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Control Strategy for a Dual-Arm Maneuverable Space Robot

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ABSTRACT: A simple strategy for the attitude control and arm coordination of a maneuverable space robot with dual arms is proposed. The basic task for the robot consists of the placement of marked rigid solid objects with specified pairs of gripping points and a specified direction of approach for gripping. The strategy consists of three phases each of which involves only elementary rotational and translational collision-free maneuvers of the robot body. Control laws for these elementary maneuvers are derived by using a body-referenced dynamic model of the dual-arm robot.

1. INTRODUCTION

In the design of orbital maneuvering vehicles (OMV) with multiple robot arms for spacecraft servicing, space station assembly and maintenance, and satellite retrieval, it is required to develop on-board feedback control systems for vehicle attitude and arm coordination [1]-[3]. In contradistinction with earth-based fixed robots, the control systems for OMV robots must take into consideration the interaction between the vehicle attitude and robot arm motions.

In this paper, we propose a simple strategy for attitude control and arm coordination of an OMV with dual robot arms whose basic task involves the placement of marked rigid solid objects. We begin with a description of the robot and its basic task to be performed. This is followed by a discussion of the basic requirements and constraints associated with the control problem. Then, the proposed control strategy for performing the basic task is presented. The paper concludes with a brief description of the work in progress.

2. ROBOT AND TASK DESCRIPTIONS

Figure 1 shows the basic configuration of the OMV dual-arm robot under consideration. For simplicity, the main frame of the OMV is represented by a maneuverable rigid body which provides a base for the robot arms. We assume that the arms have only rotary joints and their motions with respect to the base are planar.

The basic task for the OMV robot consists of placement of marked rigid objects. By a "marked object", we mean an object having a pair of specified gripping points from which the object can be grasped by the end-effectors of the dual-arm robot. Moreover, the object has a single specified direction of approach for dual-arm gripping. The gripping points are marked so that they can be viewed by a vision system. Here, we do not consider the problem of determining the optimal gripping points and direction of approach for an arbitrary shaped rigid object based on some specified criterion. To fix ideas, we shall discuss only briefly the foregoing problem for a slender rigid rod with a uniform rectangular cross-section and length L. Evidently, for such a rod, it is desirable, in most situations, to choose the gripping points p_g which are symmetrically located about the center of mass along the rod. Constraints on the admissible locations of p_g may be imposed by considering the end-effector size and the mode of gripping. Figure 2 shows two different modes for gripping the rod by a planar dual-arm robot. In the first mode, a single direction of approach for both end-effectors is specified. In the second mode, the end-effectors may approach the rod from opposite directions. The choice of the gripping mode should depend on the placement objective and gripping stability (i.e. small offset in the relative orientation between the end-effectors and the object during the approach does not result in the loss of ability to grasp the object).

3. BASIC REQUIREMENT AND CONSTRAINTS

Before discussing the problem of deriving suitable control strategies for the OMV robot to perform the basic task, we first consider the basic requirements and physical constraints associated with the problem.

Let $\Sigma_o(t)$, $\Sigma_{Ji}(t)$, $\Sigma_{Lj}(t)$ and $\Sigma_B(t)$ denote respectively the compact connected spatial domains in the Euclidean space $\mathbb{R}^3$ occupied by the marked object, i-th joint, j-th link and the OMV base at time t. Their boundaries are denoted by $\partial\Sigma_s(t)$, s = o,Ji,Lj,B. The spatial domain of the entire robot at time t is denoted by $\Sigma_R(t) = \bigcup_i(\Sigma_{Ji}(t)\cup\Sigma_{Li}(t))$. We assume that $\Sigma_o(t)$ has a pair of specified gripping points $\underline{p}_g^1(t)$ and $\underline{p}_g^2(t)$ on $\partial\Sigma_o(t)$ which are time-invariant with respect to any fixed body-frame of $\Sigma_o(t)$. We assume that the line segment $L_o(t) = co(\{\underline{p}_g^1(t),\underline{p}_g^2(t)\})$ (the convex hull of the set $\{\cdot\}$) lies in a plane $\Pi_o(t)$ with normal $\underline{n}_o(t)$ corresponding to a specified direction of approach for gripping by the end-effectors of both robot arms.

Since the OMV is to be an autonomous or self-contained system, it is natural to introduce a body coordinate system C_B which serves as the basic reference frame for the arm motions and the vision system. For convenience, the origin of C_B is fixed at the root of one of the arms. Let $\{\underline{e}_x(t),\underline{e}_y(t),\underline{e}_z(t)\}$ denote the time-dependent basis $\mathcal{B}_t$ for C_B. We align $\underline{e}_z(t)$ with the axis of the first rotary joint of arm 1 (see Fig.1).

Let $\underline{p}_R^1(t)$ and $\underline{p}_R^2(t)$ denote the base points O and O' of links 1 and 1' at time t respectively. The line segment $L_R(t) = co(\{\underline{p}_R^1(t),\underline{p}_R^2(t)\})$ and normal $\underline{n}_R(t) = \underline{e}_y(t)$ define a plane $\Pi_R(t)$, where $\underline{n}_R(t)$ specifies the heading of the OMV robot at time t. The position of the end-effector of arm i (may be taken as the tip of the second link of arm i) is denoted by $\underline{p}_E^i(t)$, and the deviations $\underline{p}_E^i(t) - \underline{p}_g^i(t)$ and $\underline{n}_R(t) - \underline{n}_o(t)$ by $\Delta\underline{p}^i(t)$ and $\Delta\underline{n}(t)$ respectively.

Now, the basic requirements and constraints associated with the control of OMV robot can be stated as follows:

(a) Before the end-effectors are in contact with the specified gripping points of the object, any OMV maneuvers and arm movements must be collision-free. This implies that

(i) $\partial\Sigma_R(t)\cap\partial\Sigma_o(t) = \emptyset$ (no robot-object collisions);

(ii) $[\bigcup_{i=1,2}(\partial\Sigma_{Ji}(t)\cup\partial\Sigma_{Li}(t))]\cap[\bigcup_{j=1',2'}(\partial\Sigma_{Jj}(t)\cup\partial\Sigma_{Lj}(t))] = \emptyset$ (no arm-arm collisions);

(iii) $\partial\Sigma_{Li}(t)\cap\partial\Sigma_B(t) = \emptyset$ and $\partial\Sigma_{Ji}(t)\cap\partial\Sigma_B(t) = \emptyset$, j=2,2' (no second link(joint)-base collisions)

at any time $t\in[0,t_1[$, where t_1 corresponds to the first time when $\Delta\underline{p}^i(t_1) = \underline{0}$, i=1,2 and $\Delta\underline{n}(t_1) = \underline{0}$ (or $\|\Delta\underline{n}(t_1)\| + \sum_i\|\Delta\underline{p}^i(t_1)\|\leq\varepsilon$, a given positive number). The foregoing conditions may be relaxed by allowing point contacts with zero velocity. In the case where a specified clearance between any two components of the robot must be maintained, we may enclose each member by a boundary layer with prescribed thickness, and impose conditions (i)-(iii) to the outside boundary of the layers.

(b) Each end-effector should tend to its designated gripping point in a smooth non-oscillatory manner during its final approach. This can be fulfilled by requiring $\|\Delta\underline{p}^i(\cdot)\|$ and $\|\Delta\underline{n}(\cdot)\|$ to be smooth strictly monotone decreasing functions of t over some subinterval $[t',t_1]\subset[0,t_1]$, and $\|\Delta\underline{p}^i(t_1)\|\leq\varepsilon_i$, i=1,2 and $\|\Delta\underline{n}(t_1)\|\leq\varepsilon_o$, where ε_o, ε_1 and ε_2 are specified nonnegative numbers. To ensure acceptable relative velocities between the end-effectors and the gripping points of the object when they are within the gripping range, additional velocity constraints: $\|\Delta\dot{\underline{p}}^i(t)\|\leq\tilde{\varepsilon}_i$, i=1,2 and $\|\Delta\dot{\underline{n}}(t_1)\|\leq\tilde{\varepsilon}_o$ may be introduced.

(c) To achieve complete autonomy of the OMV robot, the control strategies or control laws should depend only on on-board sensor data. Moreover, they should be sufficiently simple so as to permit on-board real-time implementation.

Evidently, the incorporation of the foregoing requirements and constraints into the formulation of any control problem leads to formidable difficulties. A basic difficulty is that the characterization of the class of controls which generate the collision-free maneuvers and monotone approach is not readily obtainable. In what follows, we propose a simple approach which bypasses the abovementioned difficulty.

4. PROPOSED STRATEGY

The basic idea is to decompose the robot control problem into three phases:

(P1) Alignment Phase: The objective is to maneuver the OMV so that its heading $\underline{n}_R$ is aligned with the object's gripping direction $\underline{n}_o$. Moreover, the arms are prepositioned to achieve the required end-effector orientation and position so that the object can be grasped by a subsequent straight-line translational motion of the OMV. Since it is difficult to achieve collision-free OMV maneuvers and arm movements when the object is close to the OMV robot, we propose to move the OMV sufficiently far away from the object before initiating any alignment maneuver and arm prepositioning.

(P2) Acquisition Phase: With the arms' joint angles locked in the preset values, the OMV moves along a collision-free straight-line path to rendez-vous with the object in a monotone manner. The attitude and translational motion control systems at the OMV base maintain the deviations $\|\Delta\underline{n}(t)\|,\|\Delta\dot{\underline{n}}(t)\|,\|\Delta\underline{p}^i(t)\|,\|\Delta\dot{\underline{p}}^i(t)\|$, i=1,2 within the acceptable values at all times during the rendez-vous.

(P3) Task Phase: After grasping the object by means of the end-effectors, the joint angles are again locked. Then the OMV moves to the required destination and places the object there with the specified orientation. Finally, the OMV backs away from the object along a straight-line path.

The choice of straight-line paths for translational motions is motivated from the fact that complex maneuvers of the OMV in space should be avoided, since such maneuvers could be catastrophic in case of control system failure. The proposed locking of all joints during any base attitude alignment and translational maneuvers avoids the possibility of undesirable arm motions induced by the inertial forces and moments.

In what follows, we shall present a control strategy for each phase. For simplicity, only the case with planar motion will be considered here.

4.1 Maneuvering Strategy for Alignment and Acquisition Phases

Let $\underline{\theta} = (\theta_1,\theta_2,\theta_{1'},\theta_{2'},\theta_o)$ and $\Sigma_R(\underline{\theta},\underline{r}_{co})\subset\mathbb{R}^2$ denote the spatial domain of the OMV robot corresponding to a given set of angles $\underline{\theta}$ and a specified $\underline{r}_{co}$ (position of the centroid of the OMV base relative to the inertial frame). Let $\Gamma(\underline{r}_{co}) = \bigcup_{\underline{\theta}\in\Omega_{ad}}\Sigma_R(\underline{\theta},\underline{r}_{co})$ which corresponds to the set of all points in $\mathbb{R}^2$ swept out by the robot body as $\underline{\Theta}$ varies through its admissible values specified by the set Ω_{ad} while keeping $\underline{r}_{co}$ stationary.

To simplify the ensuing development, we assume that the object Σ_o is stationary with respect to the inertial frame. If the OMV robot and the object are sufficiently close to each other initially such that $\Gamma(\underline{r}_{co}) \cap \Sigma_o \neq \emptyset$, then the objective is to move the robot to a new position specified by $\underline{r}^*_{co}$ at which $\Gamma(\underline{r}^*_{co}) \cap \Sigma_o = \emptyset$ and $\inf\{\|\underline{x} - \underline{x}'\| : \underline{x} \in \Gamma(\underline{r}_{co}), \underline{x}' \in \Sigma_o\} \geq \epsilon_A > 0$, where ϵ_A is a given nonnegative number. To avoid a complex collision-free maneuvering problem, we restrict the maneuvers to straight-line translations without robot-body rotations and with locked joint angles. Now, we give a simple collision-free maneuvering strategy by representing the robot arms Σ_{A1} and Σ_{A2} by line segments and the base Σ_B by a closed rectangle as shown in Fig.3. Moreover, the object Σ_o is assumed to be a compact convex set with interior points (If Σ_o is not convex, we consider $co(\Sigma_o)$ instead of Σ_o). We also impose the following geometric constraints:

(C1) Link-length Constraints: $\ell_1 > \ell_2$, $\ell_{1'} > \ell_{2'}$, and $\ell_1 + \ell_2 < w$, where w is the distance between the rotation axes of joints 1 and 1'. The latter condition implies that a collision between links 1 and 1' is impossible.

(C2) Joint-angle Constraints: Let $\bar{\epsilon}$ be a specified small positive angle and

$$\hat{\theta}_1 = \cos^{-1}(\ell_2/\ell_1), \quad \hat{\theta}_{1'} = \cos^{-1}(\ell_{2'}/\ell_{1'}), \quad \check{\theta}_1 = 2\tan^{-1}\left(\frac{\gamma}{s - \ell_2}\right), \quad \check{\theta}_{1'} = 2\tan^{-1}\left(\frac{\gamma'}{s' - \ell_{2'}}\right), \tag{1}$$

where

$$s = (\ell_1 + \ell_2 + h)/2, \quad s' = (\ell_{1'} + \ell_{2'} + h)/2, \quad \gamma = [(s-\ell_1)(s-\ell_2)(s-h)/s]^{\frac{1}{2}}, \quad \gamma' = [(s'-\ell_{1'})(s'-\ell_{2'})(s'-h)/s']^{\frac{1}{2}}. \tag{2}$$

We require:

$$\check{\theta}_1 - \bar{\epsilon} \leq \theta_1 \leq \pi + \hat{\theta}_1 + \bar{\epsilon}, \qquad -\check{\theta}_{1'} + \bar{\epsilon} \leq \theta_{1'} \leq \pi - \hat{\theta}_{1'} - \bar{\epsilon}, \tag{3}$$

and

$$|\theta_i| \leq \pi - \bar{\epsilon}, \quad i = 2, 2'. \tag{4}$$

Condition (3) along with constraint (C1) imply that a collision between arm 1 or arm 2 with the base is impossible. Condition (4) avoids the possibility of link 2 (link 2') folding back onto link 1 (link 1').

Let $\underline{\Theta}^o$ and $\underline{r}^o$ denote the initial set of angles $(\theta_1^o, \theta_2^o, \theta_{1'}^o, \theta_{2'}^o, \theta_o^o)$ and the position of the base-centroid respectively. Evidently, $co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$ is a closed convex polygon. The straight-line collision-free maneuvering problem can be stated as follows: Given $\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co})$ such that $\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}) \cap \Sigma_o = \emptyset$, find a direction vector $\underline{n}$ such that $\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co} + \alpha\underline{n}) \cap \Sigma_o = \emptyset$ for all real numbers $\alpha \geq 0$.

A solution to this problem is given by the following maneuvering strategy:

Case 1: If Σ_o and $co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$ have no common interior points, then we move the robot along a straight-line path in the direction $\underline{n}$ until

$$\inf\{\|\underline{x} - \underline{x}'\| : \underline{x} \in \Gamma(\underline{r}^*_{co}), \underline{x}' \in \Sigma_o\} = \epsilon_A, \tag{5}$$

where $\underline{n}$ is the normal (directed toward $co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$ of any line separating the convex sets Σ_o and $co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$, and $\underline{r}^*_{co}$ is the new position of the base-centroid (see Fig.4a).

Case 2: If Σ_o and $co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$ have common interior points, but $\Sigma_o \not\subset co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$, then there exists an "exit edge" E of the polygon $co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$ (i.e E is an edge which does not correspond to any link or edge of the base) such that $E \cap \Sigma_o \neq \emptyset$. Let $L(E)$ denote the line containing E, and $\mathbb{P}_{\underline{\nu}}$ the projection operator from $\mathbb{R}^2$ onto $L(E)$ in the direction $\underline{\nu}$. The maneuvering strategy for this case is to move the robot along a straight-line path in the direction $-\tilde{\underline{\nu}}$ until condition (5) is satisfied, where $\tilde{\underline{\nu}}$ is any direction such that

$$\mathbb{P}_{\tilde{\underline{\nu}}}(\Sigma_o \cap co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co})) \subset \mathrm{int}(E), \tag{6}$$

where $\mathrm{int}(E)$ denotes the interior of E (see Fig.4b). In general, there may exist a cone of directions $\tilde{\underline{\nu}}$ which satisfy (6).

Case 3: Suppose that $\Sigma_o \subset co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$. Let E be the exit edge of $co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$ associated with Σ_o (i.e. the exit edge associated with the closed domain $D \subset co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$ containing Σ_o, whose boundary ∂D is composed of E and one or more links and base-edges) (see Fig.4c). Suppose there exists a direction $\tilde{\underline{\nu}}$ such that

$$\mathbb{P}_{\tilde{\underline{\nu}}}(\Sigma_o) \subset \mathrm{int}(E) \tag{7}$$

and

$$co(\Sigma_o \cup \mathbb{P}_{\tilde{\underline{\nu}}}(\Sigma_o)) \cap \Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}) = \emptyset \tag{8}$$

are satisfied. Then we move the robot along a straight-line path in the direction $-\tilde{\underline{\nu}}$ until condition (5) is satisfied. Here, condition (8) implies that Σ_o can be projected into $L(E)$ in the direction $\tilde{\underline{\nu}}$ without any obstructions from any part of the robot.

Case 4: Suppose that $\Sigma_o \subset co(\Sigma_R(\underline{\Theta}^o, \underline{r}^o_{co}))$ and there does not exist a direction $\tilde{\underline{\nu}}$ such that (7) and (8) are satisfied simultaneously (see Fig.4d). Then it is impossible to achieve a collision-free straight-line maneuver for the given set of angles $\underline{\Theta}^o = (\theta_1^o, \theta_2^o, \theta_{1'}^o, \theta_{2'}^o, \theta_o^o)$. In this case, it is necessary to change $\underline{\Theta}$ until both conditions (7) and (8) are satisfied for some direction $\tilde{\underline{\nu}}$. We propose to accomplish this by altering the joint angles only without introducing a base rotation. The joint angles should be adjusted such that the sequence of domains $\{D_j, j=1,2,\ldots\}$ (defined in Case 3) generated by a sequence of joint-angle settings satisfy $D_i \subset D_{i+1}$, $i=1,2,\ldots$.

Having moved the robot sufficiently far away from the object, the next step is to perform a rotational maneuver to align the OMV heading $\underline{n}_R$ with the object's gripping direction $\underline{n}_o$ while holding all the joint angles at their initial values (see Fig.5a). This is followed by a translational maneuver along a line parallel to the line containing the gripping points $\underline{p}_g^1$ and $\underline{p}_g^2$ until a reference point on the robot base is aligned with a corresponding point on the object (see Figs 5b and 5c). Finally, the joint angles are adjusted so that the object can be grasped by the end-effectors after a straight-line translational motion of the OMV while holding the attitude of the OMV base stationary (see Fig.5d). This completes the alignment phase.

In the acquisition phase, the control system guides the OMV along a collision-free striaght-line path toward the object while keeping all the joint angles at their preset values. The translation control law should have the property that $\|\Delta \underline{p}^i(t)\|$, $i=1,2$ decrease toward zero monotonically with t as $t \to t_1$. Since the elementary maneuvers in both the alignment and acquisition phases require controlling at most two variables at a time, the control problem is greatly simplified. This aspect will be discussed in the following section.

4.2 Control Laws for Elementary Maneuvers

To derive suitable control laws for the elementary maneuvers, it is necessary to obtain first the equations of motion for the OMV robot. Here, we use the Newton-Euler formulation to obtain the equations of motion for each link. The OMV base is regarded as a rigid link between the two arms. This approach is adopted here instead of the usual Lagrangian approach because it reveals the interacting forces and moments between the arms and the OMV base.

Let the links of arm 1 (resp. arm 2) be labelled by 1 and 2 (resp. 1' and 2'). The OMV base is labelled as link 0 (see Fig.3). Adopting the notations of Asada and Slotine [3], the equations of motion for each link are given by

Link 1:

$$\underline{f}_{0,1} - \underline{f}_{1,2} - m_1 \dot{\underline{v}}_{c1} = \underline{0}, \tag{9a}$$

$$\underline{N}_{0,1} - \underline{N}_{1,2} + \underline{r}_{1,c1} \times \underline{f}_{1,2} - \underline{r}_{0,c1} \times \underline{f}_{0,1} - I_1 \dot{\underline{\omega}}_1 - \underline{\omega}_1 \times (I_1 \underline{\omega}_1) = \underline{0}; \tag{9b}$$

Link 2:

$$\underline{f}_{1,2} - m_2 \dot{\underline{v}}_{c2} = \underline{0}, \tag{10a}$$

$$\underline{N}_{1,2} - \underline{r}_{1,c2} \times \underline{f}_{1,2} - I_2 \dot{\underline{\omega}}_2 - \underline{\omega}_2 \times (I_2 \underline{\omega}_2) = \underline{0}; \tag{10b}$$

Link 1':

$$\underline{f}_{0,1'} - \underline{f}_{1',2'} - m_{1'} \dot{\underline{v}}_{c1'} = \underline{0}, \tag{11a}$$

$$\underline{N}_{0,1'} - \underline{N}_{1',2'} + \underline{r}_{1',c1'} \times \underline{f}_{1',2'} - \underline{r}_{0',c1'} \times \underline{f}_{0,1'} - I_{1'} \dot{\underline{\omega}}_{1'} - \underline{\omega}_{1'} \times (I_{1'} \underline{\omega}_{1'}) = \underline{0}; \tag{11b}$$

Link 2':

$$\underline{f}_{1',2'} - m_{2'} \dot{\underline{v}}_{c2'} = \underline{0}, \tag{12a}$$

$$\underline{N}_{1',2'} - \underline{r}_{1',c2'} \times \underline{f}_{1',2'} - I_{2'} \dot{\underline{\omega}}_{2'} - \underline{\omega}_{2'} \times (I_{2'} \underline{\omega}_{2'}) = \underline{0}; \tag{12b}$$

Link 0 (OMV Base):

$$\underline{f}_{1,0} - \underline{f}_{0,1} - m_o \dot{\underline{v}}_{co} + \underline{f}_c = \underline{0}, \tag{13a}$$

$$\underline{N}_{1,0} - \underline{N}_{0,1'} + \underline{r}_{o',co} \times \underline{f}_{0,1'} - \underline{r}_{o,co} \times \underline{f}_{1,0} - I_o \dot{\underline{\omega}}_o - \underline{\omega}_o \times (I_o \underline{\omega}_o) + \underline{N}_c = \underline{0}, \tag{13b}$$

where $\underline{v}_{ci}$ is the velocity of the centroid of link i referenced with respect to the inertial frame $\{x^o, y^o, z^o\}$; m_i is the mass of link i; $\underline{f}_{i-1,i}$ and $-\underline{f}_{i,i+1}$ are the coupling forces applied to link i by link i-1 and i+1 respectively; $\underline{\omega}_i$ and I_i are the angular velocity and centroidal inertia tensor of link i respectively; $\underline{r}_{i,ci}$ denotes the position vector from point O_i (the origin associated with joint i+1) to the centroid of link i; $\underline{N}_{i-1,i}$ is the coupling moment applied to link i by link i-1; $\underline{N}_c$ is a control torque acting on link 0; and $\underline{f}_c$ is a control force acting at the centroid of link 0.

For the case of a planar OMV robot, all the joint axes are along the z or z^o axis. Let τ_i and τ_c denote the joint torques at the i-th joint and the control torque respectively (i.e $\underline{N}_{i-1,i} = \tau_i \underline{e}_z$ and $\underline{N}_c = \tau_c \underline{e}_z$). Eliminating the forces $\underline{f}_{i,j}$ in (9b)-(13b) using (9a)-(13a), and adding (9a)-(13a) give the following equations:

$$I_1 \dot{\underline{\omega}}_1 + I_2 \dot{\underline{\omega}}_2 + m_1 \underline{r}_{o,c1} \times \dot{\underline{v}}_{c1} - m_2 (\underline{r}_{1,c1} - \underline{r}_{o,c1} - \underline{r}_{1,c2}) \times \dot{\underline{v}}_{c2} = \tau_1 \underline{e}_z, \tag{14}$$

$$I_2 \dot{\underline{\omega}}_2 - m_2 \underline{r}_{1,c2} \times \dot{\underline{v}}_{c2} = \tau_2 \underline{e}_z, \tag{15}$$

$$I_{1'} \dot{\underline{\omega}}_{1'} + I_{2'} \dot{\underline{\omega}}_{2'} + m_{1'} \underline{r}_{o',c1'} \times \dot{\underline{v}}_{c1'} - m_{2'} (\underline{r}_{1',c1} - \underline{r}_{o',c1'} - \underline{r}_{1',c2'}) \times \dot{\underline{v}}_{c1'} = \tau_{1'} \underline{e}_z, \tag{16}$$

$$I_{2'} \dot{\underline{\omega}}_{2'} - m_{2'} \underline{r}_{1',c2'} \times \dot{\underline{v}}_{c2'} = \tau_{2'} \underline{e}_z, \tag{17}$$

$$\begin{aligned} I_o \dot{\underline{\omega}}_o + I_1 \dot{\underline{\omega}}_1 + I_2 \dot{\underline{\omega}}_2 + I_{1'} \dot{\underline{\omega}}_{1'} + I_{2'} \dot{\underline{\omega}}_{2'} - m_1 (\underline{r}_{o,co} - \underline{r}_{o,c1}) \times \dot{\underline{v}}_{c1} - m_2 (\underline{r}_{1,c1} - \underline{r}_{o,c1} - \underline{r}_{1,c2} + \underline{r}_{o,co}) \times \dot{\underline{v}}_{c2} \\ - m_{1'} (\underline{r}_{o',co} - \underline{r}_{o',c1'}) \times \dot{\underline{v}}_{c1'} - m_{2'} (\underline{r}_{1',c1} - \underline{r}_{o',c1'} - \underline{r}_{1',c2'} + \underline{r}_{o',co}) \times \dot{\underline{v}}_{c2'} = \tau_c \underline{e}_z, \end{aligned} \tag{18}$$

$$m_1\dot{\underline{v}}_{c1} + m_2\dot{\underline{v}}_{c2} + m_{1'}\dot{\underline{v}}_{c1'} + m_{2'}\dot{\underline{v}}_{c2'} + m_o\dot{\underline{v}}_{co} = \underline{f}_c, \tag{19}$$

where $\underline{\omega}_i = \omega_i\underline{e}_z$. Equations (14)-(19) constitute a complete description of the motion of the planar OMV dual-arm robot.

To obtain explicit forms for (14)-(19), we choose the body coordinate system C_B with basis $\mathcal{B}_t = \{\underline{e}_x(t), \underline{e}_y(t), \underline{e}_z(t)\}$ and with the origin O on the axis of rotary joint 1. Moreover, $\underline{e}_x(t)$ is directed from point ^xO to $^yO'$. This choice of origin O is preferred over that at the base-centroid, since the latter is usally not precisely known. Now, we introduce the joint angles θ_i, i=1,2,1',2', and the OMV base attitude angle θ_o as shown in Fig. 3. Thus, the angular velocities ω_i are related to the $\dot{\theta}_j$'s by

$$\omega_1 = \dot{\theta}_o + \dot{\theta}_1, \quad \omega_2 = \dot{\theta}_o + \dot{\theta}_1 + \dot{\theta}_2, \quad \omega_{1'} = \dot{\theta}_o + \dot{\theta}_{1'}, \quad \omega_{2'} = \dot{\theta}_o + \dot{\theta}_{1'} + \dot{\theta}_{2'}. \tag{20}$$

Note that all the angles θ_i, with the exception of θ_o, are body referenced. Thus, they can be measured by means of body-reference sensors. The base attitude angle θ_o must be measured by means of an inertial-reference sensor.

Since the OMV robot is autonomous, it is natural to use the body coordinate system C_B as the basic reference frame for the arm motions and for observations of the environment from the OMV robot. Expressing the position vectors $\underline{r}_{i,cj}$ with respect to the basis $\mathcal{B}_t$, we have

$$\begin{aligned}
&\underline{r}_{o,c1} = \ell_{c1}\{(cs1)\underline{e}_x(t) + (sn1)\underline{e}_y(t)\}, && \underline{r}_{o',c1'} = \ell_{c1'}\{(cs1')\underline{e}_x(t) + (sn1')\underline{e}_y(t)\},\\
&\underline{r}_{o,co} = x_{o,co}\underline{e}_x(t) + y_{o,co}\underline{e}_y(t), && \underline{r}_{o,o'} = w\underline{e}_x(t),\\
&\underline{r}_{o',co} = (x_{o,co} - w)\underline{e}_x(t) + y_{o,co}\underline{e}_y(t), && \underline{r}_{1,c2} = \ell_{c2}\{(cs12)\underline{e}_x(t) + (sn12)\underline{e}_y(t)\},\\
&\underline{r}_{1',c2'} = \ell_{c2'}\{(cs1'2')\underline{e}_x(t) + (sn1'2')\underline{e}_y(t)\}, && \underline{r}_{1,c1} = (\ell_{c1} - \ell_1)\{(cs1)\underline{e}_x(t) + (sn1)\underline{e}_y(t)\},\\
&\underline{r}_{1',c1'} = (\ell_{c1'} - \ell_{1'})\{(cs1')\underline{e}_x(t) + (sn1')\underline{e}_y(t)\},
\end{aligned} \tag{21}$$

where $sni \overset{\Delta}{=} \sin\theta_i$, $csi \overset{\Delta}{=} \cos\theta_i$, $snij \overset{\Delta}{=} \sin(\theta_i + \theta_j)$ and $csij \overset{\Delta}{=} \cos(\theta_i + \theta_j)$. In writing down (21), we have assumed that the centroid (marked by "●" in Fig.3) of any link i of arm 1 or 2 is located inside link i along the line segment connecting the joints i and i+1. The centroid of link 0 (OMV base) specified by $\underline{r}_{o,co}$ has the time-invariant representation $(x_{o,co}, y_{o,co}, 0)$ with respect to the time dependent basis $\mathcal{B}_t$.

Let $\underline{r}_o(t) = x_o(t)\underline{e}_x(t) + y_o(t)\underline{e}_y(t)$ denote the vector directed from the origin O^o of the inertial frame to the origin O of the body coordinate system C_B. Thus,

$$\ddot{\underline{r}}_o = (\ddot{x}_o - y_o\ddot{\theta}_o - 2\dot{\theta}_o\dot{y}_o - x_o\dot{\theta}_o^2)\underline{e}_x(t) + (\ddot{y}_o + x_o\ddot{\theta}_o + 2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2)\underline{e}_y(t). \tag{22}$$

The accelerations of the link-centroids $\dot{\underline{v}}_{ci}$, i=o,1,2,1',2' can be obtained by differentiating (21) (see Appendix for their explicit expressions). Substituting the explicit expressions for $\dot{\underline{v}}_{ci}$ and $\ddot{\underline{r}}_o$ leads to the following equations of motion for the OMV robot:

$$H(\underline{q})\ddot{\underline{q}} + \underline{V}(\underline{q},\dot{\underline{q}}) = \underline{u}, \tag{23}$$

where $\underline{q} = (\theta_1, \theta_2, \theta_{1'}, \theta_{2'}, \theta_o, x_o, y_o)^T$, $\underline{u} = (u_1, \ldots, u_7)^T \overset{\Delta}{=} (\tau_1, \tau_2, \tau_{1'}, \tau_{2'}, \tau_c, f_{cx}, f_{cy})^T$; $\underline{f}_c = f_{cx}\underline{e}_x(t) + f_{cy}\underline{e}_y(t)$; $\underline{V} = (V_1, \ldots, V_7)^T$ is a vector-valued function of $\underline{q}$ and $\dot{\underline{q}}$ representing the centrifugal and Coriolis forces and moments, and $H(\underline{q})$ is a 7×7 matrix of the form:

$$H = \begin{bmatrix}
h_{11} & h_{12} & 0 & 0 & h_{15} & h_{16} & h_{17}\\
h_{12} & h_{22} & 0 & 0 & h_{25} & h_{26} & h_{27}\\
0 & 0 & h_{33} & h_{34} & h_{35} & h_{36} & h_{37}\\
0 & 0 & h_{34} & h_{44} & h_{45} & h_{46} & h_{47}\\
h_{51} & h_{52} & h_{53} & h_{54} & h_{55} & h_{56} & h_{57}\\
h_{16} & h_{26} & h_{36} & h_{46} & h_{65} & h_{66} & 0\\
h_{17} & h_{27} & h_{37} & h_{47} & h_{75} & 0 & h_{77}
\end{bmatrix} \tag{24}$$

The explicit expressions for the elements of H and the components of $\underline{V}$ are given in the Appendix.

Now, we consider the problem of deriving sutiable control laws for the elementary maneuvers using the mathematical model described by (22). Since these problems for all the elementary maneuvers are intrinsically identical, we shall discuss only a specific case to illustrate the basic ideas.

Consider the problem of deriving a control law for aligning the OMV heading $\underline{n}_R$ with a specified gripping direction $\underline{n}_o$ while keeping all the joint angles and base-centroid position stationary. In this case, we require

$$\underline{q}(t) = (\bar{\theta}_1, \bar{\theta}_2, \bar{\theta}_{1'}, \bar{\theta}_{2'}, \theta_o(t), \bar{x}_o, \bar{y}_o)^T \triangleq \tilde{\underline{q}}(t), \quad \dot{\underline{q}}(t) = (0,0,0,0,\dot{\theta}_o(t),0,0)^T \triangleq \dot{\tilde{\underline{q}}}(t), \tag{25}$$

$$\ddot{\underline{q}}(t) = (0,0,0,0,\ddot{\theta}_o(t),0,0)^T \triangleq \ddot{\tilde{\underline{q}}}(t),$$

where $\bar{x}_o, \bar{y}_o, \bar{\theta}_i$, $i=1,2,1',2'$ are specified constants. Thus, equation (23) reduces to

$$h_{55}(\tilde{\underline{q}})\ddot{\theta}_o = -V_5(\tilde{\underline{q}}, \dot{\tilde{\underline{q}}}) + \tau_c, \tag{26}$$

$$h_{i5}(\tilde{\underline{q}})\ddot{\theta}_o = -V_i(\tilde{\underline{q}}, \dot{\tilde{\underline{q}}}) + u_i, \quad i \neq 5. \tag{27}$$

Let θ_o^d denote the base angle such that $\underline{n}_o = \underline{n}_R$, and $\Delta\theta_o \triangleq \theta_o^d - \theta_o$. Equation (26) can be rewritten as:

$$h_{55}(\tilde{\underline{q}})\Delta\ddot{\theta}_o = V_5(\tilde{\underline{q}}, \dot{\tilde{\underline{q}}}) - \tau_c. \tag{28}$$

A simple control law for (28) is given by

$$\tau_c = V_5(\tilde{\underline{q}}, \dot{\tilde{\underline{q}}}) + h_{55}(\tilde{\underline{q}})(K_p \Delta\theta_o + K_r \Delta\dot{\theta}_o), \tag{29}$$

where K_p and K_r are constant feedback gains. Evidently, from (27), the required joint torques τ_i, $i=1,2,1',2'$ and the base control force $\underline{f}_c$ for keeping all joint angles and base-centroid position stationary are given by

$$u_i = V_i(\tilde{\underline{q}}, \dot{\tilde{\underline{q}}}) + h_{i5}(\tilde{\underline{q}})(K_p \Delta\theta_o + K_r \Delta\dot{\theta}_o), \quad i \neq 5. \tag{30}$$

The foregoing control laws (29) and (30) depend on the nominal values of the system parameters. It can be shown that by choosing K_p and K_r properly, such control laws remain effective in the presence of small parameter perturbations (see [3], Chapter 6). To ensure that all the joint angles and the base-centroid position remain at their specified values during the heading alignment maneuver, a suitable linear control law depending on the instantaneous deviations of the joint angles and base-centroid position from their specified values may be used.

5. CONCLUDING REMARKS

The key idea in the proposed control strategy for the dual-arm maneuverable space robot is to decompose the maneuvers into a sequence of elementary maneuvers involving at most two degrees of freedom. These elementary maneuvers are simple to perform, and they are particularly suitable for operations in a space environment in which safety is a major factor. Although the results presented here pertain only to planar motions, they are being extended presently to the general three-dimensional case. An experiment involving a planar dual-arm maneuverable robot which is levitated above ground by an air bearing is in the planning stage at this time.

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APPENDIX

By direct computation using (21) and the relations: $\dot{\underline{e}}_x(t) = \dot{\theta}_o \underline{e}_y(t)$, $\dot{\underline{e}}_y(t) = -\dot{\theta}_o \underline{e}_x(t)$, the accelerations of the link-centroids are given by

$$\dot{\underline{v}}_{co} = \ddot{\underline{r}}_o + \ddot{\underline{r}}_{o,co} = \ddot{\underline{r}}_o - (y_{o,co}\ddot{\theta}_o + x_{o,co}\dot{\theta}_o^2)\underline{e}_x(t) + (x_{o,co}\ddot{\theta}_o - y_{o,co}\dot{\theta}_o^2)\underline{e}_y(t), \tag{A-1}$$

$$\dot{\underline{v}}_{c1} = \ddot{\underline{r}}_o + \ddot{\underline{r}}_{o,c1} = \ddot{\underline{r}}_o - \ell_{c1}\{(\dot{\theta}_o + \dot{\theta}_1)^2 cs1 + (\ddot{\theta}_o + \ddot{\theta}_1) sn1\}\underline{e}_x(t) - \ell_{c1}\{(\dot{\theta}_o + \dot{\theta}_1)^2 sn1 - (\ddot{\theta}_o + \ddot{\theta}_1) cs1\}\underline{e}_y(t), \tag{A-2}$$

$$\begin{aligned}\dot{\underline{v}}_{c2} = \ddot{\underline{r}}_o + \ddot{\underline{r}}_{o,1} + \ddot{\underline{r}}_{1,c2} = \ddot{\underline{r}}_o &- \{\ell_1(\ddot{\theta}_o + \ddot{\theta}_1) sn1 + \ell_1(\dot{\theta}_o + \dot{\theta}_1)^2 cs1 + \ell_{c2}(\ddot{\theta}_o + \ddot{\theta}_1 + \ddot{\theta}_2) sn12 + \ell_{c2}(\dot{\theta}_o + \dot{\theta}_1 + \dot{\theta}_2)^2 cs12\}\underline{e}_x(t) \\ &+ \{\ell_1(\ddot{\theta}_o + \ddot{\theta}_1) cs1 - \ell_1(\dot{\theta}_o + \dot{\theta}_1)^2 sn1 + \ell_{c2}(\ddot{\theta}_o + \ddot{\theta}_1 + \ddot{\theta}_2) cs12 - \ell_{c2}(\dot{\theta}_o + \dot{\theta}_1 + \dot{\theta}_2)^2 sn12\}\underline{e}_y(t),\end{aligned} \tag{A-3}$$

$$\begin{aligned}\dot{\underline{v}}_{c1'} = \ddot{\underline{r}}_o + w\ddot{\underline{e}}_x(t) + \ddot{\underline{r}}_{o',c1'} = \ddot{\underline{r}}_o &- \{w\dot{\theta}_o^2 + \ell_{c1'}(\dot{\theta}_o + \dot{\theta}_{1'})^2 cs1' + \ell_{c1'}(\ddot{\theta}_o + \ddot{\theta}_1) sn1'\}\underline{e}_x(t) \\ &+ \{w\ddot{\theta}_o + \ell_{c1'}(\ddot{\theta}_o + \ddot{\theta}_{1'}) cs1' - \ell_{c1'}(\dot{\theta}_o + \dot{\theta}_{1'})^2 sn1'\}\underline{e}_y(t),\end{aligned} \tag{A-4}$$

$$\dot{\underline{v}}_{c2'} = \ddot{\underline{r}}_o + w\ddot{\underline{e}}_x(t) + \ddot{\underline{r}}_{o',1'} + \ddot{\underline{r}}_{1',c2'} = \ddot{\underline{r}}_o - \{w\dot{\theta}_o^2 + \ell_{1'}(\dot{\theta}_o + \dot{\theta}_{1'})^2 \mathrm{cs}1' + \ell_{1'}(\ddot{\theta}_o + \ddot{\theta}_1)\mathrm{sn}1' + \ell_{c2'}(\ddot{\theta}_o + \ddot{\theta}_{1'} + \ddot{\theta}_{2'})\mathrm{sn}1'2'$$
$$+ \ell_{c2'}(\dot{\theta}_o + \dot{\theta}_{1'} + \dot{\theta}_{2'})^2 \mathrm{cs}1'2'\}\underline{e}_x(t) + \{w\ddot{\theta}_o - \ell_{1'}(\dot{\theta}_o + \dot{\theta}_{1'})\mathrm{sn}1' - \ell_{c2'}(\dot{\theta}_o + \dot{\theta}_{1'} + \dot{\theta}_{2'})^2 \mathrm{sn}1'2' + \ell_{1'}(\ddot{\theta}_o + \ddot{\theta}_{1'})\mathrm{cs}1'$$
$$+ \ell_{c2'}(\ddot{\theta}_o + \ddot{\theta}_{1'} + \ddot{\theta}_{2'})^2 \mathrm{cs}1'2'\}\underline{e}_y(t), \qquad \text{(A-5)}$$

where $\ddot{\underline{r}}_o(t)$ is given by (22).

The elements $h_{ij}(\underline{q})$ of $H(\underline{q})$ are given by

$$h_{11} = I_1 + I_2 + m_1\ell_{c1}^2 + m_2(\ell_1^2 + 2\ell_1\ell_{c2}\mathrm{cs}2 + \ell_{c2}^2), \qquad \text{(A-6)}$$

$$h_{12} = h_{21} = I_2 + m_2\ell_{c2}(\ell_{c2} + \ell_1\mathrm{cs}2), \qquad \text{(A-7)}$$

$$h_{15} = I_1 + I_2 + m_1\{\ell_{c1}^2 + \ell_{c1}(y_o\mathrm{sn}1 + x_o\mathrm{cs}1)\} + m_2\{\ell_1^2 + 2\ell_1\ell_{c2}\mathrm{cs}2 + \ell_{c2}^2 + x_o(\ell_1\mathrm{cs}1 + \ell_{c2}\mathrm{cs}12) + y_o(\ell_1\mathrm{sn}1 + \ell_{c2}\mathrm{sn}12)\}, \qquad \text{(A-8)}$$

$$h_{16} = h_{61} = -m_1\ell_{c1}\mathrm{sn}1 - m_2(\ell_1\mathrm{sn}1 + \ell_{c2}\mathrm{sn}12), \qquad h_{17} = h_{71} = m_1\ell_{c1}\mathrm{cs}1 + m_2(\ell_1\mathrm{cs}1 + \ell_{c2}\mathrm{cs}12), \qquad \text{(A-9)}$$

$$h_{22} = I_2 + m_2\ell_{c2}^2, \quad h_{25} = I_2 + m_2\ell_{c2}(\ell_{c2} + \ell_1\mathrm{cs}2 + x_o\mathrm{cs}12 + y_o\mathrm{sn}12), \quad h_{26} = -m_2\ell_{c2}\mathrm{sn}12 = h_{62}, \quad h_{27} = m_2\ell_{c2}\mathrm{cs}12 = h_{72}, \qquad \text{(A-10)}$$

$$h_{33} = I_{1'} + I_{2'} + m_{1'}\ell_{c1'}^2 + m_{2'}(\ell_{1'}^2 + 2\ell_{1'}\ell_{c2'}\mathrm{cs}2' + \ell_{c2'}^2), \quad h_{34} = h_{43} = I_{2'} + m_{2'}\ell_{c2'}(\ell_{c2'} + \ell_{1'}\mathrm{cs}2'), \qquad \text{(A-11)}$$

$$h_{35} = I_{1'} + I_{2'} + m_{1'}\{\ell_{c1'}^2 + \ell_{c1'}(y_o\mathrm{sn}1' + (x_o + w)\mathrm{cs}1')\} + m_{2'}\{\ell_{1'}^2 + 2\ell_{1'}\ell_{c2'}\mathrm{cs}2' + \ell_{c2'}^2 + (x_o + w)(\ell_{1'}\mathrm{cs}1' + \ell_{c2'}\mathrm{cs}1'2')$$
$$+ y_o(\ell_{1'}\mathrm{sn}1' + \ell_{c2'}\mathrm{sn}1'2')\}, \qquad \text{(A-12)}$$

$$h_{36} = h_{63} = -m_{1'}\ell_{c1'}\mathrm{sn}1' - m_{2'}(\ell_{1'}\mathrm{sn}1' + \ell_{c2'}\mathrm{sn}1'2'), \qquad h_{37} = h_{73} = m_{1'}\ell_{c1'}\mathrm{cs}1' + m_{2'}(\ell_{1'}\mathrm{cs}1' + \ell_{c2'}\mathrm{cs}1'2'), \qquad \text{(A-13)}$$

$$h_{44} = I_{2'} + m_{2'}\ell_{c2'}^2, \qquad h_{45} = I_{2'} + m_{2'}\ell_{c2'}\{\ell_{c2'} + \ell_1\mathrm{cs}2' + (x_o + w)\mathrm{cs}1'2' + y_o\mathrm{sn}1'2'\}, \qquad \text{(A-14)}$$

$$h_{46} = h_{64} = -m_{2'}\ell_{c2'}\mathrm{sn}1'2', \qquad h_{47} = h_{74} = m_{2'}\ell_{c2'}\mathrm{cs}1'2', \qquad \text{(A-15)}$$

$$h_{51} = I_1 + I_2 + m_1\ell_{c1}(\ell_{c1} - x_{o,co}\mathrm{cs}1 - y_{o,co}\mathrm{sn}1) + m_2\{\ell_1^2 + 2\ell_1\ell_{c2}\mathrm{cs}2 + \ell_{c2}^2 - x_{o,co}(\ell_1\mathrm{cs}1 + \ell_{c2}\mathrm{cs}12)$$
$$- y_{o,co}(\ell_1\mathrm{sn}1 + \ell_{c2}\mathrm{sn}12)\}, \qquad \text{(A-16)}$$

$$h_{52} = I_2 + m_2\ell_{c2}(\ell_{c2} + \ell_1\mathrm{cs}2 - x_{o,co}\mathrm{cs}12 - y_{o,co}\mathrm{sn}12),$$

$$h_{53} = I_{1'} + I_{2'} + m_{1'}\ell_{c1'}\{\ell_{c1'} + (w - x_{o,co})\mathrm{cs}1' - y_{o,co}\mathrm{sn}1'\} + m_{2'}\{\ell_{1'}^2 + 2\ell_{1'}\ell_{c2'}\mathrm{cs}2' + \ell_{c2'}^2 + (w - x_{o,co})(\ell_{1'}\mathrm{cs}1'$$
$$+ \ell_{c2'}\mathrm{cs}1'2') - y_{o,co}(\ell_{1'}\mathrm{sn}1' + \ell_{c2'}\mathrm{sn}1'2')\}, \qquad \text{(A-17)}$$

$$h_{54} = I_{2'} + m_{2'}\ell_{c2'}\{\ell_{c2'} + \ell_{1'}\mathrm{cs}2' + (w - x_{o,co})\mathrm{cs}1'2' - y_{o,co}\mathrm{sn}1'2'\}, \qquad \text{(A-18)}$$

$$h_{55} = I_T + m_1\{\ell_{c1}^2 + (x_o - x_{o,co})\ell_{c1}\mathrm{cs}1 + (y_o - y_{o,co})\ell_{c1}\mathrm{sn}1 - x_o x_{o,co} - y_o y_{o,co}\} + m_2\{\ell_1^2 + 2\ell_1\ell_{c2}\mathrm{cs}2 + \ell_{c2}^2$$
$$+ (x_o - x_{o,co})(\ell_1\mathrm{cs}1 + \ell_{c2}\mathrm{cs}12) + (y_o - y_{o,co})(\ell_1\mathrm{sn}1 + \ell_{c2}\mathrm{sn}12) - x_o x_{o,co} - y_o y_{o,co}\}$$
$$+ m_{1'}\{\ell_{c1'}^2 + (x_o + 2w - x_{o,co})\ell_{c1'}\mathrm{cs}1' + (y_o - y_{o,co})\ell_{c1'}\mathrm{sn}1' + (x_o + w)(w - x_{o,co}) - y_o y_{o,co}\}$$
$$+ m_{2'}\{\ell_{1'}^2 + 2\ell_{1'}\ell_{c2'}\mathrm{cs}2' + \ell_{c2'}^2 + (x_o + 2w - x_{o,co})(\ell_{1'}\mathrm{cs}1' + \ell_{c2'}\mathrm{cs}1'2') + (y_o - y_{o,co})(\ell_{1'}\mathrm{sn}1' + \ell_{c2'}\mathrm{sn}1'2')$$
$$+ (x_o + w)(w - x_{o,co}) - y_o y_{o,co}\}, \qquad \text{(A-19)}$$

$$h_{56} = m_1(y_{o,co} - \ell_{c1}\mathrm{sn}1) + m_2(y_{o,co} - \ell_1\mathrm{sn}1 - \ell_{c2}\mathrm{sn}12) + m_{1'}(y_{o,co} - \ell_{c1'}\mathrm{sn}1') + m_{2'}(y_{o,co} - \ell_{1'}\mathrm{sn}1' - \ell_{c2'}\mathrm{sn}1'2'), \qquad \text{(A-20)}$$

$$h_{57} = m_1(\ell_{c1}\mathrm{cs}1 - x_{o,co}) + m_2(\ell_1\mathrm{cs}1 + \ell_{c2}\mathrm{cs}12 - x_{o,co}) + m_{1'}(w + \ell_{c1'}\mathrm{cs}1' - x_{o,co}) + m_{2'}(w + \ell_{1'}\mathrm{cs}1' + \ell_{c2'}\mathrm{cs}1'2'$$
$$- x_{o,co}), \qquad \text{(A-21)}$$

$$h_{65} = -m_1\ell_{c1}\mathrm{sn}1 - m_2(\ell_1\mathrm{sn}1 + \ell_{c2}\mathrm{sn}12) - m_{1'}\ell_{c1'}\mathrm{sn}1' - m_{2'}(\ell_{1'}\mathrm{sn}1' + \ell_{c2'}\mathrm{sn}1'2') - m_o y_{o,co} - m_T y_o, \qquad \text{(A-22)}$$

$$h_{66} = h_{77} = m_T, \qquad \text{(A-23)}$$

$$h_{75} = m_1\ell_{c1}\mathrm{cs}1 + m_2(\ell_1\mathrm{cs}1 + \ell_{c2}\mathrm{cs}12) + m_{1'}(w + \ell_{c1'}\mathrm{cs}1') + m_{2'}(w + \ell_{1'}\mathrm{cs}1' + \ell_{c2'}\mathrm{cs}1'2') + m_o x_{o,co} + m_T x_o, \qquad \text{(A-24)}$$

where $I_T = I_o + I_1 + I_2 + I_{1'} + I_{2'}$, $m_T = m_o + m_1 + m_2 + m_{1'} + m_{2'}$, and all the remaining h_{ij}'s are zero.

The components of $\underline{V}(\underline{q},\dot{\underline{q}})$ are given explicitly by:

$$V_1 = m_1\ell_{c1}\{(2\dot{y}_o\dot{\theta}_o + x_o\dot{\theta}_o^2)\mathrm{sn1} + (2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2)\mathrm{cs1}\} + m_2\{(2\dot{y}_o\dot{\theta}_o + x_o\dot{\theta}_o^2)(\ell_1\mathrm{sn1} + \ell_{c2}\mathrm{sn12}) + (2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2)(\ell_1\mathrm{cs1} + \ell_{c2}\mathrm{cs12}) - (2(\dot{\theta}_o + \dot{\theta}_1)\dot{\theta}_2 + \dot{\theta}_2^2)\ell_1\ell_{c2}\mathrm{sn2}\}, \tag{A-25}$$

$$V_2 = m_2\ell_{c2}\{\ell_1(\dot{\theta}_o + \dot{\theta}_1)^2\mathrm{sn2} + (2\dot{y}_o\dot{\theta}_o + x_o\dot{\theta}_o^2)\mathrm{sn12} + (2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2)\mathrm{cs12}\}, \tag{A-26}$$

$$V_3 = m_{1'}\ell_{c1'}\{(2\dot{y}_o\dot{\theta}_o + (x_o + w)\dot{\theta}_o^2)\mathrm{sn1'} + (2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2)\mathrm{cs1'}\} + m_{2'}\{(2\dot{y}_o\dot{\theta}_o + (x_o + w)\dot{\theta}_o^2)(\ell_{1'}\mathrm{sn1'} + \ell_{c2'}\mathrm{sn1'2'}) + (2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2)(\ell_{1'}\mathrm{cs1'} + \ell_{c2'}\mathrm{cs1'2'}) - (\dot{\theta}_{2'}^2 + 2(\dot{\theta}_o + \dot{\theta}_{1'})\dot{\theta}_{2'})\mathrm{sn2'}\}, \tag{A-27}$$

$$V_4 = m_{2'}\ell_{c2'}\{\ell_{1'}(\dot{\theta}_o + \dot{\theta}_{1'})^2\mathrm{sn2'} + (2\dot{y}_o\dot{\theta}_o + (x_o + w)\dot{\theta}_o^2)\mathrm{sn1'2'} + (2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2)\mathrm{cs1'2'}\}, \tag{A-28}$$

$$\begin{aligned} V_5 = {} & -m_1\{(x_{o,co} - \ell_{c1}\mathrm{cs1})(2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2 - \ell_{c1}(\dot{\theta}_o + \dot{\theta}_1)^2\mathrm{sn1}) + (y_{o,co} - \ell_{c1}\mathrm{sn1})(2\dot{y}_o\dot{\theta}_o + x_o\dot{\theta}_o^2 + \ell_{c1}(\dot{\theta}_o + \dot{\theta}_1)^2\mathrm{cs1})\} \\ & -m_2\{(x_{o,co} - \ell_1\mathrm{cs1} - \ell_{c2}\mathrm{cs12})(2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2 - \ell_1(\dot{\theta}_o + \dot{\theta}_1)^2\mathrm{sn1} - \ell_{c2}(\dot{\theta}_o + \dot{\theta}_1 + \dot{\theta}_2)^2\mathrm{sn12}) + (y_{o,co} - \ell_1\mathrm{sn1} - \ell_{c2}\mathrm{sn12})\cdot \\ & (2\dot{y}_o\dot{\theta}_o + x_o\dot{\theta}_o^2 + \ell_1(\dot{\theta}_o + \dot{\theta}_1)^2\mathrm{cs1} + \ell_{c2}(\dot{\theta}_o + \dot{\theta}_1 + \dot{\theta}_2)^2\mathrm{cs12})\} - m_{1'}\{(x_{o,co} - w - \ell_{c1'}\mathrm{cs1'})(2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2 \\ & - \ell_{c1'}(\dot{\theta}_o + \dot{\theta}_{1'})^2\mathrm{sn1'}) + (y_{o,co} - \ell_{c1'}\mathrm{sn1'})(2\dot{y}_o\dot{\theta}_o + x_o\dot{\theta}_o^2 + \ell_{c1'}(\dot{\theta}_o + \dot{\theta}_{1'})^2\mathrm{cs1'} + w\dot{\theta}_o^2)\} \\ & - m_{2'}\{(x_{c,co} - w - \ell_{1'}\mathrm{cs1'} - \ell_{c2'}\mathrm{cs1'2'})(2\dot{x}_o\dot{\theta}_o - y_o\dot{\theta}_o^2 - \ell_{1'}(\dot{\theta}_o + \dot{\theta}_{1'})^2\mathrm{sn1'} - \ell_{c2'}(\dot{\theta}_o + \dot{\theta}_{1'} + \dot{\theta}_{2'})^2\mathrm{sn1'2'}) \\ & + (y_{o,co} - \ell_{1'}\mathrm{sn1'} - \ell_{c2'}\mathrm{sn1'2'})(2\dot{y}_o\dot{\theta}_o + x_o\dot{\theta}_o^2 + \ell_{1'}(\dot{\theta}_o + \dot{\theta}_{1'})^2\mathrm{cs1'} - \ell_{c2'}(\dot{\theta}_o + \dot{\theta}_{1'} + \dot{\theta}_{2'})^2\mathrm{cs1'2'} - w\dot{\theta}_o^2)\}, \end{aligned} \tag{A-29}$$

$$V_6 = -M_T(2\dot{\theta}_o\dot{y}_o + x_o\dot{\theta}_o^2) - (m_1\ell_{c1} + m_2\ell_1)(\dot{\theta}_o + \dot{\theta}_1)^2\mathrm{cs1} - m_2\ell_{c2}(\dot{\theta}_o + \dot{\theta}_1 + \dot{\theta}_2)^2\mathrm{cs12} - (m_{1'}\ell_{c1'} + m_{2'}\ell_{1'})(\dot{\theta}_o + \dot{\theta}_{1'})^2\mathrm{cs1'} - m_{2'}\ell_{c2'}(\dot{\theta}_o + \dot{\theta}_{1'} + \dot{\theta}_{2'})^2\mathrm{cs1'2'} - (m_{1'} + m_{2'})w\dot{\theta}_o^2 - m_o x_{o,co}\dot{\theta}_o^2, \tag{A-30}$$

$$V_7 = M_T(2\dot{\theta}_o\dot{x}_o - y_o\dot{\theta}_o^2) - (m_1\ell_{c1} + m_2\ell_1)(\dot{\theta}_o + \dot{\theta}_1)^2\mathrm{sn1} - m_2\ell_{c2}(\dot{\theta}_o + \dot{\theta}_1 + \dot{\theta}_2)^2\mathrm{sn12} - (m_{1'}\ell_{c1'} + m_{2'}\ell_{1'})(\dot{\theta}_o + \dot{\theta}_{1'})^2\mathrm{sn1'} - m_{2'}\ell_{c2'}(\dot{\theta}_o + \dot{\theta}_{1'} + \dot{\theta}_{2'})^2\mathrm{sn1'2'} - m_o y_{o,co}\dot{\theta}_o^2. \tag{A-31}$$

Note that $H(\underline{q})$ is not the manipulator mass-inertia matrix. Its nonsymmetry (i.e. $h_{5i}(\underline{q}) = h_{i5}(\underline{q}), i \neq 5, i=1,\ldots,7$) is partially due to the fact that the control force $\underline{f}_c$ and the acceleration of the base-centroid are expressed in terms of the body coordinate system C_B with basis $\bar{\mathcal{B}}_t$. The moment $\underline{r}_{o,co} \times \underline{f}_c$ also attributes to the nonsymmetry of $H(\underline{q})$. It can be shown that the manipulator mass-inertia matrix $\mathbf{H}(\underline{q})$ is related to $H(\underline{q})$ by

$$\mathbf{H}(\underline{q}) = \begin{bmatrix} I_4 & 0 \\ 0 & Q \end{bmatrix} H(\underline{q}) \begin{bmatrix} I_4 & 0 \\ 0 & P \end{bmatrix}, \tag{A-32}$$

where I_4 is the 4×4 identity matrix, and

$$Q = \begin{bmatrix} 1 & -y_{o,co} & x_{o,co} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad P = \begin{bmatrix} 1 & 0 & 0 \\ y_o & \mathrm{cs0} & \mathrm{sn0} \\ -x_o & -\mathrm{sn0} & \mathrm{cs0} \end{bmatrix}. \tag{A-33}$$

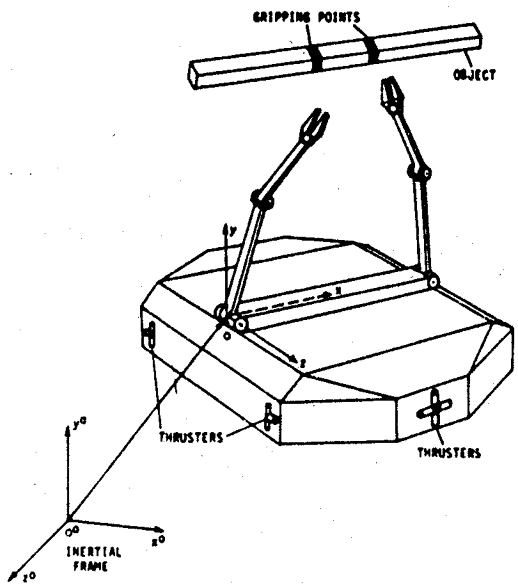

Fig.1 Sketch of a dual-arm maneuverable space robot.

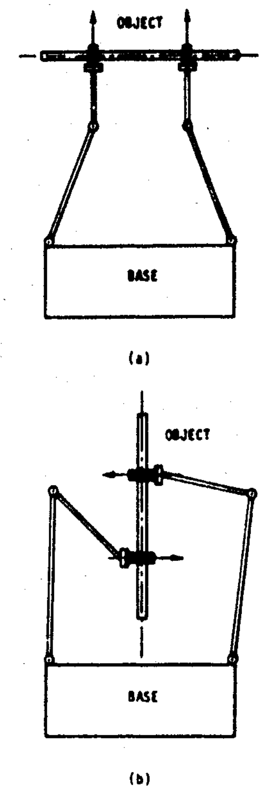

Fig.2 Modes for gripping a rod by a planar dual-arm robot. (a) gripping with a single direction of approach for both end-effectors, (b) gripping with different directions of approach for the end-effectors.

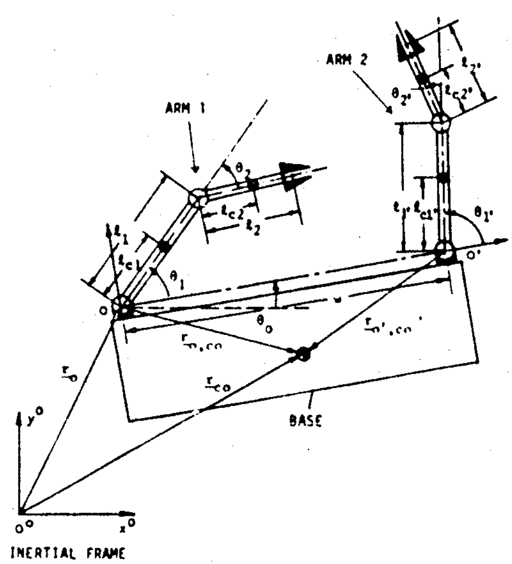

Fig.3 Simplified planar maneuverable dual-arm robot.

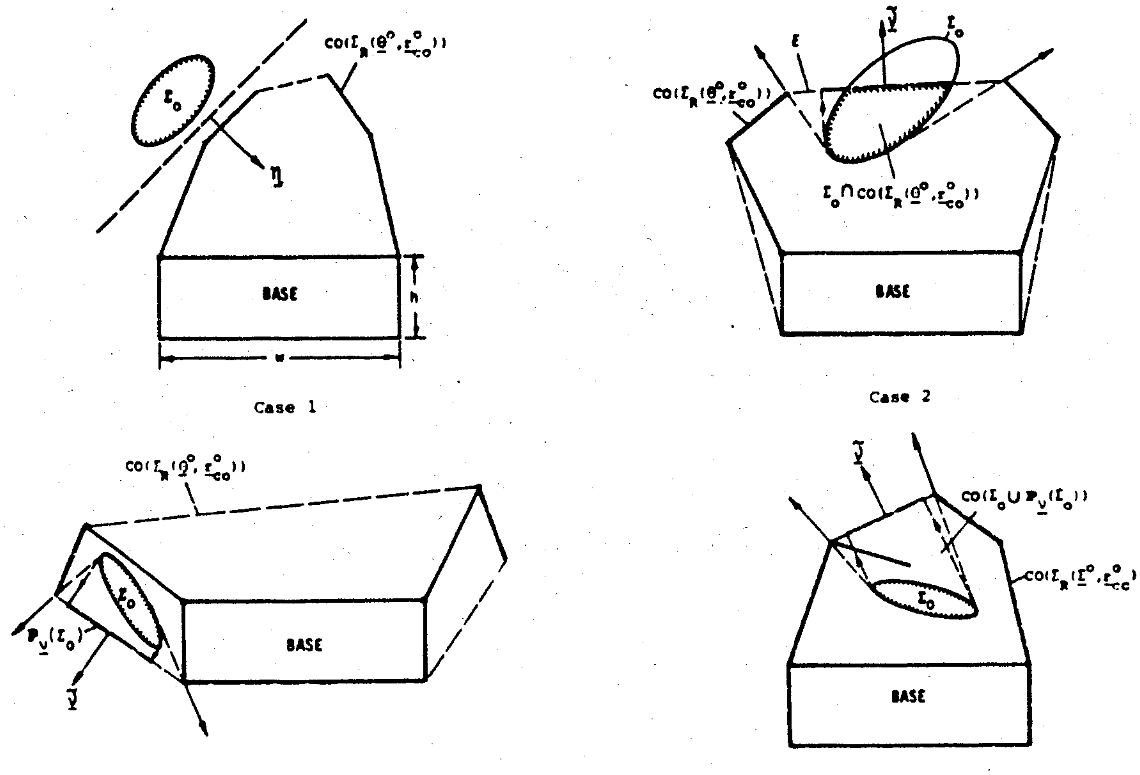

Fig.4 Relative locations of the robot
and the object.

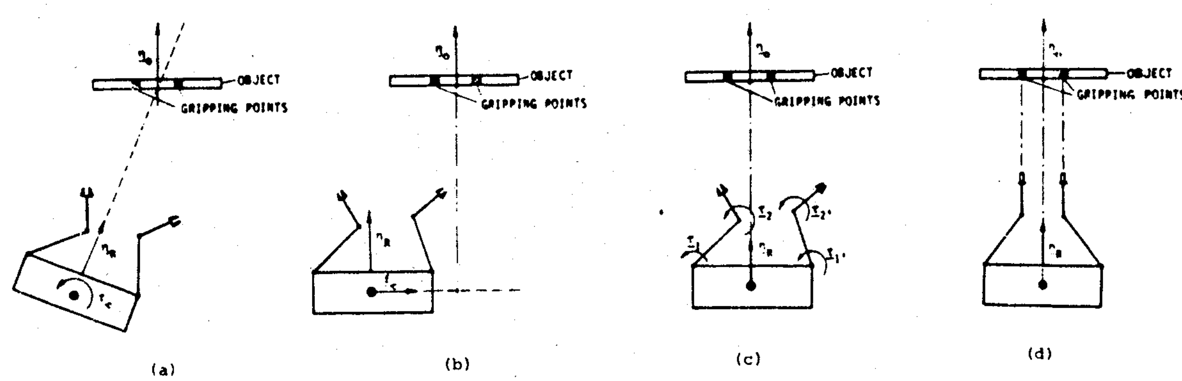

Fig.5 Elementary maneuvers for the alignment
phase.