A Square-Error-Based Regularization for Normalized LMS Algorithms

Junghsi Lee, Hsu Chang Huang, Yung Ning Yang, and Shih Quan Huang

***Abstract*—The purpose of a variable step-size normalized LMS filter is to solve the dilemma of fast convergence or low steady-state error associated with the fixed regularized NLMS. By employing the inverse of weighted square-error as the time-varying regularization parameter, we introduce a new regularization for NLMS algorithms. Extensive simulation results demonstrate that our proposed algorithm outperforms existing schemes in speed of convergence, tracking ability, and low misadjustment.**

***Index Terms*—Adaptive filters, normalized least mean square (NLMS), variable step-size NLMS, regularization parameter.**

I. Introduction

Adaptive filtering algorithms have been widely employed in many signal processing applications. Among these algorithms, the normalized least mean square (NLMS) adaptive filter is most commonly used due to its simplicity and robustness. It is well know that the stability of NLMS is controlled by a step-size parameter. This parameter also determines the speed of convergence, tracking ability and steady-state misadjustment of the filter. In practice, the NLMS is implemented by dividing the step-size parameter by the squared norm of the input vector plus a small positive constant ε called the regularization parameter. The inclusion of ε is to overcome the problem that the squared norm gets too close to zero. Since the overall step-size affects the performance of the NLMS, this regularization parameter has an effect on the convergence properties and misadjustment as well.

For the regularization parameter ε being fixed, there are conflicting objectives between fast convergence and low misadjustment. The purpose of a variable step-size NLMS (VSS-NMS) algorithm is to solve the tradeoff in the fixed regularized NLMS. Many VSS-NLMS schemes have been presented in the past two decades [1, 2, 6, 7]. In particular, quite a few time-varying regularization methods have been proposed in the past several years [3-5]. Mandic [5] presented a generalized normalized gradient descent (GNGD) algorithm which used a time-varying regularization parameter $\varepsilon(n)$. Mandic claimed that the GNGD adapts its learning rate according to the dynamics of the input signals, and its performance is bounded from below by the performance of the NLM. Combining the concepts of GNGD and NLMS, Mandic introduced another scheme which employed hybrid filters structure to further improve the steady-state misadjustment of the GNGD [4].

Choi, Shin, and Song [3] proposed a modified GNGD to resolve the limited steady-state performance of GNGD. The idea is to introduce the normalized gradient in the update process for the regularization parameter, and used simulation to demonstrate its robustness.

While most VSS-NLMS algorithms need to tune several parameters for better performance, we propose a tuning-free, simple, and robust VSS-NLMS algorithm in this paper. The idea is to introduce the inverse of weighted square-error as the regularization parameter. Our new regularized NLMS algorithm outperforms existing schemes in convergence, tracking, and misadjustment.

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II. Square-error-based regularization for NLMS Algorithm

In this section, we summarize several algorithms including NLMS, GNGD algorithm [5], the robust regularization for NLMS (RR-NLMS) [3], and presents the proposed square-error-based regularization for NLMS (SER-NLMS).

Consider the following desired signal that arise from the system identification model

$$d(n) = \mathbf{x}^T(n)\mathbf{h}(n) + v(n)\,, \tag{1}$$

where $\mathbf{h}(n)$ denotes the coefficient vector of the unknown system with length N ,

$$\mathbf{h}(n) = [h_0(n), h_1(n), \ldots, h_{N-1}(n)]^T\,. \tag{2}$$

$\mathbf{x}(n)$ is the input vector

$$\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^T\,, \tag{3}$$

and $v(n)$ is the additive noise that is independent of $x(n)$.

A. *Conventional Regularization for NLMS (ε-NLMS)*

Assume the adaptive filter has the same structure and same order as that of the unknown system. Let $\mathbf{w}(n)$ denote the coefficient vector of the adaptive filter at iteration *n*. The *a priori* estimation error is

$$e(n) = d(n) - \mathbf{x}^T(n)\mathbf{w}(n)\,. \tag{4}$$

The ε-NLMS algorithm updates $\mathbf{w}(n)$ as follows

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu}{\|\mathbf{x}(n)\|_2^2 + \varepsilon} e(n)\mathbf{x}(n)\,, \tag{5}$$

where μ is the fixed step-size, ε is a fixed small positive constant called regularization parameter. Depends on the value of ε, the overall effective step-size might become relatively large or relatively small, and this affects the convergence and racking performance.

B. GNGD algorithm [5]

The GNGD algorithm uses a time-varying regularization parameter $\varepsilon(n)$ calculated by

$$\varepsilon(n) = \varepsilon(n-1) - \rho\mu \frac{e(n)e(n-1)\mathbf{x}^T(n)\mathbf{x}(n-1)}{(\|\mathbf{x}(n-1)\|_2^2 + \varepsilon(n-1))^2}, \tag{6}$$

where ρ is an adaptation parameter needs tuning, and the initial value $\varepsilon(0)$ has to be set also.

C. RR-NLMS Algorithm [3]

The RR-NLMS algorithm calculates the time-varying regularization parameter as

$$\varepsilon'(n) = \varepsilon(n-1) - \rho\,\mathrm{sgn}\left[e(n)e(n-1)\mathbf{x}^T(n)\mathbf{x}(n-1)\right]$$
$$\varepsilon(n) = \begin{cases} \varepsilon'(n), & if\ \varepsilon'(n) \ge \varepsilon_{\min} \\ \varepsilon_{\min}, & if\ \varepsilon'(n) < \varepsilon_{\min} \end{cases}, \tag{7}$$

where sgn(x) represents the sign function, and $\varepsilon_{\min}$, a minimum allowable value of $\varepsilon(n)$, is a parameter needs tuning.

D. Proposed square-error-based regularization for NLMS algorithm

For the conventional ε-NLMS algorithm, the role of ε is to avoid the associated denominator gets too close to zero, so as to keep the filter from divergence. However, it changes the effective step-size and has an effect on convergence performance. In this paper, we propose a new regularization which uses the inverse of weighted square-error as the time-varying regularization parameter. Error signal power is obtained as

$$\sigma_e^2(n) = \lambda\sigma_e^2(n-1) + (1-\lambda)e^2(n). \tag{8}$$

And our proposed algorithm calculates the regularization parameter as

$$\varepsilon(n) = 1/\sigma_e^2(n). \tag{9}$$

We do not have to tune $\varepsilon(n)$ because, in practice, it never gets too close to zero. When $\sigma_e^2(n)$ is bigger, i.e., we need to make bigger adaptation, the regularization parameter $\varepsilon(n)$ becomes smaller, so the effective step-size is relatively large. On the other hand, the adaptive filter needs only small adjustment when the estimation error is small. In this situation, our $\varepsilon(n)$ gets larger as $\sigma_e^2(n)$ becomes smaller, and it results a relatively small value in effective step-size.

III. Simulation Results

In this section, we present the results of several experiments that compare the performance of ε-NLMS, GNGD, RR-NLMS, and our proposed regularized NLMS. The adaptive filter was used to identify a 512-tap acoustic echo impulse response. We have used the normalized squared coefficient error (NSCE) to evaluate the performance of the algorithms. The NSCE is defined as

$$NSCE(n) = 10\log_{10} \frac{\|\mathbf{h}(n) - \mathbf{w}(n)\|^2}{\|\mathbf{h}(n)\|^2} \tag{10}$$

where $\mathbf{w}(n)$ is the filter coefficient vector. We have run quite a few simulations. The results are pretty consistent. In this section, we show simulation results with the following setup: $\varepsilon_{\min} = 0.001$, $\rho = 0.15$, and $\mu = 1$.

A. AR processes

We have used AR(1) and AR(2) processes as the reference input signals. The power of each AR process is approximately 1. The acoustic echo impulse response was set to be time-varying from seconds 1.9 to 5.1. The evolution of coefficients is described by

$$\mathbf{h}(n) = \mathbf{h}_o + g(n), \tag{11}$$

where $g(n)$ is a white Gaussian noise with variance 10^{-2}. The additive noise is a white Gaussian process with zero mean and variance 10^{-3}. The NSCE curves shown here are results of ensemble averages over 20 independent runs. Figures 1 and 2 demonstrate the results of AR(1) and AR(2), respectively. The RR-NLMS has better misadjustment than that of the ε-NLMS and GNGD. Our filter exhibits even better steady-state performance than RR-NLMS does.

B. Speech Signals

In this experiment, the excitations are 8-second-long Chinese speech signals. We consider two scenarios: (a), a time-invariant system, and (b), a time-varying impulse response described in (11). The additive noise is a white Gaussian process with zero mean and variance 10^{-3}. Speech I was given in Figure 3, and results of time-invariant and time-varying system were illustrated Figures 4 and 5, respectively. The RR-NLMS has much better misadjustment than that of the ε-NLMS and GNGD. It is obvious to see that our filter converges faster than RR-NLMS.

We also used Speech II, which has more unvoiced duration. Results of Speech II were demonstrated in Figures 6, 7 and 8. Due to the unvoiced duration, ε-NLMS and GNGD performed badly. The performance of RR-NLMS degraded notably. It is clear that our filter outperformed other competing algorithms in speed of convergence, tracking ability, and steady-state misadjustment.

IV. Conclusions

This paper proposed a square-error-based regularization for NLMS algorithms. While most VSS-NLMS algorithms need to tune several parameters for better performance, our regularization algorithm is tuning-free, simple, and robust. Extensive simulation results demonstrate that our proposed algorithm outperformed other competing schemes, especially in the scenario of speech signals.

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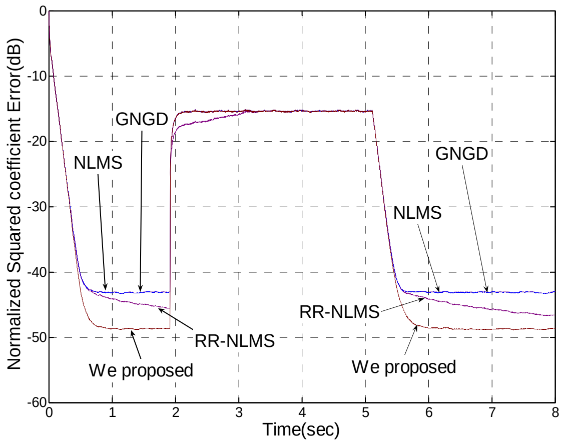

Fig. 1, NSCE curves of ε-NLMS, GNGD [5], RR-NLMS [3], and our algorithm. Input signal is AR(1).

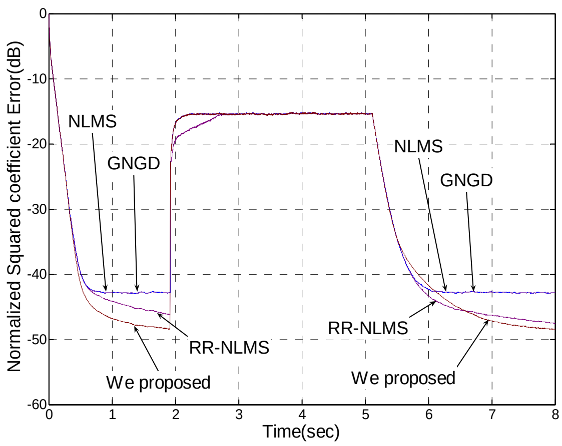

Fig. 2, NSCE curves of ε-NLMS, GNGD [5], RR-NLMS [3], and our algorithm. Input signal is AR(2).

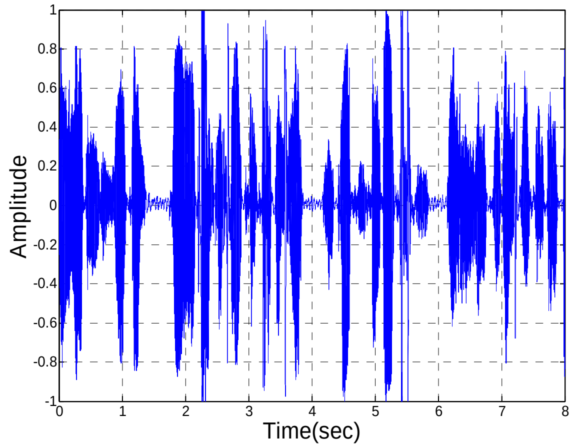

Fig. 3, Speech I wave.

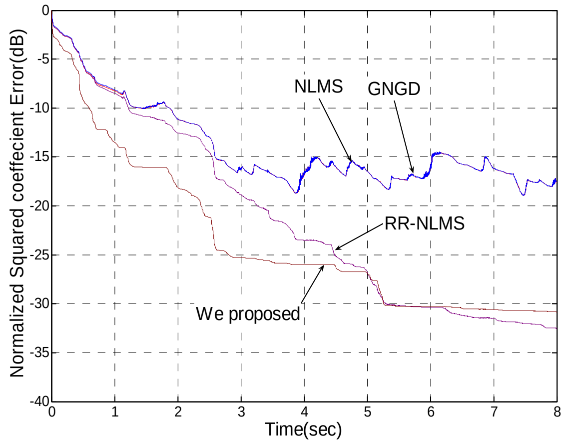

Fig. 4, NSCE curves of ε-NLMS, GNGD [5], RR-NLMS [3], and our algorithm. Time-invariant system. Input signal is Speech I.

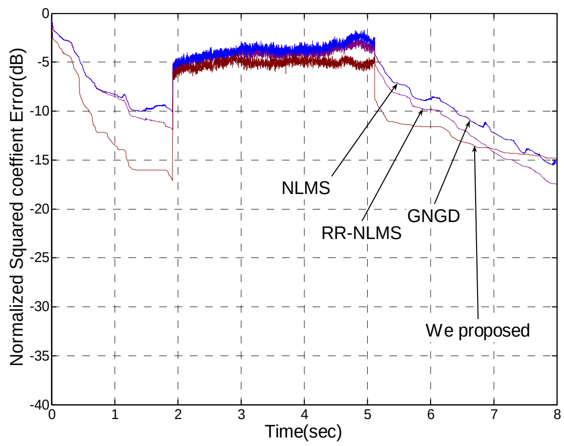

Fig. 5, NSCE curves of ε-NLMS, GNGD [5], RR-NLMS [3], and our algorithm. Time-varying system. Input signal is Speech I.

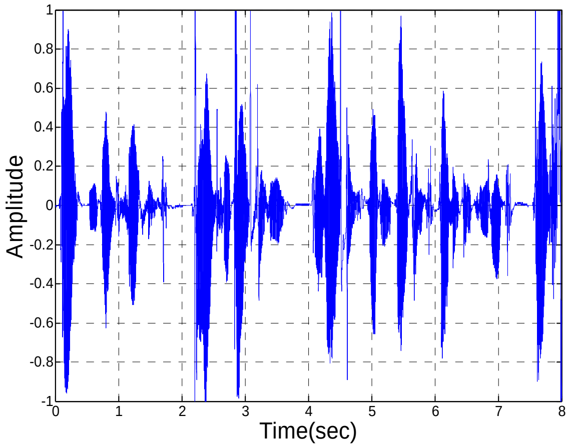

Fig. 6, Speech II wave.

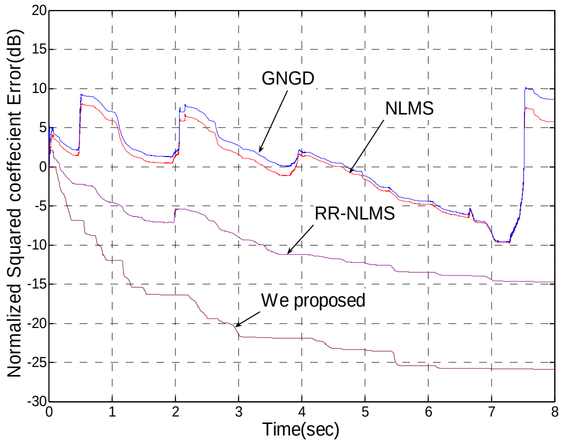

Fig. 7, NSCE curves of ε-NLMS, GNGD [5], RR-NLMS [3], and our algorithm. Time-invariant system. Input signal is Speech II.

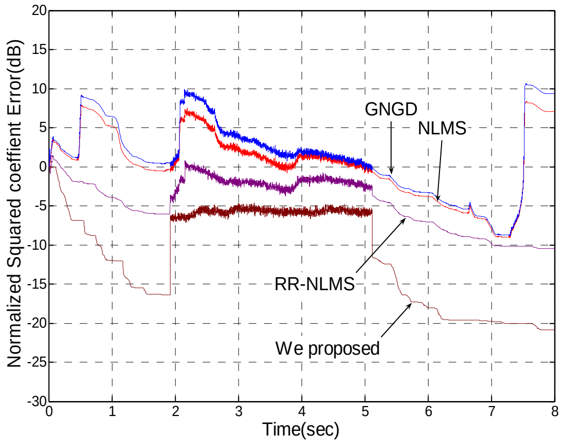

Fig. 8, NSCE curves of ε-NLMS, GNGD [5], RR-NLMS [3], and our algorithm. Time-varying system. Input signal is Speech II.

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