A spot market model for pricing derivatives in electricity markets

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Abstract.   In this article, we analyze the evolution of prices in deregulated electricity markets. We present a general model that simultaneously takes into account the following features: seasonal patterns, price spikes, mean reversion, price dependent volatilities and long-term non-stationarity. We estimate the parameters of the model using historical data from the European Energy Exchange. Finally, it is demonstrated how it can be used for pricing derivatives via Monte Carlo simulation.

1. Introduction

Contracts between electric utilities typically offer a substantial amount of flexibility in the form of complex embedded options. Demand for such optionalities arises naturally from the unpredictability of power consumption and from the optionalities inherent in power plants. In the past, there rarely was the necessity to precisely evaluate the value of these optional parts, because electricity was not a commodity which could easily be traded, and because supply of electric power was assured by utility companies under regulatory control. In fact, most counterparts did not use the flexibility of the delivery contracts in a market-orientated way. In recent years, these matters have changed dramatically. In many countries electric power markets have been liberalized and exchanges and online trading platforms for electricity contracts have been founded. Market participants now take advantage of the optionality in their contracts by optimizing against market prices and looking for arbitrage opportunities. Therefore, it has become an important task for utilities to develop new pricing models for the contracts they buy and sell and to quantify and manage the involved risks.

As an example, assume that an electric utility needs additional power at times of high demand for the first 6 months of a year. Since the utility does not know exactly when the load will be high (as it depends on uncertain factors, such as weather conditions), it signs an optional contract. One possibility is, that the utility simply buys a portfolio of call options giving the right, but not the obligation, to buy electricity each hour within the delivery period with a capacity of up to 100 MW at a fixed price of 30 EUR/MWh. This option can be viewed as a cap on an hourly power price. Another, less expensive, possibility is the purchase of a swing option. This is a contract with delivery of a certain amount of commodity on dates in the future at a stipulated constant price. The delivery dates can be nominated at short notice by the buyer within a given delivery period. In our example we assume that the utility buys a swing option, which gives the right and the obligation, to buy electricity with a maximum capacity of 100 MW, and an energy amount of 100 GWh at a fixed price, whereby the delivery may be spread over the contract period of the first half of one year. For swing options the fixed price is often specified in a way that no up front fee for the option is necessary.

To determine the fair premium for the hourly cap in the first example, or the fair fixed price for the swing option in the second example, it is necessary to analyze the traded products in the electricity markets. Commonly traded products in the electric power markets are baseload, peakload and hourly contracts. Baseload means supply with a constant capacity during the delivery period. Peakload means supply with a constant capacity during those fixed hours of a week when load usually is high. At the European Energy Exchange (EEX) for example the delivery times for peakload are defined as weekdays between 8.00 a.m. and 8.00 p.m. In the futures market contracts on both baseload and peakload are traded. The usual delivery periods are one month, one quarter and one year. In the spot market of the EEX baseload, peakload and hourly contracts up to the next weekday are traded. The underlying of the swing option and the hourly cap in our example is the hourly spot market price.

To understand the behavior of electricity prices, we have to note that electricity is scarcely storable. In most countries there are only a few reservoir power plants, and using pumps results in a loss of approximately 30% of the energy. The consequences of this non-storability can hardly be overestimated. One implication is that the relation between spot prices and futures prices cannot be described with cost of carry. The most evident result however are the enormous price fluctuations that can be observed in all electricity markets.

As shown in figure 1, those price fluctuations have a strong daily, weekly and yearly periodicity. This can be explained from a microeconomic viewpoint by looking at the market price of electricity as an equilibrium price based on supply and demand curves. Since the demand is very inelastic, the marginal costs of the supply side (described in the so called merit order curve) determine the price to a large extend. If the total load is low, the plants with the lowest variable production costs are used, if the total load is high, gas or oil fired plants with high fuel costs are additionally running. The periodicity of the total load is responsible for the periodicity of the electricity prices.
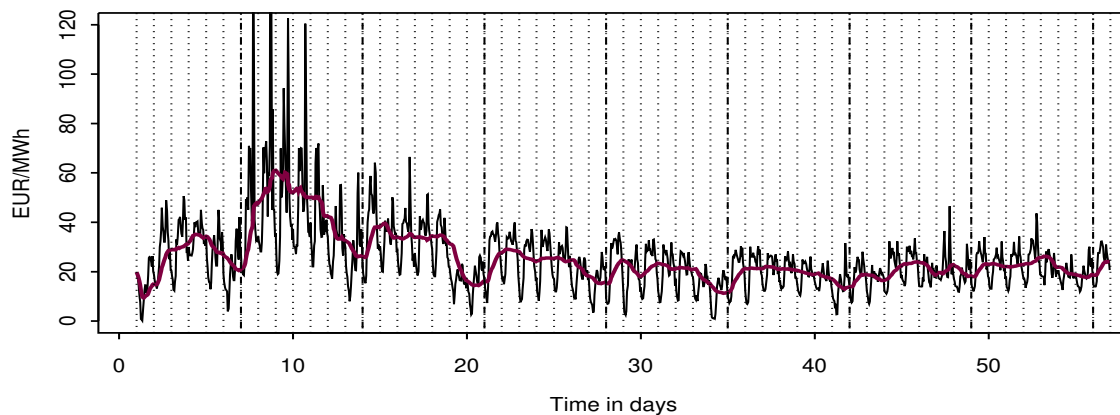
Figure 1. EEX spot prices and 24 h moving average, January 1 - February 25, 2002

The total load has a random component, depending on short term weather conditions and other uncertain parameters, but it also has a clear predictable part, and so do the prices for electricity. Most utilities have extensive data and expert knowledge to estimate the total load and therefore a good understanding for the predictable part of power prices.

It is the aim of this paper to build on these traditional methods and to combine them with modern approaches from financial mathematics. We develop a stochastic model for spot market prices of electricity that is successfully used by EnBW, Germany's third largest energy company, to price complex options such as those described in the examples above. Since we think that it is not possible to find a realistic model that allows for explicit formulas for the value of e.g. swing options, pricing algorithms have to be based on numerical methods and Monte Carlo simulation. Thus, an important requirement for the model is the availability of fast simulation schemes.

The model we describe in the subsequent sections will capture the following features observed in all known electricity markets (see e.g. [20] for a comparative study of several markets):

- **Seasonal patterns and periodicities.** All markets show seasonal patterns of electricity demand over the course of the day, week and year. These seasonal patterns are carried over to the electricity prices via the merit order curve and can therefore be fundamentally explained.

- **Price spikes.** Electricity spot prices typically exhibit extreme spikes, which are not consistent with the usual modelling via diffusion processes with normal or log-normal marginal distributions.

- **Mean reversion.** Prices have the tendency to revert rapidly from price spikes to a mean level. Characteristic times of mean reversion have a magnitude of days or at most weeks and can be explained with changes of weather conditions or recovery from power plant outages.

- **Price dependent volatilities.** It turns out that in all markets there is a strong correlation between price levels and levels of volatility.

- **Long-term nonstationarity.** Due to the increasing uncertainty about factors such as supply and demand or fuel costs in the long-term future, a nonstationary model seems more appropriate. Nonstationarity is also needed for a model to be consistent with the observed dynamics of futures prices.

The paper is organized as follows. In section 2 we give a short review of literature on models for electricity prices and compare some of the existing models with our approach. A detailed description of our model is given in section 3. The following section 4 is devoted to the statistical analysis of the data to justify our model selection. Especially, the so called price load curve is estimated and seasonal time series models are fitted to the market process and to the load process, and it is demonstrated that these models are adequate for our purposes. In section 5 we show how the long term dynamics of the price process can be estimated from futures prices using Kalman filter techniques. Finally, in section 6 some simulation results are described and it is shown how our model can be used for the pricing of derivatives.

2. A review of models for electricity prices

In the last few years there has been a rapidly increasing literature on stochastic models for prices of electricity and other commodities. Many researchers have observed that the models typically used in financial markets are inappropriate due to the special features of commodity prices and especially of electricity prices as described in the introduction. In this section we will give a short review of some of the models considered so far in the literature and compare them with our approach.

The choice of stochastic model for electricity prices depends on the time granularity that needs to be reflected in the model. Liquidly traded futures and forward contracts typically have full months, quarters or years as delivery periods, either as baseload or peakload. Price quotes for single hour deliveries are in most cases only available as day-ahead prices from the spot market. However, many structured OTC products, such as swing options, are strongly influenced by the hourly price behavior. Since due to the non-storability of electricity, spot products cannot be used for hedging purposes, the electricity market is a highly incomplete market and pure arbitrage option pricing methods fail for most structured products. Previous work has been focused mainly on either of the two following approaches:

(i) Market models for futures prices: Instead of modelling the spot price and deriving futures prices, the futures prices themselves are modelled. This approach goes back to Black's model [3], where a single futures contract is considered. Ideas from the Heath-Jarrow-Morton theory for interest rates ([17]) are used in [6], [7], [23] and [27] to model the dynamics of the whole futures price curve. Such models have the advantage that the market can be considered as being complete and standard

risk-neutral pricing may be used. Risk-neutral parameters can often be implied from traded options on futures prices. The disadvantage of such approaches is that futures prices do not reveal information about price behavior on an hourly or even daily time scale.

(ii) Spot price models: This class of models aim at capturing the hourly price behavior by fitting their model to historical spot price data. Since there is no arbitrage relation between spot prices and futures prices, additional assumptions have to be made to use this model for pricing derivatives. Usually this is done either by assuming the rational expectation hypothesis

$$F_{t,T} = \mathbb{E}[S_T \,|\, \mathcal{F}_t],$$

as done e. g. in [32] to price generation assets, or by calibrating a market price of risk for each risk factor and then changing to an equivalent martingale measure P^* under which the relation $F_{t,T} = \mathbb{E}^*[S_T \,|\, \mathcal{F}_t]$ holds.

Most models for the spot market employ at least two risk factors: one factor capturing the short-term hourly price dynamics characterized by mean reversion and very high volatility, and the other factor representing long-term price behavior observed in the futures market. Since there are no liquidly traded derivatives on a daily or hourly time scale that have a strong dependence on the short-term risk factor, it is very difficult to estimate the short-term market price of risk. In [14] the authors analyze the differences between day-ahead and real-time prices at the PJM western hub and conclude that a short-term risk premium can be observed. Similar results were obtained in [25]. Most derivatives, however, refer to the liquid day-ahead market as underlying and not to the often less liquid real-time market. Furthermore, in most European markets the real-time market is a pure OTC market, for which no regular price information exist.

The difficulties to estimate risk premia for two-factor models from historical commodity price data are well known (see e. g. [26] and [29]). Therefore, most models make certain additional assumptions. Common approaches are to assume a zero market price of risk for certain (non-hedgeable) short-term risk factors, such as jumps ([8], [9]) or to calibrate first with respect to the statistical measure using spot price data and then to imply a risk premium or risk-neutral drift from the futures market ([8], [26]). An economic reason why the statistical measure may be used for derivatives valuation in certain cases is given in [19], chap. 29 and [31]. In this paper we take a similar approach as [8] by first calibrating the statistical process parameters to the spot market and afterwards implying the risk-neutral drift and the volatility for the hedgeable long-term process from the futures market. For the non-hedgeable short-term risk factor we assume a zero market price of risk. In this way we have chosen a measure under which all liquidly traded products, which are just the futures contracts and some options on futures, are martingales and model prices for those products are consistent with market prices. When applied to non liquid options on an hourly or daily time scale, the model yields prices that are arbitrage free but not unique with respect to the choice

of martingale measure. In practice, an additional risk premium will be charged for the residual risks that cannot be hedged.

Throughout the paper we will denote by S_t the spot market price at time t. Since we are working in a deterministic interest rate framework, we will not distinguish between forward and futures prices. Therefore, single hour futures prices at time t for delivery at time T are conditional expectations under the equivalent martingale measure

$$F_{t,T} = \mathbb{E}^*[S_T \,|\, \mathcal{F}_t],$$

where $\mathcal{F}_t = \sigma(S_s : s \leq t)$ is the natural filtration generated by the price process.

Future prices for power delivery over a period $[T_1, T_2]$ are given by

$$F_{t,T_1,T_2} = \mathbb{E}^* \left[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S_T \, dT \,\bigg|\, \mathcal{F}_t \right] = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} F_{t,T} \, dT, \tag{1}$$

or, in a discrete time setting, by

$$F_{t,T_1,T_2} = \mathbb{E}^* \left[\frac{1}{T_2 - T_1} \sum_{T=T_1}^{T_2-1} S_T \,\bigg|\, \mathcal{F}_t \right] = \frac{1}{T_2 - T_1} \sum_{T=T_1}^{T_2-1} F_{t,T}. \tag{2}$$

The simplest model taking into account mean-reverting behavior is given by an Ornstein-Uhlenbeck process. Here the price process S_t is a diffusion process satisfying the stochastic differential equation

$$dS_t = -\lambda(S_t - a)dt + \sigma dW_t, \tag{3}$$

where (W_t) is a standard Brownian motion, σ the volatility of the process, and λ the velocity with which the process reverts to its long term mean a.

In electricity markets prices show strongly mean reverting behavior so that estimates for λ are quite large. Typical characteristic times for mean reversion are within a few days. Therefore this model has the major drawback that futures prices are nearly constant over time, since under the assumption of (3) the futures price is given by

$$F_{t,T} = a(1 - e^{-\lambda(T-t)}) + S_t e^{-\lambda(T-t)}.$$

For this reason several authors suggest a two factor model, see e.g. [15], [28] and [29]. In [28] a model of the form

$$dS_t = -\lambda(S_t - Y_t)dt + \sigma dW_t \tag{4}$$

is suggested, where Y_t is a Brownian motion. A similar model is given in [29], where commodity prices are described in the form

$$S_t = \exp(X_t + Y_t), \tag{5}$$

where (X_t) is an Ornstein-Uhlenbeck process responsible for the short-term variation and (Y_t) is a Brownian motion describing the long-term dynamics. The model we will introduce in (6) can be considered as an extension of the ideas of [29].

All models considered so far did not take into account seasonalities. Some authors simply neglect this serious difficulty. Others propose to use deterministic seasonalities

described by sinusoidal functions, see [1], [10], [13] and [28]. In [22] it is suggested to use equation (3) with a long run mean a_t describing the seasonal patterns. A general deterministic seasonality is proposed in [11] and [26]. Here, the spot price is modelled as

$$S_t = f(t) + X_t \quad \text{or} \quad S_t = \exp(f(t) + X_t)$$

with an arbitrary deterministic function $f(t)$ and a mean-reverting stochastic process X_t. In our approach, the deterministic component $f(t)$ is specified by the load forecast ℓ_t and additional stochastic behavior is introduced by the use of SARIMA models for the time series of load and prices.

There are also different attempts to account for price spikes. One possibility to cope with spikes is the introduction of jump terms, see [7], [11] and [20]. The main criticism for these models is that under the typical assumption of a jump-diffusion model a large upward jump is not necessarily followed by a large downward jump. Therefore some authors suggest hidden Markov models, also known as Markovian regime-switching models, where it is guaranteed that upward jumps are followed by downward jumps. Such models have been considered e.g. in [8], [9], [10], [18] and [21]. Regime-switching models are very intuitive candidates for electricity price models, since there are some clear physical reasons for switches of regimes such as forced outages of important power plants. On the other hand, it seems to be difficult to combine regime-switching with seasonalities.

Another approach is motivated by the economic background for price spikes. Prices are determined mainly by supply and demand (load). Therefore the non-linear relation between load and price should be taken into account in the model. This non-linear transformation is called the 'power stack function' in [12] and [30]. They suggest an exponential function for that purpose. A similar model for spot prices has recently been considered in [1], where the relation

$$S_t = f(X_t)$$

is suggested with X_t being an Ornstein-Uhlenbeck process, and f a power function. We also prefer an approach based on the power stack function, since there is a natural interpretation of this non-linear transform in terms of the merit order curve.

3. Description of the SMaPS model

This section is devoted to the description of our model for **S**pot **Ma**rket **P**rice **S**imulation (abbreviated as SMaPS-model). It is a general model for an electricity spot market that captures the typical features mentioned in the introduction. In this paper we calibrate it to data of the European Energy Exchange EEX and to the Spanish energy exchange Omel, but this can easily be adapted to any other market.

The spot market of the EEX is a so called 'day ahead market' where as finest granularity hourly power contracts for the 24 hours of the following day are traded. In

a closed auction a single price for the whole of Germany is determined. One possible approach used in [30] is a process with days as a time unit and with 24-dimensional data. To eliminate the weekly seasonality [30] only consider weekdays and use a principal component analysis to reduce the dimensionality. The alternative that we suppose here is the use of seasonal time series with hours as time units but with a seasonality of 24 hours as a compromise to keep the model simple but still realistic.

Therefore we describe the spot market price by a discrete time stochastic process $\{S_t, t = 0, 1, ...\}$ with hours as time units. The full model can be considered as a three factor model, based on the stochastic processes

- load process $(L_t)_{t \geq 0}$,
- short term process $(X_t)_{t \geq 0}$,
- long term process $(Y_t)_{t \geq 0}$,

and the following additional quantities

- (logarithmic) price load curves $f(t, \cdot)$, $t \geq 0$,
- the average relative availability of power plants $(v_t)_{t \geq 0}$.

The fundamental equation of our SMaPS-model can be written as

$$S_t = \exp(f(t, L_t/v_t) + X_t + Y_t), \quad t = 0, 1, 2, \ldots. \tag{6}$$

In the following we will describe the meaning of the quantities in this equation, and give reasons why we have chosen this approach.

The two factors (L_t) and (X_t) produce the short term variation of the price behavior, whereas the factor (Y_t) is responsible for the long term variation. All three stochastic factors are assumed to be stochastically independent.

The process (L_t) describes the electricity load. Most participants of the market can observe the load (which is equivalent to the demand) directly at least for a certain region, and therefore it can be assumed that they have good estimates for the load of the whole market. Consequently, the parameters of this stochastic process can be estimated directly from load data, independent of the spot market prices. We will use an approach

$$L_t = \ell_t + L'_t,$$

where ℓ_t is a deterministic load forecast and (L'_t) is a SARIMA time series model with a 24 hours seasonality.

The deterministic function v_t, $t \geq 0$, specifies the expected relative availability of power plants. In Germany, for example, maintenance of power plants is mainly done in summer, when the average load is much lower than in winter. Hence the availability of the power plants is higher in winter compared to summer. The highest availability (which is in January in Germany) is set to one. The expression L_t/v_t will be called the *adjusted load*. The statistical analysis in section 4 shows that the use of this adjusted load leads to a more realistic model.

The function $f : \mathbb{Z}_+ \times [0, \infty) \to \mathbb{R}$ depends on the actual time t and on the adjusted load L_t/v_t. It describes the non-linear relation between price and load. In technical terms, this relation is described by the so called merit order curve. However, the merit order curve at a future point of time t depends on many uncertain parameters like fuel costs, economic situation etc. Therefore we do not use a physical merit order curve in our model. Instead, we use an empirical function estimated from hourly price and load data. We will call this empirical function the *price load curve*, abbreviated as *PLC*. As will be shown in section 4, this PLC depends to some extent on weekday and daytime. Therefore the first argument of f is the time t, which enables us to use different empirical PLC's at different weekdays and daytimes. In practice, one will only use a few different PLC's differing, for example, between workdays and holidays and between peak hours and off-peak hours. This will be discussed in detail in section 4.

Summing up, the expression $f(t, L_t/v_t)$ can be considered as the component of the price which is determined by the load and the technicalities of producing electricity. However, in addition to these technical aspects there are other determinants of the price like psychological aspects of the behavior of speculators and other influences. These aspects are specified by the processes (X_t) and (Y_t).

The process (X_t) describes the residual short term market fluctuations. They are mainly due to the 'psychology of the market', as it occurs in any financial market. We mention, however, that this process also includes other risks like outages of power plants etc., for which it is too difficult to get reliable data. By definition of f as an empirically estimated price load curve, the residual process (X_t) and the load process (L_t) are uncorrelated. Therefore it is natural to assume (X_t) and (L_t) to be stochastically independent. This assumption is supported by the statistical analysis at the end of section 4. This analysis also shows that (X_t) still exhibits a 24 hours seasonality. One reason for that could be the fact that the 24 prices of the following day are settled simultaneously in one auction. Therefore, we use for (X_t) also a SARIMA time series model with a 24 hours lag.

As the futures are of a stochastic nature, too, we introduce a stochastic process (Y_t) responsible for the variation of the futures prices. The process (Y_t) will follow a random walk with drift. The term $\exp(Y_t)$ describes the long-term variation of the prices, which can not be estimated from the historical data of the spot market (the EEX, for example, only exists for about 2 years). The parameters of this process have to be estimated from data on futures, see section 5 for a detailed discussion of that topic.

4. Statistical analysis: Model selection and model fit

In this section, we discuss in more detail the statistical analysis of that parts of the SMaPS model that are related to the short term behavior. Seasonal time series models are fitted to the short term process as well as to the consumption or load process. Further, we show some results of model diagnostics indicating that the chosen models are adequate for our purposes.
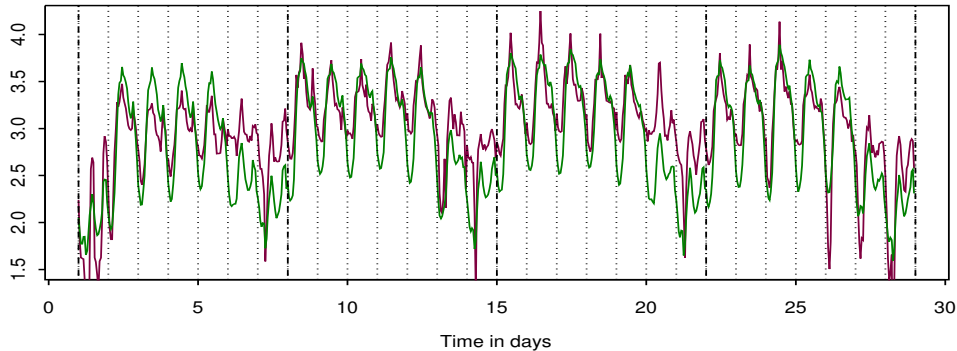
Figure 2. Logarithm of EEX spot price (red) and electric power consumption in Germany (green), April 1-28, 2002

We assume that the historical spot market process S_t satisfies equation (6) with $Y_t \equiv 0$. A justification for this assumption will be given in section 5. Hence, after specifying f in subsection 4.1, the short term process is given by

$$X_t = \ln S_t - f(t, L_t/v_t).$$

Model selection and parameter estimation for X_t are described in subsection 4.2. Since it is shown at the end of section 5 that X_t and the long term process Y_t can be fitted separately, these parameter estimates can also be used in the simulation model. Subsection 4.3 deals with the load process.

Figure 2 shows the time series (in red) of the logarithm of the hourly EEX spot prices over a period of four weeks starting April 1, 2002. The green curve is the time series of the electric power consumption in Germany (unit MWh, divided by 20000, to obtain a comparable order of magnitude).

Obviously, there is a very strong correlation between spot price and electric power consumption, and any serious model should take advantage of this fact. The low price and load on April 1, 2002, are due to the fact that Easter Monday is a public holiday in Germany.

For all statistical analyzes in the following two sections, we used EEX prices from July 1, 2001 until June 30, 2002. However, we should point out that using other time periods yields qualitatively similar results.

4.1. The empirical price load curve

This section treats the estimation of the (historical) price load curve (PLC) $f(t, x)$ which is a basic part of our spot price model. Figure 3 (left) shows a scatter plot of EEX spot prices S_t subject to the electric load L_t at time t. In red, a cubic spline is fitted to the data. We will term this curve as empirical PLC to draw a distinction to the physical merit order curve (also called marginal costs, plotted in green). The latter curve lies well below the empirical PLC.
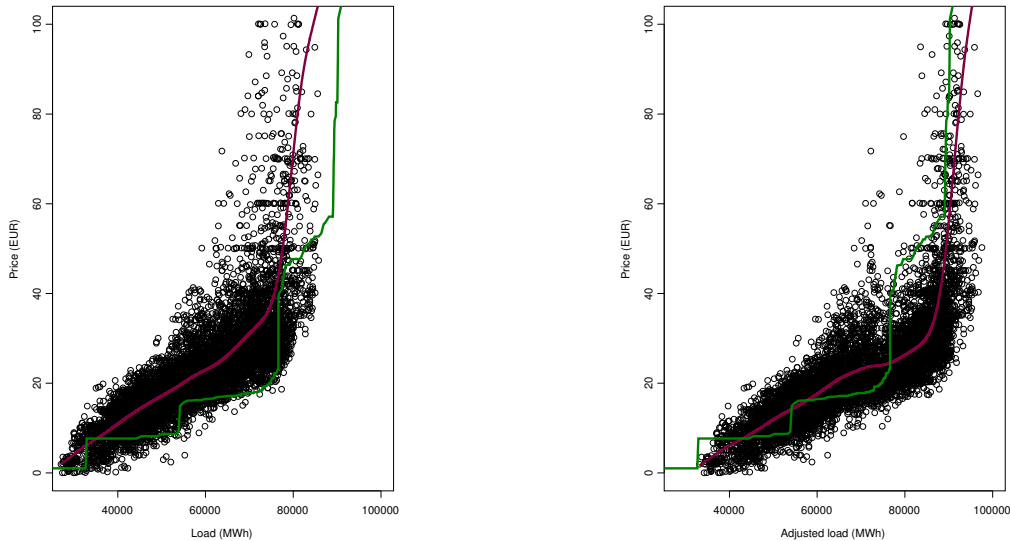
Figure 3. Left: Empirical price load curve (red), together with merit order curve (green). Right: Empirical PLC adjusted by average availability

In figure 3 (right), we used the adjusted load L_t/v_t instead of L_t (see section 3), giving better agreement with the empirical PLC. Since a dependence of spot prices on average availability is quite natural, we always use the adjusted load in our SMaPS model.

Furthermore, the shape of the PLC is influenced to some extend by the time of the day and the season. Hence, our model allows for the dependence of $f(t,x)$ on t. In practice, we distinguish simply between on-peak and off-peak hours. The price load curves for these two periods are shown in figure 4.

In our model, we fitted the PLC f to the logarithm of EEX prices. Then, $f(t,x)$ can be seen as expected value of the logarithm of spot prices at a given load x and time t, and the empirical mean of the short term process X_t should be approximately zero. We observed a mean value of $2 \cdot 10^{-8}$ and a sample standard deviation of around 0.34; hence, when modelling the short term process in the next section, it is assumed that X_t is a centered process.

Figure 5 shows the price load curve of the Spanish power market (Omel) (data from July 1 to December 31, 2001) which shows similar features as the corresponding curve of the German market.

4.2. Analysis of the short term process

After having specified f, we can look for suitable models describing the short term process $X_t = \ln S_t - f(t, L_t/v_t)$. Figure 6 shows the time series of X_t for a period of eight weeks starting April 1, 2002.

Compared with the original price process, X_t looks quite irregular; daily or weekly
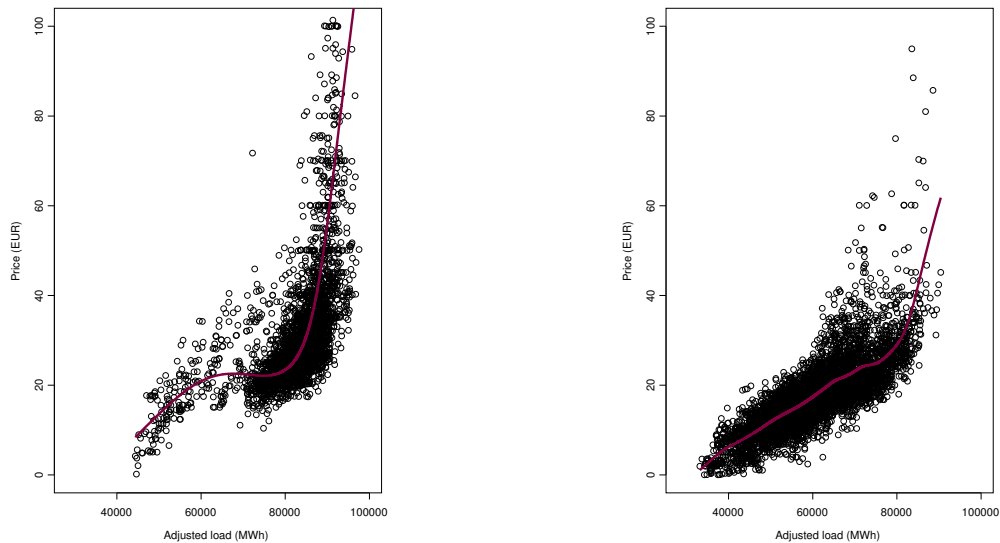
Figure 4. Empirical PLC at on-peak times (left) and off-peak times (right)
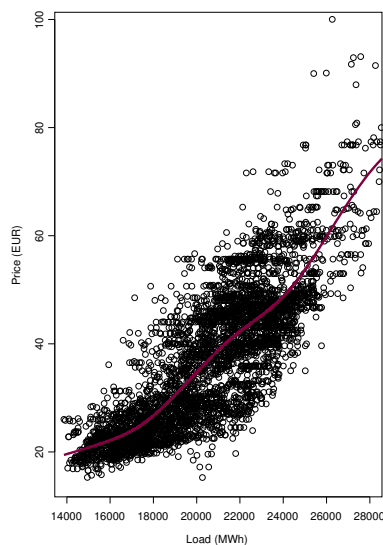

Figure 5. Empirical PLC of the Spanish power market

periods are hardly to detect. However, looking at the empirical autocorrelation function, we can still see an increased dependence at lag 24 hours (figure 7).

Figure 8 contrasts the base price (daily average price) of the short term process (in red) with the mean adjusted logarithm of the EEX base price (green) from 1/1/2002 until 6/30/2002. Figure 9 shows the seven day moving averages of the same processes.

This comparison makes clear the advantages of modelling the short term process using the PLC: price fluctuations are strongly reduced; daily or weekly periods are no more observable (even the yearly period should be eliminated). There remain only price
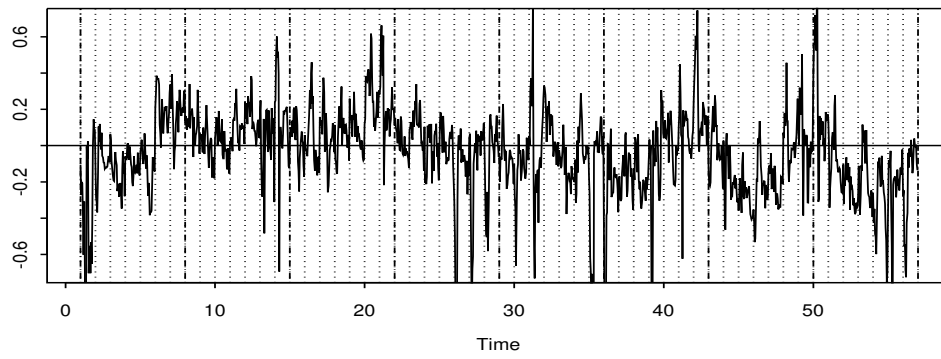
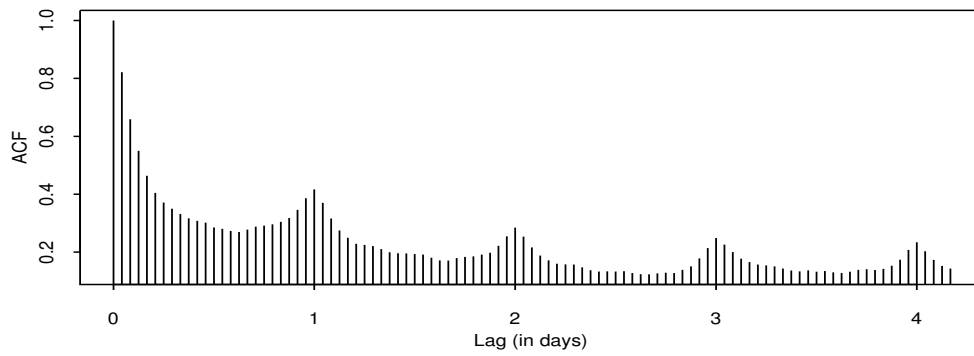**Figure 6.** Short term process X_t over an eight week period

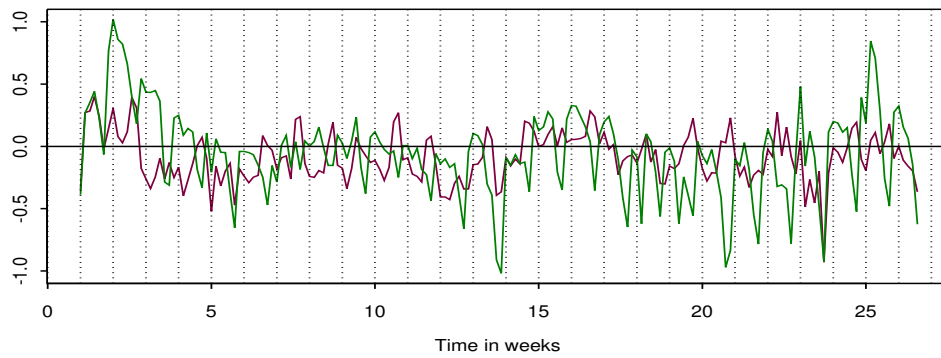**Figure 7.** Empirical autocorrelation function of the short term process X_t

Figure 8. Base price of X_t (red) compared with EEX base price (green) over a 26 week period
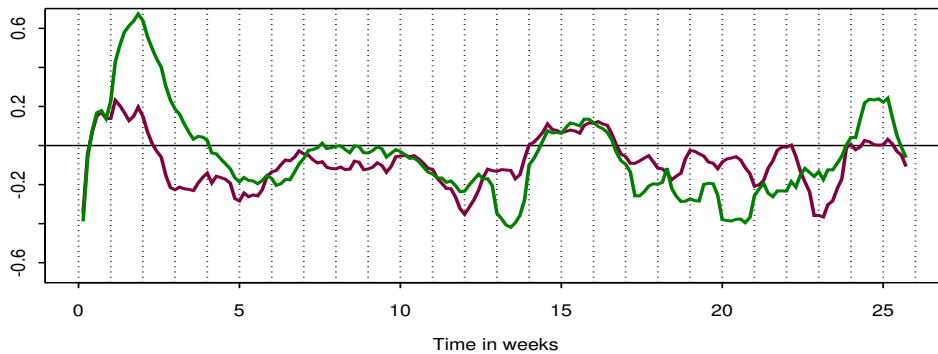
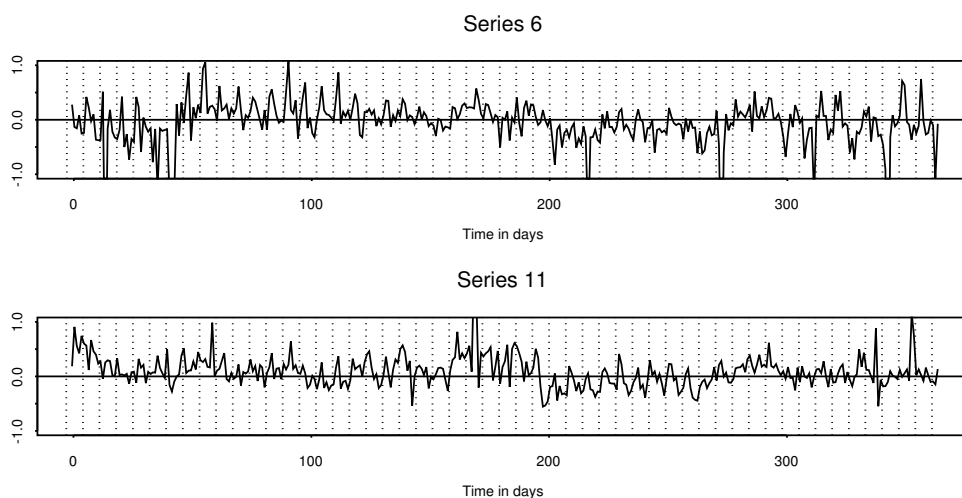**Figure 9.** Seven day moving averages of the base price of X_t (red) and EEX base (green)

Figure 10. Hourly time series 6 and 11 of short term process

movements due to the general economic conditions and market psychology.

Due to the correlations at lag 24 described above, we have chosen to model X_t as a seasonal ARIMA model with a 24 hour 'season' (see [5], p. 310, or [4], p. 300).

Since the short term process and the corresponding SARIMA model represent the core of our model, we analyze both in more detail.

To model the whole time series as SARIMA model, a certain similarity of the individual seasonal partial time series is necessary. Specifically for the short term process, this means that the hourly time series (Series 1: Hour between 0 and 1, Series 2: Hour between 1 and 2,...) can be modelled as ARIMA models with comparable parameter values.

Figure 10 shows the time series 6 and 11. Visually, one cannot see any fundamental differences, for instance in volatility. The same holds for the other hourly series.

The pertaining empirical autocorrelation functions decay fast and do not show additional distinctive features so that modelling the hourly time series as $ARMA(p,q)$
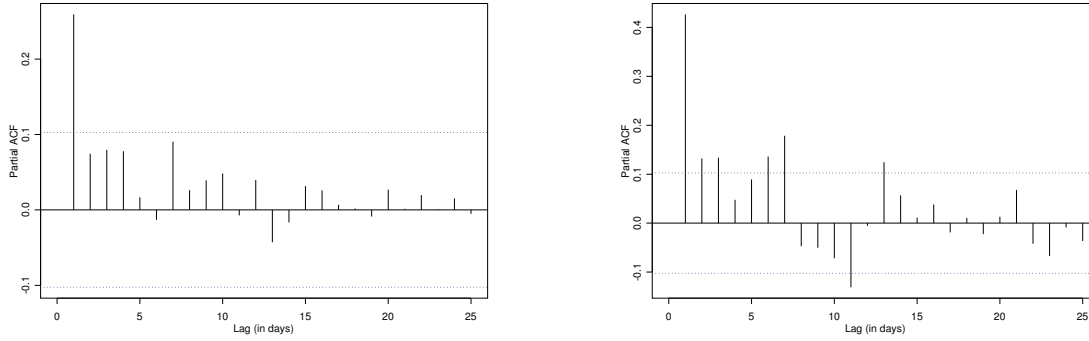
Figure 11. Partial autocorrelation function of hourly time series 6 (left) and 11 (right)

models (with low orders p, q) seems appropriate. The empirical partial autocorrelation functions (shown in figure 11 for series 6 and 11) suggest that $p = 1, q = 1$ or $p = 2, q = 0$ should suffice.

A thorough analysis shows that an $ARMA(2,0)$ or $ARMA(1,1)$ process is actually very satisfactory. With a view to the SARIMA model, we choose an $ARMA(1,1)$ process as model for the hourly time series. The estimated values of the autoregressive and moving average coefficient and of the residual standard deviation are mostly in the same order of magnitude.

In the following, we shortly discuss the fitting of a SARIMA-model for the entire short term process.

A first model which yields a satisfactory fit is a SARIMA$(1,0,1) \times (1,0,1)_{24}$-model. The use of an additional nonseasonal parameter provides some further improvements. Figure 12 (top left) shows the autocorrelation function of the residuals of a SARIMA$(2,0,1) \times (1,0,1)_{24}$-model for X_t with lag 0 to 100 hours. The use of higher nonseasonal parameters does not improve the fit.

Hence, we have the following model for the short term process. Let $\phi(z) = 1 - \phi_1 z - \phi_2 z^2$, $\Phi(z) = 1 - \Phi_1 z$, $\theta(z) = 1 - \theta_1 z$, $\Theta(z) = 1 - \Theta_1 z$. Then the definition of a SARIMA$(2,0,1) \times (1,0,1)_{24}$ models is

$$\phi(B)\Phi(B^{24})X_t = \theta(B)\Theta(B^{24})\varepsilon_t,$$

where (ε_t) is a Gaussian white noise process and B denotes the backshift operator. Written explicitly, we have

$$\begin{aligned} X_t &= \phi_1 X_{t-1} + \phi_2 X_{t-2} + \Phi_1 X_{t-24} - \Phi_1 \left(\phi_1 X_{t-25} + \phi_2 X_{t-26} \right) \\ &\quad + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \Theta_1 \varepsilon_{t-24} + \Theta_1 \theta_1 \varepsilon_{t-25}. \end{aligned}$$

Finally, figure 13 shows a plot of the standardized residuals of the SARIMA model for a period of eight weeks. At least visually, this time series looks similar to white noise.
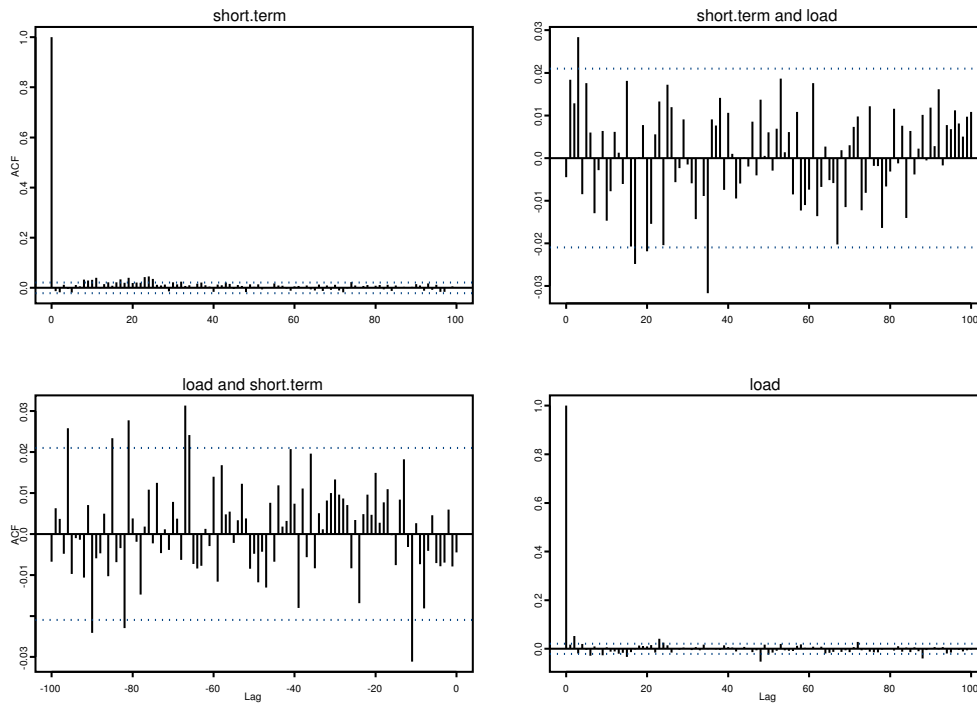
Figure 12. Empirical autocorrelation and cross-correlation functions of the residuals of a SARIMA$(2,0,1) \times (1,0,1)_{24}$ model for the short term process X_t and a SARIMA$(1,0,1) \times (1,0,1)_{24}$ model for the load process L'_t (dotted: 95% confidence bands)
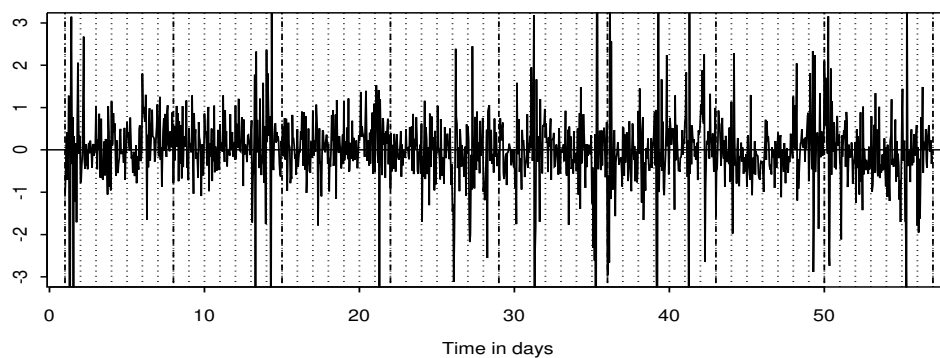

Figure 13. Standardized residuals of SARIMA model of the short term process
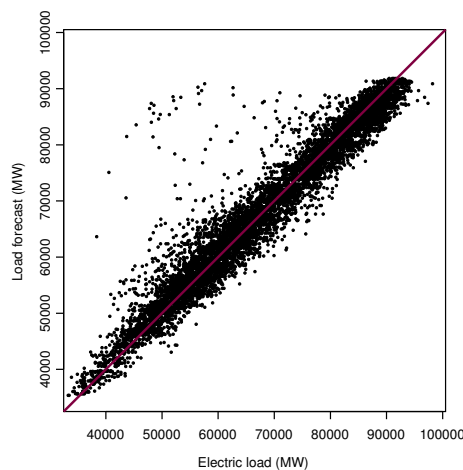
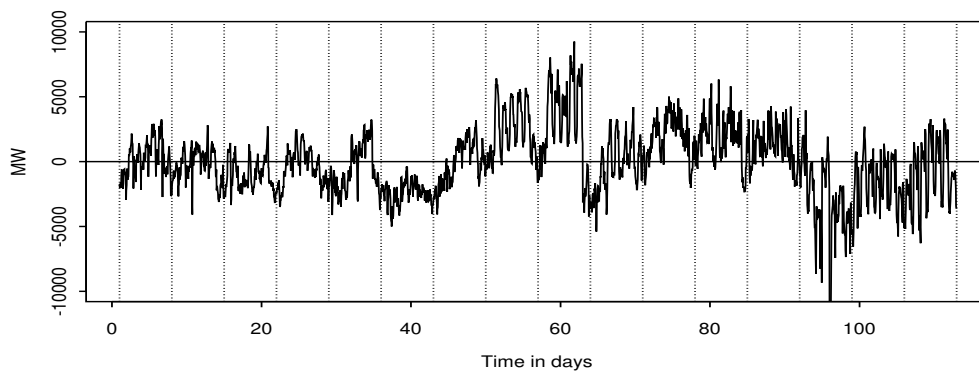**Figure 14.** Load forecast versus electric power load in Germany

Figure 15. Load process over 16 weeks, starting July 1, 2001

4.3. Analysis of the load process

In the previous section, we have seen that the actual load L_t at time t goes into the definition of the short term process. In order to make this model useful for the purpose of simulations, we need the values of L_t in the future. Since the consumption behavior does not change very much over time (up to a linear trend), this is possible to a certain extend. But there remain fluctuations due to (in the longer term) unpredictable effects such as extreme meteorological conditions.

Figure 14 shows a scatter plot of the load forecast ℓ_t based on former experience for Germany against the actual load in this area for the period from 1/1/2001 until 12/31/2001. Thereby, the effective load was adjusted by the yearly increase in power consumption. The very strong correlation between load and load forecast is evident.

The time series of the load process $L'_t = L_t - \ell_t$ over 16 weeks, starting July 1, 2001, is shown in figure 15.

Looking at the autocorrelation function of the load process, one can observe again

a strong correlation at lag 24 (hours). Therefore, a SARIMA model with a 'season' of 24 hours should again be taken into consideration. Since the load forecast can be interpreted as the expected value of the load, we assume that the process L_t' is centered. Actually, the arithmetic mean of the actual data for the period from 1/1/2001 to 12/31/2001 was very small compared to the empirical variance. Since the model building for the load process parallels the development of the short term process, we will not discuss it further.

The autocorrelation function of the residuals of a SARIMA$(1, 0, 1) \times (1, 0, 1)_{24}$-model for L_t' is plotted in figure 12 (bottom right). The top right and the bottom left part of figure 12 show the cross-correlation function between the residuals of X_t and L_t' (lag -100 to 100 hours). The majority of the empirical cross-correlations is within the 95% confidence bands. This supports our assumption that X_t and the residual load L_t' can be modelled as independent processes.

5. Nonstationarity and the long term process

5.1. Forward price dynamics and market price of risk

In the SMaPS model (6) the long term process Y_t follows a random walk with drift given by

$$Y_{t+1} = Y_t + (\mu_t - \tfrac{1}{2}\sigma_Y^2) + \sigma_Y \varepsilon_t^Y \tag{7}$$

where ε_t^Y are serially uncorrelated normally distributed random variables. Even though model (6) is a discrete time model it will be convenient to consider the continuous time extension to (7), since it corresponds better with the usual notation for financial modelling and will reveal the close relation to Black's log-normal model [3] for options on futures contracts. The continuous time extension for Y_t is given by a Brownian motion with drift

$$dY_t = (\mu_t - \tfrac{1}{2}\sigma_Y^2)\, dt + \sigma_Y\, dW_t \tag{8}$$

We now switch to an equivalent martingale measure $\mathbf{P}^*$ assuming a zero market price of risk for the non hedgeable short-term process X_t and the load process L_t. More explanation to justify this assumption was given at the beginning of section 2. The equation for the long-term process under $\mathbf{P}^*$ becomes

$$dY_t = (\mu_t^* - \tfrac{1}{2}\sigma_Y^2)\, dt + \sigma_Y\, dW_t \tag{9}$$

where $\mu_t^* = \mu_t - \lambda_t$ is the new drift including the market price of risk λ_t for Y_t. Under this new measure single hour futures prices at time t for delivery at time T are given by conditional expectations

$$F_{t,T} = \mathbb{E}^*[S_T \,|\, \mathcal{F}_t]. \tag{10}$$

For the futures price formulas for power delivery over a period we refer to equations (1) and (2).

We now derive approximations to (10) for futures contracts that have their delivery period sufficiently far in the future. If $T - t$ is large enough, one can approximate the conditional distributions of the SARIMA process variables L'_T and X_T by their stationary distributions. We then have

$$\mathbb{E}^* \left[\exp(f(T, L_T/v_T)) \mid \mathcal{F}_t\right] \approx \mathbb{E}\left[\exp(f(T, L_T/v_T))\right],$$
$$\mathbb{E}^* \left[\exp(X_T) \mid \mathcal{F}_t\right] \approx \mathbb{E}\left[\exp(X_T)\right] = e^{\text{Var}[X_T]/2}.$$

Note that by our assumption of zero market prices of risk for L_t and X_t we could set the expectations under $\mathbf{P}^*$ equal to the statistical expectations. Neglecting the approximation error, the futures prices $F_{t,T}$ can be written as

$$F_{t,T} = \hat{S}_T \mathbb{E}^* \left[\exp(Y_T) \mid \mathcal{F}_t\right] = \hat{S}_T e^{Y_t + \int_t^T \mu_s^* \, ds}, \tag{11}$$

where $\hat{S}_T$ is a deterministic "technical price"

$$\hat{S}_T = e^{\text{Var}[X_T]/2} \mathbb{E}\left[\exp(f(T, L_T/v_T))\right] \tag{12}$$

that can easily be calculated numerically by integrating over a Gaussian kernel if the process L_T is known. From equation (11) it can be seen that $F_{t,T}$, as a function of t, follows a geometric Brownian motion with SDE

$$\frac{dF_{t,T}}{F_{t,T}} = \sigma_Y \, dW_t.$$

Thus, the model reproduces Black's log-normal model for futures prices with volatility σ_Y. In the discrete time setting the integral $\int_t^T \mu_s^* \, ds$ in (11) just has to be replaced by a sum $\sum_{s=t}^{T-1} \mu_s^*$ and $F_{t,T}$ follows a geometric random walk

$$\log F_{t+1,T} = \log F_{t,T} - \tfrac{1}{2}\sigma_Y^2 + \sigma_Y \varepsilon_t^Y.$$

The futures price dynamics for contracts with arbitrary delivery periods can be derived immediately from (11) and we get

$$F_{t,T_1,T_2} = \hat{S}_{T_1,T_2} e^{Y_t + \int_t^{T_1} \mu_s^* \, ds} \tag{13}$$

where

$$\hat{S}_{T_1,T_2} = \frac{1}{T_2 - T_1} \sum_{T=T_1}^{T_2-1} \hat{S}_T e^{\int_{T_1}^T \mu_s^* \, ds}.$$

Again, F_{t,T_1,T_2} satisfies Black's model with volatility σ_Y,

$$\frac{dF_{t,T_1,T_2}}{F_{t,T_1,T_2}} = \sigma_Y \, dW_t. \tag{14}$$

The discrete versions of (13) and (14) are obtained as before.

5.2. Calibration of model parameters

To completely specify model (6), the SARIMA parameters for the processes L_t and X_t as well as the parameters for the long term process Y_t have to be calibrated. Since the process L_t is, by definition, independent from X_t and Y_t it can independently be calibrated to historical load data as shown in section 4. The processes X_t and Y_t do not immediately correspond to observable market prices even though the long term process Y_t should roughly explain the futures price dynamics whereas the short term process X_t should roughly explain the spot market. In general, futures and spot prices are functions of both X_t and Y_t. We use the following historical market data for calibration:

- Spot prices S_t as hourly time series;
- Futures settlement prices $F_{k,T_i,T_i'}$ for a number of delivery periods $[T_i, T_i']$, $1 \leq i \leq N$ and trading days $1 \leq k \leq n$.

The calibration is done in two steps:

(i) Calibration of the long term process Y_t;

(ii) Calibration of the short term process X_t.

Since we want to use the model for pricing derivatives we are mainly interested in estimating the model parameters under the pricing measure $\mathbf{P}^*$. For the long-term process those parameters can be implied from futures price data. For the short-term processes L_t and X_t we do not make a distinction between the statistical measure and the pricing measure by assumption. Those processes are thus calibrated to historical data.

5.2.1. Calibration of the long term process under $\mathbf{P}^*$ For step 1 we assume that we calibrate to futures prices having delivery periods sufficiently far in the future, so that approximation (13) is justified. In practice, one can take futures contracts with delivery period starting more than six months ahead. The assumed one-factor dynamics of Y_t has enough degrees of freedom to explain the log-normal dynamics (14) under $\mathbf{P}^*$ of one futures contract $F_{t,T_i,T_i'}$ and, via the choice of the drift function μ_t^*, today's forward curve.

Since the futures price dynamics (14) is not affected by the choice of μ_t^*, the volatility σ_Y can be calibrated directly to futures prices either as historical volatility of a certain futures contract or as an implied volatility of a traded option on a futures contract.

On the other hand, by equation (11), today's forward curve is independent of the choice of σ_Y. Rearranging (11) and solving for μ_t^*, we get an explicit expression for μ_t^* in terms of today's forward curve:

$$\mu_T^* = \frac{\partial}{\partial T} \left(\log \frac{F_{t,T}}{\hat{S}_T} \right). \tag{15}$$

In practice we can assume that μ_t^* is constant on intervals $T_i \leq t < T_{i+1}$. Its value $\mu_{T_i}^*$ on this interval can then be calculated by a discrete version of (15) as

$$\mu_{T_i}^* = \frac{1}{T_{i+1} - T_i} \left(\log \left(\frac{F_{t,T_{i+1}}}{\hat{S}_{T_{i+1}}} \right) - \log \left(\frac{F_{t,T_i}}{\hat{S}_{T_i}} \right) \right). \tag{16}$$

This formula can also be used in a discrete time setting.

5.2.2. Calibration of the short term process To calibrate the remaining parameters for the short term process we have to use spot market data. As mentioned above, we face the problem that the spot price is a function of both X_t and Y_t. A mathematical tool that can be applied in such cases is the Kalman filter (cf. [16]). It can of course be conjectured that the influence of the long term process on the hourly spot price dynamics is small compared to the short term process, so that the calibration results from the Kalman filtering method does not differ substantially from the calibration results of section 4 where only the short term process was considered. This conjecture will be confirmed later in this section. We also note that, by using historical spot market data, we now calibrate the parameters with respect to the statistical measure. When changing to the pricing measure later, the parameters for the short term processes X_t and L_t do not change by assumption and the drift μ_t of the long term process Y_t is replaced by μ_t^* as calculated in (15).

To apply the Kalman filtering technique the process equations have to be written in state space form

$$\mathbf{b}_t = \mathbf{Z}_t \mathbf{a}_t + \mathbf{d}_t + \boldsymbol{\varepsilon}_t \qquad \text{(Measurement Equation)} \tag{17}$$

$$\mathbf{a}_t = \mathbf{T}_t \mathbf{a}_{t-1} + \mathbf{c}_t + \mathbf{R}_t \boldsymbol{\eta}_t \qquad \text{(Transition Equation)} \tag{18}$$

where $\mathbf{b}_t \in \mathbb{R}^N$ are the observable variables and $\mathbf{a}_t \in \mathbb{R}^m$ are the unobservable state variables. The random variables $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\eta}_t$ are vectors of serially uncorrelated normally distributed disturbances with

$$\mathbb{E}[\boldsymbol{\varepsilon}_t] = \mathbf{0}, \quad \mathbb{E}[\boldsymbol{\eta}_t] = \mathbf{0}, \quad \text{Cov}[\boldsymbol{\varepsilon}_t] = \mathbf{H}_t \quad \text{and} \quad \text{Cov}[\boldsymbol{\eta}_t] = \mathbf{Q}_t.$$

For details about the Kalman filtering algorithm, see [16].

In our case the observable b_t, derived from the spot price, is

$$b_t = \log(S_t) - f(t, L_t/v_t).$$

For the transition equation we make use of the fact that every ARIMA process can be written in state space form by defining additional state variables that keep track of the recent history of the time series. The state space representation is not unique and here we will follow the approach of Kohn and Ansley [24]. The short term process X_t is given by some ARMA model

$$\tilde{\phi}(B) X_t = \tilde{\theta}(B) \varepsilon_t^X$$

where B is the shift operator and $\tilde{\phi}$, $\tilde{\theta}$ are polynomials. In case of a SARIMA$(1,0,1) \times (1,0,1)_{24}$ model $\tilde{\phi}$ and $\tilde{\theta}$ are of order 25. The state variables needed for the SARIMA process are

$$a_t^j = \sum_{i=j}^{p} \tilde{\phi}_i X_{t-1+j-i} + \sum_{i=j-1}^{q} \tilde{\theta}_i \varepsilon_{t-1+j-i}^X, \quad j = 1, \ldots, r = \max(p, q+1).$$

Additionally, one state variable for the long term process $a_t^{r+1} = Y_t$ is needed. Using the state space form for the ARIMA process and equation (7), the full transition equation is given by (18) with

$$\mathbf{T}_t = \begin{bmatrix} \tilde{\phi}_1 & 1 & 0 & \cdots & 0 \\ \tilde{\phi}_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \\ \tilde{\phi}_r & 0 & & 0 & 1 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}, \quad \mathbf{c}_t = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \mu_t - \frac{1}{2}\sigma_Y^2 \end{bmatrix}$$

$$\mathbf{R}_t = \begin{bmatrix} -\tilde{\theta}_1 & 0 \\ \vdots & \vdots \\ -\tilde{\theta}_r & 0 \\ 0 & \sigma_Y \end{bmatrix}, \quad \mathbf{Q}_t = \mathbf{I}$$

The measurement equation simply is

$$b_t = \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 \end{bmatrix} \mathbf{a}_t$$

The optimal parameters that maximize the likelihood function L now can be found using numerical optimization.

5.2.3. Calibration Results   We carry out the calibration using a two year history of spot market prices between July 1, 2000 and July 1, 2002. After the load process L_t and the curve $f(t, L_t/v_t)$ have been calibrated as shown in section 4 we can calculate the time series for the market deviations

$$r_t = \log(S_t) - f(t, L_t/v_t).$$

From the model assumption (6) we find

$$r_t = X_t + Y_t.$$

We assume for the short term process X_t a SARIMA$(1,0,1) \times (1,0,1)_{24}$ model

$$\phi(B)\Phi(B^{24})X_t = \theta(B)\Theta(B^{24})\varepsilon_t^X,$$

where ε_t^X is a white noise process with variance $\mathrm{Var}[\varepsilon_t^X] = \sigma_X^2$.

     The parameters for the long term process are determined as follows. The value for σ_Y is derived from the annualized implied volatility for an option on baseload delivery during the year 2003, which is 0.1. The drift parameter μ_t^* is obtained from futures prices (see (16)). If μ_t^* is assumed constant we get an annualized drift rate of 0.02, which means e. g. that the futures contract for year 2004 trades at a 2% spread above

	Φ_1	Θ_1	ϕ_1	θ_1	σ_X^2
Kalman filtering	0.971	0.876	0.801	-0.01	0.024
Simplified model $Y_t \equiv 0$	0.975	0.877	0.825	0.027	0.021

Table 1. Calibration Results

the 2003 contract. Taking into account additional futures prices, we get a more accurate time dependent representation of μ_t^*.

Now we determine the SARIMA parameters using the Kalman filtering method. The results are shown in table 1. Within the numerical accuracy the parameters of the SARIMA process are close to the parameters found for a simplified model $\log(S_t) = f(t, L_t/v_t) + X_t$ where the long term process Y_t is set to zero and the SARIMA parameters can thus be calibrated by standard methods without having to use Kalman filtering techniques. For most practical purposes, the two processes X_t and Y_t can separately be calibrated to spot market data and futures market data, respectively.

6. Results

In financial markets liquidly traded options are used to calibrate the pricing models. There is often no necessity to analyze historical data extensively. This is a significant difference to electric power markets. Since complex options on electricity, such as those described in the introduction, are embedded in delivery contracts and are not standardized, their prices are often not directly observable. Hence, inaccuracies of the pricing model can not be adjusted by calibrating the model to market data for options. The need for a simulation model to represent the market in a realistic way is therefore much higher.

Besides statistical tests, we believe that a good measure of quality of a simulation model is the view of a senior trader. Hence we give an impression of the simulation paths derived from SMaPS by comparing them with real prices from the European Energy Exchange (EEX). Figure 16 shows a simulated sample path against the real price path from the EEX during January 2002. The simulated path exhibits the typical daily and weekly seasonalities and price spikes. Of course, the days at which the most extreme price spikes occur in the simulation path are different from the ones of the real path. To explore the occurrence of price spikes over a longer time period, figure 17 shows a sample of 20 simulated paths over the period January 2002 to July 2002. These simulated paths can be compared to the real EEX prices in figure 18. It can be seen that price spikes occur quite frequently.

We now apply the simulation results to price the options described in the introduction. First we calculate the option premium of the portfolio of hourly calls. The call option gives the right to buy electricity in the first half of the year with a capacity of up to 100 MW at a fixed price K. For simplicity we assume that the value of the option is given by the expectation under the pricing measure P^*. The option
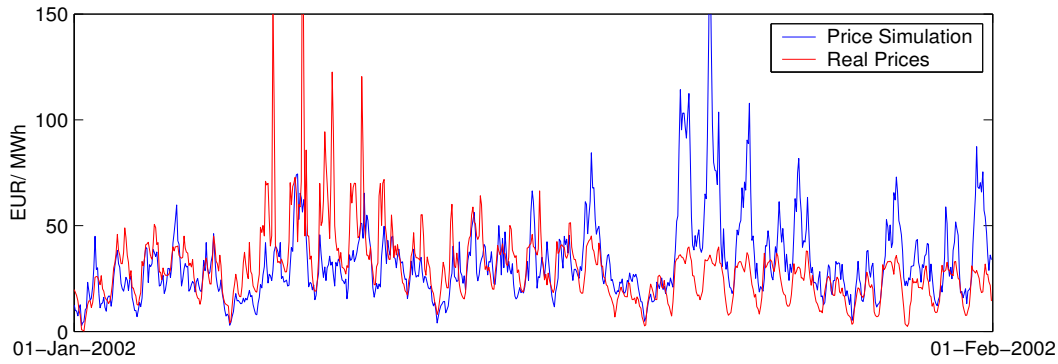
Figure 16. Hourly EEX prices vs. Simulation path (Jan. 01, 2002 - Feb. 01,2002)
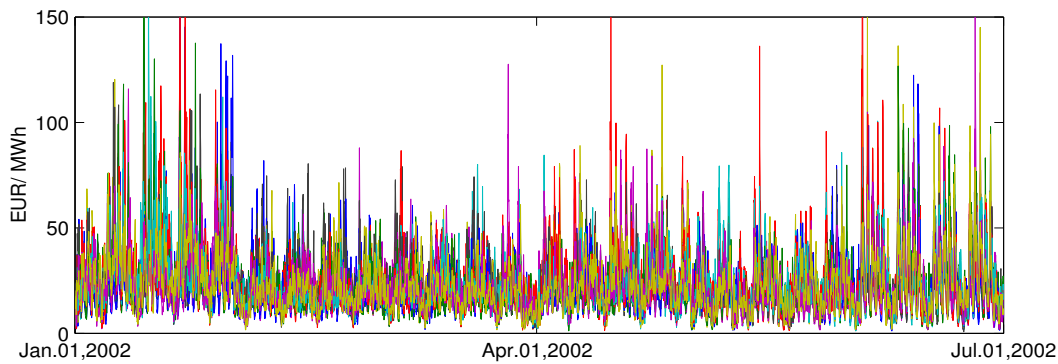

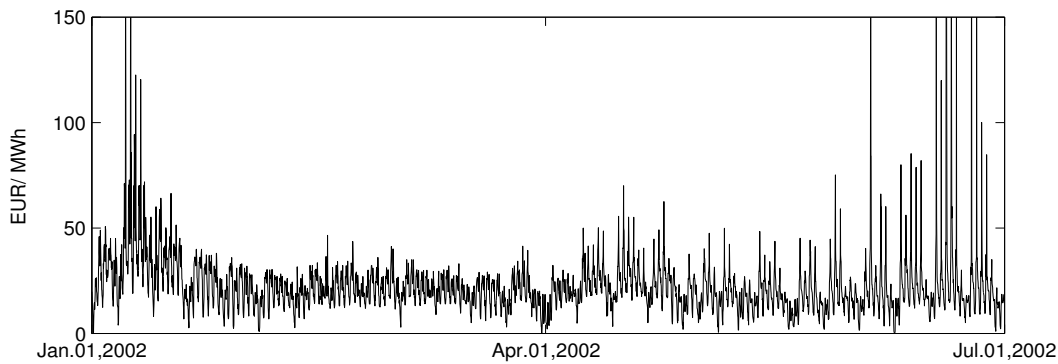**Figure 17.** 20 Simulation paths (Jan. 01, 2002 - Jul. 01, 2002)

Figure 18. Hourly EEX prices (Jan. 01, 2002 - Jul. 01, 2002)

value is then given by

$$V_{Cap} = \sum_{i=1}^{N} \mathbb{E}^* \left[e^{-r(t_i - t_0)} \max(S_{t_i} - K, 0) \right],$$

where $t_1, \ldots, t_N$ denote the hours at which the option can be exercised. In our example we have a portfolio of $N = 4344$ call options. The expected value is calculated via Monte-Carlo simulations, where each price path consists of the 4344 hours of the first half of the year. If the risk manager wants to charge more by adding some risk premium
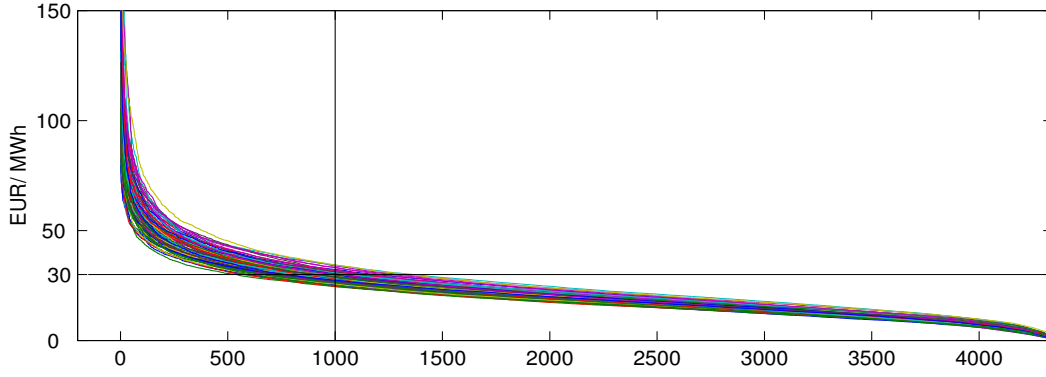
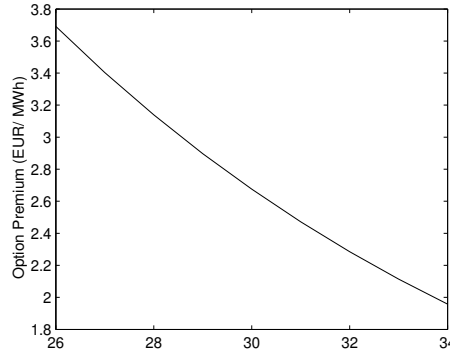**Figure 19.** Sample paths for the first half of the year in descending price order

Figure 20. Call premium MWh vs. strike price

(e.g. based on the Value at Risk, the expected shortfall or based on the use of some concave utility function) then this can be easily included in the Monte Carlo simulation.

Figure 19 shows the simulation paths sorted by the hourly price in descending order and it can be seen how often exercising the option is reasonable. The option premium as a function of the strike price is shown in figure 20.

The calculation of the fixed delivery (strike) price K_0 of the swing option in such a way, that no up front fee is necessary, is much more complicated. This option gives the right to spread the fixed energy amount $A = 100$ GWh at short notice over the contract period of the first half of the year. The capacity in this example is limited to $C = 100$ MW. Again assuming for simplicity that the risk manager is just interested in the expected value under P^*, the fair value of such a swing option with general strike price K is given by the solution of the following optimization problem

$$V_{Swing} = \max_{\phi} \left\{ \sum_{i=1}^{N} \mathbb{E}^* \left[e^{-r(t_i - t_0)} \phi(t_i) \left(S_{t_i} - K \right) \right] \right\} \tag{19}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \phi(t_i) = A, \quad 0 \le \phi(t_i) \le C \quad (i = 1, \ldots, N),$$

where $\phi(t)$ is the exercising strategy representing the capacity selected at time t. The
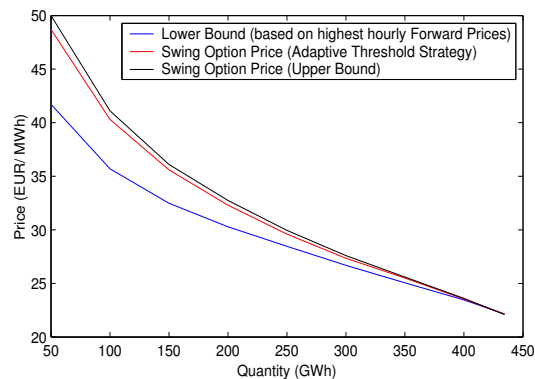
Figure 21. Fixed price of a Swing option vs. energy. Average energy price of the most expensive hours of the forward curve (Forward price) vs. energy

price K_0 now is exactly the strike price at which $V_{Swing} = 0$. From equation (19) it can be seen that a simple Monte-Carlo simulation of the price paths is not sufficient, since the optimal exercising strategy is needed as well. The problem of how to determine optimal exercising strategies for swing options will be considered in a forthcoming paper. To get an upper bound of the delivery price we can use the results from figure 19 assuming that the holder of the swing option has exercised the option at the (in our case 1000) highest prices. A lower bound can be found assuming that the holder nominates the delivery hours today according to the highest hourly forward prices. Figure 21 shows the upper and lower prices K_0 for the example swing option as a function of the total energy amount A. Between the upper and lower price is the price calculated using a more sophisticated exercising strategy based on adaptive thresholds.

Acknowledgments

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