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A SPECTROSCOPIC SURVEY OF SOUTHERN HEMISPHERE WHITE DWARFS—IV

Radial Velocities and Space Motions

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SUMMARY

New radial velocity measurements are reported for 31 southern white dwarfs. The K-term which is interpreted as the mean gravitational redshift, was estimated on the assumption of a mean solar motion for the white dwarfs. The K-term, space motions, and velocity ellipsoids were also derived for the white dwarfs from their radial velocities after combining the present data with Greenstein & Trimble's results for northern white dwarfs. With an assumed solar motion, the southern DA stars alone give a mean gravitational redshift of $+43 \pm 14\ \mathrm{km\ s^{-1}}$ which is slightly lower than previous results. The possibility that the K-term could be enlarged by white dwarfs in moving groups is suggested. For the non-DA white dwarfs, the mean gravitational redshift was found to be lower, perhaps a result of pressure shifts.

The velocity dispersions found for all the white dwarfs are comparable to the dM stars and suggests the white dwarfs are formed from old stars. The velocity dispersions for the non-DA (hydrogen-poor) stars may be slightly lower than for the DA (hydrogen-rich) white dwarfs. Some of the stellar evolutionary interpretations of these observations are briefly discussed.

1. INTRODUCTION

The lack of fundamental data on a large number of white dwarf stars stems from their intrinsic faintness and the difficulties encountered in measurement of their radial velocities. Consequently, the earliest work on the space motions of the white dwarfs relied entirely upon the proper motions. Parenago (1949) appears to have been the first to seriously investigate the kinematical properties of the white dwarfs and later investigations by Pavlovskaya (1956), Iwanowska & Opaska-Burnicka (1962), and Luyten (1963) are noteworthy early studies of the subject. Although these investigations seem to show that kinematically, the white dwarfs belong to an intermediate Population I there may also be a Population II component. Due to the statistical nature of these investigations, detailed information on the space motions of individual types of white dwarfs could not be obtained.

In more recent times we have begun to understand more about the physical properties of the white dwarfs as a larger body of spectral types, photoelectric photometry and distances have continued to become available. Greenstein & Trimble (1967) and Trimble & Greenstein (1972) have measured radial velocities for a number of northern white dwarfs. This new information makes alternate ways of studying the space motions of the white dwarfs possible using their radial velocities.

An additional important by-product of the study of the white dwarfs' radial velocities is an estimate of their mean gravitational redshift which can be compared with the theoretical mass–radius relation for electron degenerate stars and can yield a statistical mean mass for the white dwarfs. However, there are problems with the interpretation of such a number. The Trimble & Greenstein (1972) work showed how sensitive the mean radius is to the adopted temperature scale and what effect pressure shifts could have on the mean redshift. As well, selection effects might play a role, for white dwarfs selected differently in the various surveys could be physically different. In view of the importance of these results for the understanding of the white dwarfs, it was felt that an additional independent determination of the mean gravitational redshift should be made.

The present paper reports radial velocity measurements for most of the 41 southern white dwarfs reported earlier (Wegner 1973a, referred to hereafter as Paper II) and a few additional stars. This work is intended to supplement and to provide an independent check on the Greenstein & Trimble results and to further discuss some of the kinematical properties of the white dwarfs.

2. THE RADIAL VELOCITY MEASUREMENTS

2.1 *Observations*

The spectra used for the present measurements are the same ones described earlier in Paper II. They were taken at Mount Stromlo Observatory with the image-tube spectrograph at the Cassegrain focus of the 74-in. telescope during the period of 1971 April to 1972 July. The original reciprocal dispersion on the plate was about 100 Å mm^{-1} and the projected slit width was 2·8 Å. All spectra were taken on baked Eastman Kodak IIa-O plates and were generally widened to 0·4 mm. The image-tube was of the Carnegie type and consequently one suffers a loss in resolution by a factor of about two as compared to the photographic plate.

2.2 *The plate measures*

The difficulties experienced in measuring white dwarf spectra for radial velocities have been discussed in detail by Greenstein & Trimble (1967) and need not be repeated here. For the present investigation, Hβ was the only line in the spectra of the DA stars which appeared suitable for measurement. Hγ and Hδ are affected by the strong Hg I night sky emission lines from nearby Canberra and cannot be considered reliable. In actual practice, the curvature of the spectra by the image-tube was found not to be important, for Hβ could be put near the centre of the spectrum where this effect was minimized and the continuum was not sloping greatly.

About 25 per cent of the plate measurements were done at Mount Stromlo and the rest at Oxford using identical setting devices described by Gollnow (1962) and Gollnow, Rudge & Thomas (1967) that work on the principle of Tompkins & Fred (1951) and are mounted on a Zeiss–Abbé-comparator. Experience with measurement showed that the performance of these machines compares favourably with the Grant setting device which is better known in astronomical applications. The measurements presented here refer to the intensity minimum of the measured line.

All spectra were measured at least twice—once each in the direct and reverse directions and the plate measurements were reduced to wavelength in the usual way by interpolating wavelengths linearly between two known comparison lines and constructing a correction curve. If the H and He comparison lines corresponding to the stellar lines in the DA and DB stars lay on the correction curve, the radial velocity was computed differentially from the displacement of the stellar and comparison lines. Otherwise, for a spurious comparison line or in the DF stars, the correction curve was used. Some checks can be made on the internal errors of measurement using the DA star material:

(a) Only a small systematic difference between direct and reverse plate measures of -1 ± 6 km s^{-1} was found.

(b) The setting accuracy as indicated by the standard deviation or individual settings on the Hβ line is about ± 1 μ which corresponds to a velocity error of ± 6 km s^{-1}.

(c) The standard deviation for stars having more than one available spectrum gives ± 27 km s^{-1} for one spectrum. This would indicate an internal mean error

TABLE I

Radial velocity measures for the 31 southern white dwarfs (in kilometres per second)

I	II	III	IV	V
2	DA	1	+118	−9
3	DA	1	−37	−8
4	DA	2	−9·5	−9
5	DA	1	+198	−10
6	DA	1	+107	−13
7	DB	1	+61	−14·5
8*	DB	5	+40	(−3)
9	DAe	1	+57	−18
10	DF	1	−10	−16
13	DA	1	−63	−14
15	DA	1	−140	−12
16	DF	1	+11	−12
17	DA	1	+56	−11
18*	DAwk	1	+422	−11
EG76*	DA	4	+15	−5
19	DA	1	+103	−9
21	DF	1	+55	−8
22	DA	1	−12	−5·5
23	DF	1	+90	−6
24	DA	2	+108	−4
EG99*	DA	3	+37·5	−5
26	DA	1	−63	−1
27	DA	2	+58	−8
28	DA-F	1	+75	0
29	DF	1	−2	−2
30*	DA	5	+1	(−5)
31*	DA	5	+43	(+10)
32*	DA	3	+51	(+7)
34	DB	1	−27	+1·5
37	DA	1	−57	−7
40	DF	1	−20	−5
41	DA	1	+199	−4

Notes to Table I

8: Companion to HR 2274.
18: Included only for record, excluded from the analysis.
EG76: L970–30, common proper motion companion to L970–27.
EG99: W485A, member of common proper motion system.
30: Companion to HR 5864.
31: Companion to HR 6094.
32: Companion to HR 6314.

of ± 21 km s^{-1} for a single plate, a value comparable to the internal mean error of ± 13 km s^{-1} found by Greenstein & Trimble (1967) for their Palomar prime-focus photographic spectra at a reciprocal dispersion of 190 Å mm^{-1}.

For the DB and DF stars, the errors are more difficult to estimate. Each line of He I in the DB spectra gives a different velocity, an effect attributable to differences in pressure shifts and the presence of forbidden components (Greenstein & Trimble 1967). Pressure shifts could also be important in the DF stars as was pointed out by Greenstein (1972) in his work on van Maanen 2.

3. THE MEASURED RADIAL VELOCITIES

Radial velocities for the southern white dwarfs, most of which are described in Paper II and have suitable spectra available are given in Table I. Column I gives the star's list number from Paper II, or EG number from Eggen & Greenstein (1965), column II the spectral type, column III the number of plates measured, and column IV, the radial velocity with respect to the Sun. Column V gives the radial velocity, in parentheses if known from a companion, or the component of the normal solar motion, $S \cos \lambda$ (Smart 1965) which should be subtracted from the radial velocity for the determination of the mean gravitational redshift described in Section 4.1. Since distances for only a small number of these stars are presently known, U, V, W, space motions are not given, but if desired, some of them can be obtained either from Eggen & Greenstein (1965) or Wegner (1973b).

4. THE MEAN GRAVITATIONAL REDSHIFT AND SPACE FUNCTIONS

4.1 *Determination of the* K-*term using an assumed apex*

Under the most simple assumptions, the Sun is considered to be moving through the stars with velocity S. Then the radial velocity W_r is given by the usual expression:

$$W_r = K - S \cos \lambda + W_r'. \tag{4.1}$$

where W_r' is the radial component of the star's individual peculiar motion, λ is the angular distance from the apex, and here the K-term is taken to represent the gravitational redshift. Following the approach of Greenstein & Trimble (1967), the normal solar motion ($A = 270°$, $D = +29°$, $S = 19{\cdot}6$ km s^{-1}) was assumed. If the peculiar motions are taken to be at random, the mean gravitational redshift is then derived. In Table II, the K-terms derived in this way for all white dwarfs taken together, the DA stars only, and the non-DA stars are compared for the present investigation and with the results of Trimble & Greenstein (1972).

Since the Trimble & Greenstein determinations of the K-term and the results

TABLE II

Values of K (km s^{-1}) determined from the assumed space motions

Star group	This investigation K	N	Greenstein & Trimble K	N
All stars	$+40 \pm 10$	31	$+51 \pm 5$	102
All DA's	$+43 \pm 14$	21	$+57 \pm 4$	83
Non-DA's	$+34 \pm 10$	10	—	—
DB's	—	—	$+28 \pm 20$	10
DO, DA-F, DF, DG	—	—	$+27 \pm 12$	9

of this investigation listed in Table II were made in the same way, they should be directly comparable. The derived K-terms for the present investigation are systematically lower than the Greenstein & Trimble results, but due to the smaller number of stars in this investigation, both determinations marginally agree within the probable errors.

4.2 *Determination of the solar motion*, K-*term, and velocity ellipsoid from the radial velocities*

Since the space motions for the white dwarfs as found from the proper motion studies by Pavlovskaya (1956) and Iwanowska & Opaska-Burnicka (1962) ($A = 283°$, $D = +47°$, $S = 45\ \mathrm{km\ s^{-1}}$) and from the space velocities (Trimble & Greenstein 1972; Gliese 1971) differ from the mean solar motion, it was decided to determine the K-term and the kinematical properties directly using the radial velocities alone, an approach which had not been tried up until now.

Determining the K-term and the kinematical parameters for the white dwarfs using only the radial velocities has the advantage that it avoids the need for a distance scale which is necessary for application of the methods involving proper motions or space velocities. Since only 40 of the Greenstein & Trimble stars and six of the present stars have known distances, this means that all available radial velocity measures can be utilized. In the following discussion, the notation and treatment of Nordström (1936) are used and the basic ideas need not be repeated. (See also; for example, Trumpler & Weaver 1953, Ogorodnikov 1965.) The equation of condition for the least squares determination of the space motion is:

$$W_r = K + \gamma_{13}U_0'' + \gamma_{23}V_0'' + \gamma_{33}W_0'' \tag{4.2}$$

where the γ_{j3} (j = 1, 2, 3) are the direction cosines. The equation of condition for the second order central moments of the velocity ellipsoid, N_{ijk}, is:

$$M_{002} = \gamma_{13}^2N_{200} + \gamma_{23}^2N_{020} + \gamma_{33}^2N_{002} + 2\gamma_{13}\gamma_{23}N_{110} + 2\gamma_{13}\gamma_{33}N_{101} + 2\gamma_{23}\gamma_{33}N_{001}, \tag{4.3}$$

where $M_{002} = W_r'^2$. Due to the smallness of the present sample of stars, the usual simplifying assumption that $N_{101} = N_{011} = 0$ was made which corresponds to two of the axes of the velocity ellipsoid lying in the galactic plane. Finally, the axes of the velocity ellipsoid were found from the three roots, t_k, of the usual cubic secular equation which has terms involving the N_{ijk} for coefficients so that $\sigma_k^2 = t_k$.

The calculations were carried out in galactic coordinates using a machine program with the stars grouped different ways: I, All southern stars and the Greenstein & Trimble stars combined; II, The Greenstein & Trimble (GT) stars separately; III, The southern white dwarfs separately; IV, All DA stars combined; V, The DA's of GT only; VI, Southern DA's only; and VII, All non-DA stars (*i.e.* those of spectral classes DO, DB, DA-F, DF, and DG). All stars have been given unit weight.

Due to the sparsity of objects in the region $0° > \delta > -30°$, a few additional stars for which Greenstein & Trimble (1967) measured, but gave low weight, were included.

The final results for the space motions and axes of the velocity ellipsoid are given in Table III. Errors refer to the mean error from the least squares determination and have not been corrected for the influence of the radial velocity errors.

Table III

Space motion and velocity ellipsoid data derived from the white dwarf radial velocities (in kilometres per second)

Group	Spectral class	N	K	U_0''	V_0''	W_0''
I	All stars	139	$+47\pm4$	-13 ± 7	$+8\pm8$	-2 ± 8
II	All GT stars	108	$+52\pm6$	-8 ± 8	-4 ± 12	-1 ± 8
IV	DA	110	$+52\pm5$	-14 ± 8	$+7\pm9$	-7 ± 9
V	GT DA's	89	$+63\pm7$	-6 ± 8	-16 ± 13	-8 ± 9
VII	Non-DA	29	$+25\pm16$	$+14\pm16$	-2 ± 13	$+14\pm14$
—	dM (Nordström 1936)	40	$-3{\cdot}1\pm5{\cdot}7$	-20 ± 11	-6 ± 13	$+3\pm11$

Group	σ_1	σ_2	σ_3
I	63 ± 8	29 ± 15	55 ± 9
II	55 ± 6	33 ± 9	39 ± 8
IV	67 ± 9	22 ± 22	59 ± 9
V	52 ± 6	34 ± 7	39 ± 8
VII	42 ± 10	24 ± 24	46 ± 10
—	$59{\cdot}9\pm9{\cdot}3$	$8{\cdot}4\pm9{\cdot}3$	$23{\cdot}3\pm9{\cdot}3$

After evaluating the coefficients in equation (4.3), it was found that N_{110} was smaller than its mean errors, so no information could be gained on the directions of the axes of the velocity ellipsoids other than that within the errors of the determination, the major axes point toward the galactic centre. Thus the coefficients N_{ijk} were found assuming that $N_{101} = N_{011} = N_{110} = 0$. Values of σ_j are nearly identical to those found assuming N_{110} to be non-zero. In light of its large errors, the finding that σ_2 is smaller than σ_1 or σ_3 is probably not real. Such a situation is not observed for ordinary stars and would imply that the motions of the white dwarfs are in a steady-state (Woolley 1960).

The results of the analysis using the method of least squares are somewhat disappointing because of the large errors in the coefficients, particularly for the smaller groups of southern stars. Consequently, groups III and VI were excluded from further consideration, but it might be worth mentioning that the derived K-terms were $+49$ km s^{-1} and $+54$ km s^{-1} respectively.

4.3 *The mean gravitational redshift*

Using the more complicated analysis, the values of K found in Table II are confirmed, but with slightly different numerical values. It appears significant that the K-term for the DA stars in group IV ($K = +52\pm5$ km s^{-1}) is larger than for the non-DA stars in group VII ($K = +25\pm8$ km s^{-1}). Just what is the intrinsic distribution of gravitational redshifts for each of these groups and how the derived space motions are affected by a variation in the K-term from star to star cannot now be answered. The range in gravitational redshifts for stars where the K-term is known is about ±20 km s^{-1} with large uncertainty (Trimble & Greenstein 1972; Wegner 1973b), while Weidemann (1971), on the other hand, has given evidence that for the DA stars, the gravities lie in the narrow range of $\log g = 7{\cdot}90\pm0{\cdot}15$, corresponding to a velocity of $+25^{+7}_{-5}$ km s^{-1} on the assumption of helium Hamada–Salpeter (1961) configurations.

That the K-term for the northern stars is systematically larger than for the southern stars is also seen by comparing its derived values for groups I and II (all stars) and IV and V (DA stars). In both cases, the northern stars alone give systematically larger K-terms. To see how sensitive this result is to systematic errors in the velocities of the southern and northern white dwarfs, an additional solution was computed for all of the DA stars with the systematic correction of $+14$ km s^{-1} indicated in Table II added to the observed radial velocities of the southern white dwarfs. In this case, $K = +56 \pm 5$ km s^{-1} and the corresponding kinematical parameters are only altered slightly.

If significant deviations from a random velocity distribution exist for the white dwarfs near the Sun, errors could be produced in the derived K-term. Such an effect would complicate a simple explanation of any discrepancies in the observed and theoretical values of the K-term as due to pressure or magnetic field shifts and should not be ignored in future investigations. The expansion of the B stars has been discussed by numerous investigators (for example, Malmquist 1928) and is of the correct order of magnitude. Although this same effect is probably not an explanation for a large K-term in the white dwarfs, it shows that significant deviations from the true gravitational redshift might be produced by the stars' kinematics. If a number of the white dwarfs belong to moving groups, the resulting K-term might be influenced.

Several white dwarfs have been identified as members of moving groups (see Eggen & Greenstein 1965) and one such star stream containing high velocity white dwarfs is the W219 group, mentioned in Paper II. There is evidence that this group may contain a number of degenerate stars. Although most identified members are of the C2 variety, one star of spectral class DA is a possible member. For LFT 1503 (EG135), the observed proper motion given by Luyten (1955) is $\mu = 0''\cdot 84$ and $\theta = 213°$, while to be a group member, $\theta_{\rm calc} = 210°$. The predicted radial velocity of the star using the observed parallax of $+0''\cdot 076$ W(7) is $4\cdot 74\ \mu/\pi \sin \lambda = +57$ km s^{-1}. This is probably not contradicted by the observed value of $W_{\rm r} \approx +81-51 = +31$ km s^{-1} which is given by Greenstein & Trimble (1967) as being of low weight. Clube & Jones (1971) have further shown that such deviations from the usual assumptions of the Bravais method may influence the derived kinematical parameters.

4.4 *Discussion of the kinematical properties of the white dwarfs*

While earlier results for the space motions and velocity dispersions of the white dwarfs taken as a whole and the DA white dwarfs seem to be confirmed within the observational uncertainties, the new result of this investigation comes from the addition of enough southern non-DA white dwarfs that can be combined with the northern stars of this type so that a comparison of the kinematical parameters for the DA stars and the non-DA stars can be attempted.

The influence of selection effects on the results of any investigation on the white dwarfs is difficult to assess. Since the present sample of white dwarfs was found from its large proper motions, the high space motion stars are expected to have too great a weight. However, both classes of white dwarfs were found this way, so the present results are expected to reveal any differential changes in the velocity dispersions. The derived values of the space motions and velocity dispersions for the white dwarfs can be compared with Nordström's (1936) determination for the dM stars which is also given in Table III. Since these results are based on stars also found from proper motions, it is thought that they should be more directly comparable than more modern results which have been corrected for selection effects.

Johnson (1957) and Gliese (1971) pointed out that the kinematical properties of the white dwarfs and dM stars are similar. This conclusion seems to be borne out by all groups within the errors of the determination, but for the non-DA stars in group VII, the velocity dispersions seem to be somewhat lower. Because of the small number of stars used, this result should be considered only preliminary. However, the σ_j values for group I (all stars) are slightly less than for group IV (all DA's) and this might reflect the influence of the non-DA stars as well.

From the standpoint of their atmospheric abundances, it seems reasonable to consider the DA and non-DA stars as different physical groups. Bues (1970) and Strittmatter & Wickramasinghe (1971) find that the DB stars are both hydrogen and metal poor, while for the DF and DG stars similar compositions have been found (Wegner 1972). As well, Shipman (1972) finds the DA-F stars could be under-abundant in metals by a factor of up to 100. For the DA stars, on the other hand, most investigators find that hydrogen-rich models with a solar metal abundance satisfactorily reproduce the observed spectra.

Recent calculations on the hydrogen-poor white dwarfs by Bues (1973) have supported the suggestion that all non-DA stars may form cooling sequences that are different from that of the hydrogen-rich DA white dwarfs. Strittmatter & Wickramasinghe (1971) and Shipman (1972), however, have advanced the idea that all helium-rich white dwarfs evolve from the DA stars due to the depletion of hydrogen as a consequence of convective mixing. If future observations can verify the result suggested by the present study that the velocity dispersions of the hydrogen-poor and hydrogen-rich white dwarfs differ, much information regarding the formation of the white dwarfs could be gained. A smaller velocity dispersion for the non-DA's would indicate more massive parent stars and would have important astrophysical implications.

5. CONCLUSIONS

In this investigation, the following points about the white dwarfs arose:

1. The addition of new radial velocities for the southern white dwarfs to the body of data published by Greenstein & Trimble confirms the existence of a positive K-term for the white dwarfs. It appears, however, that the Greenstein & Trimble determination for the northern white dwarfs gives K slightly larger than that of the southern white dwarfs. Unfortunately, no stars were observed in common. In the GT investigations, the higher Balmer lines were measured and these lines are probably blended with each other. The influence of overlapping lines on the radial velocities of spectroscopic binaries is well known (Petrie, Andrews & Scarfe 1967) and from this standpoint, Hβ is a more suitable line to measure.

2. The large difference in the K-term for the DA and non-DA stars is confirmed. This may be a consequence of the presence of pressure shifts and forbidden components in the spectral lines of non-DA stars.

3. The dependence of the mean mass of the white dwarfs on the adopted temperature scale and measured value of K has been discussed in detail by Trimble & Greenstein (1972). If the mean radius is taken from their discussion to be given by $\log \overline{R/R_\odot} = -1.85$ or -2.0, for the southern DA stars, $\overline{M/M_\odot}$ is about 0·9 to 0·7 respectively. If the southern DA stars are assumed to obey the Hamada & Salpeter (1961) mass–radius relations for He^4 or Fe^{56}, $\overline{M/M_\odot}$ is 0·7 or 0·6 respectively.

4. Kinematical properties of the DA and non-DA white dwarfs found in this investigation suggests that the non-DA stars have lower velocity dispersions, while for all white dwarfs, the velocity dispersions are higher than would be expected if stars of all masses evolving off the Main Sequence become white dwarfs.

The time in which a broad knowledge of the kinematical properties of the white dwarfs is available is still not on hand. Further observations of a large number of these stars particularly of the hydrogen-poor varieties should be undertaken to answer some of the questions regarding their evolution and dynamics which could only be touched upon in this paper. Theoretical and laboratory studies of line broadening and pressure shift effects for white dwarfs are needed. In light of the complications involved in determining the *K*-term statistically, it appears that studying more binary pairs which have a white dwarf component would be a good approach to some aspects of this problem.

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