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Enrique G. Mendoza
And
Vivian Z. Yue

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Enrique G. Mendoza
University of Maryland and NBER

Vivian Z. Yue
New York University

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Abstract

Models of business cycles in emerging economies explain the negative correlation between country spreads and output by modeling default risk as an exogenous interest rate on working capital. Models of strategic default explain the cyclical properties of sovereign spreads by assuming an exogenous output cost of default with special features, and they underestimate debt-output ratios by a wide margin. This paper proposes a solution to this default risk-business cycle disconnect based on a model of sovereign default with endogenous output dynamics. The model replicates observed V-shaped output dynamics around default episodes, countercyclical sovereign spreads, and high debt ratios, and it also matches the variability of consumption and the countercyclical fluctuations of net exports. Three features of the model are key for these results: (1) working capital loans pay for imported inputs; (2) imported inputs support more efficient factor allocations than when these inputs are produced internally; and (3) default on the foreign obligations of firms and the government occurs simultaneously.

JEL Code: E32, E44, F32, F34

Key Words: Business cycles, sovereign default, emerging economies

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1 Introduction

Three key empirical regularities characterize the relationship between sovereign debt and economic activity in emerging economies:

(1) *Output displays V-shaped dynamics around default episodes.* Recent default episodes have been associated with deep recessions. Arellano (2007) shows that GDP deviations from trend in the quarter in which default occurred were -14 percent in Argentina, -13 percent in Russia and -7 percent in Ecuador. Using quarterly data for 39 developing countries over the 1970-2005 period, Levy-Yeyati and Panizza (2006) show that the recessions associated with defaults tend to begin prior to the defaults and generally hit bottom when the defaults take place. Tomz and Wright's (2007) study of the history of defaults for industrial and developing countries during the period 1820-2004 reports that the frequency of defaults is at its maximum when output is at least 7 percent below trend. They also found, however, that some defaults occurred with less severe recessions, or when output is not below trend in annual data.

(2) *Interest rates on sovereign debt and domestic output are negatively correlated.* Neumeyer and Perri (2005) report that the cyclical correlations between these interest rates and GDP range from -0.38 to -0.7 in five emerging economies, with an average correlation of -0.55. Uribe and Yue (2006) report correlations for seven emerging economies ranging from zero to -0.8, with an average of -0.42.[1]

(3) *External debt as a share of GDP is high on average, and high when countries default.* Foreign debt was about a third of GDP on average over the 1998-2005 period for the entire group of emerging and developing countries as defined in IMF (2006). Within this group, the highly indebted poor countries had the highest average debt ratio at about 100 percent of GDP, followed by the Eastern European and Western Hemisphere countries, with averages of about 50 and 40 percent of GDP respectively. Reinhart et al. (2003) report that the external debt ratio during default episodes averaged 71 percent of GDP for all developing countries that defaulted at least once in the 1824-1999 period. The default episodes of recent years are in line with this estimate: Argentina defaulted in 2001 with a 64 percent debt ratio, and Ecuador and Russia defaulted in 1998 with debt ratios of 85 and 66 percent of GDP respectively.

These empirical regularities have proven difficult to explain. On one hand, quantitative business cycle models can account for the negative correlation between country interest rates and output *if* the interest rate on sovereign debt is introduced as the exogenous interest rate faced by a small open economy in which firms require working capital to pay the wages bill.[2] On the other hand, quantitative models of sovereign default based on the classic setup of

[1] Neumeyer and Perri used data for Argentina, Brazil, Korea, Mexico and the Philippines. Uribe and Yue added Ecuador, Peru and South Africa, but excluded Korea.

[2] See Neumeyer and Perri (2005), Uribe and Yue (2006) and Oviedo (2005).

Eaton and Gersovitz (1981) can generate countercyclical sovereign spreads *if* the sovereign country faces stochastic shocks to an exogenous output endowment.[3] These models require exogenous output costs of default with special features in order to support non-trivial levels of debt together with observed default frequencies, but even with these costs they either produce mean debt ratios under 10 percent of GDP or underestimate default probabilities by a wide margin.[4] Thus, there is a crucial disconnect between business cycle models and sovereign default models: the former lack an explanation of the default risk premia that drive their findings, while the latter lack an explanation of the business cycle dynamics that are critical for their results.

The country risk-business cycle disconnect raises three important questions: Would a business cycle model with endogenous default risk still be able to explain the stylized facts that models with exogenous country risk have explained? Can a model of sovereign default with endogenous output dynamics produce the large output declines needed to support high ratios of defaultable debt as an equilibrium outcome? Would a model that endogenizes both country risk and output dynamics be able to mimic the V-shaped dynamics of output associated with defaults, and the countercyclical behavior of default risk?

This paper aims to answer these questions by studying the quantitative implications of a model of sovereign default with endogenous output fluctuations. The model borrows from the sovereign default literature the workhorse Eaton-Gersovitz recursive formulation of strategic default in which a sovereign borrower makes optimal default choices by comparing the payoffs of repayment and default. In addition, the model borrows from the business cycle literature a transmission mechanism that links default risk with economic activity via the financing cost of working capital. We extend the two classes of models (sovereign debt and business cycle models) by developing a framework in which the equilibrium dynamics of output and default risk are determined jointly, and influence each other via the interaction between foreign lenders, the domestic sovereign borrower, and domestic firms. In particular, a fall in productivity in our setup increases the likelihood of default and hence sovereign spreads, and this in turn increases the firms' financing costs leading to a further fall in output, which in turn feeds back into default incentives and sovereign spreads.

We demonstrate via numerical analysis that the model can explain the three key empirical regularities of sovereign debt mentioned earlier: The model mimics the V-shaped pattern of output dynamics around defaults with large recessions that hit bottom during defaults, yields countercyclical interest rates on sovereign debt, and supports high debt-GDP ratios on

[3] See, for example, Aguiar and Gopinath (2006), Arellano (2007), Yue (2006), and Bai and Zhang (2005).

[4] Arellano (2007) obtains a mean debt ratio of 6 percent of GDP assuming an output cost of default such that income is the maximum of actual output or 0.97 of average output while the economy is in financial autarky. Aguiar and Gopinath (2006) obtain a mean debt ratio of 27 percent assuming a cost of 2 percent of output per quarter, but the default frequency is only 0.02 percent (in their model without trend shocks and debt bailouts). Yue (2006) assumes the same output cost in a model with renegotiation calibrated to observed default frequencies, but obtains a mean debt ratio of 9.7 percent of output.

average and in default episodes. These results are obtained requiring only a small fraction of firms' factor costs to be paid with working capital (only 10 percent of the cost of imported inputs). Moreover, the model matches key business cycle features like the variability of consumption and the countercyclical behavior of net exports.

These results hinge on three key assumptions of the model: First, producers of final goods obtain working capital loans from abroad to finance purchases of imported intermediate goods. Second, these producers can choose optimally to employ domestic intermediate goods instead of imported inputs, but this shift entails an efficiency loss. Third, the government can divert the firms' repayment of working capital loans when it defaults on its own debt, so that both agents default on their foreign obligations at the same time, and the interest rates they face are equal at equilibrium.

The transmission mechanism that connects country risk and business cycles in our model operates as follows: Final goods producers maximize profits and choose optimally whether to use imported inputs or inputs produced in the domestic economy. These two inputs are perfect substitutes in the production technology, but imported inputs have a higher financing cost because they need to be paid in advance using working capital, while domestic inputs require costly reallocation of labor away from final goods production into intermediate goods production. Thus, a shift from imported to domestic inputs causes an efficiency loss in production of final goods due to the reallocation of labor.[5]

The choice of imported v. domestic inputs by final goods producers depends on the country interest rate (inclusive of default risk), which drives the financing cost of working capital, and on the state of total factor productivity (TFP). When the country has access to world financial markets, they choose imported intermediate goods if the country interest rate is low enough and/or TFP is high enough for the efficiency loss from using domestic inputs to exceed the higher financial cost of imported inputs. That is, final goods producers trade off the higher financing cost of imported inputs for the enhanced efficiency in the use of labor services (which are fully allocated to final goods production). In this situation, fluctuations in default risk affect the cost of working capital and thus induce fluctuations in factor demands and output. Conversely, above (below) a threshold value of the interest rate (TFP) firms choose to use domestic inputs because the financing cost of imported inputs exceeds the efficiency loss due to domestic labor reallocation, with labor services now being allocated to both final and intermediate goods production.

When the economy defaults, both the government and firms are excluded from world credit markets for some time, with an exogenous probability of re-entry as is common in the recent

[5] This efficiency loss can be modeled in different ways. We can obtain similar results as the ones shown in this paper by modeling the efficiency loss as resulting from costly sectoral reallocation of capital, given an exogenous amount of aggregate capital, or from foreign inputs that are "superior" to domestic inputs in the sense that they support higher TFP. The efficiency loss can also result from changes in capacity utilization, which can be linked to the choice of imported v. domestic inputs using Finn's (1995) setup.

quantitative studies of sovereign default. Since the probability of default depends on whether the country's value of default is higher than that of repayment, there is feedback between the economic fluctuations induced by changes in interest rate premia, default probabilities, and country risk. In particular, rising country risk in the periods leading to a default causes a decline in economic activity as the firms' financing cost increases. In turn, the expectation of lower output at higher levels of country risk alters repayment incentives for the sovereign, affecting the equilibrium determination of default risk premia.

The transmission mechanism linking country risk and business cycles generates an endogenous output cost of default that is larger in "better" states of nature (i.e., *increasing* in the state of TFP). This result follows from the efficiency loss caused by the optimal shift from imported to domestic inputs when default takes place. Since default yields an effective financing cost of working capital loans that is too high for firms to employ foreign inputs, firms always use domestic inputs when the country is in financial autarky. Before default, however, if the interest rate is low enough and/or TFP is high enough, firms operate with imported inputs, and therefore final goods production is higher than in the default scenario, in which final goods producers shift to domestic inputs. Hence, the decline in GDP at the moment of default is higher the higher TFP was just before default, and the fraction of output loss caused by a default increases with TFP. This increasing output cost of default is a key feature of the model because it implies that the option to default brings more "state contingency" into the asset market, allowing the model to produce equilibria that support significantly higher mean debt ratios than those obtained in existing models of sovereign default.

The increasing output cost of default also implies that output can fall sharply when the economy defaults, and that, because this output drop is driven by an efficiency loss due to sectoral labor reallocation, part of the output collapse will appear as a drop in the Solow residual (i.e. the fraction of aggregate GDP not accounted for by capital and labor). This is consistent with the data of emerging economies in crisis showing that a large fraction of the output collapse is attributed to the Solow residual (see Meza and Quintin (2006) and Mendoza (2007)). Moreover, Benjamin and Meza (2007) show that in Korea's 1997 crisis, the productivity drop did follow from a sectoral reallocation of labor from more to less productive sectors.

Our treatment of the financing cost of working capital differs from the treatment in Neumeyer and Perri (2005) and Uribe and Yue (2006), both of which treat the interest rate on working capital as an exogenous variable set to match the interest rate on sovereign debt. In contrast, in our setup both interest rates are driven by endogenous sovereign risk. In addition, in the Neumeyer-Perri and Uribe-Yue models, working capital loans pay the wages bill in full, while in our model firms use working capital to pay only for a fraction of imported intermediate goods. This lower working capital requirement is desirable because, at standard labor income shares, working capital loans would need to be about 2/3rds of GDP to cover the wages bill, and this is difficult to reconcile with observed ratios of bank credit to the private sector as a share of output in emerging economies, which hover around 50 percent (including *all* credit to households and firms at all maturities, not just short-term revolving loans to firms).

The rest of the paper proceeds as follows: Section 2 presents the model. Section 3 explores the model's quantitative implications for a benchmark calibration. Section 4 conducts sensitivity analysis. Section 4 concludes.

2 A Model of Sovereign Default and Business Cycles

We study a dynamic stochastic general equilibrium model of sovereign default and business cycles. There are four groups of agents in the model, three in the "domestic" small open economy (households, firms, and the sovereign government) and one abroad (foreign lenders).

2.1 Households

Households derive utility from consumption and disutility from labor. Their preferences are given by a standard time-separable utility function $E\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t - h(L_t)\right)\right]$, where $0 < \beta < 1$ is the discount factor, and c_t and L_t denote consumption and "composite" labor effort supplied in period t respectively. $u(\cdot)$ is the period utility function, which is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions. Following Greenwood, Hercowitz and Huffman (1988), we remove the wealth effect on labor supply by specifying period utility as a function of consumption net of the disutility of labor $h(L_t)$, where $h(\cdot)$ is increasing, continuously differentiable and convex. This formulation of preferences has been shown to play an important role in allowing international real business cycle models to explain observed business cycle facts, and it also simplifies the supply-side of the model by removing intertemporal considerations from the labor supply choice.

Households choose consumption and sectoral allocations of labor offered to producers of final goods and intermediate goods (L_t^f and L_t^m respectively). These sectoral labor supply allocations aggregate into a composite amount of labor effort represented by a labor transformation curve $\Psi\left(L_t^f, L_t^m\right)$, where $\Psi(\cdot)$ is a CES aggregator. L^f and L^m can thus be viewed as efficiency units of labor that households allocate across the two sectors out of a given amount of labor effort L.

Households take as given the sectoral wage rates $\left(w_t^f, w_t^m\right)$, the profits paid by firms $\left(\pi_t^f, \pi_t^m\right)$ and government transfers (T_t). Households do not borrow directly from abroad, but they are still able to smooth consumption because the government borrows, pays transfers, and makes default decisions internalizing their utility function. This assumption implies that the households' optimization problem reduces to the following static problem:

$$\max_{c_t, L_t^m, L_t^f, L_t} E\left[\sum \beta^t u\left(c_t - h\left(L_t\right)\right)\right] \quad (1)$$

$$s.t. \quad c_t = w_t^f L_t^f + w_t^m L_t^m + \pi_t^f + \pi_t^m + T_t \quad (2)$$

$$L_t = \Psi\left(L_t^m, L_t^f\right) \quad (3)$$

The optimality conditions for labor supply are:

$$w_t^f = h'\left(L_t\right) \Psi'_{L_f}\left(L_t^f, L_t^m\right) \quad (4)$$

$$w_t^m = h'\left(L_t\right) \Psi'_{L_m}\left(L_t^f, L_t^m\right) \quad (5)$$

Hence, optimal sectoral allocations of labor are obtained when the relative wage rates equal the sectoral marginal rate of transformation:

$$\frac{w_t^f}{w_t^m} = \frac{\Psi'_{L^f}(L_t^f, L_t^m)}{\Psi'_{L^m}(L_t^f, L_t^m)} \quad (6)$$

The labor disutility function is defined in isoelastic form $h(L) = \frac{L^\omega}{\omega}$ with $\omega > 1$. The period utility function takes the standard constant-relative-risk-aversion form $u\left(c, L\right) = \frac{\left(c - \frac{L^\omega}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}$ with $\sigma > 0$. The labor transformation curve is given by $\Psi\left(L_t^f, L_t^m\right) = [(L_t^f)^\upsilon + (L_t^m)^\upsilon]^{1/\upsilon}$ with $0 \leq \upsilon \leq 1$. $\upsilon = 1$ implies costless reallocation of homogenous labor, $L_t = L_t^f + L_t^m$, and $\upsilon = 0$ implies that the cost of reallocating labor across sectors is infinite. With these functional forms, the optimality condition for sectoral labor supply allocations reduces to:

$$\frac{w_t^f}{w_t^m} = \left(\frac{L_t^f}{L_t^m}\right)^{\upsilon - 1} \quad (7)$$

Hence, the elasticity of substitution between L_t^f and L_t^m is equal to $1/(\upsilon - 1)$.

2.2 Final Goods Producers

Firms are divided into a sector f of final goods producers and a sector m of producers of intermediate goods, both of which maximize profits. Firms in the f sector use labor and intermediate goods, and face Markov TFP shocks ε_t, with transition probability distribution function $\mu\left(\varepsilon_t | \varepsilon_{t-1}\right)$. The production function of the f sector is Cobb-Douglas:

$$y_t = \varepsilon_t (m_t)^{\alpha_m} (L_t^f)^{\alpha_L} k^{\alpha_k} \quad (8)$$

with $0 < \alpha_L, \alpha_m, \alpha_k < 1$ and $\alpha_L + \alpha_m + \alpha_k = 1$.

The f sector chooses optimally whether to import intermediate goods from abroad or buy them from the m sector at home. Imported inputs are sold in a competitive world market at the exogenous relative price p_m^*.[6] A fraction θ of the cost of these imported inputs needs to be paid in advance using working capital loans κ_t , which are intraperiod loans repaid at the end of the period that are offered by foreign creditors at the interest rate r_t. This interest rate is linked to the sovereign interest rate at equilibrium, as shown in the next section. Working capital loans satisfy the standard payment-in-advance condition:

$$\frac{\kappa_t}{1+r_t} \geq \theta p_m^* m_t \tag{9}$$

Profit-maximizing firms choose κ_t so that this condition holds with equality.

The profits of final goods producers when they use imported inputs are:

$$\pi_t^* = \varepsilon_t (m_t)^{\alpha_m} (L_t^f)^{\alpha_L} k^{\alpha_k} - p_m^*(1+\theta r_t) m_t - w_t^f L_t^f \tag{10}$$

Alternatively, when they use domestic intermediate goods, their profits are given by:

$$\pi_t^d = \varepsilon_t (m_t)^{\alpha_m} (L_t^f)^{\alpha_L} k^{\alpha_k} - p_m m_t - w_t^f L_t^f \tag{11}$$

where p_m is the endogenous price of intermediate goods produced at home. As noted earlier, domestic inputs do not require working capital financing. This assumption is just for simplicity, the key element for the analysis is that at high levels of country risk (including periods without access to foreign credit markets) the financing cost of foreign inputs is higher than that of domestic inputs.

Final goods producers maximize profits taking the sectoral wage rate, the interest rate, and intermediate goods prices as given, and choosing whether to use domestic or imported intermediate goods *and* the optimal amount of intermediate goods and labor to buy in each case. This is equivalent to first evaluating the profit-maximizing plans under each alternative (domestic v. imported inputs) and then choosing the one that yields higher profits:

$$\pi_t^f = \max \left[\max_{m_t, L_t^f} (\pi_t^*), \max_{m_t, L_t^f} (\pi_t^d) \right] \tag{12}$$

When imported intermediate goods are used, the optimality conditions are

$$\alpha_m \varepsilon_t (m_t)^{\alpha_m - 1} (L_t^f)^{\alpha_L} k^{\alpha_k} = p_m^*(1+\theta r_t) \tag{13}$$
$$\alpha_L \varepsilon_t (m_t)^{\alpha_m} (L_t^f)^{\alpha_{L-1}} k^{\alpha_k} = w_t^f \tag{14}$$

[6] This price can also be modeled as a terms-of-trade shock with a given stochastic process.

Alternatively, when domestic inputs are used, the optimality conditions are:

$$\alpha_m \varepsilon_t (m_t)^{\alpha_m - 1} (L_t^f)^{\alpha_L} k^{\alpha_k} = p_t^m \tag{15}$$
$$\alpha_L \varepsilon_t (m_t)^{\alpha_m} (L_t^f)^{\alpha_L - 1} k^{\alpha_k} = w_t^f \tag{16}$$

These two sets of optimality conditions are standard: Marginal products of factors of production equal the corresponding marginal costs.

2.3 Intermediate Goods Producers

Domestic inputs do not require advance payment, but in order to produce them labor has to be reallocated from the f sector to the m sector. At equilibrium, the m sector operates only if the market price of its output is positive, which occurs only if the f sector chooses to use domestic inputs.

Producers in the m sector operate with a production function given by $m = A(L_t^m)^\gamma$, with $A > 0$ and $0 \leq \gamma \leq 1$. Given the domestic price of intermediate goods and the sectoral wage rate, they choose labor demand so as to solve the following profit maximization problem:

$$\max_{L_t^m} \ \pi_t^m = p_t^m A(L_t^m)^\gamma - w_t^m L_t^m \tag{17}$$

If sector f producers find it optimal to use imported inputs, the demand for domestic intermediate goods is zero, and hence p_t^m and L_t^m are zero and the m sector is idle. If final goods producers demand domestic intermediate goods, optimal labor demand by producers of intermediate goods satisfies

$$\gamma p_t^m A(L_t^m)^{\gamma-1} = w_t^m \tag{18}$$

2.4 Competitive Equilibrium of the Private Sector

Definition 1 *A competitive equilibrium for the private sector of the economy is given by sequences of allocations* $\left[c_t, L_t, L_t^f, L_t^m, m_t, \kappa_t\right]_{t=0}^{\infty}$ *and prices* $\left[w_t^f, w_t^m, p_t^m, \pi_t^f, \pi_t^m\right]_{t=0}^{\infty}$ *such that:*

1. The allocations $\left[c_t, L_t, L_t^f, L_t^m\right]_{t=0}^{\infty}$ *solve the households' utility maximization problem.*

2. The allocations $\left[L_t^f, m_t, \kappa_t\right]_{t=0}^{\infty}$ *solve the profit maximization problem of sector f producers.*

3. The allocations $[L_t^m]_{t=0}^{\infty}$ *solve the profit maximization problem of sector m producers.*

4. The labor market-clearing conditions hold.

Standard national income accounting implies that the economy's GDP is equal to either: (a) the gross output of the f sector net of the cost of imported inputs if final goods producers use imported inputs, or (b) the gross output of the f sector if final goods producers use domestic inputs. In the first case, the m sector is not operating and GDP at factor costs follows from the definition of profits of the f sector: $w_t L_t^f + \pi_t^f = \varepsilon(m_t)^{\alpha_m}(L_t^f)^{\alpha_L} k^{\alpha_k} - p_m^*(1+\theta r_t)m_t = (1-\alpha_m)\,\varepsilon(m_t)^{\alpha_m}(L_t^f)^{\alpha_L} k^{\alpha_k}$. This excludes $(1-\alpha_m)$ of gross output of final goods because imports of intermediate goods are factor payments to foreigners. In the second case, the definitions of profits of the f and m sectors yield: $w_t^f L_t^f + w_t^m L_t^m + \pi_t^f + \pi_t^m = \varepsilon(m_t)^{\alpha_m}(L_t^f)^{\alpha_L} k^{\alpha_k}$.

A key constraint on the problem of the sovereign borrower making the default decision will be that the private-sector allocations must be a competitive equilibrium. Since the sovereign government's problem and the equilibrium of the credit market will be characterized in recursive form, it is useful to also characterize the allocations of the above competitive equilibrium in recursive form (i.e. as functions defined in the state space domain). This is done by first expressing the optimal allocations of labor and intermediate goods when sector f uses imported inputs as the following functions of r and ε:

$$m^*(r,\varepsilon) = \left[\alpha_L^{\alpha_L}\left(\varepsilon k^{\alpha_k}\right)^{\omega}\left(\frac{\alpha_m}{p_m^*(1+\theta r)}\right)^{\omega-\alpha_L}\right]^{\frac{1}{\omega(1-\alpha_m)-\alpha_L}} \tag{19}$$

$$L^{f*}(r,\varepsilon) = \left[\alpha_L\left(\varepsilon k^{\alpha_k}\right)^{\frac{1}{1-\alpha_m}}\left(\frac{\alpha_m}{p_m^*(1+\theta r)}\right)^{\frac{\alpha_m}{1-\alpha_m}}\right]^{\frac{1-\alpha_m}{\omega(1-\alpha_m)-\alpha_L}} \tag{20}$$

If sector f uses domestic inputs instead, the optimal allocations of factors of production in the f and m sectors are:

$$L^d(\varepsilon) = \left[(\alpha_L+\gamma\alpha_m)\varepsilon k^{\alpha_k} A^{\alpha_m}\left(z_{L_m}\right)^{\alpha_m\gamma}\left(z_{L_f}\right)^{\alpha_L}\right]^{1/(\omega-\alpha_L-\alpha_m\gamma)} \tag{21}$$

$$L^{f,d}(\varepsilon) = z_{L_f} L^d(\varepsilon) \tag{22}$$

$$L^{m,d}(\varepsilon) = z_{L_m} L^d(\varepsilon) \tag{23}$$

$$m^d(\varepsilon) = A\left(L^{m,d}(\varepsilon)\right)^{\gamma} \tag{24}$$

where $z_{L_m} = \left(\frac{\gamma\alpha_m}{\gamma\alpha_m+\alpha_L}\right)^{1/\nu}$ and $z_{L_f} = \left(\frac{\alpha_L}{\gamma\alpha_m+\alpha_L}\right)^{1/\nu}$. Note also that the equilibrium price of the domestic intermediate goods is $p_m(\varepsilon) = \alpha_m\varepsilon\left(m^d(\varepsilon)\right)^{\alpha_m-1}\left(L^{f,d}(\varepsilon)\right)^{\alpha_L} k^{\alpha_k}$.

It follows from the above solutions that final goods production is not affected by foreign interest rates when firms use domestic intermediate goods, because sector f is not borrowing from abroad in this case. In contrast, when producers of final goods use imported inputs, their demand for these inputs and labor *decreases* with r. Thus, in this situation, sovereign risk affects the actions of sector f firms. Because, as we show later, the interest rate on foreign working capital loans is driven by the sovereign interest rate, these firms face higher financing costs when default risk rises, and so their factor demands and output fall. One special case of this situation is the state when default occurs, in which the country has no access to working capital because effectively r has gone to infinity. In this case, firms cannot import inputs from abroad and switch to use domestic substitutes. Note, however, that the interest rate does not need to rise to infinity for the switch to occur. Firms switch to domestic inputs at a finite interest rate that is high enough for $\pi^d > \pi^*$.

Next we define the indicator function $\Phi(r, \varepsilon)$ to identify whether the f sector is using domestic or imported inputs at the current state of interest rates and TFP. In particular, $\Phi(r, \varepsilon) = 1$ if $\pi^f = \max(\pi^*)$ and $\Phi(r, \varepsilon) = 0$ if $\pi^f = \max(\pi^d)$ for a given (r, ε) pair. Hence, firms use imported (domestic) inputs when $\Phi(r, \varepsilon) = 1$ ($\Phi(r, \varepsilon) = 0$). The competitive equilibrium allocations of factor demands and working capital can now be expressed as functions of r and ε as follows:

$$\kappa(r, \varepsilon) = \Phi(r, \varepsilon)\theta p_m^* m^*(r, \varepsilon) + (1 - \Phi(r, \varepsilon)) \cdot 0 \quad (25)$$

$$m(r, \varepsilon) = \Phi(r, \varepsilon) m^*(r, \varepsilon) + (1 - \Phi(r, \varepsilon))\, m^d(\varepsilon) \quad (26)$$

$$L(r, \varepsilon) = \Phi(r, \varepsilon) L^{f*}(r, \varepsilon) + (1 - \Phi(r, \varepsilon))\, L^d(\varepsilon) \quad (27)$$

$$L^f(r, \varepsilon) = \Phi(r, \varepsilon) L^{f*}(r, \varepsilon) + (1 - \Phi(r, \varepsilon))\, L^{f,d}(\varepsilon) \quad (28)$$

$$L^m(r, \varepsilon) = \Phi(r, \varepsilon) \cdot 0 + (1 - \Phi(r, \varepsilon))\, L^{m,d}(\varepsilon) \quad (29)$$

2.5 Endogenous Output Cost of Default

The decision by firms in the f sector to shift between foreign and domestic inputs depends on the states of the interest rate and TFP. The mechanism that drives this shift can be illustrated by examining the f sector firms' optimal choice of intermediate goods using Figure 1. For simplicity, we draw this figure assuming that total labor effort L is inelastic. The demand for intermediate goods is determined by the marginal product of m. The corresponding marginal productivity curve when foreign (domestic) inputs are used is labeled εf_{m*} (εf_{md}). The marginal productivity of intermediate goods employed in final goods production is always lower when domestic inputs are used, because of the reallocation of labor from final goods production to production of intermediate goods. Given the Cobb-Douglas production function for f, the lower labor input available to the f sector when it uses domestic inputs reduces the marginal product of intermediate goods in production of final goods.[7] Moreover, because the reallocation of labor is costly, one unit of labor taken away from the f sector yields less

[7] In the model, L is elastic. Our numerical simulations show that when domestic inputs are used, L falls compared to the case when imported inputs are used. Thus, the effect illustrated in Figure 1 actually underestimates the difference in the productivity of intermediate goods under the two scenarios at work in our numerical analysis.

than one unit of labor in the m sector, and the higher this reallocation cost the lower the marginal product of domestic intermediate goods relative to that of imported intermediate goods (i.e. the larger the gap between εf_{m*} and εf_{md}).

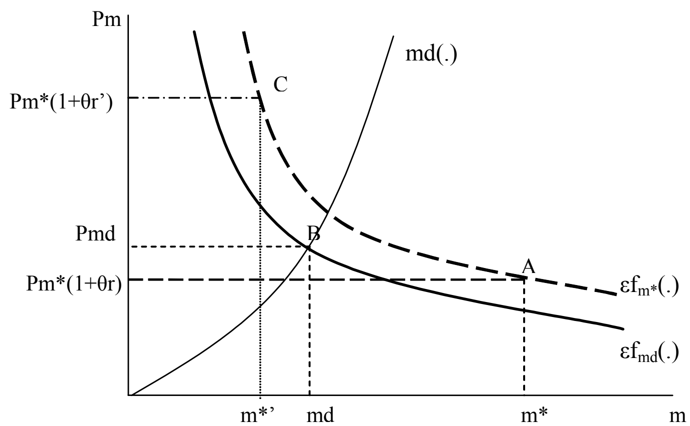


Figure 1: The Intermediate Goods Market

The supply of imported inputs is infinitely elastic at an exogenous price $p_m^*(1+\theta r)$. In contrast, the supply of domestic inputs (m_d in Figure 1) is determined by the production plans of the m sector. This supply function is given by $m^d(.) = A\left(\frac{\gamma A p^m}{w^m}\right)^{\frac{\gamma}{1-\gamma}}$.

If the interest rate is sufficiently low, the firms' optimal plans call for using imported inputs up to the amount at which the marginal product of m equals the marginal cost $p_m^*(1+\theta r)$. This is point A in Figure 1. Around point A, output fluctuates as a result of changes in r and ε. Consider first the interest rate. Given that marginal products are decreasing and continuously differentiable, it follows that as r rises the demand for imported inputs and the profits of final goods producers decline, until we reach a threshold value r' at which $\pi^* = \pi^d$. r' is an interest rate high enough for these producers to find it optimal to switch to the domestic inputs, because $r > r'$ yields $\pi^d > \pi^*$. This threshold point is shown as point C in Figure 1.

When the interest rate reaches r', final goods producers switch to domestic inputs and the equilibrium price and quantity of intermediate goods are determined at point B. Notice that, because imported inputs have higher marginal product, when interest rates are high (but not yet at r') it can be optimal for firms to use quantities of imported inputs that are lower than what they use if they operate with domestic inputs (m_d). This is because in this situation firms still make more profits with the foreign inputs than by switching to domestic inputs.

Around point B, fluctuations in output are driven by changes in ε, but output is no longer affected by the interest rate. This has two important implications. First, since in principle r' can be reached before the country defaults, high interest rates can trigger a switch to domestic inputs even before default occurs. Second, since r' is well defined and at default $r \to \infty$, firms always use domestic inputs when the economy defaults.

Productivity shocks can also cause the switch from imported to domestic inputs, even if r remains constant. As with the interest rate, there is a threshold TFP level at which final goods producers are indifferent between using imported or domestic inputs because $\pi^* = \pi^d$. For TFP shocks below this threshold, these producers opt for domestic inputs. The reason is that a low ε lowers the marginal product of imported inputs but firms still pay the extra marginal cost due to the cost of working capital. Hence, firms choose to use domestic inputs (and bear the efficiency loss) rather than paying this financing cost.

The switch from imported to domestic inputs that occurs at high interest rates has important implications for the output cost of default. In particular, it makes the cost of default an increasing function of the state of TFP. This property of the default cost can be illustrated by studying how productivity shocks affect the fraction of GDP lost in a default $1-(Y^d/Y^*)$, where Y^* and Y^d represent GDP when the economy has access to credit markets and when the economy defaults respectively (both given by the fraction $(1 - \Phi(r_t, \varepsilon_t)\alpha_m)$ of final goods production.

Figure 2 shows how Y^*, Y^d and the output loss at default change with TFP shocks for a given r. If the country defaults, exclusion from world credit markets prevents final goods producers from accessing working capital loans and forces them to switch to domestic inputs, so along the Y^d line firms always operate with domestic inputs. If the country has access to world credit markets, final goods producers choose optimally whether to use imported or domestic inputs. Hence, Y^* is produced with imported inputs as long as ε is above the threshold at which final goods producers switch to domestic inputs, and $Y^* = Y^d$ otherwise.

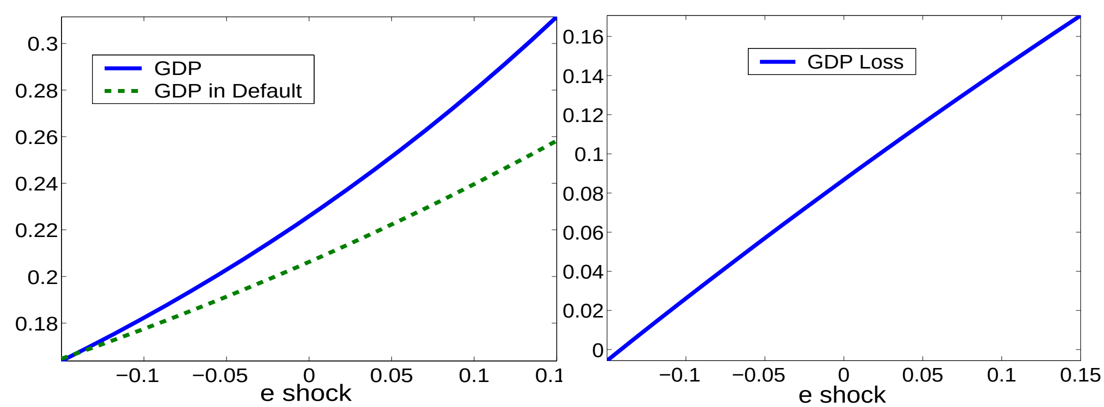


Figure 2: Output and the Output Cost of Default as Functions of TFP

As Figure 2 shows, the output cost of default increases with the size of the TFP shock, because default is accompanied by a switch from Y^* to Y^d, so default is more painful at higher levels of TFP. This property of the output cost of default is key for the model's ability to support high debt levels together with observed default frequencies, because it makes the default option more attractive to the country at lower states of productivity, and works as a desirable implicit hedging mechanism given the incompleteness of asset markets.

This finding is in line with Arellano's (2007) result showing that an exogenous default cost with similar features can allow the Eaton-Gersovitz model to support non-trivial levels of debt together with observed default frequencies. In particular, she proposed an exogenous default cost function such that below a threshold level of an output endowment default does not entail an output cost, but above that threshold default reduces the endowment to a state-invariant fraction of the long-run average of GDP. In this second range, the size of the output loss is increasing in the output realization at the time of default. Still, the mean debt ratio in her baseline calibration was only about 6 percent of GDP (assuming output at default is 3 percent below mean output), while we show later that our model with an endogenous output cost of default yields a mean debt ratio about four times larger.

2.6 The Sovereign Government

The sovereign government trades with foreign lenders one-period, zero-coupon discount bonds, so markets of contingent claims are incomplete. The face value of these bonds specifies the amount to be repaid next period and is denoted as b_{t+1}. When the country purchases bonds $b_{t+1} > 0$, and when it borrows $b_{t+1} < 0$. The set of bond face values is $B = [b_{\min}, b_{\max}] \subset R$, where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound $b_{\min} < -\frac{\overline{y}}{r}$, which is the largest debt that the country could repay with full commitment. The upper bound $b_{\max}$ is the highest level of assets that the country may accumulate.[8]

The sovereign cannot commit to repay its debt. As in the Eaton-Gersovitz model, we assume that when the country defaults it does not repay at date t and the punishment is exclusion from the world credit market in the same period. The country re-enters the credit market with an exogenous probability η, and when it does it starts with a fresh record and zero debt.[9] Also as in the Eaton-Gersovitz setup, the country cannot hold positive international assets during the exclusion period, otherwise the model cannot support equilibria with debt.

We add to the Eaton-Gersovitz setup an explicit link between default risk and private financing costs. This is done by assuming that a defaulting sovereign can divert the repayment of the firms' working capital loans to foreign lenders. Hence, both firms and government default together. This is perhaps an extreme formulation of the link between private and public borrowing costs, but we provide later some evidence in favor of this view.

[8] $b_{\max}$ exists when the interest rates on a country's saving are sufficiently small compared to the discount factor, which is satisfied in our paper since $(1 + r^*)\beta < 1$.

[9] We asbtract from debt renegotiation. See Yue (2006) for a quantitative analysis of sovereign default with renegotiation in which the length of financial exclusion is endogenous, and depends on the amount of reduced debt.

The sovereign government solves a problem akin to a Ramsey problem.[10] It chooses a debt policy (amounts and default) that maximizes the households' welfare subject to the constraints that: (a) the private sector allocations must be a competitive equilibrium; and (b) the government budget constraint must hold. The state variables are the initial foreign asset position, working capital loans as of the end of last period, and the state of TFP, denoted by the triplet $(b_t, \kappa_{t-1}, \varepsilon_t)$. The price of sovereign bonds is given by the bond pricing function $q_t(b_{t+1}, \varepsilon_t)$. Since at equilibrium the default risk premium on sovereign debt will be the same as on working capital loans, it follows that the interest rate on working capital is a function of $q_t(b_{t+1}, \varepsilon_t)$. Hence, the recursive expressions that represent the competitive equilibrium of the private sector derived earlier can be expressed as as $\kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $m(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $L^f(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $L^m(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, and $\Phi(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$.

The recursive optimization problem of the government is summarized by the following value function:

$$V(b_t, \kappa_{t-1}, \varepsilon_t) = \begin{cases} \max\left\{v^{nd}(b_t, \varepsilon_t), v^d(\kappa_{t-1}, \varepsilon_t)\right\} & \text{for } b_t < 0 \\ v^{nd}(b_t, \varepsilon_t) & \text{for } b_t \geq 0 \end{cases} \tag{30}$$

If the country has access to the world credit market at date t, the value function is the maximum of the value of continuing in the credit relationship with foreign lenders (i.e., repayment or "no default"), $v^{nd}(b_t, \varepsilon_t)$, and the value of default, $v^d(\kappa_{t-1}, \varepsilon_t)$. If the economy holds a non-negative net foreign asset position, the value function is simply the continuation value because in this case the economy is using the credit market to save, receiving a return equal to the world's risk free rate r^*.

The continuation value $v^{nd}(b_t, \varepsilon_t)$ is defined as follows:

$$v^{nd}(b_t, \varepsilon_t) = \max_{c_t, b_{t+1}} \left\{ \begin{array}{l} u(c_t - h(L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t))) \\ \qquad +\beta E\left[V(b_{t+1}, \kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), \varepsilon_{t+1})\right] \end{array} \right\} \tag{31}$$

subject to

$$\begin{aligned} & c_t + q_t(b_{t+1}, \varepsilon_t) b_{t+1} - b_t \leq \\ & \quad [1 - \Phi(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)\alpha_m] \varepsilon_t f\left(m(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), L^f(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), k\right) \end{aligned} \tag{32}$$

The constraint of this problem is the resource constraint of the economy at a competitive equilibrium. The left-hand-side is the sum of consumption and net exports, and the right-hand-side is GDP. This constraint is obtained by combining the households' budget constraint

[10] See Cuadra and Sapriza (2007) for an analysis of optimal fiscal policy as a Ramsey problem in the presence of sovereign default in an endowment economy.

(2) with the government budget constraint, $T_t = b_t - q_t\,(b_{t+1}, \varepsilon_t)\, b_{t+1}$, and noting that the firms' optimality conditions imply that total domestic factor payments, $w_t^f L_t^f + w_t^m L_t^m + \pi_t^f + \pi_t^m$, equal the fraction $(1 - \Phi(r, \varepsilon)\alpha_m)$ of gross output of final goods $\varepsilon f(m, L^f, k)$.

The resource constraint captures three important features of the model: First, the government internalizes how interest rates affect the competitive equilibrium allocations of output and factor demands. Second, the households cannot borrow from abroad, but the government internalizes their desire to smooth consumption and transfers to them an amount equal to the negative of the balance of trade (i.e. it gives the private sector the flow of resources it needs to finance the gap between GDP and consumption). Third, the working capital loans κ_{t-1} and κ_t do not enter explicitly in the continuation value or in the resource constraint, because working capital payments are included in the fraction of gross output allocated to payments of intermediate goods, $\alpha_m f(m, L^f, k)$. Still, we need to keep track of the state variable κ_t because the amount of working capital loans taken by final goods producers at date t affects the sovereign's incentive to default at $t + 1$, as explained below.

The value of default $v^d\,(\kappa_{t-1}, \varepsilon_t)$ is:

$$v^d\,(\kappa_{t-1}, \varepsilon_t) = \max_{c_t} \left\{ \begin{array}{l} u\,(c_t - h(L(\varepsilon_t))) \\ \qquad +\beta\,(1 - \eta)\, E v^d\,(0, \varepsilon_{t+1}) + \beta\eta E V\,(0, 0, \varepsilon_{t+1}) \end{array} \right\} \tag{33}$$

subject to:

$$c_t = \varepsilon_t f\left(m^d(\varepsilon_t), L^{f,d}(\varepsilon_t), k\right) + \kappa_{t-1} \tag{34}$$

Note that $v^d\,(\kappa_{t-1}, \varepsilon_t)$ takes into account the fact that in case of default at date t, the country has no access to financial markets this period, and hence the country consumes the total income given by the resource constraint in the default scenario. In this case, since firms cannot borrow to finance purchases of imported inputs, $m^d(\cdot)$, $L(\cdot)$ and $L^{f,d}\,(\cdot)$ are the competitive equilibrium allocations that correspond to the case when the f sector operates with domestic inputs. Moreover, because the defaulting government diverts the repayment of last period's working capital loans, total household income includes government transfers equal to the appropriated repayment for the amount κ_{t-1} (i.e., on the date of default, the government budget constraint is $T_t = \kappa_{t-1}$). The value of default at t also takes into account that at $t+1$ the economy may re-enter world capital markets with probability η and associated value $V\,(0, 0, \varepsilon_{t+1})$, or remain in financial autarky with probability $1-\eta$ and associated value $v^d\,(0, \varepsilon_{t+1})$.

For a debt position $b_t < 0$ and given a level of working capital κ_{t-1}, default is optimal for the set of realizations of the TFP shock for which $v^d\,(\kappa_{t-1}, \varepsilon_t)$ is at least as high as $v^{nd}\,(b_t, \varepsilon_t)$:

$$D\,(b_t, \kappa_{t-1}) = \left\{ \varepsilon_t : v^{nd}\,(b_t, \varepsilon_t) \leq v^d\,(\kappa_{t-1}, \varepsilon_t) \right\} \tag{35}$$

It is critical to note that this default set has a different specification than in the typical Eaton-Gersovitz model of sovereign default (see Arellano (2007)), because the state of working capital affects the gap between the values of default and repayment. This results in a two-dimensional default set that depends on b_t and κ_{t-1}, instead of just b_t.

Despite the fact that the default set depends on κ_{t-1}, the probability of default remains a function of b_{t+1} and ε_t only. This is because the f sector's optimality conditions imply that the next period's working capital loan κ_t depends on ε_t and the interest rate, which is a function of b_{t+1} and ε_t. Thus the probability of default at $t+1$ perceived as of date t for a country with a productivity ε_t and debt b_{t+1}, $p_t(b_{t+1}, \varepsilon_t)$, can be induced from the default set, the decision rule for working capital, and the transition probability function of productivity shocks $\mu(\varepsilon_{t+1}|\varepsilon_t)$ as follows:

$$p_t(b_{t+1}, \varepsilon_t) = \int_{D(b_{t+1}, \kappa_t)} d\mu(\varepsilon_{t+1}|\varepsilon_t) \tag{36}$$

$$\text{where } \kappa_t = \kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t) \tag{37}$$

The economy is considered to be in financial autarky when it has been in default for at least one period and remains without access to world credit markets as of date t. As noted above, the economy can exit this exclusion stage at date $t+1$ with probability η. We assume that during the exclusion stage the economy cannot build up its own stock of savings to supply working capital loans to firms, which could be used to purchase imported inputs.[11] This assumption ensures that, as long as the economy remains in financial autarky, the optimization problem of the sovereign is the same as the problem in the default period but evaluated at $\kappa_{t-1} = 0$ (i.e. $v^d(\varepsilon_t, 0)$).

We also studied an alternative setup in which we allowed for a domestic financial market to operate during the exclusion stage. In this case, households make saving plans to offer working capital loans to firms at a market-determined interest rate, and firms demand these loans if the endogenous domestic interest rate is low enough to make productions plans using foreign inputs more profitable than with domestic inputs, despite the higher financing cost of the former. In this case, domestic loans are included as an additional state variable and their interest rate is determined as an equilibrium outcome. We found, however, that for parameter values around our baseline calibration this domestic financial market is not viable: The interest rate at which households would find it optimal to accumulate savings is too high for firms to optimally choose to obtain domestic working capital loans to purchase imported inputs, instead of just using domestic inputs. Hence, the equilibrium for the model with the domestic financial market operating during the exclusion stage is the same as that for the model that simply assumes that firms operate with domestic inputs whenever they cannot

[11] Alternatively, we could assume that the default punishment includes exclusion from world capital markets and from the world market of intermediate goods.

access world credit markets.

The model preserves a standard feature of the Eaton-Gersovitz model: Given ε_t, the value of defaulting is independent of the level of debt, while the value of not defaulting increases with b_{t+1}, and consequently the default set and the equilibrium default probability grow with the country's debt. The following theorem formalizes this result:

Theorem 1 *Given a productivity shock ε and level of working capital loan κ, for $b^0 < b^1 \leq 0$, if default is optimal for b^1, then default is also optimal for b^0. That is $D\left(b^1, \kappa\right) \subseteq D\left(b^0, \kappa\right)$. The country agent's probability of default in equilibrium satisfies $p^*\left(b^0, \varepsilon\right) \geq p^*\left(b^1, \varepsilon\right)$.*

Proof. See Appendix. ■

2.7 Foreign Lenders

International creditors are risk-neutral and have complete information. They invest in sovereign bonds and in private working capital loans. Foreign lenders behave competitively and face an opportunity cost of funds equal to the world risk-free interest rate. Competition implies that they expect zero profits at equilibrium, and that the returns on sovereign debt and the world's risk-free asset are fully arbitraged:

$$q_t\left(b_{t+1}, \varepsilon_t\right) = \begin{cases} \frac{1}{1+r^*} & \text{if } b_{t+1} \geq 0 \\ \frac{\left[1-p_t\left(b_{t+1}, \varepsilon_t\right)\right]}{1+r^*} & \text{if } b_{t+1} < 0 \end{cases} \tag{38}$$

This condition implies that at equilibrium bond prices depend on the risk of default. For a high level of debt, the default probability is higher. Therefore, equilibrium bond prices decrease with indebtedness. This result, formalized in Theorem 2 below, is consistent with the empirical evidence documented by Edwards (1984).

Theorem 2 *Given a productivity shock ε and level of working capital loan κ, for $b^0 < b^1 \leq 0$, the equilibrium bond price satisfies $q^*\left(b^0, \varepsilon\right) \leq q^*\left(b^1, \varepsilon\right)$*

Proof. See Appendix. ■

The returns on sovereign bonds and working capital loans are also fully arbitraged. Because the sovereign government diverts the repayment of working capital loans when it defaults, foreign lenders assign the same risk of default to private working capital loans as to sovereign debt, and hence the no- arbitrage condition between sovereign lending and working capital loans implies:

$$r_t\left(b_{t+1}, \varepsilon_t\right) = \frac{1}{q_t\left(b_{t+1}, \varepsilon_t\right)} - 1, \text{ if } b_{t+1} < 0 \text{ and } \kappa_t > 0 \tag{39}$$

2.8 Country Risk & Private Interest Rates: Some Empirical Evidence

The result that the interest rates on sovereign debt and private working capital loans are the same raises a key empirical question: Are sovereign interest rates and the rates of interest faced by private firms closely related in emerging economies?

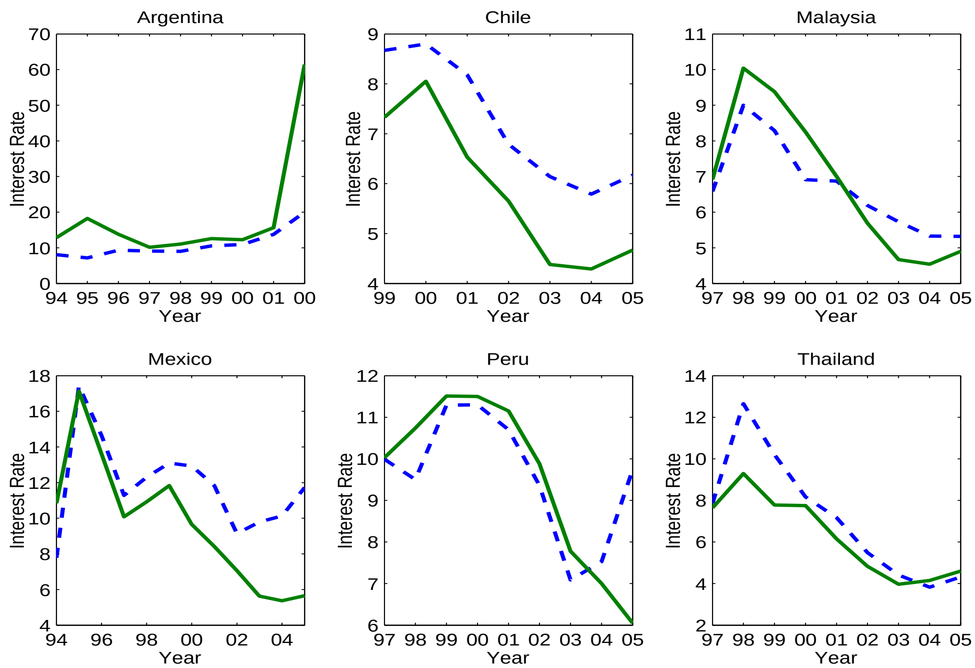
Providing a complete answer to this question is beyond the scope of this paper, but we do provide empirical evidence suggesting that indeed interest rates on loans to private firms and on sovereign bonds move together. To study this issue, we constructed country estimates of firms' financing costs that aggregate measures derived from firm-level data. We constructed a measure of firm-level effective interest rates as the ratio of a firm's total debt service divided by its total debt obligations using the *Worldscope* database, which provides the main lines of balance-sheet and cash-flow statements of publicly listed corporations. We then constructed the corresponding aggregate country measure as the median across firms.

Table 1: Sovereign Interest Rates and Firm Financing Cost

Country	Sovereign Interest Rates	Median Firm Interest Rates	Correlation
Argentina	13.32	10.66	0.87
Brazil	12.67	24.60	0.14
Chile	5.81	7.95	0.72
China	6.11	5.89	0.52
Colombia	9.48	19.27	0.86
Egypt	5.94	8.62	0.58
Malaysia	5.16	6.56	0.96
Mexico	9.40	11.84	0.74
Morocco	9.78	13.66	0.32
Pakistan	9.71	12.13	0.84
Peru	9.23	11.42	0.72
Philippines	8.78	9.27	0.34
Poland	7.10	24.27	0.62
Russia	15.69	11.86	-0.21
South Africa	5.34	15.19	0.68
Thailand	6.15	7.30	0.94
Turkey	9.80	29.26	0.88
Venezuela	14.05	19.64	0.16

The comparison of this measure of interest rates faced by private firms with the standard EMBI+ measure of interest rates on sovereign debt shows two striking facts (see Table 1): First, the two interest rates are positively correlated in most countries, with a median correlation of 0.7, and in some countries the relationship is very strong (see Figure 3).[12] Second, the effective financing cost of firms is generally *higher* than the sovereign interest rates. This fact indicates that the common conjecture that firms (particularly the large corporations covered in our data) may pay lower rates than governments with default risk is incorrect.

The study by Arteta and Hale (2007) provides further and more systematic evidence on the strong effects of sovereign debt on the terms of private-sector debt contracts of emerging economies. In particular, they show strong, systematic negative effects on private corporate bond issuance during and after default episodes.



——— Sovereign Bond Interest Rates - - - - Median Firm Financing Cost

Figure 3: Sovereign Bond Interest Rates and Median Firm Financing Costs

There is also evidence suggesting that our assumption that the government can divert the repayment of the firms' foreign obligations is realistic. In particular, it is not uncommon for the government to take over the foreign obligations of the corporate sector in actual default episodes. The following quote by the IMF historian explains how this was done in Mexico's 1982-83 default, and notes that arrangements of this type have been commonly used since then: *"A simmering concern among Mexico's commercial bank creditors was the handling of private sector debts, a substantial portion of which was in arrears...the banks and some official agencies had pressured the Mexican government to assume these debts...Known as the FICORCA scheme, this program provided for firms to pay dollar-denominated commercial debts in pesos to the central bank. The creditor was required to reschedule the debts over*

[12] Arellano and Kocherlakota (2007) document a positive correlation between private domestic lending rates and sovereign spreads using the domestic lending-deposit spread data from the Global Financial Data.

several years, and the central bank would then guarantee to pay the creditor in dollars. Between March and November 1983, close to \$12 billion in private sector debts were rescheduled under this program... FICORCA then became the prototype for similar schemes elsewhere." (Boughton (2001), Ch. 9, pp. 360-361)

2.9 Recursive equilibrium

Definition 2 *The model's recursive equilibrium is given by (i) a decision rule* $b_{t+1}(b_t, \kappa_{t-1}, \varepsilon_t)$ *for the sovereign government with associated value function* $V(b_t, \kappa_{t-1}, \varepsilon_t)$*, consumption and transfers rules* $c(b_t, \kappa_{t-1}, \varepsilon_t)$ *and* $T(b_t, \kappa_{t-1}, \varepsilon_t)$*, default set* $D(b_t, \kappa_{t-1})$ *and default probabilities* $p^*(b_{t+1}, \varepsilon_t)$*; and (ii) an equilibrium pricing function for sovereign bonds* $q^*(b_{t+1}, \varepsilon_t)$ *such that:*

1. Given $q^*(b_{t+1}, \varepsilon_t)$*, the decision rule* $b_{t+1}(b_t, \kappa_{t-1}, \varepsilon_t)$ *solves the recursive maximization problem of the sovereign government (30).*

2. The consumption plan $c(b_t, \kappa_{t-1}, \varepsilon_t)$ *satisfies the resource constraint of the economy*

3. The transfers policy $T(b_t, \kappa_{t-1}, \varepsilon_t)$ *satisfies the government budget constraint.*

4. Given $D(b_t, \kappa_{t-1})$ *and* $p^*(b_{t+1}, \varepsilon_t)$*, the bond pricing function* $q^*(b_{t+1}, \varepsilon_t)$ *satisfies the arbitrage condition of foreign lenders (38).*

Condition 1 requires that the sovereign government's default and saving/borrowing decisions be optimal given the interest rates on sovereign debt. Condition 2 requires that the private consumption allocations implied by these optimal borrowing and default choices be both feasible and consistent with a competitive equilibrium (recall that the resource constraint of the sovereign's optimization problem considers only private-sector allocations that are competitive equilibria). Condition 3 requires that the decision rule for government transfers shifts the appropriate amount of resources between the government and the private sector (i.e. an amount equivalent to net exports when the country has access to world credit markets, or the diverted repayment of working capital loans when a default occurs, or zero when the economy is in financial autarky beyond the date of default). Notice also that given conditions 2 and 3, the consumption plan satisfies the households' budget constraint. Finally, Condition 4 requires the equilibrium bond prices that determine country risk premia to be consistent with optimal lender behavior.

A solution for the above recursive equilibrium includes solutions for $\kappa(q^*(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $m(q^*(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $L^f(q^*(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $L^m(q^*(b_{t+1}, \varepsilon_t), \varepsilon_t)$ and $L(q^*(b_{t+1}, \varepsilon_t), \varepsilon_t)$. A solution for equilibrium interest rates on working capital as a function of b_{t+1} and ε_t follows from (39). Expressions for equilibrium wages, profits and the price of domestic inputs as functions of r_t and ε_t follow then from the firms' optimality conditions and the definitions of profits described earlier.

3 Quantitative analysis

3.1 Calibration

We study the quantitative implications of the model by conducting numerical simulations setting the model to a quarterly frequency and using the following benchmark calibration. The risk aversion parameter σ is set to 2 and the quarterly world risk-free interest rate r^* is set to 1 percent, which are standard values in quantitative business cycle and sovereign default studies. The productivity coefficient in production of domestic inputs A is chosen so that the average amount of domestic m (when this sector operates) is equal to the average amount of imported inputs that is used in the absence of default risk (i.e. when $r = r^*$). This calibration target for A ensures that the results are not driven by a relatively low supply of domestic intermediate goods. The curvature of aggregate labor effort in the utility function is set to $\omega = 2.1$, which implies a Frisch wage elasticity of labor supply of $1/(\omega - 1) = 0.91$, consistent with Hall's (2007) estimates for the United States. RBC models of the small open economy (e.g. Mendoza (1991) and Neumeyer and Perri (2005))) typically use $\omega = 1.45$, which originated in an older estimate of the U.S. labor supply elasticity used by Greenwood, Hercowitz and Huffman (1988), yet our main results are largely robust to this change. The probability of re-entry after default is 0.1, which implies that the country stays in exclusion for 2.5 years after default on average, in line with the finding of Gelos et al. (2003).

The share of intermediate goods in gross output α_m is set to 0.3. This parameter is difficult to set using actual data because in the model intermediate goods are either all imported or all purchased internally, but in the data the share of *total* intermediate goods often is about 40 percent of output, and only about 1/3 to 1/2 of this share corresponds to imported inputs (see Gopinath, Itskhoki, and Rigobon (2007) and Mendoza (2007)). Hence, setting $\alpha_m = 0.4$ would match the share of total intermediate goods but overestimate the fraction of them that are imported, while $\alpha_m = 0.15$ would match the share of imported inputs but underestimate the share of total intermediate goods. With this in mind, we set $\alpha_m = 0.3$ as an intermediate value and conduct sensitivity analysis later. Given this share for intermediate goods in gross output of final goods, the capital share α_k is set to 0.21 so that the capital income share in value added of the f sector ($\alpha_k/(1 - \alpha_m) = 0.3$) matches the standard 30 percent. These factor shares imply a labor share in gross output of final goods of $\alpha_L = 1 - \alpha_m - \alpha_k = 0.49$, which yields a labor share in value added $\alpha_L/(1 - \alpha_m) = 0.7$ that matches the standard 70 percent. The labor share in intermediate goods production γ is also set to 0.7, since this is also the share of labor in value added in the m sector.

Productivity shocks in final goods production follow an AR(1) process:

$$\log \varepsilon_t = \rho_\varepsilon \log \varepsilon_{t-1} + \epsilon_t \tag{40}$$

with $\epsilon_t \overset{iid}{\sim} N\left(0, \sigma_\epsilon^2\right)$. Data limitations prevent us from estimating directly this process using actual TFP data, so we set σ_ϵ^2 and ρ_ε (together with other parameters to be discussed below) using the simulated method of moments (SMM). The target moments used to set σ_ϵ^2 and ρ_ε are the variability and persistence of output, which we calibrate to quarterly data for Argentina. This facilitates comparisons with the literature on quantitative models of default, which largely focuses on data for Argentina. We use seasonally-adjusted quarterly real GDP from the Ministry of Finance (MECON) for the period 1980Q1 to 2005Q4. The standard deviation and first-order autocorrelation of the cyclical component of H-P filtered GDP are 4.7 percent and 0.79 respectively. Given these targets, the process of productivity shocks derived using SMM features $\rho_\varepsilon = 0.90$ and $\sigma_\epsilon = 1.61$ percent.

Table 2: Benchmark Model Calibration

Calibrated Parameters		*Value*	*Target statistics or source*	
CRRA risk aversion	σ	2	Standard value	
Risk-free interest rate	r^*	1%	Standard value	
Capital share in final goods gross output	α_k	0.21	Standard GDP capital share (0.3)	
Intermediate share in final gross output	α_m	0.3	National accounts	
Labor share in final goods gross output	α_L	0.49	Standard GDP labor share (0.7)	
Labor share in GDP of int. goods	γ	0.7	Standard GDP labor share (0.7)	
Labor elasticity para.	ω	2.1	Hall (2007) estimate	
Re-entry Probability	η	0.1	Length of exclusion (2.5 years)	
Intermediate goods TFP	A	1.48	Average m without default (0.096)	
Parameters set with SMM		*Value*	*Targets from Argentina's data*	
Productivity persistence	ρ_ε	0.90	GDP persistence	0.79
Productivity innovations std. dev.	σ_ϵ	1.61%	GDP std. dev.	4.70%
Time discount factor	β	0.85	Default frequency	0.69%
Labor transformation para.	ν	0.41	Output drop in default	13%
Working capital friction	θ	0.10	Trade balance volatility	2.88%

The additional parameters set using SMM are the subjective discount factor β, the curvature parameter in the labor transformation curve ν, and the share of imported inputs paid for with working capital θ. These parameters are targeted to match the frequency of default, the average fraction of output loss at default, and the volatility of the trade balance-GDP ratio. The target statistic for default frequency is 0.69 percent because Argentina has defaulted five times on its external debt since 1824 (the average default frequency is 2.78 percent annually or 0.69 percent quarterly). The average output loss at default for Argentina is 13 percent based on the cyclical position of the country's quarterly GDP around the December 2001 debt crisis. The standard deviation of the quarterly trade balance-to-GDP ratio for Argentina is 2.88 percent.

Table 2 shows the parameters of the benchmark calibration. The SMM estimate of the subjective discount factor is 0.85, which is in the range of the values used in the existing studies on sovereign default.[13] The estimate for ν is 0.41, which implies that the elasticity of substitution across L^f and L^m is -1.69. Finally, the estimate for θ implies that firms pay only 1/10 of the cost of imported inputs in advance. As noted earlier, this low θ is important because previous studies of emerging markets business cycles (e.g. Neumeyer and Perri (2005) and Uribe and Yue (2006)) assumed that 100 percent of the wages bill is paid in advance, but with a standard labor share of about 0.7, this implies that working capital financing would need to be 70 percent of GDP. This ratio exceeds estimates of the ratio of *total* bank credit to the private sector as a share of GDP in many emerging economies (which average about 50 percent of GDP).

3.2 Results of the Benchmark Simulation

This subsection examines the model's ability to account for the three key empirical regularities of sovereign debt highlighted in the Introduction: V-shaped output dynamics with deep recessions that hit bottom at times of default, countercyclical country interest rates, and high debt ratios. To explore this issue, we feed the TFP process to the model and conduct 1000 simulations, each with 500 periods and truncating the first 100 observations.

The quantitative predictions of the model approximate closely the three key stylized facts of sovereign debt, and they also match two key business cycle regularities: the cyclical variability of consumption and the correlation of net exports with GDP. Table 3 compares the moments produced by the model with moments from Argentine data. The bond spreads data are quarterly EMBI+ spreads on Argentine foreign currency denominated bonds from 1994Q2 to 2001Q4, taken from J.P. Morgan's EMBI+ dataset.

The model mimics the positive correlation between spreads and net exports, and the negative correlations of spreads and net exports with GDP. The model replicates the negative correlation between spreads and GDP because sovereign bonds have higher default risk in bad states. Several quantitative models of sovereign debt (e.g. Arellano (2007), Aguiar and Gopinath (2005), Yue (2006)) and business cycle models of emerging economies (e.g. Neumeyer and Perri (2005), Uribe and Yue (2006)) also produce countercyclical spreads, but the former treat output as an exogenous endowment and in the latter country risk is exogenous. In contrast, our model nearly matches the negative correlation between GDP and spreads in a setting in which both output and country risk are endogenous, and influence each other because of the relationship between country risk and working capital financing. Moreover, our model also produces a closer approximation to the actual correlation between bond spreads and GDP than other models of sovereign default, which do yield acyclical or

[13] The values of β used by Aguiar and Gopinath (2006), Arellano (2007), and Yue (2006) range from 0.8 to 0.953.

countercyclical spreads but miss the actual correlations by wide margins.

Table 3: Model Simulation and Statistics in the Data

Statistics	Data	Model
Corr. between Bond Spreads and GDP	-0.62	-0.48
Corr. between Bond Spreads and Trade Balance	0.68	0.39
Corr. between Trade Balance and GDP	-0.58	-0.42
Consumption Std. Dev./Output Std. Dev.	1.44	1.36
Average Debt/GDP	35%	26.05%
Bond Spreads Std. Dev.	0.78%	0.71%
Average Bond Spreads	1.86%	0.69%
Corr. between GDP and Aggregate Labor	-	0.72
Corr. between Spread and Aggregate Labor	-	-0.51
Corr. between GDP and Intermediate Goods	-	0.66
Corr. between Spread and Intermediate Goods	-	-0.51
Corr. between GDP and Defaults	-	-0.14
Fraction of defaults with GDP below trend	-	100%
Fraction of defaults with GDP 2 std dev. below trend	-	76.01%

The countercyclical net exports follow from the fact that, when the country is in a bad state, it faces higher interest rates and tends to borrow less. The country's trade balance thus increases because of the lower borrowing, leading to a negative correlation between net exports and output.

Consumption variability exceeds output variability in Argentina, and this is a common feature for emerging economies. The model is able to mimic this stylized fact because the ability to share risk with foreign lenders is negatively affected by the higher interest rates induced by increased default probabilities. The sovereign borrows less when the economy faces an adverse productivity shock, and thus households adjust consumption by more than in the absence of default risk. On the other hand, because agents are impatient, the benevolent government borrows more to increase private consumption when the productivity shock is good. Hence, the variability of consumption rises.

The model produces a debt-to-GDP ratio of 26 percent on average. This high debt ratio is mainly the result of the large output drop that occurs when the country defaults, and the fact that the size of the drop is increasing in the state of TFP. Although a 26 percent debt ratio is still below Argentina's 35 percent average debt-output ratio (based on data from the World Bank's WFD dataset for the 1980-2004 period), it is several orders of magnitude larger than the debt ratios typically obtained in quantitative models of sovereign default with exogenous output costs already targeted to improve the models' quantitative performance. For instance, Yue's (2006) model with renegotiation and an exogenous 2 percent output cost at default yields an average debt ratio of 9.7 percent. Arellano (2007) obtains a mean debt ratio of 6 percent of GDP assuming that output when the economy defaults equals the maximum of actual output or 97 percent of average output.[14]

The model also matches closely the volatility of the Argentine bond spreads observed in the data. Yet the average bond spread is lower than in the data. Because we assume a zero recovery rate on defaulted debt and risk-neutral creditors, bond spreads are linked one-to-one with default probabilities (see 38). Since the quarterly default frequency is 0.7 percent (as in the data), the model can only generate a 0.7 percent average bond spread, which is about 2/5s of the average spreads observed in the data.

Table 3 also lists the correlations of GDP and bond spreads with labor and intermediate goods. We do not have empirical counterparts for Argentina, but the model's correlations are reasonable: both labor and intermediate goods are procyclical because of the Cobb-Douglas production technology, and they share a common correlation with bond spreads because of the working capital constraint.[15]

Table 3 shows that the correlation between defaults and GDP in the model's ergodic distribution is -0.14, in line with Tomz and Wright's (2007) cross-country historical estimate for the period 1820-2004. The Table also lists the fraction of defaults that occur in "bad times," defined as periods in which GDP is either below its HP trend or below two standard deviations of its HP trend. All default events in the quarterly benchmark calibration occur when GDP is below trend, and about 3/4s occur when GDP is at least two standard deviations below trend. This seems at odds with Tomz and Wright's findings indicating that not all defaults coincide with bad times in annual data. However, if we aggregate the quarterly simulation data into an annual frequency and recalculate these statistics, we find that 22 percent of defaults occur in "good times" (i.e. with GDP above trend), 78 percent occur in bad times, and only about 6 percent of them occur when GDP is two standard deviations or more below trend. Moreover, in the sensitivity analysis of Section 4 we show that for some parameter values the model can generate default in good times even at the quarterly frequency.

3.3 Dynamics of Output Around Default Episodes

We illustrate the model's ability to match V-shaped dynamics of output around default episodes using event study techniques. Figure 4 plots the model's average path of output around default events together with the data for Argentina's HP detrended GDP around the 2001 default (1999Q1 to 2004Q2). The event window covers 12 quarters before and 10 quarters after debt defaults, with the default events normalized to date 0. We plot the

[14] As mentioned earlier, Aguiar and Gopinath (2006) obtained a higher mean debt ratio (27 percent of GDP) assuming a cost of 2 percent of output, but with a default frequency of only 0.02 percent.

[15] Using Mexican data, Mendoza (2007) reports a correlation between GDP and imported inputs of 0.91.

average for output in the model at each date $t = -12, ..., 10$ around default events in the 1000 simulations. Hence, this represents the average behavior of output around defaults in the stationary distribution of the model. Since Argentina's data is for a single default event, while the model's output dynamics correspond to the model's stochastic stationary state, we add dashed lines with one-standard-error bands around the simulation averages.

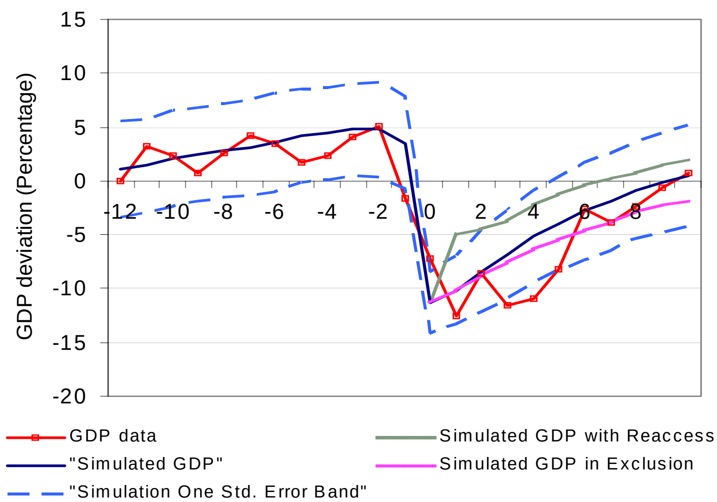


Figure 4: Output around Default Events

Figure 4 shows that the model produces a substantial output drop when the country defaults, equivalent to about 13 percent of the pre-default output level. Defaults in this baseline calibration are always triggered by adverse productivity shocks, but these shocks do not need to be unusually large. The standard deviation of the calibrated TFP process (σ_ε) is 3.69 percent. By contrast, the average decline in TFP in default events (i.e. at $t = 0$ in Figure 4) is 6 percent, which is 1.6 times the standard deviation of TFP, and hence within the two-standard-deviations range. This suggests that the model embodies a business cycle transmission mechanism that can amplify significantly the real effects of TFP shocks when these shocks trigger default.

The model displays a V-shaped recovery during the financial exclusion period. This occurs because the shock is mean-reverting, and hence TFP is likely to improve in the periods after default (for example, on average in the simulations TFP rises by 1 percent at $t = 1$). Therefore, even though the country remains financially excluded on average at dates 1 to 10, the economy recovers because TFP improves. Note that the relative magnitudes of the recession and recovery match the data quite well. The output dynamics for Argentina before and after the 2001 debt crisis are mostly within the one-standard-error bands of the model simulation.

These V-shaped dynamics are qualitatively consistent with the data of many emerging markets that suffered Sudden Stops. Calvo, Izquierdo and Talvi (2006) conducted a detailed cross-country empirical analysis of the recovery of emerging economies from Sudden Stops, and found that most recoveries are not associated with improvements in credit market access.

Further analysis of the recovery of GDP after default shows that the recovery is driven by both the direct effect of the rise in TFP after the "bad shock" at default and by the surge in output that occurs when the country re-enters credit markets. This point is illustrated in Figure 4 by the lines that show the model simulations for the path of GDP with continued exclusion for 10 quarters after default and with immediate re-entry one period after default. In the first scenario, the recovery reflects only the effect of the mean reversion of the TFP shock. GDP remains below that in the simulation average, and it is also lower than in the data starting in the 6th quarter after default. In contrast, the second scenario with immediate re-entry to international debt markets shows a big rebound in GDP at $t = 1$ because of the efficiency gain obtained as final goods producers switch back to imported inputs. The model simulation average lies below this immediate re-entry line because in the model the re-entry to credit markets is stochastic with 10 percent probability. The recovery of GDP after default is therefore influenced by both the mean reversion of the TFP shock and the re-entry to credit markets. Since re-entry has a relatively low probability, however, the model simulation for average GDP weighs more the former than the latter.

The model's output dynamics also suggest that the model can account for the seemingly dominant role of productivity shocks in explaining output collapses during financial crises in emerging markets. In particular, this can be the result of the efficiency loss caused by the sectoral reallocation of labor and the fall in use of intermediate goods when the economy defaults. To demonstrate this point, we use the model's simulated data on aggregate factor payments, GDP, and labor to compute Solow residuals in the standard way: We assume an aggregate Cobb-Douglas production function for economy-wide GDP, $gdp_t = s_t(L_t)^a k^{1-a}$, and compute the Solow residual s using the model's data for L and gdp, setting a to the model's average of the ratio of total wage payments to GDP, $[w_t^f L_t^f + w_t^m L_t^m]/gdp_t$, which is about 0.7. By construction, however, the "true" TFP driving the model is ε_t in the production function of final goods.

Figure 5 compares the quarter-on-quarter average growth rates of the Solow residual, true TFP and GDP around default events in the baseline model simulations. Clearly, there is little difference between the Solow residual and true TFP except when the economy defaults. In default events, however, the Solow residual overestimates the true adverse TFP shock by a large margin (on average, s falls by nearly twice as much as ε when the economy defaults). Moreover, a standard decomposition of the contributions of changes in TFP and factors of production to changes in GDP shows that the contribution of true TFP to the output collapse at default is about 28 percent. In contrast, the contribution of the Solow residual is nearly 66 percent, which would suggest misleadingly that the contribution of TFP shocks is 2.35 times larger than it actually is. The large difference between the two is due to the fact that the Solow residual treats the efficiency loss caused by the sectoral reallocation of labor and the lower use of intermediate goods as a reduction in TFP in final goods production.

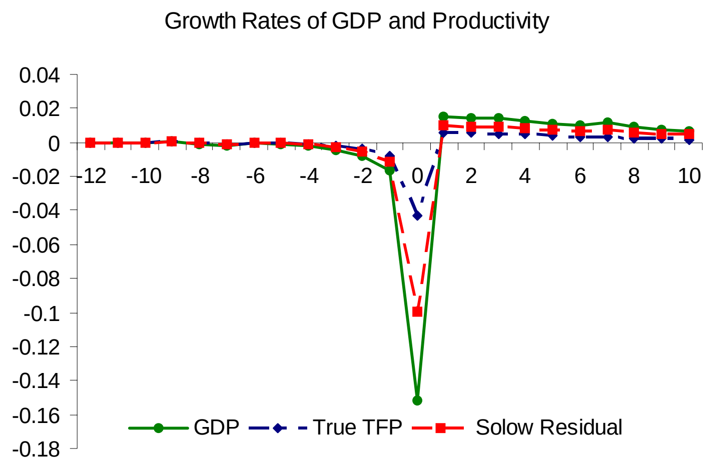


Figure 5: Growth Rates of GDP, True TFP and Solow Residual around Default

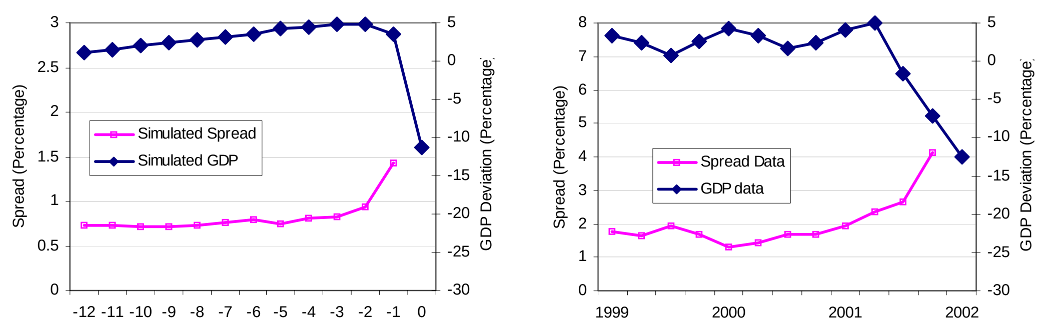


Figure 6: Dynamics of Output and Sovereign Spreads before a Debt Crisis

The model also matches nicely the dynamics of sovereign bond spreads before a debt crisis. The left panel of Figure 6 presents event windows showing the mean of simulated output and bond spreads up to 12 quarters before default events in the stationary distribution of the model. This plot clearly illustrates the negative correlation between output and bond spreads before a debt crisis. In particular, the spread increases as the country approaches a debt crisis. The average *quarterly* spread increases from 0.7 percent at $t = -12$ to almost 1.5 percent in the quarter before default. At the same time, HP detrended output starts to decline three quarters before default and suffers a sharp drop when default occurs. These features match relatively well the Argentine experience. The right-side panel of Figure 6 shows the HP detrended real GDP and EMBI+ sovereign bond spreads for Argentina from 1994Q1 to 2001 Q4. The data show a relatively stable sovereign spread before 2000 and a sharp increase in 2001, and Argentina also experienced a relatively steady output performance and then a very deep recession starting in 2001. Hence, our model seems capable of generating endogenous output and sovereign spread dynamics consistent with the data.

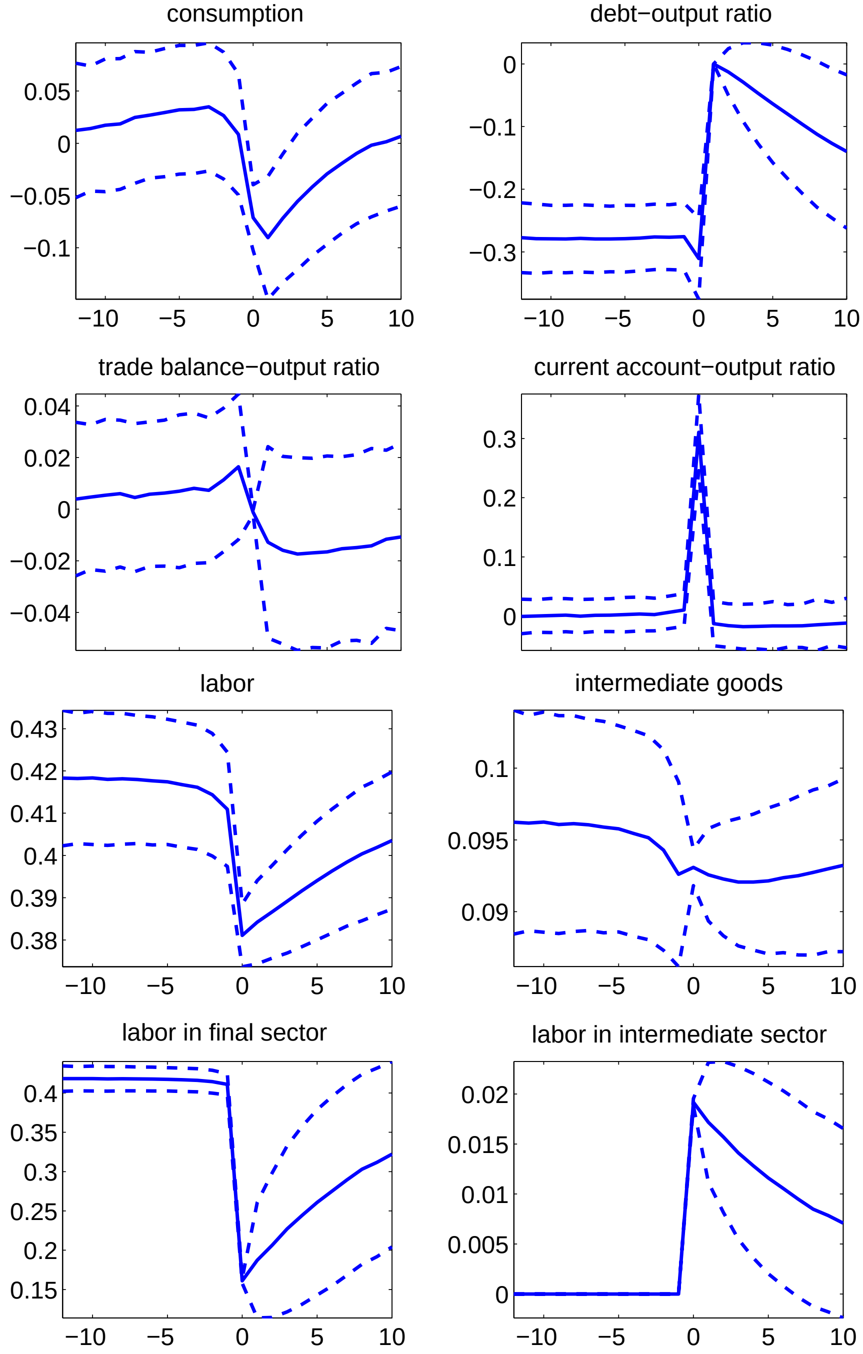


Figure 7: Macro Dynamics around Default Episodes

Figure 7 shows the event windows for the average of the model simulations of consumption, the GDP ratios of the trade balance, current account and debt, as well as labor, intermediate goods, and sectoral labor allocations (along with the corresponding one-standard-error bands). Consumption drops sharply when the government defaults and in the period that follows, and then it recovers following the V-shaped dynamics of GDP. The debt-output ratio is over 26 percent on average before default, and it increases to about 32 percent in the period just before default. The external accounts also experience sharp adjustments around default episodes. In particular, the model generates a sharp reversal in the current account. The country runs a small current account deficit on average, but default, and the loss of credit market access that it entails, produce a large jump of about 30 percentage points of GDP in the current account. Labor and the allocation of intermediate goods also fall sharply when the economy defaults. Moreover, since default triggers a shift from imported to domestic inputs in final goods production, labor is reallocated from the f sector to the m sector.

The sharp declines in GDP, consumption, labor and intermediate goods, together with the large reversal in the current account, indicate that the model yields predictions consistent with the sudden stop phenomenon observed in emerging economies around financial crises. In most of the sudden stops literature, however, the current account reversal is modeled as an exogenous shock, whereas in this model both the current account reversal and the economic collapse are endogenous. Moreover, this endogenous sudden stop is driven by default risk determined by an optimal recursive contract, instead of the ad-hoc collateral constraints emphasized in other models of endogenous sudden stops (see Mendoza (2007)).

3.4 Key Features of the Equilibrium with Default

How does the interaction between endogenous output fluctuations and endogenous default risk affect the quantitative performance of the model? We answer this question by studying the behavior of the value function when the country has access to world financial markets, the sovereign bond pricing function, the saving decision rule, and output. Figure 8 shows these equilibrium functions for high and low TFP shocks as a function of the country's foreign asset position.

The first panel of Figure 8 shows that the value function increases with the asset position for the range of asset positions higher than the value at which default is certain (for asset positions smaller than this value, the value function becomes independent of foreign assets). As the country's debt increases (i.e. assets fall) the value of default can exceed the value of not defaulting. The country is more likely to default if TFP is low because the default option is more attractive. This is because it is more painful to repay the debt in a bad state, while at the same time default does not lead to a high output loss compared to the case with a good TFP shock. The value function also differs as we vary the amount of working capital κ. The value of default increases with working capital because the government transfers the repayment of working capital loans to households when it defaults.

The second panel of Figure 8 shows that sovereign bond prices increase with asset holdings (i.e. decrease with the debt position), reflecting the standard result that default risk premia are higher at higher levels of debt. Moreover, bond prices are higher when the country experiences a good TFP shock, and higher bond prices imply lower default risk premia, lower default probabilities and lower country interest rates. Working capital financing becomes less costly as a result, leading final goods producers to increase demand for foreign inputs and produce more. This feedback from country interest rates to output dynamics also affects the country's incentives to default, reinforcing the reduction in default risk. The opposite is true when the country experiences a bad TFP shock, and this is an important result because it implies that, for any given level of debt before the country is in financial autarky, default is more likely when TFP is low than when it is high (recall that the TFP shock and asset position are the only state variables that determine bond prices in the model).

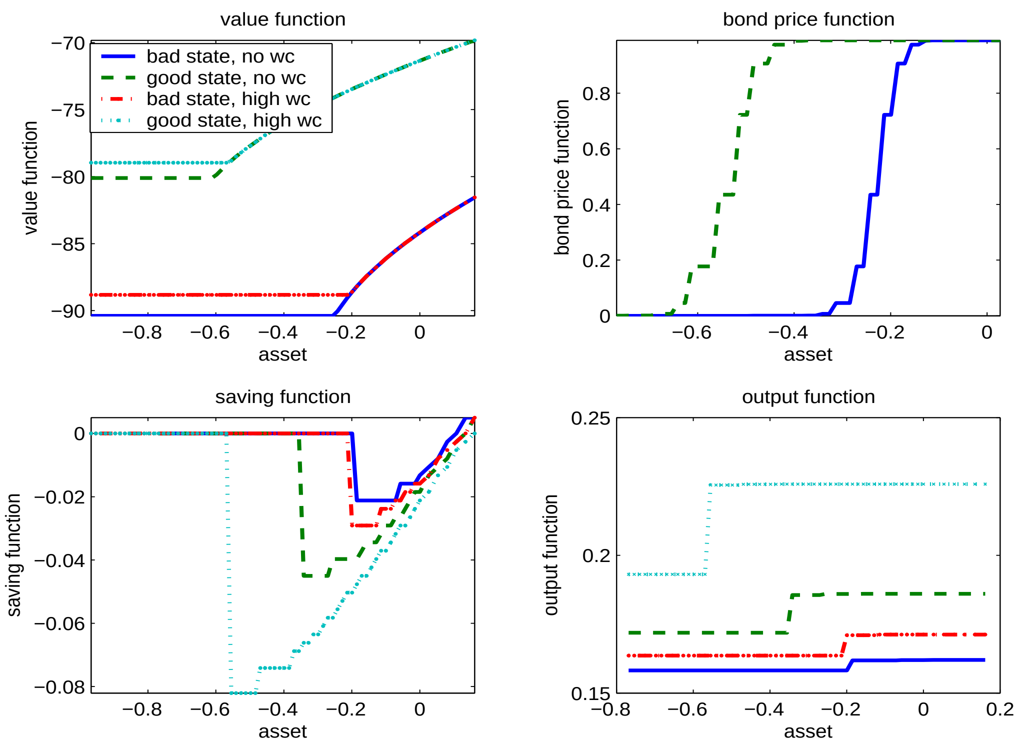


Figure 8: Value Function and Decision Rules

The lower-left panel of Figure 8 shows that the country borrows less (i.e. chooses a higher asset position) when it experiences a low TFP shock. This property of the assets decision rule is reflected in the countercyclical trade balance and the positive correlation between the trade balance and sovereign spreads documented earlier.

Finally, the lower-right panel of Figure 8 shows that the relationship between output and foreign assets follows "almost" a two-step function. The lower step corresponds to the range of high debt in which firms operate with domestic inputs, either because the country defaults or because the interest rate is sufficiently high and the state of productivity is sufficiently low. The higher step pertains to the range of debt positions when the country has access to world credit markets and firms use imported inputs. Output in this region depends on the asset position (so the output plots are not truly two step functions), but the Figure would need a finer scale for the relationship to be visible. In this region, output fluctuates with country risk because the demand for imported intermediate goods is directly affected by country spreads. When the country borrows more, default risk increases and this raises the financing cost of working capital to firms. In response, firms cut demand for imported inputs and output falls. The plot also shows that the size of the output drop at default is larger with the good productivity shock because the cost of default is increasing in TFP, as explained earlier.

4 Sensitivity Analysis

4.1 Working capital

The working capital constraint plays an important role in the quantitative performance of the model. Its relevance can be illustrated by comparing the benchmark results with the results of a simulation that abstracts from working capital (i.e., $\theta = 0$). Without working capital, the output loss at default is invariant to changes in productivity, as in the existing quantitative studies of sovereign debt that assume that income is an exogenous endowment (e.g. Aguiar and Gopinath (2006), Yue (2006)). To keep the results comparable, we introduce an exogenous output loss at default in this variant of our model and calibrate it so as to keep matching the average output loss in default of 13 percent observed in the data, which we used as a calibration target in the benchmark calibration. The other parameters are kept unchanged. The second column of Table 4 presents the simulation results for this no-working-capital case and the third column reproduces the results for the benchmark model.

Table 4: Changes in the Working Capital Constraint

Statistics	No WC	Benchmark $\theta = 0.1$	Larger WC $\theta = 0.2$	Larger WC $\theta = 0.3$
Output loss	12.5%	13.0%	12.8%	11.7%
GDP std. dev.	4.64%	4.72%	4.98%	5.00%
Default probability	0.04%	0.69%	0.81%	0.82%
Corr. between Spreads and GDP	-0.10	-0.48	-0.52	-0.45
Corr. between Spreads and TB	-0.26	0.39	0.36	0.39
Corr. between TB and GDP	-0.28	-0.42	-0.39	-0.41
Debt/GDP	6.91%	26.05%	15.79%	9.59%
Bond spreads std. dev.	0.07%	0.71%	0.76%	0.73%
Average Bond Spreads	0.04%	0.69%	0.81%	0.82%
Trade Balance std. dev.	1.80%	2.88%	2.68%	2.18%

The model without working capital performs much worse in terms of its ability to match all of the important features of the data that the benchmark model mimicked well. The frequency of defaults falls from 0.7 percent to 0.04 percent. The GDP correlations of sovereign spreads and net exports increase sharply. The correlation between spreads and net exports changes from significantly positive at 0.39 to negative at -0.26. The mean debt ratio declines by nearly 20 percentage points of GDP, and the average and standard deviation of country spreads fall by about 70 basis points. These results follow from two important differences in the model without working capital relative to our setup: First, the cost of default becomes independent of the state of nature, and second, bond spreads no longer have a direct impact on production. As a result, debt is not as good a hedging mechanism as in the benchmark model with working capital, making default more painful ex ante in the model without working capital, and thus reducing the average debt ratio. Moreover, the model without working capital cannot reproduce the V-shaped output dynamics that the benchmark model produces (see Figure 4), because it maintains the disconnect between country risk and business cycles.

We have established that removing working capital worsens significantly the quantitative performance of the model. But how sensitive are the results to the value of the working capital requirement beyond the extreme case of $\theta = 0$? To answer this question, we solved the model for $\theta = 0.2$ and 0.3 instead of 0.1 as in the benchmark case. The last two columns of Table 4 show the results for these simulations. The higher working capital requirement reduces sharply the mean debt ratio, despite very small reductions in the size of the output loss at default. In contrast, the variability of GDP, the probability of default, and the mean and standard deviation of spreads all increase as θ rises.

These changes reflect the fact that the higher θ has opposing effects on default incentives and production plans. On one hand, final goods producers are more likely to switch to domestic inputs, since higher θ increases the effective price of imported inputs, and changes in sovereign interest rates have a larger impact on production. These effects amplify the response of output to productivity shocks, making output more volatile. This result is complementary to the finding in Uribe and Yue (2006) showing that the impact of output on country interest rates magnifies business cycle volatility, and the result in Neumeyer and Perri (2005) showing that working capital loans that charge sovereign interest rates also amplify business cycle volatility. On the other hand, default leads to a lower fraction of output loss at default on average because the TFP shock that triggers default is smaller than in the benchmark case with a lower θ. Thus, the output levels before and after default are closer, generating a smaller output loss. At the same time, this lower output cost of default and a higher volatility in GDP make the sovereign exercise the default option more often, increasing the default probability and the volatility of bond spreads, and reducing the mean debt/GDP ratio. The overall quantitative effects of tightening the working capital constraint on the debt/GDP ratio and the default frequency are particularly large, and we get these results even though average sovereign spreads, and hence the average interest rate on working capital, do not deviate sharply from the one-percent risk free rate. [16]

4.2 Costly Labor Reallocation

As we explained earlier, the shift from imported to domestic inputs that occurs when the economy defaults reduces production efficiency because of the costly reallocation of labor away from final goods production. Hence, our results are likely to be sensitive to changes in the sectoral elasticity of labor, as determined by ν, because changes in this elasticity alter the size of the efficiency loss associated with default.

Table 5: Changes in Sectoral Labor Elasticity

Statistics	Lower Elasticity $\nu = 0.39$	Benchmark $\nu = 0.41$	Higher Elasticity $\nu = 0.43$
Output loss	16.1%	13.0%	11.6%
GDP std. dev.	4.71%	4.72%	4.58%
Default probability	0.34%	0.69%	0.92%
Corr. between Spreads and GDP	-0.46	-0.48	-0.35
Corr. between Spreads and TB	0.39	0.39	0.37
Corr. between TB and GDP	-0.44	-0.42	-0.43
Debt/GDP	42.23%	26.05%	9.26%
Bond spreads std. dev.	0.35%	0.71%	2.78%
Average Bond Spreads	0.34%	0.69%	0.92%

Table 5 presents simulation results comparing the benchmark case ($\nu = 0.41$, with an elasticity of substitution between L^f and L^m of -1.7) with cases in which $\nu = 0.39$ (a lower elasticity at -1.64) and $\nu = 0.43$ (a higher elasticity at -1.75). All of the other parameters are the same as in the benchmark calibration. The results show that the value of ν directly affects the fraction of output loss at default, as would be expected. The lower ν increases the loss to 16 percent, while the higher ν reduces it to about 12 percent. The lower ν yields a smaller probability of default and a much higher debt ratio, of 42 percent of GDP. The higher v increases the default probability and lowers the mean debt ratio (to around 9 percent). In addition, the volatility of spreads falls from 2.8 percent with higher ν to 0.71 percent in the benchmark case and 0.35 percent with lower ν. These results have a straightforward interpretation: lower ν increases the cost of default, and the greater default penalty makes the sovereign less likely to default and able to borrow higher amounts on average. Spreads

[16] Neumeyer and Perri (2005) and Uribe and Yue (2006) use average interest rates around 7 percent and set $\theta = 1$, and they find that the working capital constraint is important for business cycle dynamics. Oviedo (2005) also showed that obtaining significant effects of working capital in the small open economy RBC model requires high values of r^* and θ.

are also less volatile because the reduced frequency of defaults reduces the frequency of states with very high spreads. The opposite occurs when ν is higher.

Changes in ν also affect business cycle comovements but the effects are much smaller than those noted above, and they are largely non-monotonic. Lower and higher ν produce a higher correlation between GDP and spreads than in the benchmark case, but the correlation is always negative. Similarly, the correlation between net exports and GDP is always negative, but it is higher in the benchmark case than in the scenarios with lower and higher ν. The variability of GDP is almost the same in the baseline as with the lower ν, but it falls to 4.58 percent with higher ν.

The distribution of defaults across "bad times" and "good times" also changes with the value of ν. In particular, the higher value of ν shifts the distribution away from the states with larger output drops below trend. At a quarterly frequency, the model with $\nu = 0.43$ continues to generate 100 percent of the default episodes when GDP is below trend, as in the benchmark, but the fraction of defaults that occur when output is more that two standard deviations below trend falls from 76 percent in the benchmark to 50 percent. Aggregating to an annual frequency, we find that with $\nu = 0.43$ half of the defaults occur with output above trend, and no defaults occur with GDP more than two standard deviations below trend (compared with 6 percent of defaults in the benchmark case). The correlation between GDP and default is about -0.12 at both quarterly and annual frequencies.

4.3 Intermediate Inputs Share

In the benchmark calibration, we set the share of intermediate goods in final goods production α_m at 30 percent, and noted that this value is lower than the typical 40 percent share of total intermediate goods to gross output in the data but higher than the 12-15 percent share of imported intermediate goods. Hence, it is important to study how variations in the value of α_m affect our results.

Table 6 reports results for $\alpha_m = 0.25$ and 0.35, as well as the benchmark case with $\alpha_m = 0.3$. Clearly, changes in the value of α_m have important quantitative implications: Higher (lower) α_m reduces (increases) sharply the output cost of default and the mean debt ratio, while it increases (reduces) significantly the frequency of default and the standard deviation of spreads. By contrast, the volatility of output and the correlations shown in the Table are relatively unaffected.

These results are similar to what we obtained in Table 5 for changes in the elasticity of sectoral labor reallocation. This is important because it shows that we can trade off a higher (lower) value of α_m for a lower (higher) value of ν and still obtain results similar to those of the benchmark case for the output loss at default, the mean debt ratio, and the mean and standard deviation of spreads without affecting significantly the other moments.

Table 6: Changes in Share of Intermediate Goods

Statistics	Lower Interm. Share $\alpha_m = 0.25$	Benchmark $\alpha_m = 0.3$	Higher Interm. Share $\alpha_m = 0.35$
Output loss	16.1%	13.0%	7.03%
GDP std. dev.	4.45%	4.72%	4.81%
Default probability	0.32%	0.69%	2.68%
Corr. between Spreads and GDP	-0.36	-0.48	-0.42
Corr. between Spreads and TB	0.32	0.39	0.28
Corr. between TB and GDP	-0.40	-0.42	-0.42
Debt/GDP	48.67%	26.05%	7.09%
Bond spreads std. dev.	0.29%	0.71%	8.57%
Average Bond Spreads	0.32%	0.69%	2.68%

As in the case of higher ν, the model with higher α_m shifts the distribution of defaults away from the states with the lowest deviations from trend in GDP. At a quarterly frequency, the model with $\alpha_m = 0.35$ generates nearly all default episodes when GDP is below trend, but now there is a small fraction of defaults of about 0.1 percent that occur with output above trend. Thus, the model can generate defaults in good times even at the quarterly frequency. The fraction of defaults that occur when output is more that two standard deviations below trend falls from 76 percent in the benchmark to 31 percent. Aggregating to an annual frequency, $\alpha_m = 0.35$ yields almost 26 percent of defaults with output above trend, 74 percent with output below trend, and 3.4 percent of defaults occur with GDP more than two standard deviations below trend. The correlation between GDP and default is -0.22 (-0.27) at the quarterly (annual frequency).

5 Conclusions

This paper proposed a model of strategic sovereign default with endogenous output dynamics and examined its quantitative predictions. In the model, profit-maximizing producers of final goods choose optimally between imported inputs that require foreign working capital financing, or domestic inputs that do not require credit but reduce the efficiency of production via costly reallocation of labor from final goods production to production of intermediate goods. Lenders charge the same default risk premium on working capital loans as on sovereign debt because the sovereign diverts the repayment of working capital loans when the country defaults. In line with this argument, we provided evidence showing that the two interest rates are strongly correlated in the data, and that in sovereign defaults since the 1980s Debt Crisis we often observe governments taking over the foreign obligations of private firms.

The model is consistent with three key stylized facts of sovereign debt: (1) the V-shaped dynamics of output around default episodes, (2) the negative correlation between interest rates on sovereign debt and output, and (3) high debt-output ratios on average and when defaults take place. The model also replicates the observed countercyclical dynamics of net exports, the positive correlation between country spreads and GDP, and the variability of private consumption, and it is calibrated to be consistent with observed default frequencies.

The model produces endogenous output costs of default that are increasing in the state of productivity. This result follows from the fact that the financing cost of working capital when default occurs rises too much for firms to find it profitable to use imported inputs, and hence they optimally switch to domestic inputs and suffer the corresponding efficiency loss. In turn, this efficiency loss is larger the higher TFP was before the switch. This increasing endogenous output cost of default is consistent with the shape of exogenous output costs that Arellano (2007) identified as necessary in order to obtain default incentives that trigger default in bad states of nature, at non-negligible debt ratios and at realistic spreads (or default frequencies). However, the endogenous feedback between production and default in our model produces a mean debt ratio four times larger than in Arellano's endowment economy model.

Our results also show that the model can provide an explanation for the seemingly large contribution of productivity shocks to output collapses during financial crises. In particular, we showed that a standard Solow residual overestimates significantly the contribution of true TFP to the collapse of output when the economy defaults, because it masks the efficiency loss due to costly labor reallocation and reduced usage of intermediate goods as a decline in TFP.

Three features of the model are critical for the results: imported inputs require working capital, the government diverts the firms' working capital repayment when it defaults, and production with domestic inputs entails an efficiency loss. Without the first two features, output would not respond to changes in country risk. On the other hand, the model would also fail if we rely "too much" on these two features: If exclusion from world credit markets implies that firms cannot buy foreign inputs and there are no domestic inputs available, the output collapse and the associated cost of default would be unrealistically large (infinitely large if 100 percent of the cost of imported inputs requires payment in advance). In reality, firms in emerging economies facing financial crisis substitute foreign inputs with high financing costs for domestic inputs that can be employed at permissible financial terms, and/or look for alternative forms of credit, including inter-enterprise credit and internal financing using retained earnings or redirecting capital expenditures. The efficiency loss is also critical. Without it the working capital channel would not produce a sharp and sudden drop in output during periods of financial turmoil.

Our findings suggest that the model we proposed can provide a solution to the disconnect between sovereign debt models (which rely on exogenous output dynamics with particular properties to explain the stylized facts of sovereign debt) and models of emerging markets' business cycles (which assume an exogenous financing cost of working capital set to match the interest rate on sovereign debt). We acknowledge, however, that the linkages between sovereign default and private sector borrowers, and the mechanisms by which default induces economy-wide efficiency losses, should be the subject of further research. For instance, the studies by Cuadra and Sapriza (2008) and D'Erasmo (2008) show that political uncertainty can also generate high debt ratios at observed default frequencies in models of default with exogenous output dynamics. This suggests that introducing a mechanism to link political uncertainty to private sector decisions in a model with sovereign risk can be a promising line of research. Similarly, the findings of Bi (2008a and 2008b) on debt dilution effects and dynamic renegotiation in endowment economy models suggest that adding these features to default models with endogenous output dynamics can also be important. Finally, results obtained by Arellano (2007), Lizarazo (2005) and Volkan (2008) suggest that adding risk-averse foreign lenders can also contribute to produce higher debt rations and break the one-to-one link between spreads and default probabilities, so that bond spreads include an additional risk premium and can get closer to the data.

References

[1] Aguiar, Mark and Gita Gopinath, 2006, "Defaultable Debt, Interest Rates and the Current Account,", Journal of International Economics.

[2] Arellano, Cristina, 2007, "Default Risk and Income Fluctuations in Emerging Economies," American Economic Review, forthcoming.

[3] Arellano, Cristina and Narayana Kocherlakota, 2007, "Internal Debt Crises and Sovereign Defaults," Manuscript, University of Minnesota.

[4] Arteta, Carlos and Galina Hale, 2007, "Sovereign Debt Crises and Credit to the Private Sector," Journal of International Economics, Vol 74 (1), 53-69.

[5] Bai, Yan and Jing Zhang, 2006, "Financial Globalization and International Risk Sharing," Manuscript, University of Michigan and Arizona State University.

[6] Bi, Ran, 2008a, "Debt Dilution and the Maturity Structure of Sovereign Bonds," Ph.D. Dissertation, Department of Economics, University of Maryland.

[7] Bi, Ran, 2008b, "Beneficial Delays in Debt Restructuring Negotiations," Ph.D. Dissertation, Department of Economics, University of Maryland.

[8] Benjamin, David M. and Felipe Meza, 2007. "Total Factor Productivity and Labor Reallocation: the Case of the Korean 1997 Crisis," mimeo, Universidad Carlos III de Madrid.

[9] Boughton, James M., 2001, "Silent Revolution: The International Monetary Fund 1979-1989," Ch. 9, IMF Publication.

[10] Calvo, Guillermo A., Alejandro Izquierdo and Ernesto Talvi, 2006, "Phoenix Miracles in Emerging Markets: Recovering Without Credit form Systematic Financial Crises," NBER Working Paper 12101.

[11] Cuadra, Gabriel and Horacio Sapriza, 2007, "Fiscal Policy and Default Risk in Emerging Markets," Bank of Mexico Working Paper.

[12] Cuadra, Gabriel and Horacio Sapriza, 2008, "Sovereign Default, Interest Rates and Political Uncertainty in Emerging markets," Bank of Mexico Working Paper.

[13] D'Erasmo, Pablo, 2008, "Government Reputation and Debt Repayment in Emerging Economies," Ph.D. Dissertation, Department of Economics, University of Texas-Austin.

[14] Eaton, Jonathan and Mark Gersovitz, 1981, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," Review of Economic Studies, Vol 48(2), 289-309.

[15] Edwards, Sebastian, 1984, "LDC Foreign Borrowing and Default Risk: An Empirical Investigation, 1976-80," American Economic Review, Vol 74 (4), 726-734.

[16] Finn, Mary G., 1995, "Variance Properties of Solow's Productivity Residual and Their Cyclical Implications," Journal of Economics Dynamics and Control, 19, 1249-1281.

[17] Gelos, Gaston R., Ratna Sahay, Guido Sandleris, 2003, "Sovereign Borrowing by Developing Countries: What Determines Market Access?" IMF Working Paper.

[18] Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon, 2007, "Currency Choice and Exchange Rate Pass-through," Manuscript, Harvard University.

[19] Greenwood, Jeremey, Gregory W. Huffman and Zvi Hercowitz, 1988, "Investment, Capacity Utilization, and the Real Business Cycle," American Economic Association, vol. 78(3), 402-417.

[20] Hall, Robert E., 2007, "Sources and Mechanisms of Cyclical Fluctuations in the Labor Market," Manuscript, Stanford University.

[21] Levy-Yeyati, Eduardo and Ugo Panizza, 2006, "The Elusive Cost of Sovereign Default," CIF Working Paper No. 11/2006.

[22] Lizarazo, Sandra Valentina, 2005, "Default Risk and Risk Averse International Investors," mimeo, Department of Economics, ITAM, Mexico.

[23] Mendoza, Enrique G., 1991. "Real Business Cycles in a Small Open Economy". American Economic Review Vol 81(4), 797–818.

[24] Mendoza, Enrique G., 2007, "Endogenous Sudden Stops in a Business Cycle Model with Collateral Constraints: A Fisherian Deflation of Tobin's Q," NBER Working Paper No. W12564.

[25] Meza, Felipe and Erwan Quintin, 2006, "Financial Crises and Total Factor Productivity," mimeo, Universidad Carlos III de Madrid.

[26] Neumeyer, Pablo. A. and Fabrizio Perri, 2005. "Business Cycles in Emerging Economies: The Role of Interest Rates," Journal of Monetary Economics, Vol 52, Issue 2, 345-380.

[27] Oviedo, Marcelo P., 2005, "World Interest Rate, Business Cycles, and Financial Intermediation in Small Open Economies," Manuscript, Iowa State University.

[28] Reinhart Carmen. M., Kenneth S. Rogoff and M. A. Savastano, 2003, "Debt Intolerance," NBER Working Paper No. 9908.

[29] Tomz, Michael and Mark Wright, 2007, "Do Countries Default in 'Bad Times'?", mimeo, Department of Economics, UCLA

[30] Uribe, Martin and Vivian Zhanwei Yue, 2006. "Country Spreads and Emerging Countries: Who Drives Whom?" Journal of International Economics, vol 69, 6-36.

[31] Volkan, Engin, 2008, "Sovereign Default Risk, Risk-Averse Investors and Financial Contagion," PhD Dissertation, Department of Economics, University of Southern California.

[32] Yue, Vivian Zhanwei, 2006. "Sovereign Default and Debt Renegotiation," Manuscript, New York University.

Appendix

PROOF of THEOREM 1

Given a productivity shock ε and level of working capital loan κ, the utility from defaulting $v^d(\kappa, \varepsilon')$ is independent of b. We can also show that the utility from not defaulting $v^{nd}(b, \varepsilon')$ is increasing in b_{t+1}. Therefore, if $V\left(b^1, \kappa, \varepsilon'\right) = v^d(\kappa, \varepsilon')$, then it must be the case that $V\left(b^0, \kappa, \varepsilon'\right) = v^d(\kappa, \varepsilon')$. Hence, any ε' that belongs in $D\left(b^1, \varepsilon\right)$ must also belong in $D\left(b^0, \varepsilon\right)$.

Let $d^*(b, \varepsilon')$ be the equilibrium default decision rule. The equilibrium default probability is then given by

$$p(b, \varepsilon) = \int d^*\left(b, \varepsilon'\right) d\mu\left(\varepsilon' | \varepsilon\right)$$

From $D\left(b^1, \varepsilon'\right) \subseteq D\left(b^0, \varepsilon'\right)$, if $d^*\left(b^1, \varepsilon'\right) = 1$, then $d^*\left(b^0, \varepsilon'\right) = 1$. Therefore,

$$p\left(b^0, \varepsilon\right) \geq p\left(b^1, \varepsilon\right)$$

PROOF of THEOREM 2

From Theorem 1, given a productivity shock ε and level of working capital loan κ, for $b^0 < b^1 \leq 0$, $p^*\left(b^0, \varepsilon\right) \geq p^*\left(b^1, \varepsilon\right)$. The equilibrium bond price is given by

$$q\left(b', \varepsilon\right) = \frac{1 - p\left(b', \varepsilon\right)}{1 + r}$$

Hence, using Theorem 1, we obtain that:

$$q\left(b^0, \varepsilon\right) \leq q\left(b^1, \varepsilon\right)$$