

A small, structural, quarterly model for monetary policy evaluation*

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Abstract

This paper develops a small, structural model of the United States economy and estimates that model with quarterly data on output, prices, and money from 1959 through 1995. The estimates reveal that the Federal Reserve has successfully insulated the economy from the effects of exogenous demand-side disturbances, so that most of the observed variation in aggregate output reflects the impact of supply-side shocks. Indeed, the model suggests that during the sample period, Federal Reserve policy has responded efficiently to these shocks, although the rate of inflation has been, on average, too high.

1 Introduction

As its title suggests, this paper develops a small, structural model of the United States economy, estimates that model with quarterly data on output, prices, and money from 1959 through 1995, and uses the model to compare the economy's performance under the monetary policy that was actually implemented during the sample period with its hypothetical performance under a number of alternative monetary policies. Taylor (1979) and McCallum (1988) perform similar exercises. But while these authors work with models that are specified at the level of equilibrium conditions, describing the relationships between various aggregate variables using small sets of log-linear equations, the present analysis follows more recent research by King

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(1990), Hairault and Portier (1993), Leeper and Sims (1994), Cho and Cooley (1995), Kim (1995), Kimball (1995), King and Watson (1996), and Yun (1996) by constructing a dynamic, stochastic, general equilibrium monetary model that is specified at the level of preferences and technologies. The equations of the model, therefore, explicitly describe the optimizing behavior of the households and firms that populate the economy.

This new approach offers several key advantages. First, as the model's parameters ultimately describe agents' preferences and technologies, they ought to be invariant with respect to changes in the monetary policy regime. Thus, there is hope that the model is, in fact, truly structural and useful for policy evaluation. Second, by being explicit about private agents' objectives, the model provides a natural criterion with which to evaluate the performance of alternative monetary policies. Specifically, each policy may be judged by its effect on welfare, measured using a representative household's utility function rather than its effect on some arbitrarily-specified loss function that penalizes variation in aggregate output and the price level. Finally, the model essentially adds to the basic real business-cycle framework, developed by Kydland and Prescott (1982), Long and Plosser (1983), and Prescott (1986), features that allow monetary disturbances to have important short-run effects on output. Thus, the analysis reveals the extent to which insights provided by real business-cycle theories, which emphasize the effects of supply-side shocks, generalize to a setting where demand-side shocks may also play a role in accounting for aggregate fluctuations.

Indeed, the model provides estimates of the relative importance of various shocks in driving movements in output and prices in the United States economy, as well as estimates that describe how the Federal Reserve has responded to these shocks over the sample period. In the welfare analysis, these estimates take center stage since, as shown by Ireland (1996a, 1996b), once utility maximization replaces output and price-level stabilization as the fundamental goal of monetary policy, it becomes clear that policy ought to respond differently to different types of shocks.

2 The model

2.1 *The economic environment*

The model takes its principal features from those of Hairault and Portier (1993) and Kim (1995), which build, in turn, on earlier work by Rotemberg (1982) and Blanchard and Kiyotaki (1987). A representative household, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms, and a monetary authority populate an economy in which time periods are indexed by $t = 0, 1, 2, \dots$. The intermediate goods-producing firms are indexed by $i \in [0, 1]$; each produces a distinct, perishable

intermediate good. Hence, intermediate goods may also be indexed by $i \in [0, 1]$, where firm i produces good i . The model imposes enough symmetry on technologies, however, to allow the analysis to focus on the behavior of a representative intermediate goods-producing firm, identified by the generic index i .

The representative household has preferences defined over consumption of the finished good, leisure, and real cash balances, giving rise to a conventional specification for money demand as a function of aggregate consumption and the nominal interest rate. The household purchases output from the representative finished goods-producing firm and supplies labor and capital to the intermediate goods-producing firms in competitive markets. The representative finished goods-producing firm purchases the intermediate goods as inputs, with which it produces the finished good.

The representative intermediate goods-producing firm produces intermediate good i with labor and capital supplied by the representative household. Since intermediate goods substitute imperfectly for one another in the finished goods-producing firm's production function, however, the intermediate goods-producing firm sells its output in a monopolistically competitive market. Hence, as in Blanchard and Kiyotaki (1987), firm i acts as a price-setter in the market for good i . As in Rotemberg (1982), each intermediate goods-producing firm faces a cost of adjusting its nominal output price; this cost-of-price adjustment allows the monetary authority to influence aggregate output in the short run by changing the nominal money supply.

Before moving on to the details, it is worth noting that in both form and substance, this model is identical to an alternative specification in which the representative household purchases the continuum of intermediate goods directly from the intermediate goods-producing firms and combines these goods into a single composite that enters its utility. This alternative specification provides a stronger rationale for including real balances in the utility function, since it requires the household to trade with a larger number of firms. The approach taken here, however, explicitly considers the activities of the representative finished goods-producing firm, thereby facilitating the exposition by breaking the task of constructing an equilibrium for the economy into a series of smaller steps.

2.2 *The representative household*

The representative household carries M_{t-1} units of money and K_t units of capital into period t . During period t , it supplies $H_t(i)$ units of labor at the nominal wage W_t and $K_t(i)$ units of capital at the nominal rental rate R_t to each intermediate goods-producing firm $i \in [0, 1]$. The household's choices

of $H_t(i)$ and $K_t(i)$ must satisfy

$$H_t = \int_0^1 H_t(i) di, \quad (1)$$

where H_t denotes its total labor supply, and

$$K_t = \int_0^1 K_t(i) di \quad (2)$$

for all $t = 0, 1, 2, \dots$

In addition to the factor payments $W_t H_t$ and $R_t K_t$, the household receives nominal profits $D_t(i)$ as a dividend from each intermediate goods-producing firm $i \in [0, 1]$ and a lump-sum nominal transfer T_t from the monetary authority during period t . The household uses some of these funds to purchase output at the nominal price P_t from the representative finished goods-producing firm, which it divides between consumption C_t and investment I_t . It then carries M_t units of money and K_{t+1} units of capital into period $t + 1$. Thus, during each period $t = 0, 1, 2, \dots$, the household faces the budget constraint

$$\frac{M_{t-1} + T_t + W_t H_t + R_t K_t + D_t}{P_t} \geq C_t + I_t + \frac{M_t}{P_t}, \quad (3)$$

where

$$D_t = \int_0^1 D_t(i) di \quad (4)$$

denotes total nominal profits received during period t , and the capital accumulation constraint

$$(1 - \delta)K_t + I_t \geq K_{t+1}, \quad (5)$$

where $\delta \in (0, 1)$ denotes the constant depreciation rate.

The household's preferences are described by the expected utility function

$$E \sum_{t=0}^{\infty} \beta^t u(C_t, M_t/P_t, H_t), \quad (6)$$

where $\beta \in (0, 1)$ is a constant discount factor and the single-period utility function takes the form

$$u(C_t, M_t/P_t, H_t) = [\gamma/(\gamma-1)] \ln[C_t^{(\gamma-1)/\gamma} + b_t (M_t/P_t)^{(\gamma-1)/\gamma}] + \eta \ln(1-H_t) \quad (7)$$

with $\gamma > 0$ and $\eta > 0$. As in Kim (1995), the preference shock b_t translates into a shock to money demand; it follows the autoregressive process

$$\ln(b_t) = (1 - \rho_b) \ln(b) + \rho_b \ln(b_{t-1}) + \varepsilon_{bt}, \quad (8)$$

where $\rho_b \in (-1, 1)$ and the serially uncorrelated shock ε_{bt} is normally distributed with mean zero and standard deviation σ_b .

Thus, the household chooses $C_t, I_t, H_t, M_t,$ and K_{t+1} for each $t = 0, 1, 2, \dots$ to maximize its utility in equations (6) and (7) subject to the constraints in equations (3) and (5). The first-order conditions for this problem are

$$\frac{C_t^{-1/\gamma}}{C_t^{(\gamma-1)/\gamma} + b_t(M_t/P_t)^{(\gamma-1)/\gamma}} = \Lambda_t, \quad (9)$$

$$\frac{b_t(M_t/P_t)^{-1/\gamma}}{C_t^{(\gamma-1)/\gamma} + b_t(M_t/P_t)^{(\gamma-1)/\gamma}} = \Lambda_t - \beta E_t \left(\frac{\Lambda_{t+1} P_t}{P_{t+1}} \right), \quad (10)$$

$$\frac{\eta}{1 - H_t} = \frac{\Lambda_t W_t}{P_t}, \quad (11)$$

and

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} \left(\frac{R_{t+1}}{P_{t+1}} + 1 - \delta \right) \right], \quad (12)$$

as well as equations (3) and (5) with equality, where $\Lambda_t > 0$ denotes the multiplier on the budget constraint in equation (3) and $E_t(\cdot)$ denotes the household's expectation during each period $t = 0, 1, 2, \dots$

Equations (9) and (11) simply equate the marginal rate of substitution between consumption and labor to the real wage, while equation (12) equates the marginal utility cost of an additional unit of investment during period t with the discounted expected marginal utility value of its return during period $t + 1$. Finally, as in Kim (1995), equations (9) and (10) imply

$$b_t \left(\frac{C_t}{M_t/P_t} \right)^{1/\gamma} = 1 - \frac{1}{R_t^b}, \quad (13)$$

where

$$R_t^b = \frac{\Lambda_t P_t}{\beta E_t(\Lambda_{t+1}/P_{t+1})} \quad (14)$$

denotes the gross nominal yield on a one-period discount bond that costs $1/R_t^b$ dollars during period t and returns one dollar during period $t + 1$. Letting $r_t^b = R_t^b - 1$ denote the net nominal interest rate between t and $t + 1$ and using the approximation $1/R_t^b \approx 1 - r_t^b$, equation (13) may be rewritten as

$$\ln(M_t/P_t) \approx \ln(C_t) - \gamma \ln(r_t^b) + \gamma \ln(b_t), \quad (15)$$

which is a standard money-demand function with unitary scale elasticity and negative interest elasticity $-\gamma$. Thus, as indicated above, b_t represents a serially correlated shock to money demand.

2.3 *The representative finished goods-producing firm*

The representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good i during period t to produce Y_t units of the finished good according to the constant-returns-to-scale technology described by

$$\left[\int_0^1 Y_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \geq Y_t \quad (16)$$

with $\theta > 1$. Intermediate good i sells at the nominal price $P_t(i)$, while the finished good sells at the nominal price P_t ; thus, the finished goods-producing firm chooses Y_t and $Y_t(i)$ for all $i \in [0, 1]$ to maximize its profits

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad (17)$$

subject to the constraint in equation (16) during each period $t = 0, 1, 2, \dots$

The first-order conditions for this problem are equation (16) with equality and

$$Y_t(i) = [P_t(i)/P_t]^{-\theta} Y_t \quad (18)$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$. Equation (18) describes the finished goods-producing firm's demand for intermediate good i as a function of its output Y_t and the relative price $P_t(i)/P_t$.

Competition in the market for the finished good requires that the representative firm earn zero profits in equilibrium. Along with equations (16)-(18), this zero-profit condition determines P_t as

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{1/(1-\theta)} \quad (19)$$

for all $t = 0, 1, 2, \dots$

2.4 *The representative intermediate goods-producing firm*

The representative intermediate goods-producing firm hires $H_t(i)$ units of labor and $K_t(i)$ units of capital from the representative household during period t to produce $Y_t(i)$ units of output according to the constant-returns-to-scale technology described by

$$A_t K_t(i)^\alpha [g^t H_t(i)]^{1-\alpha} \geq Y_t(i) \quad (20)$$

with $\alpha \in (0, 1)$, where $g \geq 1$ denotes the gross rate of labor-augmenting technological progress. The technology shock A_t follows the autoregressive process

$$\ln(A_t) = (1 - \rho_A) \ln(A) + \rho_A \ln(A_{t-1}) + \varepsilon_{At}, \quad (21)$$

where $\rho_A \in (-1, 1)$ and the serially-uncorrelated shock ε_{At} is normally distributed with mean zero and standard deviation σ_A . The shock ε_{At} is, in addition, uncorrelated with ε_{bt} at all leads and lags.

Since intermediate goods substitute imperfectly for one another as inputs to producing the finished good, the representative intermediate goods-producing firm sells its output in an imperfectly competitive market; during each period $t = 0, 1, 2, \dots$, it sets its nominal price $P_t(i)$ subject to the requirement that it satisfy the representative finished goods-producing firm's demand in equation (18), taking P_t and Y_t as given.

In addition, each intermediate goods-producing firm faces a quadratic cost of adjusting its nominal price, given by

$$(\phi/2) \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 Y_t \quad (22)$$

during each period $t = 0, 1, 2, \dots$ this cost-of-price adjustment is measured in terms of the finished good and increases proportionally with the size Y_t of the overall economy. Rotemberg (1982) provides an interpretation of this quadratic adjustment cost; unlike more literal menu cost specifications that emphasize the fixed cost of price changes, equation (22) captures the negative effects of price changes on customer-firm relationships, which increase in magnitude with the size of the price change and with the quantity purchased. In any case, equation (22) represents a tractable way of making individual nominal goods prices—and hence the aggregate price level—respond only gradually to nominal disturbances, allowing the monetary authority to influence aggregate output in the short run.

The cost-of-price adjustment in equation (22) makes the representative intermediate goods-producing firm's problem dynamic; the firm chooses $H_t(i)$, $K_t(i)$, $Y_t(i)$, and $P_t(i)$ for each $t = 0, 1, 2, \dots$ to maximize its total market value, equal to

$$E \sum_{t=0}^{\infty} \beta^t \Lambda_t [D_t(i)/P_t], \quad (23)$$

where $\beta^t \Lambda_t / P_t$ is the marginal utility value to the representative household of an additional dollar of profits during period t and

$$D_t(i) = P_t(i)Y_t(i) - W_t H_t(i) - R_t K_t(i) - P_t(\phi/2)[P_t(i)/P_{t-1}(i) - 1]^2 Y_t \quad (24)$$

subject to the constraints in equations (18) and (20). This problem is equivalent to one of choosing $H_t(i)$, $K_t(i)$, and $P_t(i)$ to maximize

$$E \sum_{t=0}^{\infty} \beta^t \Lambda_t \left\{ \left[\frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \frac{W_t H_t(i) + R_t K_t(i)}{P_t} - (\phi/2) \right\} \quad (25)$$

$$\left\{ \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 Y_t \right\}$$

subject to the constraints

$$A_t K_t(i)^\alpha [g^t H_t(i)]^{1-\alpha} \geq [P_t(i)/P_t]^{-\theta} Y_t \quad (26)$$

for all $t = 0, 1, 2, \dots$

The first-order conditions for this problem are

$$\Lambda_t W_t / P_t = (1 - \alpha) \Xi_t A_t K_t(i)^\alpha (g^t)^{1-\alpha} H_t(i)^{-\alpha}, \quad (27)$$

$$\Lambda_t R_t / P_t = \alpha \Xi_t A_t K_t(i)^{\alpha-1} [g^t H_t(i)]^{1-\alpha}, \quad (28)$$

$$\begin{aligned} 0 = & \Lambda_t (1 - \theta) \left[\frac{P_t(i)}{P_t} \right]^{-\theta} \left(\frac{Y_t}{P_t} \right) - \Lambda_t \phi \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right] \left[\frac{Y_t}{P_{t-1}(i)} \right] \\ & + \beta \phi E_t \left\{ \Lambda_{t+1} \left[\frac{P_{t+1}(i)}{P_t(i)} - 1 \right] \left[\frac{Y_{t+1} P_{t+1}(i)}{P_t(i)^2} \right] \right\} \\ & + \Xi_t \theta \left[\frac{P_t(i)}{P_t} \right]^{-\theta-1} \left(\frac{Y_t}{P_t} \right), \end{aligned} \quad (29)$$

and equation (26) with equality for all $t = 0, 1, 2, \dots$, where $\Xi_t > 0$ denotes the multiplier on equation (26). Equations (27) and (28) equate the marginal rate of substitution between labor and capital in production to the relative factor price R_t/W_t , while equation (29) governs the adjustment of firm i 's nominal price over time. In a symmetric equilibrium, where $P_t(i) = P_t$ for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$, equations (27) and (28) also indicate that Λ_t/Ξ_t measures the gross markup of price over marginal cost. Equation (29) then shows that in the absence of costly price adjustment, when $\phi = 0$, this markup equals $\theta/(\theta - 1)$; this solution is consistent with equation (18), which implies that the elasticity of demand for good i is $-\theta$.

2.5 The monetary authority

The monetary authority manages the nominal money supply M_t by making the lump-sum transfer T_t to the representative household during each period $t = 0, 1, 2, \dots$; hence

$$M_t = M_{t-1} + T_t. \quad (30)$$

In conducting its operations, the monetary authority may respond contemporaneously to the technology and money-demand shocks; it adopts a money-supply rule of the form

$$\ln(\mu_t) = (1 - \rho_\mu) \ln(\mu) + \rho_\mu \ln(\mu_{t-1}) + \psi_A \varepsilon_{At} + \psi_b \varepsilon_{bt} + \varepsilon_{\mu t}, \quad (31)$$

where $\mu_t = M_t/M_{t-1} = 1 + T_t/M_{t-1}$ denotes the gross rate of monetary growth during period t , $\rho_\mu \in (-1, 1)$, and the serially-uncorrelated shock $\varepsilon_{\mu t}$ is normally distributed with mean zero and standard deviation σ_μ . The shock $\varepsilon_{\mu t}$ is uncorrelated with both ε_{A_t} and ε_{b_t} at all leads and lags. Thus, in the special case where $\psi_A = \psi_b = 0$, equation (31) reduces to a purely exogenous, autoregressive specification for money growth.

2.6 Symmetric equilibrium

In a symmetric equilibrium, where $P_t(i) = P_t$, $H_t(i) = H_t$, $K_t(i) = K_t$, and $Y_t(i) = Y_t$ for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$, equations (26)-(29) become

$$A_t K_t^\alpha (g^t H_t)^{1-\alpha} = Y_t \quad (32)$$

$$\Lambda_t W_t / P_t = (1 - \alpha) \Xi_t Y_t / H_t, \quad (33)$$

$$\Lambda_t R_t / P_t = \alpha \Xi_t Y_t / K_t, \quad (34)$$

and

$$\begin{aligned} 0 = & \Lambda_t (1 - \theta) - \Lambda_t \phi \left(\frac{P_t}{P_{t-1}} - 1 \right) \left(\frac{P_t}{P_{t-1}} \right) \\ & + \beta \phi E_t \left[\Lambda_{t+1} \left(\frac{P_{t+1}}{P_t} - 1 \right) \left(\frac{Y_{t+1} P_{t+1}}{Y_t P_t} \right) \right] + \Xi_t \theta. \end{aligned} \quad (35)$$

There is, in addition, an aggregate resource constraint

$$Y_t = C_t + K_{t+1} - (1 - \delta) K_t + (\phi/2) (P_t/P_{t-1} - 1)^2 Y_t, \quad (36)$$

which must hold for all $t = 0, 1, 2, \dots$

Equations (8)-(12), (21), and (31)-(36) form a system of 12 equations in the 12 variables $A_t, K_t, H_t, Y_t, \Lambda_t, W_t, P_t, \Xi_t, R_t, C_t, b_t$, and M_t . The solution to this system describes a symmetric equilibrium for the economy.

3 Solving and estimating the model

In equilibrium, most of the model's real variables inherit a deterministic trend g from the constant rate of technological progress. The nominal variables, meanwhile, must be either divided by the price level P_t or expressed in growth rates to induce stationarity. Accordingly, let $k_t = K_t/g^t$, $y_t = Y_t/g^t$, $\lambda_t = g^t \Lambda_t$, $w_t = (W_t/P_t)/g^t$, $\pi_t = P_t/P_{t-1}$, $\xi_t = g^t \Xi_t$, $r_t = R_t/P_t$, $c_t = C_t/g^t$, and $m_t = (M_t/P_t)/g^t$. Equations (8)-(12), (21), and (31)-(36) may be rewritten in terms of these transformed variables, while the definitions of π_t, m_t , and μ_t imply

$$(m_t/m_{t-1})\pi_t g = \mu_t. \quad (37)$$

Thus, equations (8)-(12), (21), and (31)-(37) form a system of 13 equations in the 13 stationary variables $A_t, k_t, H_t, y_t, \lambda_t, w_t, \pi_t, \xi_t, r_t, c_t, b_t, m_t$, and μ_t .

In the absence of shocks to technology, money demand, and money supply, the economy converges to a steady state, in which all stationary variables are constant. A log-linear approximation of equations (8)-(12), (21), and (31)-(37) around this steady state may be solved using methods outlined by Blanchard and Kahn (1980). This solution, which describes the behavior of the stationary variables as they fluctuate about their steady-state values in response to the exogenous shocks, takes the form

$$s_{t+1} = \Gamma s_t + \Omega \varepsilon_{t+1} \quad (38)$$

$$f_t = \Pi s_t, \quad (39)$$

where the elements of the matrices Γ, Ω , and Π depend on the underlying parameters of the model, where

$$s_t = [\hat{k}_t \hat{m}_{t-1} \hat{A}_t \hat{b}_t \hat{\mu}_t]', \quad (40)$$

$$\varepsilon_t = [\varepsilon_{At} \varepsilon_{bt} \varepsilon_{\mu t}]', \quad (41)$$

and

$$f_t = [\hat{\lambda}_t \hat{\xi}_t \hat{y}_t \hat{H}_t \hat{w}_t \hat{r}_t \hat{c}_t \hat{\pi}_t]', \quad (42)$$

and where each element \hat{x}_t of the vectors s_t and f_t denotes the percentage deviation $\ln(x_t/x)$ of the variable x_t from its steady-state value x .

Hansen and Sargent (1994) describe methods for estimating models with solutions of the form given by equations (38) and (39). When supplied with data on output, prices, and money, these methods apply the Kalman filter to equation (38) to construct a record of innovations $\{\varepsilon_t\}_{t=1}^T$, which can then be used to evaluate the likelihood foundation for the sample. Since the likelihood function also depends on the elements of Γ, Ω , and Π , the model's parameters may be estimated by maximizing this function. Note that the model imposes a stationary-inducing transformation on each of the series: output is detrended, and the price level and money supply expressed in growth rates, as part of the estimation procedure. Thus, when estimated, the model reduces to a constrained, first-order vector autoregression for detrended output, inflation, and money growth.

The quarterly United States data used to estimate the model run from 1959:1 through 1995:3. Output is measured by gross domestic product in constant (1987) dollars, the price level is measured by the implicit deflator for gross domestic product, and the money supply is measured by the broad aggregate M2. The use of M2 as the measure of money follows in the tradition of Friedman and Schwartz (1970) and also reflects more recent evidence,

presented by Hafer and Jansen (1991) among others, that a stable money-demand relationship exists in the postwar United States data for M2 but not for the narrower aggregate M1. All three series are seasonally adjusted; the series for output and money are converted to per capita terms by dividing by the civilian noninstitutional population, ages 16 and over.

These series for output, prices, and money were selected under the assumption that they are most informative about monetary policy and its effects on the economy. No doubt, other series would prove useful as well; in work with similar models, for instance, Kim (1995) uses data on the federal funds rate, while Rotemberg (1996) uses data on aggregate hours worked. Here, however, the model includes only three sources of shocks—technology, money demand, and money supply—so that maximum likelihood estimation with data on more than three variables requires that measurement error be introduced into the analysis. Furthermore, as shown by Kimball (1995) and King and Watson (1996), models of the type used here have difficulty explaining the observed response of short-term interest rates to monetary disturbances; indeed, work by Kim (1995) suggests that a number of extra features, including adjustment costs for both capital and nominal wages, must be added to the model to successfully account for the behavior of nominal interest rates. To avoid these complications, the analysis here focuses exclusively on data for output, prices, and money.

The data contain very little information about some of the model's parameters; values for these parameters must be fixed prior to estimation. In each case, however, guidance for choosing an appropriate parameter value may be found either in previous work with calibrated models or in other sources of data. Thus, the discount factor β , which is notoriously difficult to estimate, is set equal to 0.99 following standard practice. Values for the depreciation rate δ and the parameter α describing capital's share in production are difficult to pin down without data on capital. Thus, δ is set equal to 0.025, while α is set equal to 0.30. Similarly, the weight η on leisure in the representative household's utility function is difficult to pin down without data on hours worked; the setting $\eta = 1.63$ used here implies that in the steady state, the household spends approximately 30 percent of its time engaged in market activity.

Since the estimation procedure ultimately uses data on the growth rates, rather than the levels, of prices and money, the parameter b determining the steady-state ratio of real balances to consumption remains unidentified. Thus, b is set equal to 0.0135, matching the steady-state consumption velocity of money in the model to the average consumption velocity of M2 in the United States data, 1959:1-1995:3. Finally, the monopolistically competitive market structure in this model works primarily to lower equilibrium output below its efficient level; consequently, the parameter θ describing the elas-

ticity of demand for each intermediate good cannot be identified separately from the parameter A , which also determines the equilibrium level of output. Hence, θ is set equal to 6, so that the gross steady-state markup of price over marginal cost in the model matches Rotemberg and Woodford's (1992) benchmark value of 1.2, chosen based on their survey of empirical studies of the markup in the United States economy. The results presented below are quite robust to the choice of θ , however, with all figures and tables virtually unchanged when the model is reestimated under the alternative settings $\theta = 11$ (corresponding to a markup of 1.1) and $\theta = 3.5$ (corresponding to a markup of 1.4).

Table 1 lists the maximum likelihood estimates of the model's remaining 13 parameters, along with their standard errors; the Appendix provides the corresponding estimates of the matrices Γ, Ω , and Π from equations (38) and (39). The estimate of γ implies an interest elasticity of M2 demand equal to -0.159, while the estimates $\rho_b = 0.998$ and $\sigma_b = 0.102$ show that money-demand shocks tend to be large and highly persistent. The estimate $g = 1.00367$ implies that the annual trend rate of real, per capita output growth falls just short of 1.5 percent, while the estimate of A simply matches the levels of per capita output in the model and the data. The estimates $\rho_A = 0.974$ and $\sigma_A = 0.00633$ lie close to those used in the real business-cycle literature.

Table 1:
Maximum Likelihood Estimates and Standard Errors

Parameter	Description	Estimate	Standard Error
γ	Interest Elasticity of Money Demand	0.159	0.0934
ρ_b	Persistence of Money-Demand Shocks	0.998	0.00986
σ_b	Standard Deviation of Money-Demand Shocks	0.102	0.0411
g	Rate of Technological Progress	1.00367	0.000287
A	Level of Output	1197.7	6.69
ρ_A	Persistence of Technology Shocks	0.974	0.0140
σ_A	Standard Deviation of Technology Shocks	0.00633	0.000467
ϕ	Price-Adjustment Cost	4.05	0.905
μ	Steady-State Money Growth Rate	1.0148	0.000623
ψ_A	Policy Response to Technology Shocks	0.0872	0.0427
ψ_b	Policy Response to Money-Demand Shocks	0.0558	0.0237
ρ_μ	Persistence of Money-Supply Shocks	0.687	0.0589
σ_μ	Standard Deviation of Money-Supply Shocks	0.00216	0.000420

Equation (36) reveals that the share of aggregate output devoted to costly price changes is $(\phi/2)(\pi_t - 1)^2$, where π_t denotes the gross, quarterly rate

of inflation during period t . Since the model's steady-state inflation rate is determined as $\pi = \mu/g$, the estimate $\mu = 1.0148$ implies $\pi = 1.0111$, which translates into an average annual inflation rate of about 4.5 percent. Hence, the estimate $\phi = 4.05$ implies that the steady-state costs of price adjustment are quite small, equal to less than 0.025 percent of aggregate output. Still, the costs of price adjustment are large enough to allow monetary shocks to have important short-run effects on output. Figure 1 plots the impulse response of detrended output to a one-percent money-supply shock at $t = 1$ using the set of estimated parameter values. Output increases by more than 1.6 percent during the period of the shock. As in Hairault and Portier (1993), however, this specification does not allow for much persistence in the output effects of monetary shocks.

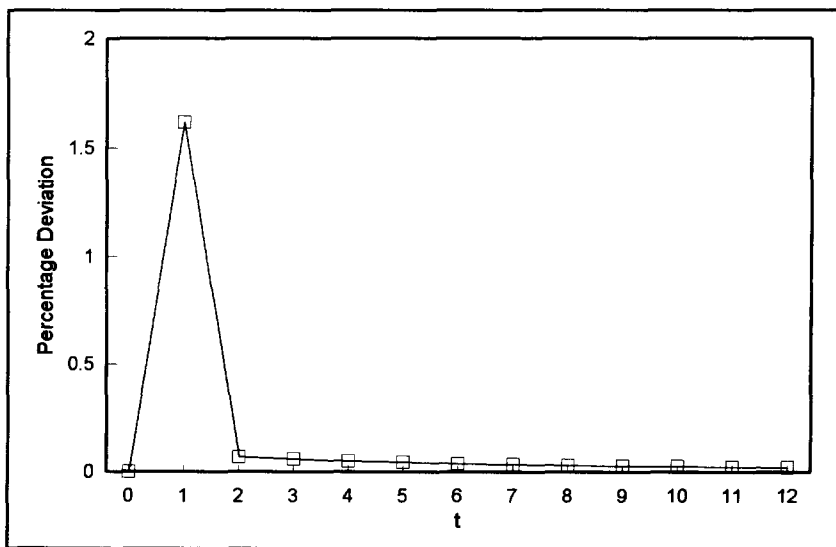
Finally, the estimates $\psi_A = 0.0872$ and $\psi_b = 0.0558$ indicate that over the sample period, the Federal Reserve has responded to both positive technology shocks, which increase aggregate output, and positive money-demand shocks, which decrease aggregate output, by increasing the rate of monetary growth. In other words, over the sample period, the Federal Reserve has responded procyclically to supply-side shocks and countercyclically to demand-side shocks. The variation in money growth not explained by the Federal Reserve's response to shocks, measured by the estimate $\sigma_\mu = 0.00216$, is small, although the estimate $\rho_\mu = 0.687$ indicates that shocks to the rate of money growth tend to persist.

Since, as noted above, the model implies that detrended output, inflation, and money growth follow a constrained, first-order vector autoregression, the model's restrictions may be tested by comparing its fit to that of an unconstrained vector autoregression for these three variables. The unconstrained model has 19 parameters: the trend for output; the constant term and the coefficients on lagged output, inflation, and money growth in each equation; and the six elements of the error covariance matrix. The constrained model has only 13 parameters; it imposes six constraints on the estimated model. Thus, if L^u denotes the maximized value of the log-likelihood function for the unconstrained model, and if L^c denotes the maximized value of the log-likelihood function for the constrained model, the likelihood ratio statistic $2(L^u - L^c)$ has a chi-square distribution with six degrees of freedom under the null hypothesis that the restrictions hold.

In the data, $L^u = 2044$, while $L^c = 1979$. The 0.1 percent critical value for a $\chi^2(6)$ random variable is 22.5. Thus, the likelihood ratio test easily rejects the model's restrictions. Given the number of restrictions relative to the number of free parameters, however, it is hardly surprising that the constrained model fails to fit the data along some dimensions. More important is the fact that the constrained model's restrictions allow the maximum likelihood procedure to obtain reasonable and precise estimates of the parameters

Figure 1

Impulse Response of Detrended Output to a One Percent Money Supply Shock



shown in Table 1.

4 Welfare analysis

Figures 2 and 3 foreshadow all of the results that follow. They display the impulse responses for detrended output to one-percent technology and money-demand shocks at $t = 1$ in three economies. The first economy, labeled “estimated,” sets all parameters equal to their estimated values. The second economy, labeled “exogenous money,” sets the parameters ψ_A and ψ_b equal to zero, so that the money supply no longer responds to the technology and money-demand shocks; all other parameters remain at their estimated values. Finally, the third economy, labeled “flexible prices,” sets the cost of adjustment parameters ϕ , in addition to the money-supply parameters ψ_A and ψ_b , equal to zero, so that nominal prices are free to adjust immediately to the shocks; again, all other parameters remain at their estimated values.

The figures show that in the flexible price case, output jumps in response to a positive technology shock before returning gradually to its steady-state level. Output responds very little to the money-demand shock, however, since nominal prices fall freely to accommodate the increase in real money demand. Nominal prices rise gradually as b_t returns to its steady-state level; this small increase in inflation acts as a distortionary tax on real balances, causing output to remain slightly below its steady-state level for many periods after the shock.

In the exogenous money economy, where prices are sticky but the monetary authority fails to respond to the shocks, part of the increase in output following a positive technology shock is delayed; nominal prices cannot fall fast enough to generate the appropriate increase in demand during period $t = 1$. Furthermore, with nominal prices unable to immediately adjust, the positive shock to money demand causes output to fall sharply.

Thus, the two figures reveal that in the estimated economy, monetary policy allows output to move as it would in an economy without nominal price rigidity. The increase in money growth following both types of shocks permits output to rise farther after a positive technology shock and helps to insulate the economy from the real effects of a positive money-demand shock.

Tables 2 and 3 decompose the forecast error variance of detrended output, inflation, and money growth at various horizons in the estimated and exogenous money economies. Since, in the estimated economy, monetary policy works to offset the real effects of money-demand shocks, panel A of Table 2 shows that technology shocks account for most of the observed variation in output, even at short horizons; the fraction of the total variance attributable to technology shocks approaches 75 percent at the one-quarter-ahead horizon and exceeds 90 percent at the one-year-ahead horizon. Panel C of Table

Figure 2

Impulse Response of Detrended Output to a One Percent Technology Shock

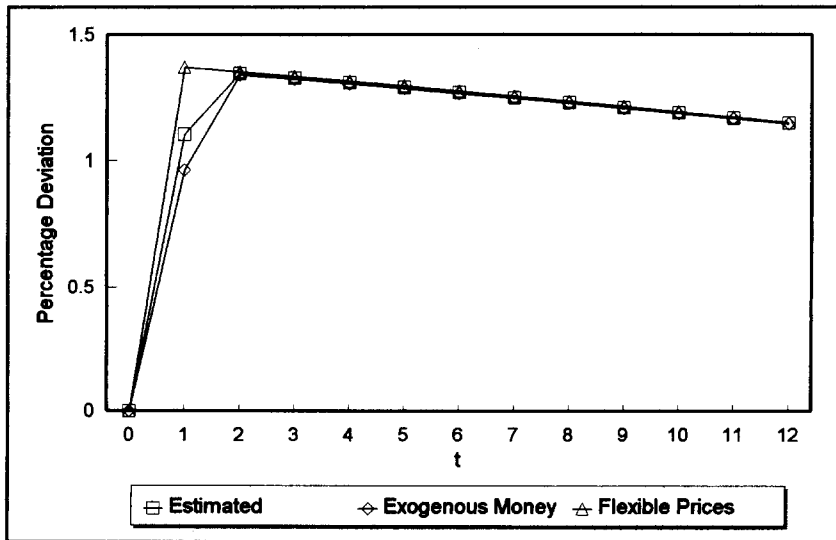
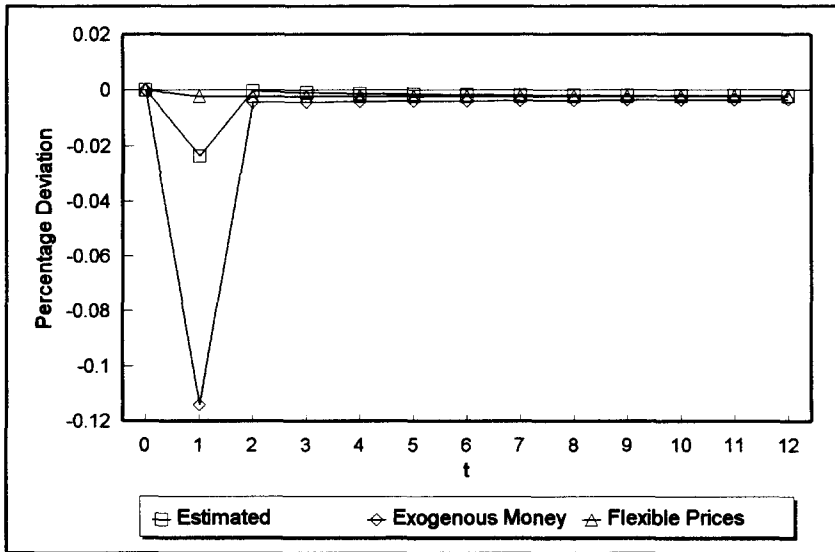


Figure 3

Impulse Response of Detrended Output to a One Percent Money Demand Shock



2 shows that most changes in money growth reflect the deliberate policy response to money-demand disturbances; money-supply shocks account for only 12.6 percent of all observed variation in M2 growth.

Panel A of Table 3 shows that in the exogenous money economy, money-demand shocks replace technology shocks as the dominant source of short-run output fluctuations, accounting for 73 percent of the one-quarter-ahead forecast error variance in detrended output. In fact, output becomes far more volatile without the estimated policy response to shocks: the one-quarter-ahead variance increases by a factor of nearly 2.8 moving from Table 2 to Table 3, while the one-year-ahead variance rises by a factor of 1.4.

Thus, the results displayed in Figures 2 and 3 and Tables 2 and 3 indicate that the Federal Reserve has successfully insulated the economy from exogenous demand-side disturbances over the sample period, so that most of the observed fluctuations in aggregate output simply reflect the economy's efficient response to supply-side shocks. Table 4 shows the effects of the estimated policy on welfare, measured using the representative household's utility function.

Panel A compares the estimated policy with three alternatives. The first alternative is the "exogenous" policy considered above, which sets ψ_A and ψ_b equal to zero, so that money growth does not respond to the technology and money-demand shocks. The second and third are constant money-growth policies, although each involves a different interpretation of the money-supply shock $\varepsilon_{\mu t}$. The policy labeled "constant I" interprets $\varepsilon_{\mu t}$ as an inevitable monetary control error, which the Federal Reserve cannot avoid even if it adopts a constant money-growth-rate rule. Hence, this policy sets ρ_μ , ψ_A , and ψ_b equal to zero, but keeps σ_μ at its estimated value, so that serially uncorrelated control errors cause money growth to fluctuate randomly about its target rate μ . The policy labeled "constant II," on the other hand, interprets $\varepsilon_{\mu t}$ as reflecting deliberate policy actions taken by the Federal Reserve over the sample period that are not captured by the estimation procedure as responses to technology or money-demand shocks. Under this interpretation, the Federal Reserve could keep money growth absolutely constant, if it so desired. Hence, this policy sets σ_μ , as well as ρ_μ , ψ_A , and ψ_b , equal to zero. In the welfare analysis, all other parameters are set at their estimated values.

Table 4 shows the value of the representative household's expected utility under each monetary policy, along with a measure of the welfare cost of the alternative monetary policies: the permanent percentage increase in consumption that makes the representative household as well off under each alternative policy as it is under the estimated rule. Thus, a positive reading from this measure indicates that the estimated policy improves on the alternative, while a negative reading indicates that the estimated policy's performance falls short of the alternative's.

Table 2:
Forecast Error Variance Decomposition Under
the Estimated Monetary Policy
Panel A. Detrended Output

Quarters Ahead	Total Variance	Percentage Due To		
		Technology	Money Demand	Money Supply
1	0.000066	73.1	8.6	18.3
2	0.000139	87.1	4.1	8.8
3	0.000210	91.4	2.7	5.8
4	0.000279	93.5	2.1	4.4
8	0.000534	96.6	1.1	2.3
12	0.000757	97.6	0.8	1.6
20	0.001113	98.3	0.6	1.1
∞	0.001912	98.2	1.1	0.6

Panel B. Inflation

Quarters Ahead	Total Variance	Percentage Due To		
		Technology	Money Demand	Money Supply
1	0.000034	16.7	6.7	76.7
2	0.000036	15.9	10.5	73.6
3	0.000037	15.7	12.2	72.1
4	0.000037	15.6	13.0	71.4
8	0.000037	15.6	13.8	70.7
12	0.000038	15.6	13.8	70.5
20	0.000038	15.6	13.9	70.5
∞	0.000038	16.2	14.2	69.5

Panel C. Money Growth

Quarters Ahead	Total Variance	Percentage Due To		
		Technology	Money Demand	Money Supply
1	0.000037	0.8	86.6	12.6
2	0.000054	0.8	86.6	12.6
3	0.000062	0.8	86.6	12.6
4	0.000066	0.8	86.6	12.6
8	0.000070	0.8	86.6	12.6
12	0.000070	0.8	86.6	12.6
20	0.000070	0.8	86.6	12.6
∞	0.000070	0.8	86.6	12.6

Table 3:
Forecast Error Variance Decomposition Under
the Exogenous Monetary Policy
Panel A. Detrended Output

Quarters Ahead	Total Variance	Percentage Due To		
		Technology	Money Demand	Money Supply
1	0.000183	20.3	73.0	6.7
2	0.000255	42.8	52.4	4.8
3	0.000325	55.1	41.1	3.8
4	0.000394	62.9	34.0	3.1
8	0.000648	77.3	20.8	1.9
12	0.000871	83.1	15.5	1.4
20	0.001226	87.9	11.1	1.0
∞	0.002025	91.9	7.5	0.6

Panel B. Inflation

Quarters Ahead	Total Variance	Percentage Due To		
		Technology	Money Demand	Money Supply
1	0.000264	5.2	84.8	10.0
2	0.000264	5.2	84.7	10.1
3	0.000264	5.3	84.7	10.1
4	0.000265	5.3	84.6	10.1
8	0.000265	5.3	84.6	10.1
12	0.000265	5.3	84.6	10.1
20	0.000265	5.3	84.6	10.1
∞	0.000265	5.4	84.5	10.1

Panel C. Money Growth

Panel C. Money Growth

Quarters Ahead	Total Variance	Percentage Due To		
		Technology	Money Demand	Money Supply
1	0.000004	0.0	0.0	100.0
2	0.000006	0.0	0.0	100.0
3	0.000007	0.0	0.0	100.0
4	0.000008	0.0	0.0	100.0
8	0.000008	0.0	0.0	100.0
12	0.000008	0.0	0.0	100.0
20	0.000008	0.0	0.0	100.0
∞	0.000008	0.0	0.0	100.0

Table 4:

Expected Utility and Welfare Cost of Various Monetary Policies
 Panel A. Positive Steady-State Inflation

Policy	Expected Utility	Welfare Cost (Percentage of Consumption)
Estimated	929.36599	0.00000
Exogenous	929.35472	0.01153
Constant I	929.35611	0.01010
Constant II	929.35656	0.00985

Panel B. Zero Steady-State Inflation

Policy	Expected Utility	Welfare Cost (Percentage of Consumption)
Estimated	929.72720	-0.36563
Exogenous	929.71651	-0.35480
Constant I	929.71820	-0.35651
Constant II	929.71842	-0.35674

As suggested by the impulse response and variance decompositions, panel A of Table 4 indicates that the estimated policy outperforms all three alternatives. The estimated policy provides only small welfare gains, however; the representative household requires only a 0.01-percent increase in consumption to make it as well off under a constant money-growth rule as it is under the estimated policy.

All of these results appear quite general. Using a model of nominal price rigidity similar to the one developed here, Ireland (1996b) shows analytically that the optimal monetary policy calls for a procyclical response to technology shocks and a countercyclical response to money-demand shocks; these policy responses allow aggregate output to fluctuate as it would in an economy without nominal price rigidity. Ireland (1996a), meanwhile, establishes that the welfare cost of adopting a constant money-growth rule rather than the optimal policy is always quite small.

According to panel B of Table 4, however, larger welfare gains are offered by policies that decrease the average rate of inflation. The four policies shown there are the same as in panel A, except that each sets μ equal to the estimated value of g , so that the steady-state rate of inflation equals zero. Thus, the representative household would be willing to permanently sacrifice 0.366 percent of its consumption to live under a policy that is identical to the estimated policy but lowers inflation to zero.

These results indicate that while monetary policy has responded efficiently to both demand-side and supply-side shocks over the same period, the rate of inflation has been, on average, too high.

5 Conclusion

When estimated with quarterly United States data on output, prices, and money from 1959 through 1995, the small, structural model developed here suggests that the Federal Reserve has successfully insulated the economy from the real effects of exogenous, demand-side disturbances over the sample period. Thus, when compared with alternatives that make no attempt to respond to these shocks, actual Federal Reserve policy has both dampened the magnitude of short-run output fluctuations and increased welfare.

The model also indicates that additional welfare gains could be achieved by lowering the average rate of inflation to zero. Overall, therefore, the results suggest that the best way to improve monetary policy in the United States would be to impose constraints that provide for long-run price stability while still allowing the Federal Reserve to respond to transitory shocks as it sees fit. The existing Humphrey-Hawkins procedures, for instance, could be replaced with new legislation that lists price stability as the single long-run objective of monetary policy, leaving the Federal Open Market Committee to react to

economic news on a meeting-by-meeting basis as it does today.

Finally, although the model used here departs from the real business-cycle framework by allowing monetary disturbances to play a role in explaining aggregate fluctuations, the analysis supports some of the insights provided by real business-cycle theories. Most significantly, the results confirm the real business-cycle theorists' claim that shocks to technology account for a very large fraction of the observed variation in aggregate output in the United States economy. The results qualify this statement in one important way, however, by indicating that the relative importance of supply-side shocks partly reflects the Federal Reserve's success at offsetting the effects of exogenous demand-side disturbances.

Appendix

This Appendix presents the matrices Γ , Ω , and Π from equation (38) that describe the model's solution and are implied by the maximum-likelihood estimates of the parameters shown in Table 1; they are

$$\Gamma = \begin{bmatrix} 0.884 & 0.101 & 0.0772 & -0.0159 & 0.244 \\ 0.442 & 0.0506 & 0.585 & 0.148 & -1.38 \\ 0 & 0 & 0.974 & 0 & 0 \\ 0 & 0 & 0 & 0.998 & 0 \\ 0 & 0 & 0 & 0 & 0.687 \end{bmatrix}, \quad (43)$$

$$\Omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.0872 & 0.0558 & 1 \end{bmatrix}, \quad (44)$$

and

$$\Pi = \begin{bmatrix} -0.442 & -0.0506 & -0.585 & 0.00563 & -0.137 \\ -0.838 & 0.747 & -1.04 & -0.119 & 1.69 \\ -0.217 & 0.717 & 0.963 & -0.114 & 1.62 \\ -0.738 & 1.02 & -0.0522 & -0.163 & 2.31 \\ 0.125 & 0.491 & 0.563 & -0.0755 & 1.13 \\ -1.61 & 1.51 & 0.511 & -0.238 & 3.44 \\ 0.442 & 0.0506 & 0.585 & -0.00929 & -0.0199 \\ -0.442 & 0.949 & -0.585 & -0.148 & 2.38 \end{bmatrix}, \quad (45)$$

Note that these matrices can be used to generate the impulse responses of any of the variables in the vectors s_t and f_t , defined by equations (40) and (42), to any of the innovations in the vector ε_t , defined by equation (41).

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