A social capital index

Enrique González-Arangüena* **Anna Khmelnitskaya†**
Conrado Manuel‡ **Mónica del Pozo§**

December 16, 2011

Abstract

In this paper we propose a social capital measure for individuals belonging to a social network. To do this, we use a game theoretical approach and so we suppose that these individuals are also involved in a cooperative TU-game modelling the economic or social interests that motivate their interactions. We propose as a measure of individual social capital the difference between the Myerson and the Shapley values of actors in the social network and explore the properties of such a measure. This definition is close to our previous measure of centrality (Gómez et al., 2003) and so in this paper we also study the relation between social capital and centrality, finding that this social capital measure can be considered as a vector magnitude with two additive components: centrality and positional externalities. Finally, several real political examples are used to show the agreement of our conclusions with the reality in these situations.

Keywords: social capital, centrality, TU game, Shapley value, Myerson value

JEL Classification Number: C71
Mathematics Subject Classification 2010: 91A12, 91A43

1 Introduction

During last two decades the concept of social capital has been actively popularized by scholars, especially economists, sociologists and political scientists. This notion initially appeared to highlight the importance of social network relations as a valuable resource for social and economic affairs. However, the current meaning of social capital is wider and nowadays it is usually assumed that social capital describes circumstances in which individuals can use membership in groups and networks to secure benefits. This formulation treats social capital as a resource available to individuals (as economic capital or human capital) that cannot be evaluated without knowledge of the society in which they operate and that has value modifying

*E. González-Arangüena, Departamento de Estadística e I.O. III. Escuela Universitaria de Estadística, Universidad Complutense de Madrid, Spain, e-mail: egaran@estat.ucm.es

†A.B. Khmelnitskaya, Saint-Petersburg State University, Faculty of Applied Mathematics, Russia, e-mail: a.khmelnitskaya@math.utwente.nl

‡C. Manuel, Departamento de Estadística e I.O. III. Escuela Universitaria de Estadística, Universidad Complutense de Madrid, Spain, e-mail: conrado@estat.ucm.es

§M. del Pozo, Departamento de Estadística e I.O. III. Escuela Universitaria de Estadística, Universidad Complutense de Madrid, Spain, e-mail: mpozojuan@telefonica.net

individual and collective productivity. The term social capital is introduced in Salisbury (1969). Bourdieu (1972) gives a more precise definition and Putnam(2000) and Putnam and Feldstein (2003) popularized it. Today the concept of social capital is widely used to explain or to understand the form in which social networks produce returns in different environments. Knack and Keefer (1997), Burt (1997), Portes (1998), Dasgupta and Serageldin (2000), Flap (2001) and Van Emmerik (2006) are contributions in this direction, to name only a few. How to measure social capital is a point of controversy that affects even to its denomination. Some scholars doubt on the appropriateness of the term when we have problems to measure it in a quantitative manner. The oldest method to measure social capital is the Name/Generator Interpreter (McCallister and Fischer, 1978). Later, Lin and Dumin (1986) and Lin et al. (2001) introduced the Position Generator method. Snijders (1999) and Van der Gaag and Snijders (2005) proposed the so called Resource Generator method trying to avoid the disadvantages of previous ones. Our main concern in this paper is to introduce a numerical measure of individual social capital for a society composed by a finite set of individuals. We will use a game theoretical point of view and thus we will assume that interests of individuals (players) are represented by means of a cooperative game with transferable utility and players' personal bilateral relations (social network) are introduced via a graph.

Given a cooperative game and a social network among players we define an *index of a player's individual social capital* as an excess of the Myerson value over the Shapley value of the player which, in turn, equals to the Shapley value of the player in the game being the difference between Myerson restricted game and the given game itself. The consideration of the difference between the Myerson and Shapley values of a player provides a tool for revealing the influence of the player's social network relations to the outcome of the game. Remark that the so defined social capital index is conceptually close to the centrality measure introduced in Gómez et al. (2003). But while in the above mentioned paper the authors define the centrality measure only for evaluation of a player's positional importance in a graph avoiding a priori differences among players and thus using a symmetric game, we define the social capital index as an index of player's relational importance admitting that they possibly have different cooperative abilities. Moretti et al. (2010) use a very related approach to measure centrality and power of genes in biological networks.

In the paper we study general properties of the social capital index and reveal its upper and lower bounds. We show that, given a game and a communication graph, the social capital of a player reaches its maximum when the communication graph is a star and the player is the hub of this star, while the social capital of a player is minimal when the player is an isolated point in a communication graph. Being these results not surprising we try to analyze the impact that can the network have in powering players with poor expectations in the game and also the possibility that a peripheral or isolated position in the graph destroys most of the advantages of being the more powerful player in the game. Another aim of this paper is to relate the proposed social capital measure with the centrality measures defined in Gómez et al. (2003). We prove that social capital of each player can be viewed as a two dimensional vector, one of the dimensions being the centrality of his position in the network and the other the externalities he obtains because of being in such a position. The computations done for two real-life examples: (i) 2009 Basque Country Parliament elections, when the Basque Nationalist Party winning the maximal number of seats finally was not included in to the majority due to its weak communication ability on the political spectrum, and (*ii*) 1983 Italian Parliament elections, when Bettino Craxi from the Italian Socialist Party, which got in the Lower Chamber only 73 seats from the total amount of 630 but had very strong central position, became the Prime Minister, clearly show the coincidence of our theoretical predictions to the historical facts in these two situations.

The structure of the paper is as follows: basic definitions and notation are given in Sect. 2; in Sect. 3 we introduce and study the social capital index for general case. Sect. 4 uncovers the relation between the introduced social capital index and centrality measures studied in Gómez et al. (2003). Discussion of practical examples is presented in Sect. 5. A final section includes conclusions and some possible future research.

2 Preliminaries

A *cooperative game with transferable utility* (*TU game*) is a pair $\langle N, v\rangle$, where $N = \{1,\ldots,n\}$ is a finite set of $n \geq 2$ players and $v\colon 2^N \to \mathbb{R}$ is a *characteristic function*, defined on the power set of N, satisfying $v(\emptyset) = 0$. A subset $S \subseteq N$ (or $S \in 2^N$) of s players is called a *coalition*, and the associated real number $v(S)$ presents the *worth* of S. The set of all games with fixed N we denote by $\mathcal{G}_N$. For simplicity of notation and if no ambiguity appears, we write v instead of $\langle N, v\rangle$ when refer to a game. A *value* is a mapping $\xi\colon \mathcal{G}_N \to \mathbb{R}^n$ that assigns to every $v \in \mathcal{G}_N$ a vector $\xi(v) \in \mathbb{R}^n$; the real number $\xi_i(v)$ represents the *payoff* to player i in v. A *subgame* of v with a player set $T \subseteq N$, $T \neq \emptyset$, is a game $v|_T$ defined as $v|_T(S) = v(S)$, for all $S \subseteq T$. A game v is *symmetric*, if the worth of each coalition depends only on its cardinality. A game v is *nonnegative*, if $v(S) \geq 0$ for all $S \subseteq N$. A game v is *monotonic*, if $v(S) \leq v(T)$ for all $S \subseteq T \subseteq N$. It is easy to see that every monotonic game is nonnegative. A game v is *superadditive*, if $v(S \cup T) \geq v(S) + v(T)$ for all $S, T \subseteq N$, such that $S \cap T = \emptyset$. A player $i \in N$ is a *null-player* in game $v \in \mathcal{G}_N$, if he adds nothing when he joins any coalition not containing him, i.e., $v(S \cup \{i\}) = v(S)$ for all $S \subset N \setminus \{i\}$. A player i is a *veto-player* in the game $v \in \mathcal{G}_N$, if $v(S) = 0$, for every $S \subseteq N \setminus \{i\}$. A game $v \in \mathcal{G}_N$ is a *veto-rich* game, if it has at least one veto-player. For any game $v \in \mathcal{G}_N$, by v_0 we denote its 0-normalization, i.e., $v_0(S) = v(S) - \sum_{i\in S} v(\{i\})$ for all $S \subseteq N$. The cardinality of a given set A we denote by $|A|$ along with lower case letters like $n = |N|$, $m = |M|$, $n_k = |N_k|$, and so on.

It is well known (Shapley, 1953) that *unanimity games* $\{u_T\}_{\substack{T\subseteq N\\ T\neq\emptyset}}$, defined as $u_T(S) = 1$ if $T \subseteq S$, and $u_T(S) = 0$ otherwise, create a basis in $\mathcal{G}_N$, i.e., every $v \in \mathcal{G}_N$ can be uniquely presented in the linear form $v = \sum_{T\subseteq N, T\neq\emptyset} \lambda^v_T\, u_T$, where $\lambda^v_T = \sum_{S\subseteq T} (-1)^{t-s}\, v(S)$, for all $T \subseteq N$, $T \neq \emptyset$. Following Harsanyi (1959) the coefficient λ^v_T is referred to as a *dividend* of coalition T in game v.

The *Shapley value* (Shapley, 1953) of a game $v \in \mathcal{G}_N$ can be given by

$$Sh_i(v) = \sum_{S\subseteq N\setminus\{i\}} \frac{s!\,(n-s-1)!}{n!}\big(v(S\cup\{i\}) - v(S)\big), \qquad \text{for all } i \in N.$$

In what follows, for simplicity, we will use notation $p_s = \dfrac{s!\,(n-s-1)!}{n!}$.

In this paper we study cooperative games among the players that belong to some social network represented via an undirected graph, i.e., we consider cooperative games with limited cooperation possibilities given in terms of undirected graphs as introduced in Myerson (1977). An *undirected graph* (N,γ) is a collection of unordered pairs of nodes/players $\gamma \subseteq \gamma_N^c$, where $\gamma_N^c = \{\,\{i,j\} \,|\, i,j \in N,\ i \neq j\}$ is the complete undirected graph without loops on N and an unordered pair $\{i,j\}$ represents a *link* between $i,j \in N$. A pair $\langle v,\gamma\rangle$ of a game $v \in \mathcal{G}_N$ and a social network (a graph) γ on N composes a *game with cooperation (graph) structure*, or simply a Γ-*game*. The set of all Γ-games with fixed N we denote $\mathcal{G}_N^\Gamma$. A Γ-*value* is a mapping $\xi\colon \mathcal{G}_N^\Gamma \to \mathbb{R}^n$ that assigns to every $\langle v,\gamma\rangle \in \mathcal{G}_N^\Gamma$ a vector $\xi(v,\gamma) \in \mathbb{R}^n$.

For any social network γ on N and any coalition $S \subseteq N$, the graph γ restricted to S is $(S,\gamma|_S)$, where $\gamma|_S = \{\{i,j\} \in \gamma \,|\, i,j \in S\}$. A sequence of different nodes $\{i_1,\ldots,i_k\} \subseteq N$ is a *path* in a graph γ, if $\{i_h,i_{h+1}\} \in \gamma$, for all $h = 1,\ldots,k-1$. Two nodes $i,j \in N$ are *connected* in γ, if there exists a path $\{i_1,\ldots,i_k\}$ in γ with $i_1 = i$ and $i_k = j$. A social network is *connected* if any two actors in it are connected. Given a graph γ, a coalition $S \subseteq N$ is said to be *connected* if $(S,\gamma|_S)$ is connected. For a given social network γ and a coalition $S \subseteq N$, denote by S/γ the set of all *components* (maximally connected subcoalitions) of S. Notice that S/γ provides a partition of S.

Following Myerson (1977), we assume that for a given $\langle v,\gamma\rangle \in \mathcal{G}_N^\Gamma$, cooperation is possible only among connected players and consider a *restricted game* $v^\gamma \in \mathcal{G}_N$ defined as

$$v^\gamma(S) = \sum_{C \in S/\gamma} v(C), \qquad \text{for all } S \subseteq N.$$

The *Myerson value* (Myerson, 1977) is defined for any $\langle v,\gamma\rangle \in \mathcal{G}_N^\Gamma$ with arbitrary undirected graph γ as the Shapley value of the restricted game v^γ:

$$\mu_i(v,\gamma) = Sh_i(v^\gamma), \qquad \text{for all } i \in N.$$

3 An index of social capital

For every Γ-game $\langle v,\gamma\rangle \in \mathcal{G}_N^\Gamma$ we define a *social capital index* of a player $i \in N$ as

$$SC_i(v,\gamma) = \mu_i(v,\gamma) - Sh_i(v) = Sh_i(v^\gamma - v). \tag{1}$$

Due to its definition via the Shapley and Myerson values, the social capital index satisfies

(*i*) *Linearity*: for any two Γ-games $\langle v,\gamma\rangle, \langle v',\gamma\rangle \in \mathcal{G}_N^\Gamma$ with the same graph γ and all $\alpha,\beta \in \mathbb{R}$ it holds that for every $i \in N$

$$SC_i(\alpha\, v + \beta\, v', \gamma) = \alpha\, SC_i(v,\gamma) + \beta\, SC_i(v',\gamma);$$

(*ii*) *Fairness*: for any $\langle v,\gamma\rangle \in \mathcal{G}_N^\Gamma$, for any link $\{i,j\} \in \gamma$, $i,j \in N$, it holds that

$$SC_i(v,\gamma) - SC_i(v,\gamma\backslash\{i,j\}) = SC_j(v,\gamma) - SC_j(v,\gamma\backslash\{i,j\}).$$

(*iii*) *Balanced Contributions*: for any $\langle v, \gamma \rangle \in \mathcal{G}_N^\Gamma$ and for any pair of players $i, j \in N$, we have

$$SC_i(v, \gamma) - SC_i(v, \gamma_{-j}) = SC_j(v, \gamma) - SC_j(v, \gamma_{-i}),$$

γ_{-k} being the set of links of γ not incident in a given player $k \in N$, i.e.: $\gamma_{-k} = \gamma \backslash \{\{k, l\} | \{k, l\} \in \gamma\}$. Notice that k becomes an isolated actor in γ_{-k}.

Proposition 1 *For any* Γ*-game* $\langle v, \gamma \rangle \in \mathcal{G}_N^\Gamma$ *the* SC*-index meets the following properties:*

(*i*) *Shift Independence:* $SC(v, \gamma) = SC(v_0, \gamma)$;

(*ii*) *Non-Positivity in Total for superadditive games: if* v *is superadditive, then*

$$\sum_{i \in N} SC_i(v, \gamma) \leq 0;$$

(*iii*) *Neutrality for connected graphs: if graph* γ *is connected, then*

$$\sum_{i \in N} SC_i(v, \gamma) = 0;$$

(*iv*) *Link Monotonicity for superadditive games: if* v *is superadditive, then for any link* $\{i, j\} \in \gamma$, $i, j \in N$, *it holds*

$$SC_i(v, \gamma) \geq SC_i(v, \gamma \backslash \{i, j\}).$$

Proof.
(*i*) It is enough to show that $v_0^\gamma - v_0 = v^\gamma - v$. Indeed, for all $S \subseteq N$ it holds that

$$(v_0^\gamma - v_0)(S) = v_0^\gamma(S) - v_0(S) = v^\gamma(S) - \sum_{i \in S} v^\gamma(\{i\}) - [v(S) - \sum_{i \in S} v(\{i\})] =$$

$$= v^\gamma(S) - v(S) = (v^\gamma - v)(S).$$

(*ii*) $\sum\limits_{i \in N} SC_i(v, \gamma) = \sum\limits_{i \in N} \mu_i(v, \gamma) - \sum\limits_{i \in N} Sh_i(v) = \sum\limits_{C_k \in N/\gamma} v(C_k) - v(N) \leq 0.$

The last inequality is due to the superadditivity of v.

(*iii*) If γ is connected then $N/\gamma = \{N\}$ and $\sum\limits_{C_k \in N/\gamma} v(C_k) = v(N)$. Whence repeating the same arguments as in (*ii*) we obtain the desirable equality.

(*iv*) $SC_i(v, \gamma) - SC_i(v, \gamma \backslash \{i, j\}) = \mu_i(v, \gamma) - \mu_i(v, \gamma \backslash \{i, j\}) =$

$$= \sum_{S \subseteq N \backslash \{i\}} p_s[v^\gamma(S \cup \{i\}) - v^\gamma(S)] - \sum_{S \subseteq N \backslash \{i\}} p_s[v^{\gamma \backslash \{i,j\}}(S \cup \{i\}) - v^{\gamma \backslash \{i,j\}}(S)] =$$

$$= \sum_{S \subseteq N \backslash \{i,j\}} p_{s+1}[v^\gamma(S \cup \{i\} \cup \{j\}) - v^\gamma(S \cup \{j\})] -$$

$$-\sum_{S\subseteq N\setminus\{i,j\}} p_{s+1}[v^{\gamma\setminus\{i,j\}}(S\cup\{i\}\cup\{j\}) - v^{\gamma\setminus\{i,j\}}(S\cup\{j\})] =$$

$$= \sum_{S\subseteq N\setminus\{i,j\}} p_{s+1}[v^{\gamma}(S\cup\{i\}\cup\{j\}) - v^{\gamma\setminus\{i,j\}}(S\cup\{i\}\cup\{j\})].$$

The first of two latter equalities follows from the fact that for all $S \subseteq N \setminus \{i,j\}$, it holds that $v^{\gamma}(S) = v^{\gamma\setminus\{i,j\}}(S)$ whereas the second one holds true because for all $S \subseteq N \setminus \{i,j\}$, $v^{\gamma}(S\cup\{j\}) = v^{\gamma\setminus\{i,j\}}(S\cup\{j\})$. Therefore,

$$SC_i(v,\gamma) - SC_i(v,\gamma\setminus\{i,j\}) =$$

$$= \sum_{S\subseteq N\setminus\{i,j\}} p_{s+1}\left[\sum_{T\in(S\cup\{i\}\cup\{j\})/\gamma} v(T) - \sum_{T\in(S\cup\{i\}\cup\{j\})/(\gamma\setminus\{i,j\})} v(T)\right].$$

To complete the proof, notice that the last expression is nonnegative because of superadditivity of v and due to the fact that for each $S \subseteq N \setminus \{i,j\}$, the partition of $(S\cup\{i\}\cup\{j\})$ done by $\gamma\setminus\{i,j\}$ is finer than that done by γ. ∎

The following theorem shows that in a monotonic game with a star structure social network, the player in the hub of the star possesses the highest value of social capital among the other players.

Theorem 1 *For any monotonic* Γ*-game* $\langle v,\sigma^i\rangle \in \mathcal{G}_N^{\Gamma}$ *with graph* σ^i *on* N *being the star with the hub at* $i \in N$*, it holds that*

$$SC_i(v,\sigma^i) \geq SC_j(v,\sigma^i), \qquad \textit{for all } j \in N\setminus\{i\}.$$

Proof. Due to the shift independence of SC-index and also because 0-normalization v_0 inherits monotonicity from v, without loss of generality we may assume game v to be 0-normalized. Then for any $S \subseteq N\setminus\{i\}$, it holds that $v^{\sigma^i}(S\cup\{i\}) = v(S\cup\{i\})$ and $v^{\sigma^i}(S) = \sum_{j\in S} v(\{j\}) = 0$. Whence it follows that for the hub i of the star σ^i it holds that

$$SC_i(v,\sigma^i) = Sh_i(v^{\sigma^i}) - Sh_i(v) =$$

$$= \sum_{S\subseteq N\setminus\{i\}} p_s(v(S\cup\{i\}) - v^{\sigma^i}(S)) - \sum_{S\subseteq N\setminus\{i\}} p_s(v(S\cup\{i\}) - v(S)) = \sum_{S\subseteq N\setminus\{i\}} p_s v(S), \quad (2)$$

and for every satellite $j \in N\setminus\{i\}$ in σ^i we have

$$SC_j(v,\sigma^i) = \sum_{S\subseteq N\setminus\{i,j\}} [p_s(v^{\sigma^i}(S\cup\{j\}) - v^{\sigma^i}(S)) + p_{s+1}(v^{\sigma^i}(S\cup\{i,j\}) - v^{\sigma^i}(S\cup\{i\}))] -$$

$$- \sum_{S\subseteq N\setminus\{i,j\}} [p_s((S\cup\{i,j\}) - v(S)) + p_{s+1}(v(S\cup\{i,j\}) - v(S\cup\{i\}))] =$$

$$= - \sum_{S\subseteq N\setminus\{i,j\}} p_s(v(S\cup\{j\}) - v(S)).$$

Since every monotonic game is nonnegative, for all $j \in N\setminus\{i\}$ it holds that

$$\sum_{S\subseteq N\setminus\{i\}} p_s v(S) \geq 0 \geq - \sum_{S\subseteq N\setminus\{i,j\}} p_s(v(S\cup\{j\}) - v(S)).$$

∎

Remark 1 From shift independence property of the social capital index, since every nonnegative superadditive game is monotonic and every 0-normalized superadditive game is nonnegative, Theorem 1 holds true for every superadditive game.

While the previous theorem compares the social capital of different players in monotonic games with star shaped social networks, the next one compares the social capital of the same player in different social networks. It shows that in a superadditive game the best position for a player (in terms of social capital) is to take the central position among the others, i.e., the social capital of a player reaches its maximum when the network contains only the links connecting him with all the others.

Theorem 2 *For any two* Γ*-games* $\langle v, \gamma\rangle, \langle v, \sigma^i\rangle \in \mathcal{G}_N^\Gamma$ *with the same superadditive game* v *and graph* σ^i *on* N *being the star with the hub at* $i \in N$*, it holds that*

$$SC_i(v, \sigma^i) \geq SC_i(v, \gamma).$$

Proof. Due to the shift independence of SC-index and also because 0-normalization v_0 inherits superadditivity from v, without loss of generality we may assume game v to be 0-normalized, and therefore, equality (2) holds true.

On the other hand, given any graph γ on N from superaddiivity of $v$it follows that for all $S \subseteq N \setminus \{i\}$, $v^\gamma(S \cup \{i\}) \leq v(S \cup \{i\})$. Wherefrom we obtain that

$$SC_i(v, \gamma) = \sum_{S \subseteq N \setminus \{i\}} p_s(v^\gamma(S \cup \{i\}) - v^\gamma(S)) - \sum_{S \subseteq N \setminus \{i\}} p_s(v(S \cup \{i\}) - v(S)) \leq$$

$$\leq \sum_{S \subseteq N \setminus \{i\}} p_s(v(S) - v^\gamma(S)).$$

Since v^γ inherits properties of superadditivity and 0-normalization from v, we have that for all $S \subseteq N$, $v^\gamma(S) \geq 0$. Whence it follows that

$$SC_i(v, \gamma) \leq \sum_{S \subseteq N \setminus \{i\}} p_s(v(S)) \overset{(2)}{=} SC_i(v, \sigma^i).$$

∎

The meaning of Theorem 3 is that playing a superadditive game for any player the worst option is to be isolated.

Theorem 3 *For any two* Γ*-games* $\langle v, \gamma\rangle, \langle v, \gamma^i\rangle \in \mathcal{G}_N^\Gamma$ *with the same superadditive game* v *and graph* γ^i *on* N *in which* $i \in N$ *is an isolated node, it holds that*

$$SC_i(v, \gamma^i) \leq SC_i(v, \gamma).$$

Proof. Similarly as in the proof of Theorem 2, without loss of generality we may assume game v to be 0-normalized. Therefore, for any graph γ on N, v^γ is superadditive and 0-normalized as well. Hence,

$$Sh_j(v^\gamma) \geq 0, \qquad \text{for all } \ j \in N, \tag{3}$$

and in particular, $Sh_i(v^\gamma) \geq 0$. The assumption that i is an isolated node in γ^i, involves that

$$SC_i(v, \gamma^i) = Sh_i(v^{\gamma^i}) - Sh_i(v) = -Sh_i(v). \tag{4}$$

Then from (3) and (4) we obtain

$$SC_i(v, \gamma^i) \leq Sh_i(v^\gamma) - Sh_i(v) = SC_i(v, \gamma).$$

∎

The next corollary provides upper and lower bounds of the SC-index for superadditive games.

Corollary 1 For any Γ-game $\langle v, \gamma \rangle \in \mathcal{G}_N^\Gamma$,with a superadditive game v, it holds that

$$-Sh_i(v) \leq SC_i(v, \gamma) \leq \sum_{S \subseteq N \setminus \{i\}} p_s \, v(S), \qquad \text{for all } \; i \in N.$$

Proof. Due to Theorem 2, for any $i \in N$,

$$SC_i(v, \gamma) \leq SC_i(v, \sigma^i) \overset{(2)}{=} \sum_{S \subseteq N \setminus \{i\}} p_s v(S).$$

Next, from superadditivity of v and because, again without loss of generality we may assume v to be 0-normalized, the inequality (3) holds true, and therefore, for all $i \in N$,

$$SC_i(v, \gamma) = \mu_i(v, \gamma) - Sh_i(v) = Sh_i(v^\gamma) - Sh_i(v) \overset{(3)}{\geq} -Sh_i(v).$$

∎

It is already proved above that in a monotonic game with a star shaped social network the player located at the hub of the star has at least the same social capital as the others. Besides, in a superadditive game a particular player has maximal social capital with respect to the full range of all possible social networks in the star shaped social in which he is the hub. Now we show that in superadditive games the player with minimal marginal contributions in the star shaped social network in which he is in the hub possesses the maximal social capital among all the players with respect to all possible networks. This theorem establishes a very important fact: that even being in a weak position in the game, a player can possess the highest possible value of social capital using his emotional-social intelligence to create a good set of relations (specifically, if he has relations with all the others and these are all the existing links).

Theorem 4 *For any two Γ-games $\langle v, \gamma \rangle, \langle v, \sigma^i \rangle \in \mathcal{G}_N^\Gamma$ with the same superadditive game v and graph σ^i on N being the star with the hub at $i \in N$, if marginal contributions of any $j \in N$, $j \neq i$, to all coalitions $S \subseteq N \setminus \{i, j\}$ are not less than marginal contributions of i, i.e.,*

$$v(S \cup \{j\}) \geq v(S \cup \{i\}), \qquad \textit{for all } \; S \subseteq N \setminus \{i, j\},$$

then

$$SC_i(v, \sigma^i) \geq SC_j(v, \gamma).$$

Proof. Similarly as in the proof of Theorem 2, without loss of generality we may assume v to be 0-normalized. Consider a star σ^j on N with the hub at $j \in N$, $j \neq i$. Since (2) holds true we obtain that

$$SC_i(v,\sigma^i) - SC_j(v,\sigma^j) \overset{(2)}{=} \sum_{S\subseteq N\setminus\{i\}} p_s v(S) - \sum_{S\subseteq N\setminus\{j\}} p_s v(S) =$$

$$= \sum_{S\subseteq N\setminus\{i,j\}} p_s v(S) + \sum_{S\subseteq N\setminus\{i,j\}} p_{s+1} v(S\cup\{j\}) - \sum_{S\subseteq N\setminus\{i,j\}} p_s v(S) - \sum_{S\subseteq N\setminus\{i,j\}} p_{s+1} v(S\cup\{i\}) =$$

$$= \sum_{S\subseteq N\setminus\{i,j\}} p_{s+1}[v(S\cup\{j\}) - v(S\cup\{i\})].$$

By hypothesis the latter expression is nonnegative. Wherefrom and also using Theorem 2 we obtain that for any graph γ on N, for all $j \in N$, $j \neq i$, it holds that

$$SC_i(v,\sigma^i) \geq SC_j(v,\sigma^j) \geq SC_j(v,\gamma).$$

∎

The next theorem proves that a veto-player in a superadditive game being isolated in a network is able to obtain at most the lowest value of the social capital among all other players with respect to all possible social networks. In a sense, this theorem is a reverse to the previous one stating that even a veto-player possesses a poor social capital being disconnected with all other players.

Theorem 5 *For any two Γ-games $\langle v,\gamma\rangle, \langle v,\gamma^i\rangle \in \mathcal{G}_N^\Gamma$ with the same super-additive veto-rich game v with player $i \in N$ being a veto-player and graph γ^i on N, in which i is an isolated node, it holds that*

$$SC_i(v,\gamma^i) \leq SC_j(v,\gamma), \qquad \textit{for all } j \in N.$$

Proof. Every veto-player in any veto-rich game appears to be a veto-player in the 0-normalization of this game as well. Then similarly as in the proof of Theorem 2, without loss of generality we may assume v to be 0-normalized. Whence it follows that $Sh_i(v) \geq Sh_j(v)$ for all $j \in N$, $j \neq i$. Indeed,

$$Sh_i(v) - Sh_j(v) = \sum_{S\subseteq N\setminus\{i\}} p_s[v(S\cup\{i\}) - v(S)] - \sum_{S\subseteq N\setminus\{j\}} p_s[v(S\cup\{j\}) - v(S)] =$$

$$= \sum_{S\subseteq N\setminus\{i,j\}} p_s[v(S\cup\{i\}) - v(S)] + \sum_{S\subseteq N\setminus\{i,j\}} p_{s+1}[v(S\cup\{i\}\cup\{j\}) - v(S\cup\{j\})] -$$

$$- \sum_{S\subseteq N\setminus\{i,j\}} p_s[v(S\cup\{j\}) - v(S)] - \sum_{S\subseteq N\setminus\{i,j\}} p_{s+1}[v(S\cup\{i\}\cup\{j\}) - v(S\cup\{i\})] =$$

$$= \sum_{S\subseteq N\setminus\{i,j\}} (p_s + p_{s+1})(v(S\cup\{i\}) - v(S\cup\{j\})) \geq 0.$$

The last inequality holding because $v(S\cup\{j\}) = 0$ for all $S \subset N \setminus \{i,j\}$, as i is a veto-player in v. (Notice that, in a 0-normalized and superadditive game v, $v(S) \geq 0$ for all $S \subset N$).

Since the hypothesis of the theorem is stronger than the hypothesis of Theorem 3, inequality (3) and equality (4) hold true. Therefore, for any $j \in N$ we obtain that for any graph γ on N it holds that

$$SC_i(v, \gamma^i) = -Sh_i(v) = -\max_{k \in N} Sh_k(v) \leq Sh_j(v^\gamma) - Sh_j(v) = SC_j(v, \gamma).$$

∎

4 Social capital versus centrality

The social capital index defined above has close affinities with the centrality measure of a node in an undirected communication graph as introduced in Gómez et al. (2003). However, in the model of centrality the only main entity is a communication graph and the main goal is the definition of a numerical value of a node due to its positional importance in the graph. The centrality of a node is defined as the difference between the Myerson and Shapley values in an a priori chosen symmetric game with respect to the given communication graph. In this case a symmetric game treating players symmetrically brings no additional information concerning the individual feathers of the participants but only plays an instrumental role serving as a tool for the definition. In fact Gómez et al. (2003) define not one but a family of centrality measures with respect to the subclass of symmetric games because different symmetric games may cause different rankings of nodes in the graph. Different to the model of centrality, in our model of social capital there are two main entities: a game representing cooperative abilities of the players and a social network representing the relations among them. The next proposition shows that every TU game can be decomposed into a sum of a symmetric game and its orthogonal complement. Then while a given game in combination with a given graph respond for the value of social capital index, its symmetric component together with the same graph are responsible for the corresponding value of centrality. The example below gives insight into the difference between the social capital index determined by a given game and the corresponding centrality measure determined by the symmetric component of this game.

Since for every game $v \in \mathcal{G}_N$, $v(\emptyset) = 0$, the space $\mathcal{G}_N$ of all TU games with the fixed player set N can be naturally identified with the usual Euclidean space $\mathbb{R}^{2^n-1}$ of vectors v, $v = \{v(T)\}_{\substack{T \subseteq N \\ T \neq \emptyset}}$. Denote by $\mathcal{S}_N$ the subset of symmetric games in $\mathcal{G}_N$, $\mathcal{S}_N = \{v \in \mathcal{G}_N \mid v(T) = v(T'), \text{for all}, T, T' \subseteq N, t = t'\}$. $\mathcal{S}_N$ is a linear subspace in $\mathcal{G}_N$ of dimension n and therefore, $\mathcal{G}_N = \mathcal{S}_N \bigoplus \mathcal{S}_N^\perp$, where $\mathcal{S}_N^\perp = \{v \in \mathcal{G}_N \mid v \perp w \text{ for all} w \in \mathcal{S}_N\}$ is the orthogonal complement to $\mathcal{S}_N$ in $\mathcal{G}_N$ with respect to Euclidean inner product.

Proposition 2 *Every game $v \in \mathcal{G}_N$ can be uniquely presented as a sum*

$$v = v_S + v_S^\perp,$$

with

$$v_S(T) = v_S(t) = \frac{1}{\binom{n}{t}} \sum_{\substack{R \subseteq N \\ r = t}} v(R), \qquad \textit{for all} \quad T \subseteq N,$$

and

$$\sum_{\substack{T\subseteq N\\ t=k}} v_S^{\perp}(T) = 0, \qquad \textit{for all} \quad k = 1, ...n,$$

where $v_S \in \mathcal{S}_N$ *is the symmetric component of* v *and* $v_S^{\perp} \in \mathcal{S}_N^{\perp}$ *is the orthogonal complement to* v_S *with respect to the inner product of* R^{2^n-1}.

Proof. It is not difficult to see that the games $v_k = \sum_{t=k} u_T$, $k = 1, ..., n$, are symmetric and linear independent. The dimension of $\mathcal{S}_N$ is equal to n, and therefore, games v_k, $k = 1, ..., n$, create a (nonorthogonal) basis in $\mathcal{S}_N$. $\mathcal{S}_N^{\perp} = \{v \in \mathcal{G}_N \mid v \perp w,\ \forall w \in \mathcal{S}_N\}$. In particular, $v \in \mathcal{S}_N^{\perp}$, if and only if $\langle v, v_k\rangle = 0$ for all $k = 1, ...n$.

$$\langle v, v_k\rangle = \sum_{T\subseteq N} \left[v(T) \sum_{r=k} u_R(T)\right] = \sum_{r=k}^{n} \binom{r}{k} \sum_{\substack{T\subseteq N\\ t=r}} v(T).$$

Whence it follows that $v \in \mathcal{S}_N^{\perp}$, if and only if for all $k = 1, ...n$,

$$\sum_{r=k}^{n} \binom{r}{k} \sum_{\substack{T\subseteq N\\ t=r}} v(T) = 0. \tag{5}$$

For $k = n$, (5) reduces to

$$v(N) = 0.$$

For $k = n - 1$, (5) reduces to

$$n \cdot \sum_{\substack{T\subseteq N\\ t=n-1}} v(T) + v(N) = 0,$$

wherefrom together with latter equality it follows that

$$\sum_{\substack{T\subseteq N\\ t=n-1}} v(T) = 0.$$

Repeating the same procedure for all $k = n - 2, ..., 1$, we obtain that $v \in \mathcal{S}_N^{\perp}$, if and only if

$$\sum_{\substack{T\subseteq N\\ t=k}} v(T) = 0, \qquad \text{for all } \ k = 1, ...n. \tag{6}$$

Since $\mathcal{G}_N = \mathcal{S}_N \bigoplus \mathcal{S}_N^{\perp}$, every game $v \in \mathcal{G}_N$ can be uniquely decomposed as

$$v = v_S + v_S^{\perp},$$

where $v_S \in \mathcal{S}_N$ and $v_S^{\perp} \in \mathcal{S}_N^{\perp}$. Whence, $v - v_S \in \mathcal{S}_N^{\perp}$, and therefore, for all $k = 1, ...n$,

$$\sum_{\substack{T\subseteq N\\ t=k}} (v - v_S)(T) \overset{(6)}{=} 0.$$

Since for every symmetric game the worth of each coalition depends only on its size, for all $T \subseteq N$, we obtain that

$$v_S(T) = \frac{1}{\binom{n}{t}} \sum_{\substack{R\subseteq N\\ r=t}} v(R), \qquad \text{for all} \quad T \subseteq N.$$

∎

Remark 2 As a consequence of previous proposition and using the linearity of our social capital measure we have, for each $i \in N$:

$$SC_i(v, \gamma) = SC_i(v_S + v_S^{\perp}, \gamma) = SC_i(v_S, \gamma) + SC_i(v_S^{\perp}, \gamma)$$

Being $v_S \in \mathcal{S}_N$, $SC_i(v_S, \gamma)$ coincides with the centrality measure defined by Gómez et al. (2003), $\kappa(v_S, \gamma)$, for nodes in γ and using in this case the game v_S, derived from v erasing the *a priori* differences among players. On the other hand, $SC_i(v_S^{\perp}, \gamma)$ represents the externalities obtained by player i when his network of relations is γ and the played game is the one that offers to each coalition S the discrepancy between its v-value and the mean of the v-values of all coalitions with the same cardinality that S. So, we will call $SC_i(v_S^{\perp}, \gamma)$ the positional externalities, $PE_i(v, \gamma)$ of player i and thus social capital of each actor it is assumed to be a vectorial magnitude with two components: one of them is the centrality of his position in the social network to which he belongs and the other corresponds to the the externalities that this position produces to him.

Example 1 Consider the following situation when two persons, seller 1 ($S1$) and seller 2 ($S2$), are going to sell their houses and there is another person, a buyer (B), who is ready to buy any one of these houses. The buyer prefers to buy more expensive and better house of seller 2, if he has a choice. The problem of this particular market situation is that in a big city without a mediator (M, a real estate agency) both sides cannot meet each other to start negotiations. This situation can be modelled as a 4-person TU game with the player set $N = \{S1, S2, M, B\}$ and the characteristic function

$$v(T) = \begin{cases} 1, & \text{if } \{S1, B\} \subseteq T \text{ and } S2 \notin T, \\ 2, & \text{if } \{S2, B\} \subseteq T, \\ 0, & \text{otherwise.} \end{cases}$$

The graph representing communication abilities of the players is given by Figure 1.

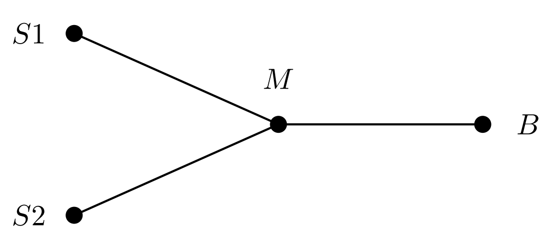

Figure 1

It is not difficult to see that M is a null-player in game v, and in case when there is no restrictions on communication, the Shapley value assigns to M nothing and shares the profits of the game among other participants,

$$Sh(v) = \left(\frac{1}{6}, \frac{4}{6}, 0, \frac{7}{6}\right).$$

In case of limited communication represented by the given graph we obtain the following values of the players social capital

$$SC = \Big(-\frac{1}{12}, -\frac{3}{12}, \frac{9}{12}, -\frac{5}{12}\Big).$$

Then, if we separate the symmetric component v_S of v, [1]

$$v_S = \{0, 0, \frac{1}{2}, \frac{5}{4}, 2\}.$$

we obtain also the corresponding to v centralities of the players in graph γ,

$$\kappa = \Big(-\frac{1}{16}, -\frac{1}{16}, \frac{3}{16}, -\frac{1}{16}\Big),$$

whereas restricting us to the difference game $v - v_S$ we obtain the positional externalities

$$PE = \Big(-\frac{1}{48}, -\frac{9}{48}, \frac{27}{48}, -\frac{17}{48}\Big).$$

As we can see, the mediator M, simultaneously being a null-player in v but the hub of the star presenting given relations' network, is the only one whose social capital and centrality are positive due to his extreme importance in the communication structure.

Besides, it is necessary to emphasize that, while the centrality assigns equal values to all other players, $S1$, $S2$ and B, which have symmetric positions in the communication graph being the satellites of the hub M, the social capital index takes care over these players' importance in the game as well. Namely, the social capital of the second seller $S2$ who provides the highest value to the market is the highest among the rest of the players. Moreover, it is worth noticing that the social capital index, that takes into account the asymmetry of the players in the game, increases the value of the social capital of the mediator M in comparison to his centrality value. Looking to positional externalities, we can see that, in this case, among the three players with symmetric positions in the graph, the better is the power (the Shapley value) of a player in the game, the worse is his positional externality.

Remark 3 It can be thought that the positional externalities, PE, are in fact a measure of social capital by themselves. Nevertheless, the monotonicity and superadditivity of game v, needed to guarantee some particular properties to the social capital measure, are not, in general, inherited by game $v_S^{\perp}$. Thus, PE does not serve as a measure of social capital. In the following example, an actor can have maximal social capital but minimal externalities. Let us consider the game (N, v) with $N = \{1, 2, 3\}$, $v(\{1, 2\}) = 1$, $v(\{2, 3\}) = 1$, $v(\{1, 3\}) = 0$, $v(N) = 2$, and the graph $\gamma = \{\{1, 2\}, \{2, 3\}\}$, the star with hub at node 2. Then:

$$PE = \frac{2}{3}\left(\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$$

$$SC = (0, 0, 0)$$

and thus, the positional externalities are not maximal in the hub of the star.

[1] Since in v_S the worth of each coalition depends only on its size, we write $v_S = \{v_S(0), v_S(1), v_S(2), v_S(3), v_S(4)\}$

5 Examples

5.1 Basque Country Parliament 2009

The Basque Country (Euskadi) is one of the Spain seventeen autonomous communities. Its Parliament is constituted by 75 members. Following last elections, in March 2009, the composition of this Parliament is as appears in Table 1:

	Party	Seats
1	PNV (Basque Nationalist Party)	30
2	PSE (Euskadi's Socialist Party)	25
3	PP (Popular Party)	13
4	ARALAR	4
5	EA (Euzko Alkartasuna)	1
6	EB-B (Ezker Batua-Berdeak)	1
7	UPD (Union, Progress and Democracy)	1

Table 1: Basque Country Parliament 2009

Taking into account the alignment of these parties in the political spectrum (given mainly by the left-right and the nationalist-not nationalist axis) and the history of its relations in the most recent years, the affinities among these parties can plausibly be represented by the graph γ (Fig 2). The existence of a possible link between PSE and EB parties was, at that time, a controversial issue but, as we will see, it is irrelevant for our purposes.

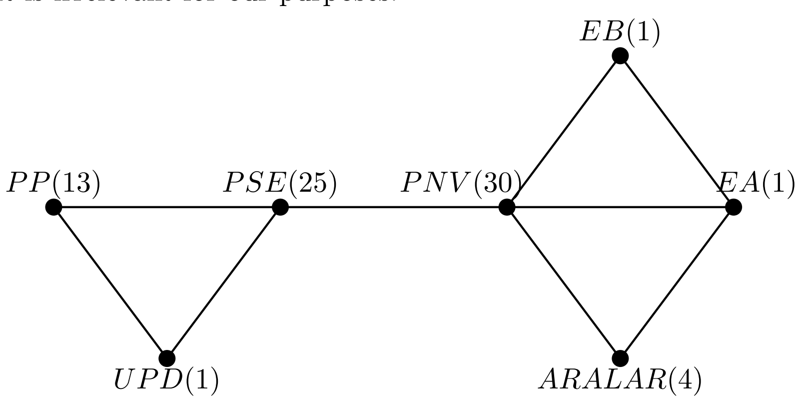

Figure 2: Basque Country Parliament 2009

To form a government, any party or coalition must obtain the vote of, at least, 38 parliamentarians.

We can consider that the situation (the government formation) is well represented by the weighted majority game:

$\{q; p_1, p_2, \ldots, p_7\}$, with: $q = 38$; $p_1 = 30$, $p_2 = 25$, $p_3 = 13$,....

and: $v(S) = \begin{cases} 1 & \sum_{i \in S} p_i \geq 38 \\ 0 & \text{otherwise.} \end{cases}$

The minimal winning coalitions in this game are: $\{1, 2\}$ (or PNV-PSE), $\{1, 3\}$ (or PNV-PP), and $\{2, 3\}$ (or PSE-PP). Because of the parliamentary arithmetic, the remaining four parties are dummies in the described game.

We can then write the game v as: $v = u_{\{1,2\}} + u_{\{1,3\}} + u_{\{2,3\}} - 2u_{\{1,2,3\}}$, and we will obtain:

$$Sh(v) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0\right).$$

Given the graph γ of the relations, we obtain the graph-restricted game:

$$v^{\gamma} = u_{\{1,2\}} + u_{\{2,3\}} - u_{\{1,2,3\}},$$

as minimal winning coalition (in the original game) PNV-PP is not feasible in the restricted one.

Therefore:

$$\mu(v, \gamma) = Sh(v^{\gamma}) = \left(\frac{1}{6}, \frac{2}{3}, \frac{1}{6}, 0, 0, 0, 0\right),$$

and:

$$SC(v, \gamma) = Sh(v - v^{\gamma}) = \left(-\frac{1}{6}, \frac{1}{3}, -\frac{1}{6}, 0, 0, 0, 0\right),$$

maximal Social Capital corresponding, obviously, to the PSE, due to its location in the relations graph. So, model permits us to predict a crucial role in the government formation for the socialist party PSE. In fact, PSE formed a minority govern with the external support of PP and UPD parties, avoiding the most voted party, the nationalist PNV, do it.

5.2 Italian Elections 1983

Another good example in which a small but central party that got not too many seats in a parliament finally became the most important is italian elections of 1983. Following these elections the Prime-Minister, Bettino Craxi, was chosen from Italian Socialist Party (PSI) that got only 73 seats from a total amount of 630. But PSI had a very strong bargaining power being between the two biggest parties, Christian Democrats (DC), with 225 seats, and Italian Communist Party (PCI), with 198 seats. The complete data about 1983 elections are presented in Table 2.

The possible links between parties are given in Figure 3.

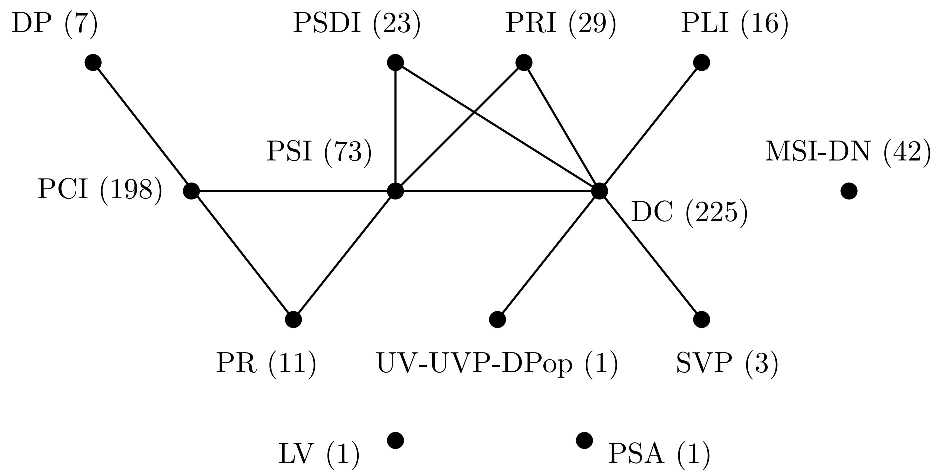

Figure 2

	Party	Seats	$Sh(v)$	$Sh(v^\gamma)$	$SC(v,\gamma)$
1	Democrazia Cristiana (DC)	225	0.3450	0.2369	-0.1081
2	Partito Comunista Italiano (PCI)	198	0.2436	0.0798	-0.1638
3	Partito Socialista Italiano (PSI)	73	0.1787	0.5202	0.3415
4	Movimiento Sociale It. (MSI-DN)	42	0.0689	0	-0.0689
5	Partito Repubblicano Italiano (PRI)	29	0.0517	0.0978	0.0281
6	Partito Socialista Dem. It. (PSDI)	23	0.0403	0.0443	0.0061
7	Partito Liberale Italiano (PLI)	16	0.0318	0.0107	-0.0211
8	Partito Radicale (PR)	11	0.0155	0.0131	-0.0024
9	Democrazia Proletaria (DP)	7	0.0130	0.0095	-0.0035
10	Sudtiroler Volkspartei (SVP)	3	0.0059	0.0036	-0.0023
11	Liga Veneta (LV)	1	0.0019	0	-0.0019
12	Partito Sardo d'Azione (PSA)	1	0.0019	0	-0.0019
13	UV-UVP-DPop	1	0.0019	0	-0.0019

Table 2: Social Capital in Italian Elections 1983

The total number of seats was 630. So the ruling coalitions should contain at least 316 members. Considering the corresponding weighted majority game representing the govern formation, minimal winning coalitions in the graph-restricted game, v^γ are: $\{DC, PCI, PSI\}$, $\{DC, PSI, PRI\}$, $\{DC, PSI, PSDI\}$, $\{PCI, PSI, PRI, PSDI\}$ and $\{PCI, PSI, PRI, PR, DP\}$.

Italian Socialist Party appear in all of them, whereas the two most voted parties, Christian Democrats and Communists appear only in three of them. As a consequence, PSI is a dictator in this restricted game: it is not possible to form government ignoring it. Detailed computations give us results appearing in Table 1.

In spite of PSI had around half the power (measured in terms of Shapley value) in the original game than the most voted party, DC, PSI had, by far, the greatest Social Capital and, finally, Bettino Craxi, a PSI member, was chosen as Prime-Minister.

Our conclusions appear to be in a good agrement with historical facts. The traditional ruling coalition, known as Pentapartito (five parties), was $\{DC, PLI, PSI, PRI, PSDI\}$ with $225+16+73+28+23 = 366$ seats. But there was another possible winning coalition: $\{PCI, PSI, DP, PR, PSDI, PRI\}$, with $198 + 73 + 7 + 11 + 23 + 29 = 341$ seats. The latter coalition never formed before on the country government level but it happened to be formed on the levels of local governments. This fact provide PSI with very strong bargaining power and finally the prime-minister Bettino Craxi, member of PSI, was chosen.

6 Conclusions

In this paper we introduce a measure of social capital for players involved in a cooperative TU game that simultaneously belong to a social network. The game represents the economic interests that motivate the interactions among actors and the social network the existing relations among them. The proposed measure assumes that the social capital is the difference between the Shapley value of the game when considering the restrictions in the cooperation given by the network (Myerson value) and the Shapley value of the game when the network is ignored. Some properties are proved to show the behavior of this measure and how the network influences the outcome that players can obtain. In particular, upper and lower bounds for the measure are obtained. Moreover, for a monotonic game, a player being the hub of a star shaped network has always the greater social capital among all the others. If the game is also superadditive this higher social value is even the maximum social capital considering the game as fixed and the network as variable. On the other hand, being isolated in the network generates the poorest social capital when the game is superadditive.

The influence of the network can be so important that the player involved in a superadditive game and having minimal marginal contributions in it becomes the one that possesses the maximal social capital among all the players with respect to all possible networks when he is able to create a set of relations around him such that he is directly related with all people and the rest are only directly connected with him (i.e., being the hub of a star). On the other hand, a very powerful player (a veto player) in a superadditive game becomes the one with the poorest social capital if all the others break their relations with him (isolating him).

In a previous paper (Gómez et al., 2003) we introduced a measure of centrality for individuals in a social network from a game theoretical point of view. Another aim of the present paper is to establish a possible relation between centrality and social capital. The obtained results show that the defined centrality in Gómez et al. (2003) can be considered as a component of the social capital, which can be additively decomposed in that centrality plus positional externalities. Given the game players are involved in, the centrality of the position of an actor in the social network is the variation that this position produces in his outcome in the game obtained from the initial one when deleting the a priori differences among players. The positional externalities obtained because of the position are the variation (debt to the restrictions in the communications imposed for the graph) of the Shapley value in the game in which coalitions earn excess (positive or negative) with respect to the egalitarian situation.

Finally, we think that it could be interesting to explore the existence of appealing properties of the measure we have called positional externalities and also to explore how previous ideas can be used to introduce a measure of social capital for directed or weighted social networks. For example, the definition of centrality for nodes in directed social networks introduced in Pozo et al. (2011) can be related, we think, with an appropriate social capital measure.

Acknowledgements We want to thank Javier Castro and Michela Chessa for helpful comments and calculations of the Shapley value in the political examples. This research has been supported by the "Plan Nacional de I+D+i" of the Spanish Government, under the project MTM2008-06778-C02-02/MTM. The research was done during Anna Khmelnitskaya 2011 research stay at the School of Statistics of the Complutense University of Madrid, under the research grant of the Insituto de Matemática Interdisciplinar (IMI), whose hospitality and support are highly appreciated.

Bibliography

Bourdieu, P., 1972. Esquisse d'une theorie de la pratique, principes de trois estudes d'ethnologie kabyle, Eng. Outline of a Theory of Practice, Cambridge University Press 1977.

Burt, R.S., 1997. A note on social capital and network content. Social networks, 19, 355-373.

Dasgupta, P., Serageldin, I. (eds.), 2000. Social capital: a multifaceted perspective. Washington, D.C.: World Bank.

Flap, H., Völker, B., 2001. Goal specific social capital and job satisfaction. Effects of different types of networks on instrumental and social aspects of work. Social Networks 23, 297-320.

Gómez, D., González-Arangüena, E., Manuel, C., Owen, G., del Pozo, M., Tejada, J., 2003. Centrality and power in social networks: a game theoretic approach, Mathematical Social Sciences, 46, 27–54.

Harsanyi, J.C., 1959. A bargaining model for cooperative n-person games, in: Tucker AW, Luce RD (eds.) Contributions to the theory of games IV, Princeton University Press, Princeton, NJ, 325–355.

Knack, S., Keefer, P., 1997. Does social capital have an economic payoff? A cross-country investigation. Quartely Journal of Economics, 112(4), 1251–1288.

Lin, N., Dumin, M., 1986. Access to occupations through socialties. Social Networks 8, 365–385.

Lin, N., Fu, Y., Hsung, R., 2001. The Position Generator: measurement techniques for social capital. In: N. Lin, K. Cook, R.S. Burt (Eds) , Social Capital: Theory and Research, Aldine De Gruyter, New York, 57–84.

McCallister, L., Fischer, C., 1978. A procedure for surveying personal networks. Sociological Methods and Research, 7, 131–148.

Moretti, S., Fragnelli, V., Patrone, F. Bonassi, S., 2010. Using coalitional games on biological networks to measure centrality and power of genes, Bioinformatics, 26, 2721–2730.

Myerson, R.B., 1977. Graphs and cooperation in games, Mathematics of Operations Research, 2, 225–229.

Portes, A., 1998. Social capital: its origins and applications in modern sociology. Annual Review of Sociology, 24, 1–24.

Pozo, M. del, Manuel, C., González Arangüena, E., Owen, G., 2011. Centrality in directed social networks. A game theoretic approach. Social Networks, 33(3), 191–200.

Putnam, R. D., 2000. Bowling alone: the collapse and revival of American community. Simon and Schuster.

Putnam, R.D., Feldstein, L., Cohen, D. 2003. Better Together: Restoring the American Community. New York: Simon and Schuster.

Salisbury, R.H. 1969. An exchange theory of interest groups. Midwest journal of political science 13, 1–32.

Shapley, L.S., 1953. A value for n-person games, in: Tucker, A.W., Kuhn, H.W. (eds.). Contributions to the theory of games II, Princeton University Press, Princeton, NJ, 307–317.

Snijders, T.A.B., 1999. Prologue to the measurement of social capital. La Revue Tocqueville, 20, 27–44.

Van Der Gaag, M., Snijders, T.A.B., 2004. Proposals for the measurement of individual social capital. In: Flap, H., Völker, B., (Eds), Creation and Returns of Social Capital, Routledge, 199–218.

Van Der Gaag, M., Snijders, T. A. B., 2005. The Resource Generator: social capital quantification with concrete items. Social Networks 25, 1–29.

Van Emmerick, I.J.H., 2006. Gender differences in the creation of different types of social capital: A multilevel study. Social Networks 28, 24-37.