−0.4595	0.9461	−0.4340	0.9077	16	5.64	4.78	2.89	2.21
CX	−0.4592	0.9468	−0.4252	0.9123	16	5.83	4.84	3.05	2.43
CD	−0.3124	0.9119	−0.3064	0.9149	16	5.08	4.69	2.64	2.19
CS	−0.4503	0.9422	−0.4161	0.9215	16	5.76	4.63	2.89	2.15
CO	−0.0001	0.8804	0.0000	0.8954	16	4.68	5.00	2.22	2.43
CUX	−0.4399	0.9061	−0.4094	0.8873	15	5.79	4.50	2.98	2.00
CTX	−0.6370	0.8949	−0.6019	0.8851	15	5.48	4.49	2.96	2.09
CTD	−0.4747	0.8741	−0.4586	0.8682	15	5.69	4.28	3.13	1.87
CTS	−0.6579	0.8974	−0.6368	0.8717	15	5.70	4.67	3.06	2.24
CDS	−0.4823	0.9175	−0.4602	0.8969	15	5.62	4.86	3.04	2.13
CDO	−0.0902	0.8157	−0.0736	0.8509	15	4.77	4.63	2.28	2.26
UTXDO	−0.1953	0.7299	−0.1877	0.7491	13	4.88	4.52	2.46	2.01
CUTXDO	−0.2823	0.7402	−0.2798	0.7571	12	5.16	4.29	2.52	1.85
CUNDD*O	−0.0101	0.8142	−0.0047	0.8329	12	5.14	4.76	2.49	2.30

Table 4*b*

Monte Carlo Summary Statistics for t_c^* (corrected t_c)

Tests for 2nd Order Serial Correlation

Sample size = 20; Replications = 10,000

model	sample mean	sample variance	asymptotic approximation to mean	asymptotic approximation to variance	degrees of freedom	10% two-tailed test actual percentage in lower tail	10% two-tailed test actual percentage in upper tail	5% two-tailed test actual percentage in lower tail	5% two-tailed test actual percentage in upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.2177	0.9690	−0.2156	0.9550	16	4.98	5.34	2.39	2.84
T	−0.2054	0.9305	−0.2066	0.9596	16	4.16	5.08	1.99	2.77
X	−0.2154	0.9304	−0.2103	0.9562	16	4.41	5.10	2.22	2.66
D	−0.1625	0.9487	−0.1620	0.9697	16	4.54	5.35	2.13	2.76
S	−0.2407	0.9547	−0.2297	0.9556	16	4.70	5.26	2.03	2.77
O	−0.2022	0.9391	−0.2156	0.9550	16	4.48	5.46	1.95	2.85
CU	−0.2097	0.9534	−0.1871	0.9657	15	4.57	4.75	2.20	2.50
CN	−0.1454	0.9576	−0.1321	0.9379	15	4.90	5.53	2.55	2.84
CT	−0.4065	0.9350	−0.3894	0.9322	15	5.28	4.82	2.75	2.18
CX	−0.4114	0.8969	−0.3721	0.9337	15	5.16	4.21	2.71	1.93
CD	−0.3220	0.9612	−0.3000	0.9635	15	4.91	5.14	2.41	2.58
CS	−0.3665	0.9515	−0.3543	0.9640	15	4.90	4.98	2.40	2.66
CO	−0.4691	0.8826	−0.4444	0.9043	15	4.58	4.99	2.14	2.55
CUX	−0.4438	0.9162	−0.4207	0.9311	14	4.79	4.80	2.40	2.36
CTX	−0.5220	0.9577	−0.4732	0.9470	14	5.63	4.67	2.77	2.29
CTD	−0.4619	0.9437	−0.4384	0.9502	14	5.40	4.60	2.82	2.21
CTS	−0.5376	0.9346	−0.5252	0.9333	14	5.24	5.06	2.75	2.61
CDS	−0.4174	0.9591	−0.4091	0.9852	14	4.62	4.95	2.36	2.60
CDO	−0.5822	0.9106	−0.5447	0.9067	14	5.22	4.53	2.74	2.22
UTXDO	−0.8383	0.8802	−0.7781	0.8784	12	6.00	3.95	3.23	1.75
CUTXDO	−0.8955	0.8896	−0.8279	0.9020	11	5.89	3.69	3.17	1.82
CUNDD*O	−0.7277	0.9273	−0.6680	0.9084	11	5.88	4.40	3.26	2.15

Table 4c

Monte Carlo Summary Statistics for t_c^* (corrected t_c)

Tests for 4th Order Serial Correlation

Sample size = 20; Replications = 10,000

	sample		asymptotic approximation to		degrees of	10% two-tailed test actual percentage in		5% two-tailed test actual percentage in	
model	mean	variance	mean	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.2022	0.9991	−0.1936	0.9708	14	4.89	5.37	2.46	2.67
T	−0.1802	1.0048	−0.1792	0.9774	14	5.08	5.37	2.65	2.58
X	−0.1852	1.0130	−0.1867	0.9709	14	5.19	5.61	2.51	2.83
D	−0.1631	1.0100	−0.1305	0.9742	14	5.64	4.93	2.87	2.36
S	−0.2049	1.0143	−0.2010	0.9855	14	4.91	5.36	2.43	2.83
O	−0.1981	1.0065	−0.1936	0.9708	14	5.38	5.42	2.75	2.95
CU	−0.2160	1.0561	−0.1915	0.9815	13	5.60	5.75	2.86	2.97
CN	−0.2639	0.9846	−0.2480	0.9486	13	5.54	5.14	2.79	2.64
CT	−0.2758	1.0246	−0.2751	0.9953	13	5.16	5.52	2.69	2.72
CX	−0.2630	1.0051	−0.2587	0.9845	13	5.16	5.46	2.35	2.43
CD	−0.2052	1.0226	−0.1994	0.9926	13	5.48	5.11	2.75	2.51
CS	−0.2317	1.0838	−0.2292	1.0196	13	5.06	5.48	2.74	2.97
CO	−0.4140	0.9932	−0.4009	0.9375	13	5.46	5.19	2.89	2.33
CUX	−0.3254	1.0369	−0.3085	0.9996	12	5.41	5.05	2.98	2.57
CTX	−0.2380	1.0704	−0.2385	1.0242	12	4.96	5.49	2.48	2.85
CTD	−0.2753	1.0926	−0.2699	1.0295	12	5.29	5.45	2.95	2.58
CTS	−0.2678	1.0553	−0.2544	1.0333	12	5.33	4.88	2.85	2.39
CDS	−0.1703	1.0911	−0.1714	1.0394	12	5.37	5.38	2.68	2.76
CDO	−0.4372	0.9861	−0.4169	0.9569	12	5.46	4.77	2.69	2.34
UTXDO	−0.5873	1.1247	−0.5512	1.0251	10	6.41	4.76	3.59	2.22
CUTXDO	−0.5183	1.1364	−0.4851	1.0127	9	6.78	5.12	3.63	2.42
CUNDD*O	−0.4339	1.1970	−0.3896	1.0729	9	6.77	5.43	3.62	2.68

Table 4d
Monte Carlo Summary Statistics for t_c^* (corrected t_c)
Tests for 8th Order Serial Correlation
Sample size = 20; Replications = 10,000

	sample		asymptotic approximation to		degrees of	10% two-tailed test actual percentage in		5% two-tailed test actual percentage in	
model	mean	variance	mean	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.0939	1.1216	−0.1005	1.0101	10	6.04	5.87	3.06	3.10
T	−0.1135	1.1503	−0.0885	1.0078	10	6.49	5.97	3.46	3.20
X	−0.1055	1.1824	−0.0997	1.0100	10	6.33	6.38	3.56	3.19
D	−0.0833	1.1460	−0.0753	1.0057	10	6.26	5.83	3.42	3.14
S	−0.0780	1.1336	−0.0596	1.0036	10	6.12	5.63	3.18	2.94
O	−0.1066	1.1570	−0.1005	1.0101	10	6.11	5.73	3.10	3.18
CU	−0.0547	1.2231	−0.0723	1.0141	9	6.14	6.52	3.29	3.78
CN	−0.1477	1.1926	−0.1265	1.0123	9	6.53	6.10	3.79	3.19
CT	0.0144	1.1807	0.0251	0.9983	9	6.36	6.23	3.43	3.39
CX	0.0146	1.2248	0.0134	0.9980	9	6.64	6.50	3.66	3.44
CD	0.0048	1.2196	0.0053	1.0084	9	6.54	6.61	3.49	3.53
CS	0.0150	1.2376	0.0200	1.0004	9	6.88	6.66	3.71	3.59
CO	−0.2202	1.1753	−0.2108	1.0222	9	6.93	5.45	3.57	2.89
CUX	0.0855	1.2342	0.0791	1.0012	8	6.73	6.67	3.73	3.65
CTX	0.0781	1.2657	0.0716	1.0041	8	6.56	6.91	3.38	3.71
CTD	0.0625	1.2545	0.0713	1.0038	8	6.85	6.65	3.77	3.36
CTS	0.0743	1.2850	0.0730	1.0042	8	7.09	6.85	3.79	3.82
CDS	0.0087	1.2484	0.0261	1.0021	8	6.96	6.49	3.55	3.52
CDO	−0.1270	1.2604	−0.1075	1.0195	8	7.18	6.18	3.86	3.36
UTXDO	0.0607	1.4572	0.0567	1.0078	6	7.92	8.14	4.83	4.44
CUTXDO	0.0204	1.6465	0.0269	1.0057	5	9.11	8.15	5.22	4.71
CUNDD*O	0.0361	1.5355	0.0467	1.0116	5	8.27	7.66	5.01	4.44

Table 4e
Monte Carlo Summary Statistics for t_c^* (corrected t_c)
Tests for 12th Order Serial Correlation
Sample size = 20; Replications = 10,000

model	sample mean	sample variance	asymptotic approximation to mean	asymptotic approximation to variance	degrees of freedom	10% two-tailed test actual percentage in lower tail	10% two-tailed test actual percentage in upper tail	5% two-tailed test actual percentage in lower tail	5% two-tailed test actual percentage in upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	0.0169	1.4524	0.0	1.0	6	7.56	8.24	4.26	4.42
T	0.0109	1.3904	0.0	1.0	6	7.79	7.94	4.11	4.59
X	0.0022	1.4620	0.0	1.0	6	7.65	8.00	4.34	4.52
D	−0.0105	1.3923	0.0	1.0	6	7.56	7.68	4.40	4.33
S	−0.0300	1.3938	0.0	1.0	6	7.99	7.47	4.56	4.08
O	0.0033	1.3944	0.0	1.0	6	7.54	7.69	4.18	4.25
CU	0.0074	1.5982	0.0	1.0	5	8.67	8.99	4.91	5.05
CN	−0.0074	1.6794	0.0	1.0	5	9.34	8.77	5.39	5.00
CT	0.0152	1.5204	0.0	1.0	5	8.10	8.67	4.60	4.96
CX	−0.0093	1.6346	0.0	1.0	5	8.62	8.31	4.79	4.69
CD	0.0161	1.4774	0.0	1.0	5	7.99	8.23	4.64	4.61
CS	−0.0101	1.6573	0.0	1.0	5	9.15	8.88	5.38	4.86
CO	0.0056	1.4309	0.0	1.0	5	7.79	7.91	4.01	4.29
CUX	0.0173	1.8059	0.0	1.0	4	9.87	10.23	5.43	6.03
CTX	−0.0010	1.7781	0.0	1.0	4	9.74	9.57	5.49	5.23
CTD	−0.0188	1.9061	0.0	1.0	4	10.02	9.28	6.00	5.30
CTS	0.0006	1.8016	0.0	1.0	4	9.96	9.84	5.84	5.63
CDS	−0.0037	1.8322	0.0	1.0	4	10.12	9.73	5.70	5.52
CDO	0.0215	1.8124	0.0	1.0	4	9.32	9.38	5.32	5.46
UTXDO	0.0182	7.3619	0.0	1.0	2	idof	idof	idof	idof
CUTXDO	1.6809	1.6E4	0.0	1.0	1	idof	idof	idof	idof
CUNDD*O	−0.0345	1.3E3	0.0	1.0	1	idof	idof	idof	idof

idof = inadequate degrees of freedom to make calculation.

Table 5a

Monte Carlo Summary Statistics for t_c^* (corrected t_c)

Tests for 1st Order Serial Correlation

Sample size = 60; Replications = 5,000

model	sample		asymptotic approximation to		degrees of	10% two-tailed test actual percentage in		5% two-tailed test actual percentage in	
	mean	variance	mean	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.1438	1.0027	−0.1291	0.9833	57	4.86	5.32	2.30	2.72
T	−0.1355	0.9631	−0.1280	0.9836	57	4.78	5.06	2.08	2.88
X	−0.1230	0.9862	−0.1272	0.9838	57	5.22	5.16	2.46	2.62
D	−0.1180	0.9771	−0.1002	0.9839	57	4.62	4.88	2.14	2.58
S	−0.1398	0.9838	−0.1299	0.9837	57	4.78	4.90	2.20	2.74
O	0.1020	0.9993	0.1291	0.9833	57	5.06	4.96	2.46	2.88
CU	−0.1463	1.0006	−0.1453	0.9871	56	4.86	5.44	2.48	2.88
CN	−0.1326	0.9977	−0.1330	0.9846	56	4.72	5.42	2.32	2.74
CT	−0.2444	0.9858	−0.2559	0.9673	56	5.04	5.14	2.52	2.50
CX	−0.2176	0.9766	−0.2240	0.9758	56	4.54	4.98	2.22	2.68
CD	−0.1869	0.9956	−0.1598	0.9748	56	4.94	5.38	2.42	2.78
CS	−0.2761	0.9924	−0.2487	0.9722	56	5.36	5.28	2.34	2.92
CO	0.0033	0.9960	0.0000	0.9661	56	4.60	5.42	2.18	3.02
CUX	−0.2610	0.9685	−0.2284	0.9699	55	5.36	4.78	2.56	2.38
CTX	−0.3634	0.9913	−0.3534	0.9592	55	5.28	5.30	2.48	2.84
CTD	−0.2974	0.9974	−0.2867	0.9579	55	5.16	5.60	2.76	2.84
CTS	−0.4014	0.9609	−0.3779	0.9555	55	4.88	4.96	2.30	2.64
CDS	−0.2776	0.9302	−0.2637	0.9652	55	4.60	4.46	2.22	2.34
CDO	−0.0295	0.9736	−0.0302	0.9572	55	4.80	5.48	2.28	3.02
UTXDO	−0.1756	0.9377	−0.1491	0.9356	53	4.74	4.92	2.34	2.54
CUTXDO	−0.2030	0.9548	−0.2185	0.9286	52	4.58	5.48	2.40	2.88
CUNDD*O	−0.0190	0.9570	−0.0249	0.9573	52	4.98	5.30	2.38	2.58

Table 5b
Monte Carlo Summary Statistics for t_c^* (corrected t_c)
Tests for 2nd Order Serial Correlation
Sample size = 60; Replications = 5,000

model	sample mean	sample variance	asymptotic approximation to mean	asymptotic approximation to variance	degrees of freedom	10% two-tailed test actual percentage in lower tail	10% two-tailed test actual percentage in upper tail	5% two-tailed test actual percentage in lower tail	5% two-tailed test actual percentage in upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.1238	0.9490	−0.1279	0.9837	56	4.32	5.24	2.00	2.76
T	−0.1197	0.9492	−0.1259	0.9842	56	3.76	5.52	1.42	2.92
X	−0.1246	0.9537	−0.1244	0.9843	56	4.12	5.24	1.78	2.96
D	−0.1122	0.9673	−0.0989	0.9880	56	4.46	5.00	1.98	2.84
S	−0.1335	0.9887	−0.1271	0.9858	56	4.34	5.38	1.84	3.12
O	−0.1187	0.9486	−0.1279	0.9837	56	3.94	5.52	1.70	3.08
CU	−0.1225	0.9497	−0.1146	0.9876	55	4.58	5.02	2.08	2.42
CN	−0.1102	0.9630	−0.1166	0.9828	55	4.34	5.64	1.90	2.90
CT	−0.2429	0.9253	−0.2488	0.9693	55	4.22	4.74	1.84	2.80
CX	−0.1854	0.9925	−0.1888	0.9840	55	4.74	5.82	2.10	3.12
CD	−0.1593	0.9915	−0.1633	0.9878	55	4.26	5.72	1.98	3.22
CS	−0.2366	0.9368	−0.2259	0.9847	55	3.96	4.72	1.92	2.92
CO	−0.2647	0.9055	−0.2580	0.9668	55	4.10	5.12	1.56	2.56
CUX	−0.1991	0.9151	−0.2106	0.9796	54	3.76	5.14	1.76	2.86
CTX	−0.3211	0.8930	−0.3128	0.9691	54	4.08	4.70	1.36	2.38
CTD	−0.3248	0.9436	−0.2860	0.9729	54	4.68	4.74	2.08	2.70
CTS	−0.3724	0.9428	−0.3495	0.9697	54	4.36	4.82	1.78	2.54
CDS	−0.2494	0.9630	−0.2500	0.9880	54	3.90	5.16	1.92	2.86
CDO	−0.3058	0.8926	−0.2953	0.9705	54	4.12	4.30	1.68	2.20
UTXDO	−0.4163	0.8515	−0.4303	0.9534	52	3.70	5.04	1.46	2.12
CUTXDO	−0.4697	0.9176	−0.4895	0.9528	51	3.96	5.22	1.62	2.94
CUNDD*O	−0.3376	0.8934	−0.3294	0.9808	51	4.08	4.38	1.94	1.96

Table 5c
Monte Carlo Summary Statistics for t_c^* (corrected t_c)
Tests for 4th Order Serial Correlation
Sample size = 60; Replications = 5,000

model	sample mean	sample variance	asymptotic approximation to mean	asymptotic approximation to variance	degrees of freedom	10% two-tailed test actual percentage in lower tail	10% two-tailed test actual percentage in upper tail	5% two-tailed test actual percentage in lower tail	5% two-tailed test actual percentage in upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.0974	0.9768	−0.1252	0.9845	54	4.48	5.52	2.00	2.98
T	−0.1352	1.0112	−0.1214	0.9856	54	5.08	5.24	2.64	2.90
X	−0.1256	1.0067	−0.1192	0.9852	54	4.86	5.48	2.40	2.84
D	−0.0689	0.9577	−0.0854	0.9880	54	4.36	4.98	2.12	2.52
S	−0.0885	0.9530	−0.1148	0.9903	54	4.22	5.14	1.68	2.68
O	−0.1160	0.9531	−0.1252	0.9845	54	4.42	5.10	2.16	2.36
CU	−0.1026	0.9877	−0.1124	0.9798	53	5.02	5.28	2.44	2.86
CN	−0.1366	1.0134	−0.1275	0.9903	53	4.70	5.50	2.26	2.78
CT	−0.2456	0.9742	−0.2334	0.9738	53	4.96	5.50	2.52	2.74
CX	−0.1478	1.0131	−0.1340	0.9955	53	4.70	5.44	2.16	2.72
CD	−0.0965	0.9694	−0.1158	0.9958	53	4.38	5.08	1.88	2.70
CS	−0.1792	0.9943	−0.1605	1.0091	53	4.76	4.82	2.36	2.68
CO	−0.2689	0.9607	−0.2527	0.9684	53	5.02	5.26	2.48	2.56
CUX	−0.1684	0.9940	−0.1553	0.9960	52	4.46	5.36	2.12	2.66
CTX	−0.2564	0.9714	−0.2462	0.9847	52	5.00	4.94	1.96	2.40
CTD	−0.2229	0.9566	−0.2271	0.9851	52	4.44	5.02	2.38	2.42
CTS	−0.2778	0.9910	−0.2724	0.9987	52	4.78	5.12	2.06	2.72
CDS	−0.1420	0.9786	−0.1425	1.0164	52	4.38	5.10	1.78	2.64
CDO	−0.2665	0.9918	−0.2448	0.9797	52	4.80	5.48	2.20	2.76
UTXDO	−0.3823	0.9285	−0.3637	0.9707	50	4.78	4.62	1.84	2.08
CUTXDO	−0.3788	0.9464	−0.3781	0.9777	49	4.26	4.94	2.04	2.66
CUNDD*O	−0.2033	0.9988	−0.2068	0.9993	49	4.48	5.38	2.14	2.62

Table 5d

Monte Carlo Summary Statistics for t_c^* (corrected t_c)

Tests for 8th Order Serial Correlation

Sample size = 60; Replications = 5,000

	sample		asymptotic approximation to		degrees of	10% two-tailed test actual percentage in		5% two-tailed test actual percentage in	
model	mean	variance	mean	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.1290	0.9310	−0.1185	0.9869	40	4.26	4.44	2.16	2.24
T	−0.0965	0.9677	−0.1118	0.9888	40	4.10	5.12	1.86	3.04
X	−0.1205	0.9877	−0.1083	0.9862	40	4.56	5.58	2.12	2.86
D	−0.0732	0.9839	−0.0702	0.9865	40	4.38	5.08	2.24	2.80
S	−0.0689	0.9563	−0.0834	0.9826	40	4.22	5.24	1.84	3.04
O	−0.1280	0.9606	−0.1185	0.9869	40	4.44	4.84	2.14	2.74
CU	−0.1288	0.9292	−0.1382	0.9784	39	3.84	4.80	1.76	2.50
CN	−0.0835	0.9754	−0.0924	0.9842	39	4.40	5.00	1.72	2.82
CT	−0.1895	0.9372	−0.1968	0.9851	39	4.26	4.74	1.96	2.68
CX	−0.0733	0.9449	−0.0777	0.9921	39	4.08	5.00	1.98	2.34
CD	−0.0579	0.9908	−0.0743	0.9919	39	4.30	5.22	2.10	3.00
CS	−0.0254	0.9763	−0.0252	0.9862	39	4.28	5.46	2.32	2.70
CO	−0.2635	0.9866	−0.2393	0.9733	39	5.18	4.98	2.48	2.80
CUX	−0.1029	0.9638	−0.0790	0.9850	38	4.74	5.00	2.26	2.40
CTX	−0.1558	0.9591	−0.1581	0.9903	38	4.64	5.06	2.14	2.54
CTD	−0.1453	0.9791	−0.1550	0.9909	38	4.72	5.20	2.04	2.80
CTS	−0.1117	0.9951	−0.1059	0.9861	38	4.94	5.18	2.36	2.80
CDS	0.0177	0.9976	0.0085	0.9930	38	4.44	5.22	2.08	2.98
CDO	−0.2060	0.9319	−0.1950	0.9783	38	4.34	5.02	2.02	2.82
UTXDO	−0.2523	0.9393	−0.2504	0.9688	36	4.64	4.44	2.32	2.34
CUTXDO	−0.2528	0.9338	−0.2320	0.9721	35	5.32	4.30	2.44	1.96
CUNDD*O	−0.1433	0.9669	−0.1447	0.9791	35	4.68	5.04	2.38	2.52

Table 5*e*

Monte Carlo Summary Statistics for t_c^* (corrected t_c)

Tests for 12th Order Serial Correlation

Sample size = 60; Replications = 5,000

	sample		asymptotic approximation to		degrees of	10% two-tailed test actual percentage in		5% two-tailed test actual percentage in	
model	mean	variance	mean	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.1069	0.9413	−0.1094	0.9907	46	4.10	4.82	1.76	2.60
T	−0.0759	0.9507	−0.1008	0.9929	46	3.82	5.42	1.54	2.98
X	−0.1029	0.9453	−0.0988	0.9893	46	4.18	4.94	1.70	2.70
D	−0.0545	0.9681	−0.0640	0.9890	46	3.72	5.94	1.48	3.42
S	−0.0850	0.9424	−0.0838	0.9885	46	4.00	4.88	1.84	2.78
O	−0.1036	0.9766	−0.1094	0.9907	46	3.68	5.40	1.44	3.34
CU	−0.0756	0.9383	−0.0602	0.9918	45	4.28	5.04	1.78	2.58
CN	−0.1348	0.9578	−0.1230	0.9897	45	4.62	5.04	2.00	2.84
CT	−0.1474	0.9734	−0.1520	0.9985	45	4.20	5.20	1.98	3.08
CX	−0.0662	0.9944	−0.0660	0.9906	45	4.68	5.60	2.12	2.98
CD	−0.0583	1.0116	−0.0727	0.9949	45	4.68	5.70	1.92	3.12
CS	−0.0409	0.9894	−0.0320	0.9930	45	4.24	5.58	1.92	2.94
CO	−0.2210	0.9124	−0.2212	0.9810	45	4.06	4.60	1.94	2.24
CUX	−0.0733	0.9667	−0.0680	0.9866	44	4.04	5.60	1.66	2.78
CTX	−0.1292	0.9754	−0.1088	0.9970	44	4.34	5.16	2.00	2.48
CTD	−0.1071	0.9904	−0.1151	1.0019	44	4.28	5.28	1.78	3.04
CTS	−0.0783	0.9996	−0.0756	1.0019	44	4.34	5.48	1.68	3.04
CDS	0.0102	0.9850	−0.0057	0.9970	44	4.12	5.76	1.88	3.08
CDO	−0.1847	0.9361	−0.1844	0.9854	44	3.92	5.22	1.62	2.94
UTXDO	−0.2010	0.9475	−0.1998	0.9921	42	4.14	4.66	1.86	2.48
CUTXDO	−0.1700	0.9359	−0.1850	0.9922	41	3.70	5.06	1.80	2.30
CUNDD*O	−0.1463	0.9600	−0.1452	0.9988	41	4.52	4.86	2.32	2.46

Table 6*a*
Monte Carlo Summary Statistics for t_c and t_c^*
Tests for 1st Order Serial Correlation
Various smaller and larger degrees of freedom; Replications = 5,000

Model CT

degrees of freedom	sample		asymptotic approximation to		10% two-tailed test: actual percentage					
					uncorrected		corrected		mean-corrected	
	mean	variance	mean	variance	lower	upper	lower	upper	lower	upper
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
7	−0.6356	0.9001	−0.5657	0.8533	8.60	0.50	6.50	3.42		
9	−0.5776	0.9134	−0.5270	0.8687	8.92	0.60	6.22	4.34		
10	−0.5493	0.9034	−0.5103	0.8757	8.84	0.52	6.24	4.06		
11	−0.5422	0.9254	−0.4949	0.8823	9.02	0.70	6.26	4.60		
12	−0.4850	0.8886	−0.4807	0.8883	8.06	0.60	5.60	4.16		
13	−0.5096	0.8920	−0.4677	0.8938	8.80	0.82	5.66	4.56		
14	−0.4808	0.9197	−0.4556	0.8988	8.66	0.94	5.34	4.30		
15	−0.4633	0.8894	−0.4444	0.9034	8.04	1.06	5.04	4.82		
16	−0.4780	0.9283	−0.4340	0.9077	9.33	0.92	5.99	4.40		
56	−0.2444	0.9858	−0.2559	0.9673	7.28	2.54	5.04	5.14	4.34	4.50
116	−0.2047	1.0126	−0.1818	0.9835	7.20	3.34	5.20	4.98	4.88	4.84
236	−0.1233	0.9727	−0.1288	0.9917	6.10	3.58	4.66	5.04	4.52	4.96

Table 6*b*
Monte Carlo Summary Statistics for t_c and t_c^*
Tests for 1st Order Serial Correlation
Various smaller and larger degrees of freedom; Replications = 5,000

Model CD

degrees of freedom	sample		asymptotic approximation to		10% two-tailed test: actual percentage					
					uncorrected		corrected		mean-corrected	
	mean	variance	mean	variance	lower	upper	lower	upper	lower	upper
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
7	−0.1440	1.0175	−0.1210	0.9286	4.36	1.98	6.18	4.72		
9	−0.1451	0.9926	−0.1365	0.9517	5.14	2.24	6.32	4.88		
10	−0.1453	0.9708	−0.1250	0.9484	4.40	2.40	5.40	5.24		
11	−0.1514	0.9878	−0.1201	0.9510	4.86	2.44	5.86	4.52		
12	−0.1633	0.9529	−0.1369	0.9555	4.66	2.16	5.30	4.46		
13	−0.1754	0.9555	−0.1545	0.9524	5.10	2.56	5.38	4.96		
14	−0.2220	0.9241	−0.2083	0.9248	5.30	1.68	5.16	4.52		
15	−0.3118	0.8987	−0.2782	0.9254	6.42	1.40	5.16	4.34		
16	−0.2994	0.9231	−0.3064	0.9149	6.24	1.71	5.00	5.01		
56	−0.1869	0.9956	−0.1598	0.9748	6.42	3.46	4.94	5.38	4.24	4.80
116	−0.1226	0.9810	−0.1103	0.9878	5.62	3.58	4.60	4.76	4.38	4.56
236	−0.0951	0.9665	−0.0771	0.9940	5.72	3.92	4.68	4.76	4.58	4.70

Table 7*a*

Monte Carlo Summary Statistics for t_c Corrected Only for Mean

Tests for 8th Order Serial Correlation

Sample size = 20; Replications = 10,000

	sample		asymptotic approximation to		degrees of	sizes with mean-correction only: 10% two-tailed test, actual percentage in		5% two-tailed test, actual percentage in	
model	mean	variance	mean	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.0939	1.1216	−0.1005	1.0101	10	4.44	4.21	2.14	1.95
T	−0.1135	1.1503	−0.0885	1.0078	10	4.70	4.31	2.40	1.98
X	−0.1055	1.1824	−0.0997	1.0010	10	4.90	4.57	2.45	2.19
D	−0.0833	1.1460	−0.0753	1.0057	10	4.59	4.41	2.10	1.97
S	−0.0780	1.1336	−0.0596	1.0036	10	4.46	4.18	2.20	1.98
O	−0.1066	1.1570	−0.1005	1.0101	10	4.49	4.27	2.19	2.27
CU	−0.0547	1.2231	−0.0723	1.0141	9	4.46	4.83	2.09	2.47
CN	−0.1477	1.1926	−0.1265	1.0123	9	4.89	4.29	2.57	1.93
CT	0.0144	1.1807	0.0251	0.9983	9	4.39	4.38	2.27	2.19
CX	0.0146	1.2248	0.0134	0.9980	9	4.74	4.49	2.24	2.14
CD	0.0048	1.2196	0.0053	1.0084	9	4.68	4.47	2.28	2.15
CS	0.0150	1.2376	0.0200	1.0002	9	4.86	4.57	2.33	2.23
CO	−0.2202	1.1753	−0.2108	1.0222	9	4.86	4.04	2.30	1.91
CUX	0.0855	1.2342	0.0791	1.0012	8	4.54	4.49	2.21	2.25
CTX	0.0781	1.2657	0.0716	1.0041	8	4.33	4.58	2.05	2.20
CTD	0.0625	1.2545	0.0713	1.0038	8	4.70	4.34	2.26	2.09
CTS	0.0743	1.2850	0.0730	1.0042	8	4.76	4.88	2.37	2.28
CDS	0.0087	1.2484	0.0261	1.0021	8	4.55	4.39	2.13	2.08
CDO	−0.1270	1.2604	−0.1075	1.0195	8	4.98	4.52	2.54	2.03
UTXDO	0.0607	1.4572	0.0567	1.0078	6	5.17	4.85	2.89	2.26
CUTXDO	0.0204	1.6465	0.0269	1.0057	5	5.11	4.55	2.36	2.40
CUNDD*O	0.0361	1.5355	0.0467	1.0116	5	4.94	4.37	2.50	2.06

Table 7*b*
Monte Carlo Summary Statistics for t_c Corrected Only for Mean
Tests for 12th Order Serial Correlation
Sample size = 20; Replications = 10,000

model	sample mean	sample variance	asymptotic approximation to mean	asymptotic approximation to variance	degrees of freedom	sizes with mean-correction only: 10% two-tailed test, actual percentage in lower tail	10% two-tailed test, actual percentage in upper tail	5% two-tailed test, actual percentage in lower tail	5% two-tailed test, actual percentage in upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	0.0169	1.4524	0.0	1.0	6	4.59	4.87	2.11	2.35
T	0.0109	1.3904	0.0	1.0	6	4.45	4.85	2.07	2.30
X	0.0022	1.4620	0.0	1.0	6	4.78	4.85	2.22	2.49
D	−0.0105	1.3923	0.0	1.0	6	4.77	4.63	2.28	2.14
S	−0.0300	1.3938	0.0	1.0	6	4.84	4.41	2.32	2.10
O	0.0033	1.3944	0.0	1.0	6	4.40	4.62	2.31	2.30
CU	0.0074	1.5982	0.0	1.0	5	4.75	4.94	2.48	2.41
CN	−0.0074	1.6794	0.0	1.0	5	5.25	4.87	2.63	2.43
CT	0.0152	1.5204	0.0	1.0	5	4.41	4.83	2.12	2.27
CX	−0.0093	1.6346	0.0	1.0	5	4.62	4.52	2.23	2.30
CD	0.0161	1.4774	0.0	1.0	5	4.51	4.46	2.16	2.15
CS	−0.0101	1.6573	0.0	1.0	5	5.20	4.74	2.62	2.37
CO	0.0056	1.4309	0.0	1.0	5	3.90	4.19	1.85	1.99
CUX	0.0173	1.8059	0.0	1.0	4	4.45	4.79	2.03	2.30
CTX	−0.0010	1.7781	0.0	1.0	4	4.42	4.26	2.24	2.07
CTD	−0.0188	1.9061	0.0	1.0	4	4.83	4.42	2.59	2.10
CTS	0.0006	1.8016	0.0	1.0	4	4.87	4.50	2.32	2.20
CDS	−0.0037	1.8322	0.0	1.0	4	4.56	4.53	2.20	2.17
CDO	0.0215	1.8124	0.0	1.0	4	4.38	4.49	2.08	2.12
UTXDO	0.0182	7.3619	0.0	1.0	2	4.33	4.95	1.99	2.28
CUTXDO	1.6809	1.6E4	0.0	1.0	1	4.95	4.86	2.54	2.44
CUNDD*O	−0.0345	1.3E3	0.0	1.0	1	5.08	5.08	2.57	2.60

Table 8a

Monte Carlo Summary Statistics for t_c^* (coefficient-corrected t_c)

Tests for 1st Order Serial Correlation

Sample size = 20; Replications = 10,000

	sample			asymptotic	degrees	sizes with mean-correction only			
						10% two-tailed test		5% two-tailed test	
				approx. to	of	actual percentage in		actual percentage in	
model	mean	variance	skewness	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.0090	0.9591	0.0218	0.9504	17	4.98	4.94	2.37	2.52
T	−0.0097	0.9559	0.0220	0.9529	17	4.95	5.04	2.47	2.46
X	−0.0097	0.9790	0.0774	0.9516	17	5.38	5.30	2.53	2.76
D	−0.0343	0.9464	0.0056	0.9526	17	5.32	4.46	2.50	2.34
S	−0.0151	0.9768	0.1007	0.9481	17	4.96	5.29	2.48	2.77
O	0.0277	0.9513	0.1833	0.9504	17	4.36	5.51	1.92	3.01
CU	−0.0321	1.0070	0.0367	0.9647	16	5.66	5.03	2.69	2.54
CN	0.0010	0.9853	0.0623	0.9693	16	4.78	5.08	2.29	2.48
CT	−0.0238	0.9249	−0.0517	0.9077	16	5.11	4.80	2.58	2.34
CX	−0.0319	0.9256	−0.0034	0.9123	16	5.47	4.86	2.63	2.49
CD	−0.0056	0.9050	0.0104	0.9149	16	4.91	4.83	2.42	2.26
CS	−0.0320	0.9207	0.0229	0.9215	16	5.32	4.64	2.43	2.25
CO	−0.0001	0.8804	0.0552	0.8954	16	4.68	5.00	2.22	2.43
CUX	−0.0291	0.8821	−0.0952	0.8873	15	5.42	4.51	2.67	2.12
CTX	−0.0290	0.8549	0.0098	0.8851	15	4.85	4.59	2.40	2.20
CTD	−0.0146	0.8492	−0.1248	0.8682	15	5.18	4.39	2.78	1.94
CTS	−0.0141	0.8513	−0.0644	0.8717	15	4.96	4.62	2.30	2.36
CDS	−0.0196	0.8932	−0.0413	0.8969	15	5.26	4.93	2.64	2.28
CDO	−0.0170	0.8156	0.0323	0.8509	15	4.75	4.67	2.25	2.28
UTXDO	−0.0091	0.7248	−0.0520	0.7491	13	4.79	4.56	2.35	2.09
CUTXDO	−0.0045	0.7292	−0.0847	0.7571	12	4.88	4.27	2.29	1.89
CUNDD*O	−0.0054	0.8142	−0.0610	0.8329	12	5.13	4.76	2.49	2.30

Table 8*b*

Monte Carlo Summary Statistics for t_c^* (coefficient-corrected t_c)

Tests for 2nd Order Serial Correlation

Sample size = 20; Replications = 10,000

	sample			asymptotic	degrees	sizes with mean-correction only			
						10% two-tailed test		5% two-tailed test	
				approx. to	of	actual percentage in		actual percentage in	
model	mean	variance	skewness	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.0015	0.9660	0.1609	0.9550	16	4.83	5.40	2.31	2.90
T	0.0014	0.9285	0.2227	0.9596	16	4.01	5.13	1.91	2.87
X	−0.0047	0.9281	0.1919	0.9562	16	4.30	5.15	2.13	2.76
D	−0.0001	0.9477	0.2042	0.9697	16	4.46	5.43	2.08	2.85
S	−0.0103	0.9511	0.2077	0.9556	16	4.48	5.27	1.90	2.86
O	0.0138	0.9364	0.1531	0.9550	16	4.25	5.57	1.82	2.96
CU	−0.0221	0.9500	0.1566	0.9657	15	4.45	4.79	2.15	2.54
CN	−0.0131	0.9556	0.1323	0.9379	15	4.85	5.55	2.46	2.86
CT	−0.0148	0.9179	0.0196	0.9322	15	4.91	4.83	2.52	2.26
CX	−0.0375	0.8808	−0.0001	0.9337	15	4.81	4.23	2.40	2.05
CD	−0.0202	0.9531	0.1264	0.9635	15	4.67	5.27	2.11	2.65
CS	−0.0102	0.9387	0.1906	0.9640	15	4.60	4.96	2.17	2.69
CO	−0.0214	0.8660	0.2079	0.9043	15	4.15	5.05	1.90	2.60
CUX	−0.0210	0.8966	0.1495	0.9311	14	4.40	4.85	2.14	2.47
CTX	−0.0431	0.9315	0.0394	0.9470	14	5.13	4.70	2.40	2.38
CTD	−0.0200	0.9196	0.0166	0.9502	14	4.88	4.60	2.44	2.31
CTS	−0.0066	0.9057	0.1034	0.9333	14	4.49	5.11	2.16	2.67
CDS	−0.0058	0.9396	0.1553	0.9852	14	4.23	4.95	2.05	2.74
CDO	−0.0318	0.8809	0.0911	0.9067	14	4.69	4.65	2.25	2.29
UTXDO	−0.0425	0.8012	−0.0084	0.8784	12	4.71	3.82	2.22	1.83
CUTXDO	−0.0461	0.7917	0.0574	0.9020	11	4.37	3.62	2.06	1.81
CUNDD*O	−0.0485	0.8680	0.0502	0.9084	11	4.91	4.44	2.38	2.08

Table 8c

Monte Carlo Summary Statistics for t_c^* (coefficient-corrected t_c)

Tests for 4th Order Serial Correlation

Sample size = 20; Replications = 10,000

	sample			asymptotic	degrees	sizes with mean-correction only			
						10% two-tailed test		5% two-tailed test	
				approx. to	of	actual percentage in		actual percentage in	
model	mean	variance	skewness	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	−0.0064	0.9958	0.0464	0.9708	14	4.81	5.43	2.34	2.74
T	0.0008	1.0014	0.0713	0.9774	14	4.99	5.52	2.49	2.70
X	0.0031	1.0096	0.1196	0.9709	14	5.00	5.77	2.42	2.90
D	−0.0314	1.0074	0.0579	0.9742	14	5.49	4.93	2.77	2.38
S	−0.0021	1.0099	0.0974	0.9855	14	4.77	5.52	2.24	2.86
O	−0.0024	1.0029	0.0832	0.9708	14	5.22	5.51	2.64	2.97
CU	−0.0217	1.0522	0.1558	0.9815	13	5.47	5.69	2.60	3.03
CN	−0.0134	0.9791	0.0572	0.9486	13	5.25	5.26	2.62	2.79
CT	0.0032	1.0167	0.1271	0.9953	13	5.01	5.59	2.44	2.80
CX	−0.0010	0.9965	0.0842	0.9845	13	4.88	5.49	2.16	2.55
CD	−0.0034	1.0156	0.0435	0.9926	13	5.26	5.33	2.60	2.60
CS	−0.0001	1.0754	0.0982	1.0196	13	4.93	5.66	2.51	3.03
CO	−0.0060	0.9694	−0.0320	0.9375	13	4.91	5.36	2.44	2.61
CUX	−0.0123	1.0232	0.0473	0.9996	12	5.21	5.10	2.77	2.63
CTX	0.0038	1.0628	0.0923	1.0242	12	4.76	5.61	2.24	2.91
CTD	−0.0003	1.0829	0.0266	1.0295	12	5.00	5.57	2.66	2.76
CTS	−0.0097	1.0421	−0.0039	1.0333	12	5.09	5.11	2.71	2.57
CDS	0.0038	1.0860	0.0637	1.0394	12	5.23	5.50	2.49	2.84
CDO	−0.0128	0.9611	0.0563	0.9569	12	4.94	4.93	2.39	2.49
UTXDO	−0.0200	1.0699	−0.0921	1.0251	10	5.44	4.99	2.93	2.57
CUTXDO	−0.0176	1.0804	−0.0812	1.0127	9	5.75	5.13	2.93	2.70
CUNDD*O	−0.0336	1.1714	0.0582	1.0729	9	6.12	5.59	2.97	2.81

Table 8d

Monte Carlo Summary Statistics for t_c^* (coefficient-corrected t_c)

Tests for 8th Order Serial Correlation

Sample size = 20; Replications = 10,000

	sample			asymptotic	degrees	sizes with mean-correction only: 10% two-tailed test		5% two-tailed test	
				approx. to	of	actual percentage in		actual percentage in	
model	mean	variance	skewness	variance	freedom	lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C	0.0101	1.1209	−0.0193	1.0101	10	5.84	5.97	2.98	3.12
T	−0.0213	1.1494	−0.0116	1.0078	10	6.39	6.09	3.41	3.27
X	−0.0018	1.1808	0.0213	1.0100	10	6.24	6.42	3.47	3.28
D	−0.0047	1.1450	−0.0418	1.0057	10	6.13	5.90	3.33	3.11
S	−0.0159	1.1334	−0.0218	1.0036	10	5.92	5.69	3.16	2.97
O	−0.0022	1.1570	0.0066	1.0101	10	6.03	5.84	3.06	3.25
CU	0.0210	1.2223	0.0662	1.0141	9	5.97	6.59	3.22	3.87
CN	−0.0154	1.1894	−0.0574	1.0123	9	6.30	6.32	3.60	3.30
CT	−0.0118	1.1808	−0.0745	0.9983	9	6.38	6.22	3.44	3.36
CX	0.0005	1.2249	−0.0389	0.9980	9	6.65	6.48	3.70	3.44
CD	−0.0007	1.2195	−0.0914	1.0084	9	6.54	6.59	3.48	3.52
CS	−0.0059	1.2373	0.0030	1.0004	9	6.91	6.61	3.74	3.58
CO	0.0012	1.1679	−0.0313	1.0222	9	6.60	5.81	3.29	3.04
CUX	0.0020	1.2334	−0.0641	1.0012	8	6.73	6.47	3.79	3.57
CTX	−0.0001	1.2821	0.0135	1.0041	8	7.02	6.72	3.62	3.52
CTD	−0.0124	1.2539	−0.0887	1.0038	8	6.88	6.54	3.88	3.30
CTS	−0.0030	1.2833	−0.0229	1.0042	8	7.22	6.75	3.89	3.73
CDS	0.0061	1.2731	−0.0260	1.0021	8	6.89	6.89	3.54	3.75
CDO	−0.0131	1.2569	−0.1138	1.0195	8	6.95	6.37	3.69	3.44
UTXDO	−0.0014	1.4571	−0.1368	1.0078	6	7.99	8.04	4.89	4.33
CUTXDO	−0.0099	1.6458	0.0156	1.0057	5	9.19	8.13	5.28	4.61
CUNDD*O	−0.0153	1.5368	−0.2845	1.0116	5	8.37	7.56	5.09	4.33

Table 10a
Monte Carlo Summary Statistics for $\hat{t}_c^*$
Tests for 1st Order Serial Correlation
Various smaller degrees of freedom; Replications = 5,000

Model CT

degrees of freedom	sample			asymptotic approx. to variance	10% two-tailed test: actual percentage			
					uncorrected		corrected	
	mean	variance	skewness		lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
7	−0.0580	0.8221	−0.1213	0.8533	2.46	1.50	5.56	3.72
9	−0.0429	0.8568	−0.1712	0.8687	2.84	2.32	5.34	4.44
10	−0.0331	0.8582	−0.0954	0.8757	3.12	2.22	5.56	4.22
11	−0.0415	0.8842	−0.0573	0.8823	3.22	2.58	5.58	4.64
12	−0.0020	0.8538	−0.1543	0.8883	3.08	2.58	5.14	4.18
13	−0.0384	0.8624	−0.0426	0.8938	3.22	2.70	5.10	4.64
14	−0.0226	0.8931	−0.0639	0.8988	3.12	2.78	4.90	4.38
15	−0.0171	0.8686	0.0587	0.9034	3.04	3.42	4.66	4.90
16	−0.0266	0.9172	−0.0646	0.9077	3.36	3.08	5.06	4.72

Table 10b
Monte Carlo Summary Statistics for $\hat{t}_c^*$
Tests for 1st Order Serial Correlation
Various smaller degrees of freedom; Replications = 5,000

Model CD

degrees of freedom	sample			asymptotic approx. to variance	10% two-tailed test: actual percentage			
					uncorrected		corrected	
	mean	variance	skewness		lower tail	upper tail	lower tail	upper tail
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
7	−0.0225	1.0138	−0.1684	0.9286	3.72	2.60	6.14	4.76
9	−0.0081	0.9873	−0.1681	0.9517	3.90	3.08	6.16	4.84
10	−0.0201	0.9680	−0.0189	0.9484	3.48	3.28	5.30	5.36
11	−0.0312	0.9847	−0.0587	0.9510	3.78	2.82	5.82	4.52
12	−0.0263	0.9503	−0.0349	0.9555	3.68	3.14	5.18	4.54
13	−0.0207	0.9531	0.0515	0.9524	3.62	3.52	5.28	5.02
14	−0.0140	0.9170	−0.1333	0.9248	3.50	3.02	5.02	4.60
15	−0.0334	0.8900	−0.1065	0.9254	3.62	2.88	4.96	4.44
16	−0.0100	0.9021	0.0194	0.9149	3.54	2.88	5.02	4.44