INTERNATIONAL FOOD POLICY RESEARCH INSTITUTE

sustainable options for ending hunger and poverty

A Standard Computable General Equilibrium (CGE) Model in GAMS

Hans Lofgren

Rebecca Lee Harris

Sherman Robinson

With assistance from

Marcelle Thomas

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MICROCOMPUTERS IN POLICY RESEARCH 5

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PREFACE

Over the past decade, the increasing power and reliability of microcomputers and the development of sophisticated software designed specifically for use with them has led to significant changes in the way quantitative food policy analysis is conducted. These changes cover most aspects of the analysis, ranging from the collections and analysis of socioeconomic data to the conduct of model-based policy simulations. The venue of the computations has shifted from off-site mainframes dependent on highly trained operators and significant capital investment in supporting equipment, to desktop and laptop computers dependent only on the occasional availability of electricity. This means that it is now feasible to quickly transfer new techniques between IFPRI and its collaborators in developing countries, that the costs of policy analysis have been substantially reduced, and that a new level of complexity and accuracy in policy analysis is now possible.

As with any new technology, however, substantial costs in time and money are involved in learning the most efficient ways of using this new technology and then transmitting these lessons to others. This series, Microcomputers in Policy Research, represents IFPRI's ongoing collective experience in adapting microcomputer technology for use in food policy analysis in developing countries. Publication decisions are made on the basis of a review by an external referee. The manuals in the series are primarily for the purpose of sharing these lessons with potential users in developing countries, although persons and institutions in developed countries may also find them useful. The series is designed to provide hands-on methods for quantities food policy analysis. In our opinion, examples provide the best and clearest form of instruction; therefore, examples—including actual software codes wherever relevant—are used extensively throughout this series.

Computable general equilibrium (CGE) models are used widely in policy analysis, especially in developed-country academic settings. The purpose of the fifth volume in the series, A Standard Computable General Equilibrium (CGE) Model in GAMS, by Hans Lofgren, Rebecca Lee Harris, and Sherman Robinson, with assistance from Marcelle Thomas and Moataz El-Said, is to contribute to and facilitate the use of this class of models in developing countries. The volume includes a detailed presentation of a static "standard" CGE model and its required database. The model is written for application at the country level; however, only minimal changes are needed before it can be applied to a region within a country (such as a village) or to a farm household involved in production and consumption activities. The model incorporates features developed over recent years through IFPRI's research projects. These features—of particular importance in developing countries—include household consumption of nonmarketed ("home") commodities, explicit treatment of transaction costs for commodities that enter the market sphere, and a separation between production activities and commodities that permits any activity to produce multiple commodities and any commodity to be produced by multiple activities. The manual discusses the implementation of the model in GAMS (the General Algebraic Modeling System) and is accompanied by a CD-ROM that includes the GAMS files for the model, sample databases, simulations, solution reports, and a social accounting matrix (SAM)

aggregation program. Although the volume provides a standardized framework for analysis, the analyst is not forced to make "one-size-fits-all" assumptions. The GAMS code is written to give the analyst considerable flexibility in model specification.

Howarth Bouis and Hans Lofgren, Series Editors

1. INTRODUCTION

Over the past 25 years, computable general equilibrium (CGE) models have become a standard tool of empirical economic analysis. In recent years, improvements in model specification, data availability, and computer technology have improved the payoffs and reduced the costs of policy analysis based on CGE models, paving the way for their widespread use by policy analysts throughout the world. The purpose of this manual is to contribute to and facilitate the use of CGE models, making them accessible to a wider group of economists. The manual includes a detailed presentation of a static, "standard" CGE model implemented in a computer modeling language called GAMS (General Algebraic Modeling System). It also provides a sample database in an accompanying CD-ROM.[1]

Although most CGE models have been developed for countries, the basic framework applies, and has been applied, in settings ranging from the world (divided into multiple regions) to disaggregated regions within a country, such as villages, and even to households. In most applications, the markets and prices in the model represent actual markets with money used as a medium of exchange. However, especially in household models, they may be viewed as "implicit" markets where the solution wages and prices represent "shadow prices" or "exchange values." Our standard CGE model is written for application at the country level and has been implemented with a number of country data sets, but only minimal changes are needed to apply the model to a region within a country or to a producer-consumer household.

The standard model includes a number of features designed to reflect the characteristics of developing countries. The specification follows the neoclassical-structuralist modeling tradition presented in Dervis et al. (1982). It incorporates additional features developed in recent years in research projects conducted at IFPRI. These features, of particular importance in developing countries, include household consumption of nonmarketed (or "home") commodities, explicit treatment of transaction costs for commodities that enter the market sphere, and a separation between production activities and commodities that permits any activity to produce

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The authors would like to thank Ed Taylor for a constructive review, and Renger van Nieuwkoop and Jennifer Chung-I Li for useful comments.

[1]We assume that the reader has a basic familiarity with CGE modeling using GAMS. Brooke et al. (1998) is the basic reference on the GAMS software; it also includes a self-contained tutorial. The basics of GAMS-based CGE modeling are summarized in Robinson et al. (1999). Lofgren (2000a, 2000b) presents a set of hands-on exercises in CGE modeling with GAMS. Extensive treatments of CGE methods are found in Dervis et al. (1982), Robinson (1989), Shoven and Whalley (1992), Dixon et al. (1992), and Ginsburgh and Keyzer (1997). References to and examples of CGE-based analyses of food policy in developing countries are found in the Trade and Macroeconomics Division section of the IFPRI website (www.ifpri.org).

multiple commodities and any commodity to be produced by multiple activities.

The CD-ROM provided includes the GAMS files for the CGE model, sample databases, simulations, solution reports, and a social accounting matrix (SAM) aggregation program. In the GAMS code, the model is explicitly linked to a file for country data, including a "standard" SAM that follows the format required for the standard CGE model and a set of elasticities. Optionally, the user may provide quantity data for primary factors (for example, labor types) that appear in the SAM. In the model code, this data set is used to define model parameter values in a manner that assures that the base solution to the model exactly reproduces the values in the SAM. In other words, the model is "calibrated" to the SAM. It is, moreover, straightforward for users to develop new data sets for other applications.

The CGE model and the accompanying GAMS code are written to give analysts considerable flexibility. He or she can choose between alternative treatments for macroeconomic balances and for factor markets. It is also possible to exclude various features that appear in the standard model, such as home consumption and transaction costs. The country database to which the model should be applied can incorporate a wide range of policy tools as well as any desired degree of disaggregation of production activities, commodities, households, and enterprises. Flexibility in terms of model structure and the fact that model parameters are derived from an empirical database (which may be very detailed) permit the analyst to capture country-specific aspects of economic structure and functioning. Hence, although the manual provides a standardized framework for analysis, the analyst is not forced to make "one-size-fits-all" assumptions.

We consider this CGE model and the accompanying computer code as work in progress and encourage readers and users to send us their comments. A number of extensions are possible. For example, users may be interested in adding alternative treatments of production technology or a more detailed treatment of policy tools. However, when new features are added, there is a tradeoff between additional versatility and additional complexity. Unless the new features are of general interest, they should preferably be added in the context of specific applications that use the current, relatively simple model as their starting point. As noted earlier, the model can be easily adapted for application to regions within a country or to a household that is involved in production and consumption. More fundamental changes would be needed to make it dynamic or to turn it into a world model.[2]

The remainder of this manual is organized as follows: Chapter 2 describes the standard SAM. Chapter 3 provides an overview of the features of the CGE model, followed by an equation-by-equation description in Chapter 4. Chapter 5 describes the structure of the GAMS files for the standard CGE model and its database and discusses how they may be used for policy analysis. The appendixes include the mathematical model statement in summary form and core sections of the GAMS code for the model.

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[2]To apply the model to a region or a household, the only changes needed involve the addition of new rules for closing the accounts for the government and the rest of the world (now representing the economy outside the region or the household). The database (including the SAM) should then represent a region or a farm household.

2. THE SOCIAL ACCOUNTING MATRIX

A social accounting matrix (SAM) is a comprehensive, economywide data framework, typically representing the economy of a nation.[3] More technically, a SAM is a square matrix in which each account is represented by a row and a column. Each cell shows the payment from the account of its column to the account of its row. Thus, the incomes of an account appear along its row and its expenditures along its column. The underlying principle of double-entry accounting requires that, for each account in the SAM, total revenue (row total) equals total expenditure (column total).[4]

Table 1 shows an aggregated SAM with verbal explanations in the cells instead of numbers. With one exception, it has all of the features required for implementation with the standard CGE model. The exception is that in the standard SAM, taxes have to be paid to tax accounts, disaggregated by tax type, each of which forwards its revenues to the core government account. The tax types are divided into direct taxes (on domestic non-government institutions and factors), commodity sales taxes, import taxes, export taxes, activity taxes, and value-added taxes. Also note that, in the standard SAM, payments are not permitted in the blank cells of Table 1. Any original SAM that includes such payments should be restructured before being implemented with the standard CGE model.[5]

Table 2 shows a real-world standard SAM for Zimbabwe in which the tax accounts are treated in the required manner.[6] In addition, it has multiple accounts for activities, commodities, factors, and domestic non-

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[3]For general discussions of SAMs, see Pyatt and Round (1985) and Reinert and Roland-Holst (1997); for perspectives on SAM-based modeling, see Pyatt (1988) and Robinson and Roland-Holst (1988).

[4]The GAMS program checks that the SAM that is entered is balanced (meaning the row and column totals are equal for each account). If the absolute value of the sum of account imbalances exceeds a cutoff point, an optimization program is used to estimate a balanced SAM. The program, which minimizes the entropy distance of the cells of the estimated SAM from those of the initial SAM subject to the constraint that row and column totals are equal, is primarily intended to remove rounding errors. For SAM estimation in GAMS in a setting with substantial imbalances in raw data (not only rounding errors), see Robinson and El-Said (2000) and Robinson, Cattaneo, and El-Said (2001).

[5]One common case would be payments from the government to factors (for the labor services provided by government employees). To restructure the SAM to work with the standard model, the preferred approach is to reallocate such payments to a commodity for government services that pays a government service activity which, in turn, pays the labor account.

[6]For other examples of SAMs that have the required structure, see the "data sets" page on IFPRI's website (www.ifpri.org).

government institutions. In each category, the GAMS code can handle any desired disaggregation, including having just a single account. In any real-world application, the preferred disaggregation of the SAM and the CGE model depends on data availability and the purposes of the analysis. It is typically preferable to include relatively detailed treatment in areas of interest while keeping the database relatively aggregated in other areas.[7]

With regard to the structure of the standard SAM, a number of features are noteworthy. First, the standard SAM distinguishes between accounts for "activities" (the entities that carry out production) and "commodities." The receipts are valued at producer prices in the activity accounts and at market prices (including indirect commodity taxes and transaction costs) in the commodity accounts. The commodities are activity outputs, either exported or sold domestically, and imports. This separation of activities from commodities is preferred because it permits activities to produce multiple commodities (for example, a dairy activity may produce the commodities cheese and milk) while any commodity may be produced by multiple activities (for example, activities for small-scale and large-scale maize production may both produce the same maize commodity). In the commodity columns, payments are made to domestic activities, the rest of the world, and various tax accounts (for domestic and import taxes). This treatment provides the data needed to model imports as perfect or imperfect substitutes vis-à-vis domestic production.[8]

Second, the matrix explicitly associates trade flows with transactions (trade and transportation) costs, also referred to as marketing margins. For each commodity, the SAM accounts for the costs associated with domestic, import, and export marketing. For domestic marketing of domestic output, the marketing margin represents the cost of moving the commodity from the producer to the domestic demander. For imports, it represents the cost of moving the commodity from the border (adding to the c.i.f. price) to the domestic demander, while for exports, it shows the cost of moving the commodity from the producer to the border (reducing the price received by producers relative to the f.o.b. price). The Zimbabwe SAM in Table 2 shows how these transaction costs appear in commodity and activity accounts in the standard SAM: A services activity, in Table 2 called transportation (account 4), produces a commodity (account 8) that, like other commodities, may be purchased for intermediate use by activities and for final use by institutions. However, the transportation commodity also receives payments from three special accounts, representing the transaction costs associated with domestic sales, imports, and exports (accounts 10-12).[9] These special accounts are paid by the accounts

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[7]The CD-ROM that accompanies this manual includes a program for aggregating an existing SAM.

[8]In addition, our model code makes it possible to treat selected imports as separate, "noncomparable" commodities (not produced domestically). In the commodity rows, such import commodities receive payments from one or more domestic users. In the columns, these payments would be passed on to the accounts for the rest of the world, import marketing margins, and relevant taxes. The columns for this category of imports do not have any payments to domestic activities.

Table 1—The Basic SAM structure used in the CGE model

	Expenditures								
Receipts	**Activities**	**Commodities**	**Factors**	**Households**	**Enterprises**	**Government**	**Savings-Investment**	**Rest of the World (ROW)**	**Total**
Activities		Marketed outputs		Home-consumed outputs					Activity income (gross output)
Commodities	Intermediate inputs	Transaction costs		Private consumption		Government consumption	Investment	Exports	Demand
Factors	Value-added							Factor income from ROW	Factor income
Households			Factor income to households	Interhousehold transfers	Surplus to households	Transfers to households		Transfers to households from ROW	Household income
Enterprises			Factor income to enterprises			Transfers to enterprises		Transfers to enterprises from ROW	Enterprise income
Government	Producer taxes, value-added tax	Sales taxes, tariffs, export taxes	Factor income to government, factor taxes	Transfers to government, direct	Surplus to Government, direct enterprise taxes			Transfer to Government from ROW	Government income
Savings-Investment				Household savings	Enterprise savings	Government savings		Foreign savings	Savings
Rest of the World (ROW)		Imports	Factor income to ROW		Surplus to ROW	Government transfers to ROW			Foreign exchange outflow
Total	Activity	Supply expenditures	Factor expenditures	Household expenditures	Enterprise expenditures	Government expenditures	Investment	Foreign exchange inflow	

Table 2—Standard SAM for Zimbabwe, 1991

	Activities					Commodities				Transaction costs			Factors			Households		Other institutions								
Category	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**Total**
Activities																										
1. Agriculture, large-scale	0	0	0	0	0	5,250	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5,250
2. Agriculture, small-scale	0	0	0	0	0	670	0	0	0	0	0	0	0	0	0	685	0	0	0	0	0	0	0	0	0	1,355
3. Industry	0	0	0	0	0	0	17,859	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17,859
4. Transportation	0	0	0	0	0	0	0	8,263	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8,263
5. Other services	0	0	0	0	0	0	0	0	15,781	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15,781
Commodities																										
6. Agriculture	244	7	2,697	0	97	0	0	0	0	0	0	0	0	0	0	616	629	0	0	0	0	0	2,964	0	-30	7,223
7. Industry	1,145	165	5,859	3,075	3,619	0	0	0	0	0	0	0	0	0	0	5,236	7,256	0	310	0	0	0	2,513	3,399	-494	32,083
8. Transportation	38	29	176	183	282	0	0	0	0	3,444	1,689	986	0	0	0	605	662	0	169	0	0	0	0	0	0	8,263
9. Other services	715	30	281	421	1,685	0	0	0	0	0	0	0	0	0	0	1,913	2,561	0	4,295	0	0	0	1,598	2,784	0	16,283
Transaction costs																										
10. Domestic sales	0	0	0	0	0	657	2,788	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3,444
11. Imports	0	0	0	0	0	9	1,680	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,689
12. Exports	0	0	0	0	0	580	406	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	986
Factors																										
13. Labor	755	684	2,936	2,447	6,028	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12,851
14. Capital	1,719	260	5,386	1,950	3,524	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12,839
15. Land	458	137	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	594
Households																										
16. Rural	0	0	0	0	0	0	0	0	0	0	0	0	1,605	1,979	594	0	259	5,526	1,113	0	0	0	102	0	0	11,179
17. Urban	0	0	0	0	0	0	0	0	0	0	0	0	11,220	127	0	0	0	3,306	346	0	0	0	0	0	0	14,998
Other institutions																										
18. Enterprise	0	0	0	0	0	0	0	0	0	0	0	0	0	10,733	0	0	0	0	1,209	0	0	0	0	0	0	11,942
19. Government	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3,727	1,478	1,861	291	0	0	7,357
20. Direct taxes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	709	1,351	1,667	0	0	0	0	0	0	0	3,727
21. Indirect taxes	176	43	524	188	546	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,478
22. Import tariffs	0	0	0	0	0	10	1,800	0	51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,861
23. Rest of world	0	0	0	0	0	48	7,550	0	450	0	0	0	26	0	0	0	0	535	418	0	0	0	0	0	0	9,027
24. Savings–Investment	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,415	2,280	908	-504	0	0	0	1,559	0	0	5,658
25. Stock change	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-525	0	-525
Total	5,250	1,355	17,859	8,263	15,781	7,223	32,083	8,263	16,283	3,444	1,689	986	12,851	12,839	594	11,179	14,998	11,942	7,357	3,727	1,478	1,861	9,027	5,658	-525	0

Source: Thomas and Bautista (1999).
Note: Data are presented in million Zambian dollars.

for marketed agricultural and industrial commodities (accounts 6 and 7). Thus the total value of each commodity includes these transaction costs. The standard CGE model will also work with SAMs without this treatment of (and these accounts for) transaction costs.

Third, as noted, the government is disaggregated into a core government account and different tax accounts, one for each tax type. This disaggregation is often necessary because the economic interpretation of some payments may otherwise be ambiguous. In any given application, the SAM may exclude any (or all) of the individual tax accounts. In the SAM, payments between the government and other domestic institutions are reserved for transfers.

Fourth, the domestic nongovernment institutions in the SAM consist of households and enterprises. The enterprises earn factor incomes (reflecting their ownership of capital and/or land). They may also receive transfers from other institutions. Their incomes are used for direct taxes, savings, and transfers to other institutions. As opposed to households, enterprises do not consume. Assuming that the relevant data are available, it is preferable to have one or more accounts for enterprises when these have tax obligations and a savings behavior that are independent of the household sector. The enterprise sector should be disaggregated in a manner that captures differences across enterprises in terms of tax rates, savings rates, and the shares of retained earnings that are received by different household types. For example, in some settings it may be appropriate to disaggregate enterprises into the categories nonagricultural (meaning earnings from nonagricultural capital), small-scale agricultural (earnings from land and capital controlled by small farmers), and large-scale agricultural (earnings from land and capital of large farmers). Technically, the standard CGE model requires that the SAM have at least one household account; enterprise accounts are not necessary.

Finally, the SAM distinguishes between home consumption, which is activity-based, and households' marketed consumption, which is commodity-based. Home consumption, which in the SAM appears as household payments to activities, is valued at producer prices—that is, without marketing margins and the sales taxes that may be imposed on marketed commodities.[10] Household consumption of marketed commodities appears as payments from household accounts to commodity accounts, the values of which include marketing margins and commodity taxes. The standard CGE model also accepts a SAM without (explicit) home consumption.

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[9]The distinction between intermediate use of transportation services and their use in output marketing (giving rise to transaction costs) is that intermediate input use is part of the production process whereas use in marketing is incurred only if the output is actually marketed (as opposed to being home-consumed). Input-output tables typically include information on marketing margins but in a less (or differently) disaggregated format than that proposed for the standard model SAM. Hence, additional data and analysis may be needed if the model user wishes to construct a SAM with the proposed treatment of marketing margins.

[10]In the model, home consumption demand is for the commodity output(s) of the activities that, in the SAM, receive payments from households (compare with footnote 7 and equations 18 and 34 in Chapter 4).

3. OVERVIEW OF THE STANDARD CGE MODEL

The standard CGE model explains all of the payments recorded in the SAM. The model therefore follows the SAM disaggregation of factors, activities, commodities, and institutions. It is written as a set of simultaneous equations, many of which are nonlinear. There is no objective function. The equations define the behavior of the different actors. In part, this behavior follows simple rules captured by fixed coefficients (for example, ad valorem tax rates). For production and consumption decisions, behavior is captured by nonlinear, first-order optimality conditions—that is, production and consumption decisions are driven by the maximization of profits and utility, respectively. The equations also include a set of constraints that have to be satisfied by the system as a whole but are not necessarily considered by any individual actor. These constraints cover markets (for factors and commodities) and macroeconomic aggregates (balances for Savings–Investment, the government, and the current account of the rest of the world).

This chapter summarizes the basic characteristics of the model. Unlike the more detailed presentation in Chapter 4, it uses no mathematical notation.

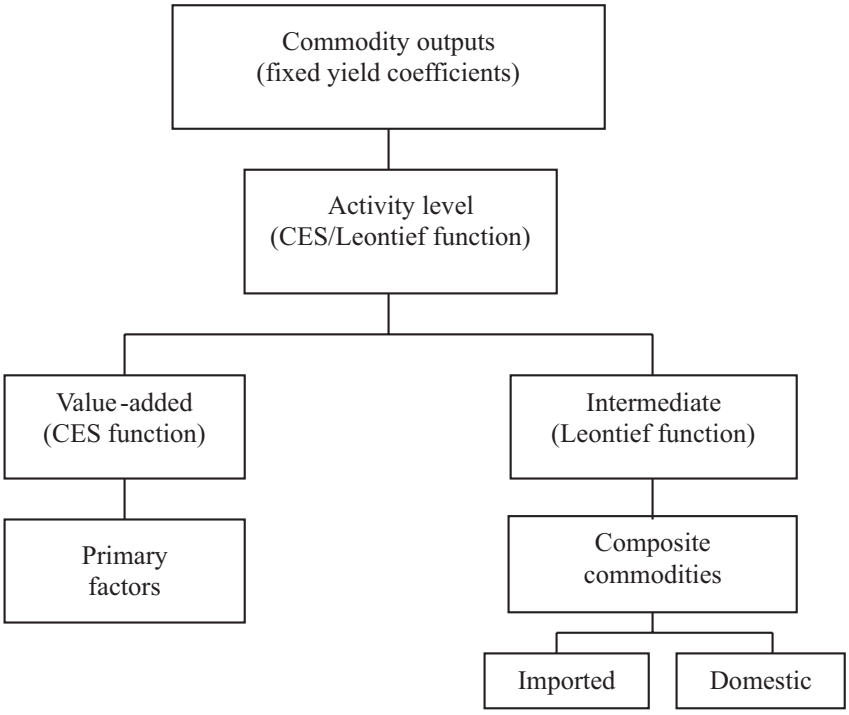
ACTIVITIES, PRODUCTION, AND FACTOR MARKETS

Each producer (represented by an activity) is assumed to maximize profits, defined as the difference between revenue earned and the cost of factors and intermediate inputs. Profits are maximized subject to a production technology, the structure of which is shown in Figure 1. At the top level, the technology is specified by a constant elasticity of substitution (CES) function or, alternatively, a Leontief function of the quantities of value-added and aggregate intermediate input. The Leontief alternative is the default. The CES alternative may be preferable for particular sectors if empirical evidence suggests that available techniques permit the aggregate mix between value-added and intermediate inputs to vary. Value-added is itself a CES function of primary factors whereas the aggregate intermediate input is a Leontief function of disaggregated intermediate inputs.

Each activity produces one or more commodities according to fixed yield coefficients. As noted, a commodity may be produced by more than one activity. The revenue of the activity is defined by the level of the activity, yields, and commodity prices at the producer level.

As part of its profit-maximizing decision, each activity uses a set of factors up to the point where the marginal revenue product of each factor is equal to its wage (also called factor price or rent). Factor wages may differ across activities, not only when the market is segmented but also for mobile factors. In the latter case, the model incorporates discrepancies that stem from exogenous causes (for example, wage differences across activities resulting from considerations such as status, comfort, or health risks).

Figure 1—Production technology



The user can choose between alternative factor market closures (mechanisms for equilibrating supplies and demands in factor markets). According to the default closure, the quantity supplied of each factor is fixed at the observed level. An economywide wage variable is free to vary to assure that the sum of demands from all activities equals the quantity supplied. Each activity pays an activity-specific wage that is the product of the economywide wage and an activity-specific wage (distortion) term. For the default closure, the latter terms are fixed.

Alternatively, it is possible to assume that a factor is unemployed and the real wage is fixed. This assumption may, for example, be appropriate in settings where there is considerable unemployment for a given labor category. Compared with the default closure, the only change is that the economywide wage variable is fixed (or exogenized) while the supply variable is "flexed" (or endogenized). Each activity is free to hire any desired quantity at its fixed, activity-specific wage (which, implicitly, is indexed to the model numéraire). In this setting, the supply variable is superfluous; it merely records the total quantity demanded.

Under a third closure, the factor market is segmented and each activity is forced to hire the observed, base-year quantity—that is, the factor is activity-specific. This closure may be preferred in short-run analyses or when there are significant quality differences between the units of a factor that are used in different activities—for example, units of non-agricultural capital used in different industrial and service activities. For this case, the quantities of activity-specific factor demands and the

economywide wage are fixed while the activity-specific wage terms and the supply variables are flexible.

INSTITUTIONS

In the CGE model, institutions are represented by households, enterprises, the government, and the rest of the world.

The households (disaggregated as in the SAM) receive income from the factors of production (directly or indirectly via the enterprises) and transfers from other institutions. Transfers from the rest of the world to households are fixed in foreign currency. In fact, all transfers between the rest of the world and domestic institutions and factors are fixed in foreign currency. The households use their income to pay direct taxes, save, consume, and make transfers to other institutions. In the basic model version, direct taxes and transfers to other domestic institutions are defined as fixed shares of household income whereas the savings share is flexible for selected households. The treatment of direct tax and savings shares is related to the choice of closure rule for the government and savings–investment balances. This topic is discussed further in the final section of this chapter, on macroeconomic balances. The income that remains after taxes, savings, and transfers to other institutions is spent on consumption.

Household consumption covers marketed commodities, purchased at market prices that include commodity taxes and transaction costs, and home commodities, which are valued at activity-specific producer prices.[11] Household consumption is allocated across different commodities (both market and home commodities) according to linear expenditure system (LES) demand functions, derived from maximization of a Stone–Geary utility function (for details, see Blonigen et al. 1997, 223–225, and Dervis et al. 1982, 482–485).

Instead of being paid directly to the households, factor incomes may be paid to one or more enterprises. Enterprises may also receive transfers from other institutions. Enterprise incomes are allocated to direct taxes, savings, and transfers to other institutions. Enterprises do not consume. Apart from this, the payments to and from enterprises are modeled in the same way as the payments to and from households.

The government collects taxes and receives transfers from other institutions. In the basic model version, all taxes are at fixed *ad valorem* rates. The government uses this income to purchase commodities for its consumption and for transfers to other institutions. Government consumption is fixed in real (quantity) terms whereas government transfers to domestic institutions (households and enterprises) are CPI-indexed. Government savings (the difference between government income and spending) is a flexible residual.

The final institution is the rest of the world. As noted, transfer payments between the rest of the world and domestic institutions and factors are all fixed in foreign currency. Foreign savings (or the current account

[11]In the standard SAM, home consumption is only disaggregated by activity and household, not by commodity, activity, and household. When households consume from activities that produce multiple outputs, extraneous, non-SAM data are needed to allocate home consumption across the commodities produced by each relevant multiple-output activity.

deficit) is the difference between foreign currency spending and receipts. Commodity trade with the rest of the world is discussed in the next section. Thereafter, the final section of this chapter discusses the rules for clearing the macroeconomic balances (the macroclosures)—that is, how equilibrium is achieved in the balances for the government, the rest of the world, and the Savings–Investment account (where institutional savings are aggregated and allocated to domestic investment).

COMMODITY MARKETS

With the exception of home-consumed output, all commodities (domestic output and imports) enter markets. Figure 2 shows the physical flows for marketed commodities along with the associated quantity and price variables as defined in the model equations discussed in the following section.

Domestic output may be sold in the market or consumed at home. For marketed output, the first stage in the chain consists of generating aggregated domestic output from the output of different activities of a given commodity. These outputs are imperfectly substitutable as a result of, for example, differences in timing, quality, and distance between the locations of activities. A CES function is used as the aggregation function. The demand for the output of each activity is derived from the problem of minimizing the cost of supplying a given quantity of aggregated output subject to this CES function. Activity-specific commodity prices serve to clear the implicit market for each disaggregated commodity.

At the next stage, aggregated domestic output is allocated between exports and domestic sales on the assumption that suppliers maximize sales revenue for any given aggregate output level, subject to imperfect transformability between exports and domestic sales, expressed by a constant elasticity of transformation (CET) function. In the international markets, export demands are infinitely elastic at given world prices. The price received by domestic suppliers for exports is expressed in domestic currency and adjusted for the transaction costs (to the border) and export taxes (if any). The supply price for domestic sales is equal to the price paid by domestic demanders minus the transaction costs of domestic marketing (from the supplier to the demander) per unit of domestic sales. If the commodity is not exported, total output is passed to the domestic market.

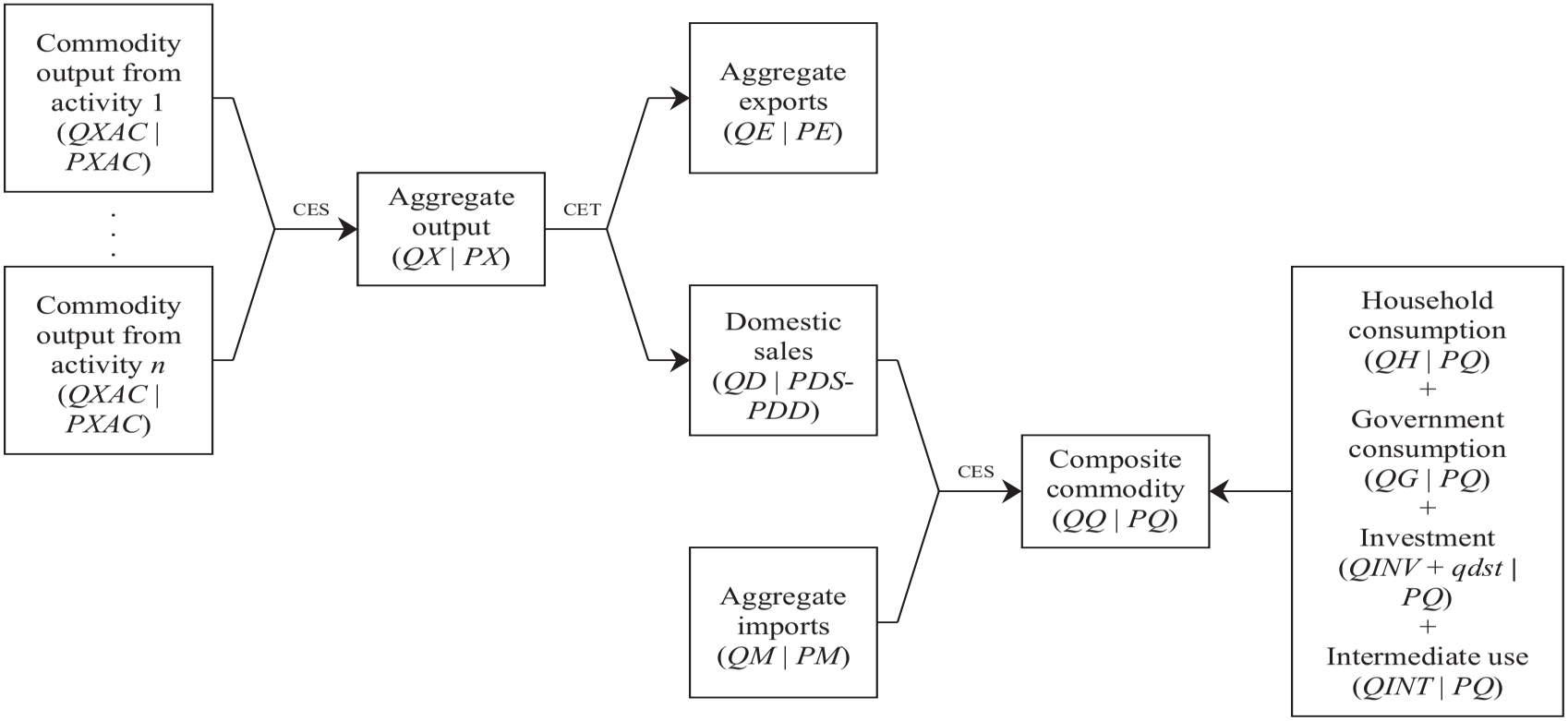
Domestic demand is made up of the sum of demands for household consumption, government consumption, investment (the determination of which is discussed below), intermediate inputs, and transactions (trade and transportation) inputs.

To the extent that a commodity is imported, all domestic market demands are for a composite commodity made up of imports and domestic output, the demands for which are derived on the assumption that domestic demanders minimize cost subject to imperfect substitutability. This is also captured by a CES aggregation function.[12] Total market demand is directed to imports for commodities that lack domestic production and to domestic output for non-imported commodities.

The derived demands for imported commodities are met by international supplies that are infinitely elastic at given world prices. The import

[12]This function is also referred to as an Armington function, named after Paul Armington who introduced imperfect substitutability between imports and domestic commodities in economic models (Armington 1969).

Figure 2—Flows of marketed commodities



Note: CES is constant elasticity of substitution; CET is constant elasticity of transformation.

prices paid by domestic demanders also include import tariffs (at fixed *ad valorem* rates) and the cost of a fixed quantity of transactions services per import unit, covering the cost of moving the commodity from the border to the demander.[13] Similarly, the derived demand for domestic output is met by domestic suppliers. The prices paid by the demanders include the cost of transactions services, in this case reflecting that the commodity was moved from the domestic supplier to the domestic demander. The prices received by domestic suppliers are net of these transaction costs. Flexible prices equilibrate demands and supplies of domestically marketed domestic output.

Compared with the alternative assumptions of perfect substitutability and transformability, the assumptions of imperfect transformability (between exports and domestic sales of domestic output) and imperfect substitutability (between imports and domestically sold domestic output)

Table 3—Alternative closure rules for macrosystem constraints

Constraint		
Government	**Rest of the World**	**Savings–Investment**
GOV-1: Flexible government savings; fixed direct tax rates	ROW-1: Fixed foreign savings; flexible real exchange rate	SI-1: Fixed capital formation; uniform MPS point change for selected institutions
GOV-2: Fixed government savings; uniform direct tax rate point change for selected institutions	ROW-2: Flexible foreign savings; fixed real exchange rate	SI-2: Fixed capital formation; scaled MPS for selected institutions
GOV-3: Fixed government savings; scaled direct tax rates for selected institutions		SI-3: Flexible capital formation; fixed MPS for all non government institutions
		SI-4: Fixed investment and government consumption absorption shares (flexible quantities); uniform MPS point change for selected institutions
		SI-5: Fixed investment and government consumption absorption shares (flexible quantities); scaled MPS for selected institutions

Notes: For the specified closure rules, the choice for one of the three constraints does not constrain the choice for the other two. MPS is marginal propensity to save.

[13]Note that these transaction costs are not ad valorem. The rates—the ratio between the margin and the price without the margin—change with changes in the prices of transactions services and/or the commodities that are marketed.

permit the model to better reflect the empirical realities of most countries. The assumptions used give the domestic price system a degree of independence from international prices and prevent unrealistic export and import responses to economic shocks. At the disaggregated commodity level, these assumptions allow for a continuum of tradability and two-way trade, which is commonly observed even at very fine levels of disaggregation.

MACROECONOMIC BALANCES

The CGE model includes three macroeconomic balances: the (current) government balance, the external balance (the current account of the balance of payments, which includes the trade balance), and the Savings–Investment balance. In the GAMS code, the user chooses among a relatively large number of pre-programmed alternative closure rules for these balances. The choices made have no influence on the solution to the base simulation but will typically influence the results for other simulations. The closures are summarized in Table 3.[14]

For the government balance, the default closure (GOV-1) is that government savings (the difference between current government revenues and current government expenditures) is a flexible residual while all tax rates are fixed. Under the two alternative government closures, the direct tax rates of domestic institutions (households and enterprises) are adjusted endogenously to generate a fixed level of government savings. For the first of these alternative closures (GOV-2), the base-year direct tax rates of selected domestic nongovernment institutions (households and enterprises) are adjusted endogenously by the same number of percentage points. For the second (GOV-3), the rates of selected institutions are multiplied by a flexible scalar.[15] For these three government closures, government consumption is fixed, either in real terms or as a share of nominal absorption, depending on the treatment of the Savings–Investment balance, discussed below. In other words, we do not specify a closure where government savings and direct tax rates are both fixed and government consumption is the adjusting variable.

For the external balance, which is expressed in foreign currency, the default closure (ROW-1) is that the real exchange rate is flexible while foreign savings (the current account deficit) is fixed. Given that all other items are fixed in the external balance (transfers between the rest of the world and domestic institutions), the trade balance is also fixed. If, *ceteris paribus*, foreign savings are below the exogenous level, a depreciation of

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[14]Macroclosure of CGE models is a contentious topic with a large literature. For summaries, see Robinson (1991), Rattsø (1982), and Taylor (1990).

[15]The difference between these two closures in terms of simulated changes in post-tax incomes may be substantial, as illustrated by an example with two institutions—an enterprise and a household that each, under base conditions, have incomes of 200 and face direct tax rates of 20 percent and 10 percent, respectively. Assume that total direct tax collection has to increase from 60 to 90 to reach a fixed level of government savings (assuming, for simplicity, no income changes). Under the first closure, the rates would increase by 7.5 percentage points for both entities, to 27.5 percent for the enterprise and 17.5 percent for the household. The payments would increase by 15 percentage points for both. Under the second closure, the new tax rates would be 30 percent and 15 percent (multiplying both base rates by 1.5), respectively. The tax payments increase by 20 percentage points for the enterprise and 10 percentage points for the household.

the real exchange rate would correct this situation by simultaneously (i) reducing spending on imports (a fall in import quantities at fixed world prices) and (ii) increasing earnings from exports (an increase in export quantities at fixed world prices). Under an alternative closure (ROW-2), the real exchange rate (indexed to the model numéraire) is fixed while foreign savings (and the trade balance) is flexible.[16]

For the Savings–Investment balance, closures are either investment-driven (the value of savings adjusts) or savings-driven (the value of investment adjusts). The default closure (SI-1) is investment-driven. Real investment quantities are fixed. In order to generate savings that equal the cost of the investment bundle, the base-year savings rates of selected nongovernment institutions are adjusted by the same number of percentage points. Implicitly, it is assumed that the government is able to implement policies that generate the necessary private savings to finance the fixed real investment quantities.

Four additional closures are also specified. The first alternative (SI-2) is also investment-driven. It differs from the default in that, instead of adjusting base-year savings rates by a fixed number of percentage points, the rates of selected institutions are multiplied by a scalar (compare with the above discussion of the treatment of direct tax rates under alternative government closures). The second alternative (SI-3) is savings-driven. All nongovernment savings rates are fixed. The quantity of each commodity in the investment bundle is multiplied by a flexible scalar to ensure that the investment cost equals the savings value.

The last two alternatives (SI-4 and SI-5) are "balanced" closures, which may be viewed as variants of investment-driven closures although they also impose an adjustment rule for government consumption. Under these, adjustments in absorption are spread across all of its components (household consumption, investment, and government consumption).[17] The nominal absorption shares of investment and government consumption are fixed at base levels, although this could be generalized. (Except for SI-4 and SI-5, government consumption is fixed in real terms.) Given this specification, the residual share for household consumption is also fixed. For the first balanced closure (SI-4), the savings rates of selected institutions are adjusted by an equal number of percentage points (compare with SI-1). For the second balanced closure (SI-5), the savings rates of selected institutions are scaled so as to generate enough savings to finance investment (compare with SI-2). The balanced closures are compatible with any combination of the pre-programmed closures for the government and the rest of the world.

The appropriate choice between the different macroclosures depends on the context of the analysis. Given that this is a single-period model, a closure combining fixed foreign savings, fixed real investment, and fixed real government consumption may be preferable for simulations that

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[16]For a discussion of the real exchange rate in neoclassical, trade-focused CGE models, see Devarajan et al. (1993).

[17]Under the other investment-driven closures, the quantities of investment and government consumption are both fixed. Hence, household consumption is the only part of absorption that adjusts (in response to changes in savings rates). Under the savings-driven closure, the bulk of the adjustment is carried by investment.

explore the equilibrium welfare changes of alternative policies. In terms of the rules in Table 3, this closure combines ROW-1 with SI-1 or SI-2 and any one of the three specified government closures. In the literature on macroclosures, this is known as "Johansen closure."[18] Such a closure avoids the misleading welfare effects that appear when foreign savings and real investment change in simulations with a single-period model—*ceteris paribus*, for the simulated period, increases in foreign savings and decreases in investment raise household welfare (and vice versa for decreases in foreign savings and increases in investment). This result is misleading because the analysis does not capture welfare losses in later periods that arise from a larger foreign debt and a smaller capital stock. With regard to government consumption, the model does not capture its direct and indirect welfare contributions; to avoid misleading results, it is also preferable in welfare analysis to keep this variable fixed.

Another macroclosure often used in applied work is the savings-driven "neoclassical closure" in which investment is determined by the sum of private, government, and foreign savings. It is distinguished from the Johansen closure in that it uses SI-3 instead of SI-1 or SI-2. Both the savings-driven neoclassical closure and the investment-driven Johansen closure seem extreme when looking at the historical experience of countries adjusting to macroshocks. If the analysis aims at capturing the likely effects of an exogenous shock or policy change in a given (historical, current, or future) setting, perhaps in order to explore the role for complementary policies, it is generally preferable to impose a closure that more closely mimics the real world, with simultaneous adjustments in the three components of absorption. Under these circumstances, a macroscenario that incorporates a balanced closure (in Table 3, SI-4 or SI-5) is a useful option.

The Johansen, neoclassical, and balanced closures all assume no link between macrovariables and aggregate employment. If full-employment is assumed in the factor markets, these closures will yield different effects of shocks on the composition of aggregate demand, but with little or no effect on aggregate GDP. It is also feasible in the standard model to specify a "Keynesian" closure in which aggregate employment is linked to macrovariables through a Keynesian multiplier process. This closure is an example of a structuralist macromodel of the type advocated by Lance Taylor (1990). In this Keynesian closure, investment is fixed in real terms. In the labor market (in one of the labor markets if labor is disaggregated), it is assumed that the real wage is flexible in a setting with unemployment. Adjustment in the real wage induces firms to change their labor demand and employment sufficiently to generate incomes and savings that are needed to finance the fixed quantity of real investment. In this model, an increase in exogenous real investment (or in real government expenditure) will generate a fall in the wage, an increase in employment, an increase in income, and an increase in savings to finance the increased investment. In the context of the standard model, the easiest way to implement this closure is to (i) introduce a modified investment-driven macroclosure that is identical to SI-1 except that the MPS adjustment variable

[18]A closure of this type was used in the first CGE model, developed by Leif Johansen (1960).

is fixed; and (ii) for one labor type, introduce a modified version of the default factor-market closure where not only the wage variable, WF, but also the labor supply variable, QFS, is flexible.

Finally, it is often informative to explore the impact of experiments under a set of alternative macroclosures. The results provide important insights into the real-world tradeoffs that are associated with alternative macroeconomic adjustment patterns.

4. MATHEMATICAL MODEL STATEMENT

This chapter presents the mathematical model statement equation by equation. In its mathematical form, the CGE model is a system of simultaneous, nonlinear equations. The model is square—that is, the number of equations is equal to the number of variables. In this class of models, this is a necessary (but not a sufficient) condition for the existence of a unique solution. The chapter divides the equations into four blocks: prices, production and trade, institutions, and system constraints. New items (sets, parameters, and variables) are defined the first time that they appear in the equations. Table 4 summarizes the notational principles. Parameter and variable names are chosen to facilitate interpretation; most importantly, commodity and factor quantities start with q, commodity prices with p, and factor prices with w.

Table 4—Notational principles

Item	Notation
Endogenous variables	Upper-case Latin letters without a bar
Exogenous variables	Upper-case Latin letters with a bar
Parameters	Lower-case Latin letters (with or without a bar) or lower-case Greek letters (with or without superscripts)
Set indices	Lower-case Latin letters as subscripts to variables and parameters
Notes:	Exogenous variables are fixed in the basic model version but may be endogenous in versions with different treatments of macro- or factor-market closures.

Notes: For the specified closure rules, the choice for one of the three constraints does not constrain the choice for the other two. MPS is marginal propensity to save.

PRICE BLOCK

The price system of the model is rich, primarily because of the assumed quality differences among commodities of different origins and destinations (exports, imports, and domestic outputs used domestically). The price block consists of equations in which endogenous model prices are linked to other prices (endogenous or exogenous) and to nonprice model variables.

Import Price

$$PM_c = pwm_c \cdot (1 + tm_c) \cdot EXR + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c'c}$$

$$\begin{bmatrix} \text{import} \\ \text{price} \\ (LCU) \end{bmatrix} = \begin{bmatrix} \text{import} \\ \text{price} \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} \text{tariff} \\ \text{adjust-} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ (LCU \text{ per} \\ FCU) \end{bmatrix} + \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{import unit} \end{bmatrix} \qquad c \in CM \quad (1)$$

where

$c \in C$	=	a set of commodities (also referred to as *c'* and *C'*),
$c \in CM\ (\subset C)$	=	a set of imported commodities,
$c \in CT\ (\subset C)$	=	a set of domestic trade inputs (distribution commodities),
PM_c	=	import price in LCU (local-currency units) including transaction costs,
pwm_c	=	c.i.f. import price in FCU (foreign-currency units),
tm_c	=	import tariff rate,
EXR	=	exchange rate (LCU per FCU),
PW_c	=	composite commodity price (including sales tax and transaction costs), and
$icm_{c'c}$	=	quantity of commodity c' as trade input per imported unit of c.

The import price in LCU (local-currency units) is the price paid by domestic users for imported commodities (exclusive of the sales tax). Equation (1) states that it is a transformation of the world price of these imports, considering the exchange rate and import tariffs plus transaction costs (the cost of trade inputs needed to move the commodity from the border to the demander) per unit of the import. For all commodities, the market price paid by domestic commodity demanders is the composite price, *PQ*; in this equation, *PQ* applies only to payments for trade inputs. The domain of the equation is the set of imported commodities (a subset of the commodity set). The model includes one equation like (1) for every imported commodity.

Note that the notational principles make it possible to distinguish between variables (upper-case Latin letters) and parameters (lower-case Latin letters). This means that the exchange rate and the domestic import price are flexible, while the tariff rate and the world import price are fixed. The fixedness of the world import price stems from the "small-country" assumption. That is, for all its imports, the assumed share of world trade for the modeled country is so small that it faces an infinitely elastic supply curve at the prevailing world price.

Export Price

$$PE_c = pwe_c \cdot (1 - te_c) \cdot EXR - \sum_{c' \in CT} PQ_{c'} \cdot ice_{c'c}$$

$$\begin{bmatrix} \text{export} \\ \text{price} \\ (LCU) \end{bmatrix} = \begin{bmatrix} \text{export} \\ \text{price} \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} \text{tariff} \\ \text{adjust-} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ (LCU \text{ per} \\ FCU) \end{bmatrix} - \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{export unit} \end{bmatrix} \qquad c \in CE \quad (2)$$

where

$c \in CE\ (\subset C)$	=	a set of exported commodities (with domestic production),
PE_c	=	export price (LCU),
pwe_c	=	f.o.b. export price (FCU),
te_c	=	export tax rate,
$ice_{c'c}$	=	quantity of commodity *c'* as trade input per exported unit of *c*.

The export price in LCU is the price received by domestic producers when they sell their output in export markets. This equation is similar in structure to the import price definition. The main difference is that the tax and the cost of trade inputs reduce the price received by the domestic producers of exports (instead of adding to the price paid by domestic demanders of imports). The domain of the equation is the set of exported commodities, all of which are produced domestically.[19]

Demand Price of Domestic Non traded Goods

$$PDD_c = PDS_c + \sum_{c' \in CT} PQ_{c'} \cdot icd_{c'c}$$

$$\begin{bmatrix} \text{domestic} \\ \text{demand} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{domestic} \\ \text{supply} \\ \text{price} \end{bmatrix} + \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{unit of} \\ \text{domestic sales} \end{bmatrix} \qquad c \in CD \quad (3)$$

where

$c \in CD\ (\subset C)$	=	a set of commodities with domestic sales of domestic output,
PDD_c	=	demand price for commodity produced and sold domestically,
PDS_c	=	supply price for commodity produced and sold domestically, and
$icd_{c'c}$	=	quantity of commodity *c'* as trade input per unit of *c* produced and sold domestically.

The model includes distinct prices for domestic output that is used domestically. In the presence of transaction costs, it is necessary to distinguish between prices paid by demanders and those received by suppliers. Equation (3) defines the demand prices as the supply price plus the cost of trade inputs per unit of domestic sales of the commodity in question.

Absorption

$$PQ_c \cdot (1 - tq_c) \cdot QQ_c = PDD_c \cdot QD_c + PM_c \cdot QM_c$$

$$\begin{bmatrix} \text{absorption} \\ \text{(at demand} \\ \text{prices net of} \\ \text{sales tax)} \end{bmatrix} = \begin{bmatrix} \text{domestic demand price} \\ \text{times} \\ \text{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \text{import price} \\ \text{times} \\ \text{import quantity} \end{bmatrix} \qquad c \in (CD \cup CM) \quad (4)$$

where

QQ_c	=	quantity of goods supplied to domestic market (composite supply),
QD_c	=	quantity sold domestically of domestic output,
QM_c	=	quantity of imports of commodity, and
tq_c	=	rate of sales tax (as share of composite price inclusive of sales tax).

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[19]The model does not include any commodities that are imported for immediate re-export. As long as such trade uses domestic factors (and, possibly, intermediate inputs), it can be handled without any changes in model structure by including an activity in the SAM that imports a nonproduced commodity and exports all of its output.

Absorption is total domestic spending on a commodity at domestic demander prices. Equation (4) defines it exclusive of the sales tax. Absorption is expressed as the sum of spending on domestic output and imports at the demand prices, *PDD* and *PM*. The prices *PDD* and *PM* include the cost of trade inputs but exclude the commodity sales tax (compare with equations 1 and 3).

The equation as a whole applies to all commodities that are imported and/or have domestic sales of domestic output (the union of the sets *CD* and *CM*). It does not apply to commodities for which the entire output volume is exported. Each of the two terms on the right-hand side applies only to its relevant set (*CD* and *CM*, respectively). In the *GAMS* code, *PM* and *QM* are fixed at zero for commodities that are not elements in the set *CM*; similarly *PDD* and *QD* are fixed at zero for commodities that are not elements in the set *CD*. This approach is followed throughout: all variables that should be excluded from the model are fixed at zero. The equation would be transformed into an explicit definition of absorption at market prices or of the composite price (the price paid by domestic demanders, inclusive of the sales tax) if it were divided by $(1-tq)$ or $(1-tq).QQ$.

Marketed Output Value

$$PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c$$

$$\begin{bmatrix} \text{producer price} \\ \text{times marketed} \\ \text{output quantity} \end{bmatrix} = \begin{bmatrix} \text{domestic supply price} \\ \text{times} \\ \text{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \text{export price} \\ \text{times} \\ \text{export quantity} \end{bmatrix} \qquad c \in CX \quad (5)$$

where

PX_c	=	aggregate producer price for commodity,
QX_c	=	aggregate marketed quantity of domestic output of commodity,
QE_c	=	quantity of exports, and
$c \in CX\ (\subset C)$	=	a set of commodities with domestic output.

For each domestically produced commodity, the marketed output value at producer prices is stated as the sum of the values of domestic sales and exports.[20] Domestic sales and exports are valued at the prices received by the suppliers, PDS and PE, both of which have been adjusted downwards to account for the cost of trade inputs (compare with equations 2 and 3).

The domain limitation to domestically produced commodities (the elements in the set *CX*) has to be stated explicitly given that the model includes a category of imported commodities without domestic production. The domestic part applies only to elements in *CD* whereas the export part applies only to elements in *CE*. In the GAMS code, the variables PE and *QE* are fixed at zero for commodities that are not elements in the set *CE*. *PX* and *QX* are referred to as "aggregate" values since they may apply to an aggregation of output from different domestic producers of the same commodity. By dividing through by *QX*, this equation could be rewritten as an explicit definition of *PX*.

[20]This value excludes the value of home-consumed output.

Activity Price

$$PA_a = \sum_{c \in C} PXAC_{a\,c} \cdot \theta_{a\,c}$$

$$\begin{bmatrix} \text{activity} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{producer prices} \\ \text{times yields} \end{bmatrix} \qquad a \in A \quad (6)$$

where

$a \in A$	=	a set of activities,
PA_a	=	activity price (gross revenue per activity unit),
$PXAC_{a\,c}$	=	producer price of commodity c for activity a, and
$\theta_{a\,c}$	=	yield of output c per unit of activity a.

The gross revenue per activity unit, the activity price, is the return from selling the output or outputs of the activity, defined as yields per activity unit multiplied by activity-specific commodity prices, summed over all commodities. This allows for the fact that activities may produce multiple commodities.

Aggregate Intermediate Input Price

$$PINTA_a = \sum_{c \in C} PQ_c \cdot ica_{c\,a} \qquad a \in A \quad (7)$$

$$\begin{bmatrix} \text{aggregate} \\ \text{intermediate} \\ \text{input price} \end{bmatrix} = \begin{bmatrix} \text{intermediate input cost} \\ \text{per unit of aggregate} \\ \text{intermediate input} \end{bmatrix}$$

where

$PINTA_a$	=	aggregate intermediate input price for activity a, and
$ica_{c\,a}$	=	quantity of c per unit of aggregate intermediate input a.

The activity-specific aggregate intermediate input price shows the cost of disaggregated intermediate inputs per unit of aggregate intermediate input. It depends on composite commodity prices and intermediate input coefficients, which show the quantity of input commodity c per unit of aggregate intermediate input (not per unit of output).

Activity Revenue and Costs

$$PA_a \cdot (1 - ta_a) \cdot QA_a = PVA_a \cdot QVA_a + PINTA_a \cdot QINTA_a$$

$$\begin{bmatrix} \text{activity price} \\ \text{(net of taxes)} \\ \text{times activity level} \end{bmatrix} = \begin{bmatrix} \text{value-added} \\ \text{price times} \\ \text{quantity} \end{bmatrix} + \begin{bmatrix} \text{aggregate} \\ \text{intermediate} \\ \text{input price times} \\ \text{quantity} \end{bmatrix} \qquad a \in A \quad (8)$$

where

ta_a	=	tax rate for activity,
QA_a	=	quantity (level) of activity,
QVA_a	=	quantity of (aggregate) value-added,
$QINTA_a$	=	quantity of aggregate intermediate input, and
PVA_a	=	price of (aggregate) value-added.

For each activity, total revenue net of taxes is fully exhausted by payments for value-added and intermediate inputs. Given the above definitions of *PA* and *PINTA*, equation (8) implicitly defines the value-added price, *PVA*.

Consumer Price Index

$$\overline{CPI} = \sum_{c \in C} PQ_c \cdot cwts_c \qquad (9)$$

$$\begin{bmatrix} consumer \\ price\ index \end{bmatrix} = \begin{bmatrix} prices\ times \\ weights \end{bmatrix}$$

where

$cwts_c$ = weight of commodity c in the consumer price index, and

$\overline{CPI}$ = consumer price index (exogenous variable).

Producer Price Index for Nontraded Market Output

$$DPI = \sum_{c \in C} PDS_c \cdot dwts_c \qquad (10)$$

$$\begin{bmatrix} producer\ price\ index \\ for\ non\text{-}traded\ outputs \end{bmatrix} = \begin{bmatrix} prices\ times \\ weights \end{bmatrix}$$

where

$dwts_c$ = weight of commodity *c* in the producer price index, and

DPI = producer price index for domestically marketed output.

Equations (9) and (10) define the consumer price index and the producer price index for domestically marketed output. The *CPI* is fixed and functions as the numéraire in the basic model version; alternatively, the *DPI* may be fixed. A numéraire is required since the model is homogeneous of degree zero in prices—a doubling of the value of the numéraire would double all prices but leave all real quantities unchanged. All simulated price and income changes should be interpreted as changes vis-à-vis the numéraire price index.

PRODUCTION AND TRADE BLOCK

The production and trade block covers four categories: domestic production and input use; the allocation of domestic output to home consumption, the domestic market, and exports; the aggregation of supply to the domestic market (from imports and domestic output sold domestically); and the definition of the demand for trade inputs that is generated by the distribution process.

Production is carried out by activities that are assumed to maximize profits subject to their technology, taking prices (for their outputs, intermediate inputs, and factors) as given. In other words, it acts in a perfectly competitive setting. The CGE model includes the first-order conditions for profit-maximization by producers. As noted in the preceding section (see Figure 1), two alternative specifications are permitted at the top level of the technology nest: the activity level is either a CES or a Leontief

function of the quantities of value-added and aggregate intermediate input use.[21]

CES Technology: Activity Production Fucntion

$$QA_a = \alpha_a^a \cdot \left(\delta_a^a \cdot QVA_a^{-\rho_a^a} + (1-\delta_a^a) \cdot QINTA_a^{-\rho_a^a}\right)^{-\frac{1}{\rho_a^a}}$$

$$\begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix} = CES \begin{bmatrix} \text{quantity of aggregate value added,} \\ \text{quantity of aggregate intermediate input} \end{bmatrix} \qquad a \in ACES \quad (11)$$

CES Technology: Value-Added–Intermediate-Input Ratio

$$\frac{QVA_a}{QINTA_a} = \left(\frac{PINTA_a}{PVA_a} \cdot \frac{\delta_a^a}{1-\delta_a^a}\right)^{\frac{1}{1+\rho_a^a}}$$

$$\begin{bmatrix} \text{value-added} - \\ \text{intermediate-} \\ \text{input quantity} \\ \text{ratio} \end{bmatrix} = f \begin{bmatrix} \text{intermediate-input} : \\ \text{value-added} \\ \text{price ratio} \end{bmatrix} \qquad a \in ACES \quad (12)$$

where

$a \in ACES(\subset A)$	=	a set of activities with a CES function at the top of the technology nest,
α_a^a	=	efficiency parameter in the CES activity function,
δ_a^a	=	CES activity function share parameter, and
ρ_a^a	=	CES activity function exponent.

The user specifies the activities, if any, that belong to the set *ACES*. If not in *ACES*, an activity belongs to the set *ALEO*, which is introduced below. Activities in *ACES* have CES technology at the top level of the technology nest. In other words, the activity level is a CES function of value-added and aggregate intermediate input use (equation 11). The optimal mix of intermediate inputs and value-added is a function of the relative prices of value-added and the aggregate intermediate input (equation 12).[22] Below, in equation (18), the activity level determines the quantity of commodity outputs produced by each activity. The exponent,

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[21]For the alternative with CES technology at the top of the technology nest, the profit-maximization problem, which applies to each relevant activity, a, is as follows: maximize $PA_a \cdot (1 - ta_a) \cdot QA_a - \sum_{c \in C} PQ_c \cdot QINT_{a\,c} - \sum_{f \in F} WF_f \cdot WFDIST_{f\,a} \cdot QF_{f\,a}$ subject to equations (11), (15), and (17); equations (7), (8), (11), (12), (15), (16), and (17) are the first-order conditions. For the alternative with Leontief technology at the top, the profit-maximization problem includes equations (13) and (14) among its constraints but excludes equation 11; the Leontief first-order conditions include equations (13) and (14) instead of equations (11) and (12). In both optimization problems, all quantities are variables whereas the decisionmakers view all other items as parameters or exogenous variables (including all prices and wages).

[22]In general, when writing nonlinear equations to be solved numerically, it is good practice to avoid division by a variable that the solver treats as possibly going to zero. Accordingly, in the GAMS version of equation (12), QINTA was moved to the right-hand side. Parallel adjustments were made in equations (22) and (25).

ρ_a^a, is a transformation of the elasticity of substitution between value-added and the aggregate intermediate input: the higher this elasticity, the smaller the value of ρ_a^a and the larger the optimal change in the ratios between the quantities of value-added and the intermediate input aggregate in response to changes in their relative prices.[23]

Leontief Technology: Demand for Aggregate Value-Added

$$QVA_a = iva_a \cdot QA_a \qquad a \in ALEO \quad (13)$$

$$\begin{bmatrix} \text{demand for} \\ \text{value added} \end{bmatrix} = f \begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix}$$

Leontief Technology: Demand for Aggregate Intermediate Input

$$QINTA_a = inta_a \cdot QA_a \qquad a \in ALEO \quad (14)$$

$$\begin{bmatrix} \text{demand for aggregate} \\ \text{intermediate input} \end{bmatrix} = f \begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix}$$

where

$a \in ALEO(\subset A)$ = a set of activities with a Leontief function at the top of the technology nest,

iva_a = quantity of value-added per activity unit, and

$inta_a$ = quantity of aggregate intermediate input per activity unit.

For the alternative model version with a Leontief function at the top of the technology nest, equations (11) and (12) are replaced by equations (13) and (14) where the demands for value-added and the aggregate intermediate inputs are defined as Leontief functions of the activity level. Each activity is an element in either *ACES* or *ALEO*, but not both.

Value-Added and Factor Demands

$$QVA_a = \alpha_a^{va} \cdot \left(\sum_{f \in F} \delta_{f\,a}^{va} \cdot QF_{f\,a}{}^{-\rho_a^{va}} \right)^{-\frac{1}{\rho_a^{va}}} \qquad a \in A \quad (15)$$

$$\begin{bmatrix} \text{quantity of aggregate} \\ \text{value added} \end{bmatrix} = CES \begin{bmatrix} \text{factor} \\ \text{inputs} \end{bmatrix}$$

Factor Demand

$$WF_f \cdot \overline{WFDIST}_{f\,a} = PVA_a \left(1 - tva_a\right) \cdot QVA_a \cdot \left(\sum_{f \in F'} \delta_{f\,a}^{va} \cdot QF_{f\,a}{}^{-\rho_a^{va}} \right)^{-1} \cdot \delta_{f\,a}^{va} \cdot QF_{f\,a}{}^{-\rho_a^{va}-1} \qquad \begin{matrix} a \in A \\ f \in F \end{matrix} \quad (16)$$

$$\begin{bmatrix} \text{marginal cost of} \\ \text{factor } f \text{ in activity } a \end{bmatrix} = \begin{bmatrix} \text{marginal revenue product} \\ \text{of factor } f \text{ in activity } a \end{bmatrix}$$

[23]For CES functions, $\sigma = \frac{1}{1+\rho}$, where is the elasticity of substitution and the exponent.

where

$f \in F(=F')$	=	a set of factors,
tva_a	=	rate of value-added tax for activity a,
a_a^{va}	=	efficiency parameter in the CES value-added function,
$\delta_{f\,a}^{va}$	=	CES value-added function share parameter for factor f in activity a,
$QF_{f\,a}$	=	quantity demanded of factor f from activity a,
ρ_a^{va}	=	CES value-added function exponent,
$\overline{WF_f}$	=	average price of factor, and
$\overline{WFDIST}_{fa}$	=	wage distortion factor for factor f in activity a (exogenous variable).

Equation (15) states that, for each activity, the quantity of value-added is a CES function of disaggregated factor quantities. According to equation (16), activities demand factors at the point where the marginal cost of each factor (defined on the left-hand side as the activity-specific factor price) is equal to the marginal revenue product (net of intermediate input costs) of the factor. In the GAMS code, the domain of equation (16) is limited to the factor-activity combinations that appear in the base-year SAM. Similar domain restrictions apply to other equations that are defined over mappings between multiple indices (for example, equation 17). The exponent, ρ_a^{va}, is a transformation of the elasticity of factor substitution: the higher this elasticity, the smaller the value of ρ_a^{va} and the larger the optimal change in the ratios between different factor quantities in response to changes in relative factor prices (compare with footnote 16).

The fact that the average factor price is an endogenous variable while the activity-specific "wage-distortion" factor is exogenous reflects the treatment of factor markets in the basic model version (see equation 39 below).

Disaggregated Intermediate Input Demand

$$QINT_{c\,a} = ica_{c\,a} \cdot QINTA_a \qquad a \in A,\ c \in C \qquad (17)$$

$$\begin{bmatrix} \text{intermediate demand} \\ \text{for commodity } c \\ \text{from activity } a \end{bmatrix} = f \begin{bmatrix} \text{aggregate intermediate} \\ \text{input quantity} \\ \text{for activity } a \end{bmatrix}$$

where

$QINT_{c\,a}$	=	quantity of commodity c as intermediate input to activity a.

For each activity, the demand for disaggregated intermediate inputs is determined via a standard Leontief formulation as the level of aggregate intermediate input use times a fixed intermediate input coefficient.

Commodity Production and Allocation

$$QXAC_{a\,c} + \sum_{h \in H} QHA_{a\,c\,h} = \theta_{a\,c} \cdot QA_a \qquad a \in A,\ c \in CX \qquad (18)$$

$$\begin{bmatrix} \text{marketed quantity} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix} + \begin{bmatrix} \text{household home} \\ \text{consumption} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix} = \begin{bmatrix} \text{production} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix}$$

where

$QXAC_{a\,c}$ = marketed output quantity of commodity c from activity a, and

$QHA_{a\,c\,h}$ = quantity of household home consumption of commodity c from activity a for household h.

On the right-hand side, production quantities, disaggregated by activity, are defined as yields times activity levels. On the left-hand side, these quantities are allocated to market sales and home consumption. Note that this equation permits (i) any commodity to be produced by one or more activities and (ii) any activity to produce one or more commodities.[24]

Output Aggregation Function

$$QX_c = \alpha_c^{ac} \cdot \left(\sum_{a \in A} \delta_{a\,c}^{ac} \cdot QXAC_{a\,c}^{\;-\rho_c^{ac}} \right)^{-\frac{1}{\rho_c^{ac}-1}} \qquad c \in CX \quad (19)$$

$$\begin{bmatrix} \text{aggregate} \\ \text{marketed} \\ \text{production of} \\ \text{commodity c} \end{bmatrix} = CES \begin{bmatrix} \text{activity-specific} \\ \text{marketed} \\ \text{production of} \\ \text{commodity c} \end{bmatrix}$$

where

α_c^{ac} = shift parameter for domestic commodity aggregation function,

$\delta_{a\,c}^{ac}$ = share parameter for domestic commodity aggregation function, and

ρ_c^{ac} = domestic commodity aggregation function exponent.

First-Order Condition for Output Aggregation Function

$$PXAC_{a\,c} = PX_c \cdot QX_c \left(\sum_{a \in A'} \delta_{a\,c}^{ac} \cdot QXAC_{a\,c}^{-\rho_c^{ac}} \right)^{-1} \cdot \delta_{a\,c}^{ac} \cdot QXAC_{a\,c}^{-\rho_c^{ac}-1} \qquad \begin{matrix} a \in A \\ c \in CX \end{matrix} \quad (20)$$

$$\begin{bmatrix} \text{marginal cost of com-} \\ \text{modity c from activity a} \end{bmatrix} = \begin{bmatrix} \text{marginal revenue product of} \\ \text{commodity c from activity a} \end{bmatrix}$$

Aggregate marketed production of any commodity is defined as a CES aggregate of the marketed output levels of the different activities producing the commodity (equation 19). The optimal quantity of the commodity from each activity source is inversely related to the activity-specific price (equation 20). *QX* appears as the output, sold at the price, *PX*, and produced with the inputs, *QXAC*, that are purchased at the prices, *PXAC*.

More specifically, the choice between commodities from different sources is cast as an optimization problem. Equations (19) and (20) are the

[24]In the SAM, home consumption is represented by payments from households to activities. For the case of home consumption out of activities with multiple outputs, it is necessary to complement the information in the SAM with data on the allocation of consumption across the different activity outputs.

first-order conditions for maximizing profits from selling the aggregate output, *QX*, at the price, *PX*, subject to the aggregation function and the disaggregated commodity prices, *PXAC*. A decline in the price, *PXAC*, of one activity relative to others would shift demand in its favor without totally eliminating demand for other, higher-price sources. The degree of substitutability between different producers depends on the value of ρ_c^{ac}, which is a transformation of the elasticity of substitution (compare with footnote 16). Its values, and those of the elasticity, are restricted to assure that the corresponding isoquant is convex to the origin. In terms of production economics, this is equivalent to a diminishing technical rate of substitution.

It should be noted that, for the case where there is a single producer of a given commodity, the value of the share parameter, $\delta_{a\,c}^{ac}$, would be unity and, as a result, *QXAC* = *QX* and *PXAC* = *PX*, irrespective of the values for the elasticity and the exponent.

Output Transformation (CET) Function

$$QX_c = \alpha_c^t \cdot \left(\delta_c^t \cdot QE_c^{\rho_c^t} + (1-\delta_c^t)\cdot QD_c^{\rho_c^t}\right)^{\frac{1}{\rho_c^t}} \qquad c \in (CE \cap CD) \quad (21)$$

$$\begin{bmatrix}\text{aggregate marketed}\\ \text{domestic output}\end{bmatrix} = CET\begin{bmatrix}\text{export quantity, domestic}\\ \text{sales of domestic output}\end{bmatrix}$$

where

α_c^t	=	a CET function shift parameter,
δ_c^t	=	a CET function share parameter, and
ρ_c^t	=	a CET function exponent.

Equations (21) and (22) address the allocation of marketed domestic output, defined in equation (19), to two alternative destinations: domestic sales and exports. Equation (21) reflects the assumption of imperfect transformability between these two destinations. The CET function, which applies to commodities that are both exported and sold domestically, is identical to a CES function except for negative elasticities of substitution. The elasticity of transformation between the two destinations is a transformation of ρ_c^t, for which the lower limit is one. The values are restricted to assure that the isoquant corresponding to the output transformation function is concave to the origin.[25]

Export-Domestic Supply Ratio

$$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c}\cdot\frac{1-\delta_c^t}{\delta_c^t}\right)^{\frac{1}{\rho_c^t - 1}} \qquad c \in (CE \cap CD) \quad (22)$$

$$\begin{bmatrix}\textit{export-domestic}\\ \textit{supply ratio}\end{bmatrix} = f\begin{bmatrix}\textit{export-domestic}\\ \textit{price ratio}\end{bmatrix}$$

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[25]For CET functions, $\Omega = \frac{1}{1+\rho}$, where is the elasticity of transformation and p the exponent. As Ω varies from zero to infinity, the value of ρ_c^t varies from infinity to one. In equation (22), as ptc approaches one from above, the elasticity of the *QE-QD* ratio with respect to changes in the *PE-PDD* ratio increases.

Equation (22) defines the optimal mix between exports and domestic sales. Equations (5), (21), and (22) constitute the first-order conditions for maximization of producer revenues given the two prices and subject to the CET function and a fixed quantity of domestic output. Note that equation (22) assures that an increase in the export-domestic price ratio generates an increase in the export-domestic supply ratio (that is, a shift toward the destination that offers the higher return).

Output Transformation for Domestically Sold Outputs Without Exports and for Exports Without Domestic Sales

$$QX_c = QD_c + QE_c \qquad c \in (CD \cap CEN) \cup (CE \cap CDN) \quad (23)$$

$$\begin{bmatrix} \text{aggregate} \\ \text{marketed} \\ \text{domestic output} \end{bmatrix} = \begin{bmatrix} \text{domestic market} \\ \text{sales of domestic} \\ \text{output [for} \\ c \in (CD \cap CEN)] \end{bmatrix} + \begin{bmatrix} \text{exports [for} \\ c \in (CE \cap CDN)] \end{bmatrix}$$

where

$c \in CEN\ (\subset C)$ = non-exported commodities (complement of CE), and

$c \in CDN\ (\subset C)$ = commodities without domestic market sales of domestic output (complement of CD).

This equation replaces the CET function for domestically produced commodities that do not have both exports and domestic sales. It allocates the entire output volume to one of these two destinations.

Composite Supply (Armington) Function

$$QQ_c = \alpha_c^q \cdot \left(\delta_c^q \cdot QM_c^{-\rho_c^q} + (1-\delta_c^q) \cdot QD_c^{-\rho_c^q}\right)^{-\frac{1}{\rho_c^q}} \qquad c \in (CM \cap CD) \quad (24)$$

$$\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = f \begin{bmatrix} \text{import quantity, domestic} \\ \text{use of domestic output} \end{bmatrix}$$

where

α_c^q = an Armington function shift parameter,

δ_c^q = an Armington function share parameter, and

ρ_c^q = an Armington function exponent.

Imperfect substitutability between imports and domestic output sold domestically is captured by a CES aggregation function in which the composite commodity that is supplied domestically is "produced" by domestic and imported commodities entering this function as "inputs." When the domain of this function is limited to commodities that are both imported and produced domestically, it is often called an "Armington" function, named after the originator of the idea of using a CES function for this purpose. The elasticity of substitution between commodities from these two sources is a transformation of for which the lower limit is minus one (compare with footnote 16).

Import-Domestic Demand Ratio

$$c \in (CM \cap CD) \quad (25)$$

Equation (25) defines the optimal mix between imports and domestic output. Its domain is thus limited to imports with domestic production. Note that the equation assures that an increase in the domestic-import price ratio generates an increase in the import-domestic demand ratio (that is, a shift away from the source that becomes more expensive).[26] Together, equations (4), (24), and (25) constitute the first-order conditions for cost-minimization given the two prices and subject to the Armington function and a fixed quantity of the composite commodity.

Composite Supply for Non-imported Outputs and Non-produced Imports

$$QQ_c = QD_c + QM_c \qquad c \in (CD \cap CMN) \cup (CM \cap CDN) \quad (26)$$

$$\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = \begin{bmatrix} \text{domestic use of} \\ \text{marketed domestic} \\ \text{output [for} \\ c \in (CD \cap CMN)] \end{bmatrix} + \begin{bmatrix} \text{imports [for} \\ c \in (CM \cap CDN)] \end{bmatrix}$$

where
$c \in CMN\ (\subset C)$ = a set of non-imported commodities.

The Armington function is replaced by equation (26) for the union of commodities that have either imports or domestic sales of domestic output but not both. For any commodity in this category, it imposes equality between "composite supply" and one of the variables on the right-hand side.

Demand for Transactions Services

$$QT_c = \sum_{c' \in C'} \left(icm_{c\,c'} \cdot QM_{c'} + ice_{c\,c'} \cdot QE_{c'} + icd_{c\,c'} \cdot QD_{c'} \right)$$

$$\begin{bmatrix} \text{demand for} \\ \text{transactions} \\ \text{services} \end{bmatrix} = \begin{bmatrix} \text{sum of demands} \\ \text{for imports, exports,} \\ \text{and domestic sales} \end{bmatrix} \qquad c \in CT \quad (27)$$

where
QT_c = quantity of commodity demanded as transactions service input.

Total demand for trade inputs is the sum of the demands for these inputs that are generated by imports (from moving commodities from the border to domestic demanders), exports (from moving commodities from domestic producers to the border), and domestic market sales (from moving commodities from domestic producers to domestic demanders). In all three cases, fixed quantities of one or more transactions service inputs are required per unit of the traded commodity.

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[26]See footnote 16 for the definition of the elasticity of substitution. In equation (25), as the value of ρ_c^q approaches minus one from above, the elasticity of the import-domestic demand ratio with respect to changes in the *PDD-PM* ratio increases.

INSTITUTION BLOCK

Factor Income

$$YF_f = \sum_{a \in A} WF_f \cdot \overline{WFDIST}_{f\,a} \cdot QF_{f\,a} \qquad f \in F \quad (28)$$

$$\begin{bmatrix} \text{income of} \\ \text{factor } f \end{bmatrix} = \begin{bmatrix} \text{sum of activity payments} \\ \text{(activity-specific wages} \\ \text{times employment levels)} \end{bmatrix}$$

where

YF_f = income of factor f.

Institutional Factor Incomes

$$YIF_{i\,f} = shif_{i\,f} \cdot \left[\left(1 - tf_f\right) \cdot YF_f - trnsfr_{row\,f} \cdot EXR \right] \qquad i \in INSD,\ f \in F \quad (29)$$

$$\begin{bmatrix} \text{income of} \\ \text{institution } i \\ \text{from factor } f \end{bmatrix} = \begin{bmatrix} \text{share of income} \\ \text{of factor } f \text{ to} \\ \text{institution } i \end{bmatrix} \cdot \begin{bmatrix} \text{income of factor } f \\ \text{(net of tax and} \\ \text{transfer to RoW)} \end{bmatrix}$$

where

$i \in INS$ = a set of institutions (domestic and rest of the world),
$i \in INSD(\subset INS)$ = a set of domestic institutions,
$YIF_{i\,f}$ = income to domestic institution i from factor f,
$shif_{i\,f}$ = share of domestic institution i in income of factor f,
tf_f = direct tax rate for factor f, and
$trnsfr_{i\,f}$ = transfer from factor f to institution i.

Equation (28) defines the total income of each factor. In equation (29), this income is split among domestic institutions in fixed shares after payment of direct factor taxes and transfers to the rest of the world.[27] The latter are fixed in foreign currency and transformed into domestic currency by multiplying by the exchange rate. This equation makes reference to the set of domestic institutions (households, enterprises, and the government), a subset of the set of institutions, which also includes the rest of world.

Income of domestic, Nongovernment Institutions

$$YI_i = \sum_{f \in F} YIF_{i\,f} + \sum_{i' \in INSDNG'} TRII_{i\,i'} + trnsfr_{i\,gov} \cdot \overline{CPI} + trnsfr_{i\,row} \cdot EXR \qquad i \in INSDNG \quad (30)$$

$$\begin{bmatrix} \text{income of} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{factor} \\ \text{income} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from other domestic} \\ \text{non-government} \\ \text{institutions} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from} \\ \text{government} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from} \\ \text{RoW} \end{bmatrix}$$

where

$i \in INSDNG(=INSDNG' \subset INSD)$
= a set of domestic nongovernment institutions,
YI_i = income of institution i (in the set INSDNG), and
$TRII_{ii'}$ = transfers from institution i' to i (both in the set INSDNG).

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[27] To assure that the total factor income is distributed, it is necessary that $\sum_{i \in INSD} shift_{if} = 1$.

Domestic nongovernment institutions form a subset of the set of domestic institutions. The total income of any domestic nongovernment institution is the sum of factor incomes (defined in equation 29), transfers from other domestic nongovernment institutions (defined below in equation 31), transfers from the government (indexed to the CPI), and transfers from the rest of the world.[28]

Infra-Institutional Transfers

$$TRII_{i\,i'} = shii_{i\,i'} \cdot \left(1 - MPS_{i'}\right) \cdot \left(1 - TINS_{i'}\right) \cdot YI_{i'} \qquad \begin{matrix} i \in INSDNG \\ i' \in INSDNG' \end{matrix} \quad (31)$$

$$\begin{bmatrix} \text{transfer from} \\ \text{institution } i' \text{ to } i \end{bmatrix} = \begin{bmatrix} \text{share of net income} \\ \text{of institution } i' \\ \text{transfered to } i \end{bmatrix} \cdot \begin{bmatrix} \text{income of institution} \\ i', \text{ net of savings and} \\ \text{direct taxes} \end{bmatrix}$$

where

$shii_{ii}$ = share of net income of i' to i ($i' \in INSDNG'$; $i \in INSDNG$),

MPS_i = marginal propensity to save for domestic nongovernment institution (exogenous variable), and

$TINS_i$ = direct tax rate for institution i ($i \in INSDNG$).

Transfers between domestic nongovernment institutions are paid as fixed shares of the total institutional incomes net of direct taxes and savings. The values of *MPS* and *TINS* are defined in separate equations, discussed below.

Household Consumption Expenditures

$$EH_h = \left(1 - \sum_{i \in INSDNG} shii_{i\,h}\right) \cdot \left(1 - MPS_h\right) \cdot \left(1 - TINS_h\right) \cdot YI_h \qquad h \in H \quad (32)$$

$$\begin{bmatrix} \text{household income} \\ \text{disposable for} \\ \text{consumption} \end{bmatrix} = \begin{bmatrix} \text{household income, net of direct} \\ \text{taxes, savings, and transfers to} \\ \text{other non-government institutions} \end{bmatrix}$$

where

$i \in H (\subset INSDNG)$

= a set of households, and

EH_h = household consumption expenditures.

Among the domestic nongovernment institutions, only households demand commodities. In equation (32), the total value of consumption spending is defined as the income that remains after direct taxes, savings, and transfers to other domestic nongovernment institutions.

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[28]The fact that government transfers are indexed to the CPI makes the model homogeneous of degree zero in prices. This indexing influences the results when the DPI is the model numéraire. If the CPI is the numéraire, it has no effect.

Household Consumption Spending on Marketed Commodities

$$PQ_c \cdot QH_{c\,h} = PQ_c \cdot \gamma^m_{c\,h} + \beta^m_{c\,h} \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot \gamma^m_{c'\,h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a\,c'} \cdot \gamma^h_{a\,c'\,h} \right)$$

$$\begin{bmatrix} \text{household consumption} \\ \text{spending on market} \\ \text{commodity } c \end{bmatrix} = f \begin{bmatrix} \text{total household consumption} \\ \text{spending, market price of c, and other} \\ \text{commodity prices (market and home)} \end{bmatrix} \qquad \begin{matrix} c \in C \\ h \in H \end{matrix} \quad (33)$$

where

$QH_{c\,h}$ = quantity of consumption of marketed commodity c for household h,

$\gamma^m_{c\,h}$ = subsistence consumption of marketed commodity c for household h,

$\gamma^h_{a\,c\,h}$ = subsistence consumption of home commodity c from activity a for household h, and

$\beta^m_{c\,h}$ = marginal share of consumption spending on marketed commodity c for household h.

Household Consumption Spending on Home Commodities

$$PXAC_{a\,c} \cdot QHA_{a\,c\,h} = PXAC_{a\,c} \cdot \gamma^h_{a\,c\,h} + \beta^h_{a\,c\,h} \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot \gamma^m_{c'\,h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a\,c'} \cdot \gamma^h_{a\,c'\,h} \right) \qquad \begin{matrix} a \in A \\ c \in C \\ h \in H \end{matrix} \quad (34)$$

$$\begin{bmatrix} \text{household consumption} \\ \text{spending on home commodity} \\ \text{c from activity a} \end{bmatrix} = f \begin{bmatrix} \text{total household consumption spending,} \\ \text{producer price, and other} \\ \text{commodity prices (market and home)} \end{bmatrix}$$

where

$\beta^h_{a\,c\,h}$ = marginal share of consumption spending on home commodity c from activity a for household h.

It is assumed that each household maximizes a "Stone–Geary" utility function subject to a consumption expenditure constraint. The resulting first-order conditions, equations (33) and (34), are referred to as LES (linear expenditure system) functions since spending on individual commodities is a linear function of total consumption spending, EH. Two functions are needed since household consumption is for two types of commodities: (i) consumption of marketed commodities (purchased at market prices; equation 33) and (ii) consumption of home production (valued at their opportunity cost, the activity-specific producer price not including marketing costs; equation 34). Explicit demand functions may be derived by dividing both sides of each equation by the relevant price.

Investment Demand

$$QINV_c = \overline{IADJ} \cdot \overline{qinv}_c \qquad c \in C \quad (35)$$

$$\begin{bmatrix} \text{fixed investment} \\ \text{demand for} \\ \text{commodity } c \end{bmatrix} = \begin{bmatrix} \text{adjustment factor} \\ \text{times} \\ \text{base-year fixed} \\ \text{investment} \end{bmatrix}$$

where

$QINV_c$	=	quantity of fixed investment demand for commodity,
$\overline{IADJ}$	=	investment adjustment factor (exogenous variable), and
$\overline{qinv_c}$	=	base-year quantity of fixed investment demand.

Fixed investment demand is defined as the base-year quantity multiplied by an adjustment factor. For the basic model version, the adjustment factor is exogenous, in effect also making the investment quantity exogenous. Inventory investment is also included in the model, but is treated as an exogenous demand (see equation 40 below).

Government Consumption Demand

$$QG_c = \overline{GADJ} \cdot \overline{qg}_c$$

$$\begin{bmatrix} \text{government} \\ \text{consumption} \\ \text{demand for} \\ \text{commodity } c \end{bmatrix} = \begin{bmatrix} \text{adjustment factor} \\ \text{times} \\ \text{base-year government} \\ \text{consumption} \end{bmatrix} \qquad c \in C \quad (36)$$

where

QG_c	=	government consumption demand for commodity,
$\overline{GADJ}$	=	government consumption adjustment factor (exogenous variable), and
$\overline{qg}_c$	=	base-year quantity of government demand.

Similarly, government consumption demand, in which the main component tends to be the services provided by the government labor force, is also defined as the base-year quantity multiplied by an adjustment factor. This factor is also exogenous and, hence, the quantity of government consumption is fixed.

Government Revenue

$$\begin{aligned} YG = & \sum_{i \in INSDNG} TINS_i \cdot YI_i + \sum_{f \in F} tf_f \cdot YF_f + \sum_{a \in A} tva_a \cdot PVA_a \cdot QVA_a \\ & + \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a + \sum_{c \in CM} tm_c \cdot pwm_c \cdot QM_c \cdot EXR + \sum_{c \in CE} te_c \cdot pwe_c \cdot QE_c \cdot EXR \\ & + \sum_{c \in C} tq_c \cdot PQ_c \cdot QQ_c + \sum_{f \in F} YIF_{gov\,f} + trnsfr_{gov\,row} \cdot EXR \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = & \begin{bmatrix} \text{direct taxes} \\ \text{from} \\ \text{institutions} \end{bmatrix} + \begin{bmatrix} \text{direct taxes} \\ \text{from} \\ \text{factors} \end{bmatrix} + \begin{bmatrix} \text{value-} \\ \text{added} \\ \text{tax} \end{bmatrix} \\ & + \begin{bmatrix} \text{activity} \\ \text{tax} \end{bmatrix} + \begin{bmatrix} \text{import} \\ \text{tariffs} \end{bmatrix} + \begin{bmatrix} \text{export} \\ \text{taxes} \end{bmatrix} \\ & + \begin{bmatrix} \text{sales} \\ \text{tax} \end{bmatrix} + \begin{bmatrix} \text{factor} \\ \text{income} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from} \\ \text{RoW} \end{bmatrix} \end{aligned} \qquad (37)$$

where

YG	=	government revenue.

Total government revenue is the sum of revenues from taxes, factors, and transfers from the rest of the world.

Government Expenditure

$$EG = \sum_{c \in C} PQ_c \cdot QG_c + \sum_{i \in INSDNG} trnsfr_{i\ gov} \cdot \overline{CPI} \tag{38}$$

$$\begin{bmatrix} \text{government} \\ \text{spending} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{transfers to domestic} \\ \text{non-government} \\ \text{institutions} \end{bmatrix}$$

where

EG = government expenditures.

Total government spending is the sum of government spending on consumption and transfers.

SYSTEM CONSTRAINT BLOCK

Factor Markets

$$\sum_{a \in A} QF_{f\ a} = \overline{QFS}_f \qquad f \in F \tag{39}$$

$$\begin{bmatrix} \text{demand for} \\ \text{factor } f \end{bmatrix} = \begin{bmatrix} \text{supply of} \\ \text{factor } f \end{bmatrix}$$

where

$\overline{QFS}_f$ = quantity supplied of factor (exogenous variable).

Equation (39) imposes equality between the total quantity demanded and the total quantity supplied for each factor. In the basic model version, all demand variables are flexible while the supply variable is fixed. The factor wage, WF_f, is the equilibrating variable that assures that this equation is satisfied: an increase in WF_f raises the wage paid by each activity, $WF_f \cdot \overline{WFDIST}_{f\,a}$, which is inversely related to the quantities of factor demand, $QF_{f\,a}$. All factors are mobile between the demanding activities.

Two other factor-market closures are programmed in the GAMS version. To specify the case with unemployment at a given wage for a factor, the supply variable for the factor is unfixed (QFS_f) while its economywide wage is fixed ($\overline{WF}_f$). The model remains square (one endogenous variable is added but another is removed). Each activity is free to employ the quantity it desires ($QF_{f\,a}$) at a fixed wage ($\overline{WF}_f \cdot \overline{WFDIST}_{f\,a}$). The free supply variable, QFS_f, records the total employment level.

Alternatively, to specify the case of a fully segmented factor market with fixed factor demands (for example, short-run fixity of nonagricultural capital use), the variables for factor demand and the economywide wage are fixed (written $\overline{QF}_{f\,a}$ and $\overline{WF}_f$) while the variables for supply and wage distortions are unfixed (written QFS_f and $WFDIST_{f\,a}$). The model again remains square—that is, the economywide wage variable and a set of activity-specific factor-demand variables are fixed while the supply variable and a set of activity-specific wage-distortion variables are unfixed.

Activity-specific wages, $\overline{WF}_f \cdot WFDIST_{fa}$, vary to assure that the fixed activity-specific employment level, $\overline{QF}_{fa}$, is consistent with profit-maximization (compare with equation 16). Also for this formulation, the endogenous aggregate factor supply variable merely records the total employment level.

Composite Commodity Markets

$$QQ_c = \sum_{a \in A} QINT_{ca} + \sum_{h \in H} QH_{ch} + QG_c + QINV_c + qdst_c + QT_c \qquad c \in C \quad (40)$$

$$\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = \begin{bmatrix} \text{intermediate} \\ \text{use} \end{bmatrix} + \begin{bmatrix} \text{household} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix} + \begin{bmatrix} \text{trade} \\ \text{input use} \end{bmatrix}$$

where

$qdst_c$ = quantity of stock change.

Equation (40) imposes equality between quantities supplied (from equations 24, 25, and 26) and demanded of the composite commodity. The demand side includes endogenous terms (from equations 17, 27, 33, 35, and 36) and a new exogenous term for stock change. Among the endogenous terms, *QG* and *QINV* are fixed in the basic model version (compare with equations 35 and 36). The composite commodity supply, *QQ*, drives demands for domestic marketed output, *QD*, and imports, *QM*. The market-clearing variables are the quantities of import supply, for the import side, and the two interrelated domestic prices, *PDD* and *PDS*, for domestic market output.

Current-Account Balance for the Rest of the World, in Foreign Currency

$$\sum_{c \in CM} pwm_c \cdot QM_c + \sum_{f \in F} trnsfr_{row\,f} = \sum_{c \in CE} pwe_c \cdot QE_c + \sum_{i \in INSD} trnsfr_{i\,row} + \overline{FSAV} \quad (41)$$

$$\begin{bmatrix} \text{import} \\ \text{spending} \end{bmatrix} + \begin{bmatrix} \text{factor} \\ \text{transfers} \\ \text{to RoW} \end{bmatrix} = \begin{bmatrix} \text{export} \\ \text{revenue} \end{bmatrix} + \begin{bmatrix} \text{institutional} \\ \text{transfers} \\ \text{from RoW} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix}$$

where

$\overline{FSAV}$ = foreign savings (FCU) (exogenous variable).

The current-account balance, which is expressed in foreign currency, imposes equality between the country's spending and its earning of foreign exchange. For the basic model version, foreign savings is fixed; the (real) exchange rate (*EXR*) serves the role of equilibrating variable to the current-account balance. The fact that all items except imports and exports are fixed means that, in effect, the trade deficit also is fixed. Alternatively, the exchange rate may be fixed and foreign savings unfixed. In this case, the trade deficit is free to vary.

Government Balance

$$YG = EG + GSAV$$

$$\begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{expenditures} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix} \tag{42}$$

where

$GSAV$ = government savings.

The government balance imposes equality between current government revenue and the sum of current government expenditures (not including government investment) and savings. Savings may be negative. The alternative mechanisms for maintaining this balance are associated with equation (43).

Direct Institutional Tax Rates

$$TINS_i = \overline{tins}_i \cdot \left(1 + \overline{TINSADJ} \cdot tins01_i\right) + \overline{DTINS} \cdot t_i$$

$$\begin{bmatrix} \text{direct tax} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix} \qquad i \in INSDNG \tag{43}$$

where

$TINS_i$ = rate of direct tax on domestic institutions i,

$\overline{tins}_i$ = exogenous direct tax rate for domestic institution i,

$\overline{TINSADJ}$ = direct tax scaling factor (= 0 for base; exogenous variable),

$tins01_i$ = 0–1 parameter with 1 for institutions with potentially flexed direct tax rates, and

$\overline{DTINS}_i$ = change in domestic institution tax share (= 0 for base; exogenous variable).

Equation (43) defines the direct tax rates of domestic nongovernment institutions. For the basic model version, all variables on the right-hand side are fixed, in effect fixing the values for the direct tax rate variable for all institutions. In this setting, government savings is the endogenous variable that clears the government balance.

In the GAMS implementation of the standard model, two alternative closure rules are coded for the government balance (see *Macroeconomic Balances* in Chapter 3). For both alternatives, government savings is fixed. In the first case, *DTINS* is the flexible variable that clears the government balance by scaling the base-year tax rates of each tax-paying institution. In this setting, the rates will change by a uniform number of (percentage) points for all institutions with a value of 1 for the parameter *tins*01 (that is, for all institutions with potentially flexed direct tax rates). Hence, the initial tax rate has no impact on the rate change. In the second case, *TINSADJ* becomes a variable while *DTINS* is fixed. For this closure, the changes in *TINS* are relatively large for institutions with relatively large base-year rates (if they have a value of 1 for *tins*01).

Notice that when *GSAV* is fixed for the two alternative closure rules, another variable is made endogenous, thus maintaining a model with an equal number of variables and equations. The choice between alternative closure rules should depend on the empirical context. For example, if the government pursues a policy of raising effective direct tax rates to maintain fixed savings in a setting with reduced other revenues and/or increased government spending, will it raise rates for all or only a subset of the nongovernment institutions? For the targeted institutions, will the government aim at uniform point increases or will it raise rates in proportion to current rates?

Institutional Savings Rates

$$MPS_i = \overline{mps}_i \cdot \left(1 + \overline{MPSADJ} \cdot mps01_i\right) + DMPS \cdot mps01_i \qquad i \in INSDNG \quad (44)$$

$$\begin{bmatrix} \text{savings} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix}$$

where

$\overline{mps_i}$	=	base savings rate for domestic institution i,
$\overline{MPSADJ}$	=	savings rate scaling factor (= 0 for base),
$MPS01_i$	=	0-1 parameter with 1 for institutions with potentially flexed direct tax rates, and
$DMPS$	=	change in domestic institution savings rates (= 0 for base; exogenous variable).

Equation (44) defines the savings rates of domestic nongovernment institutions. Its structure is the same as that of equation (43). Whether one or none of the variables *MPSADJ* and *DMPS* is flexible depends on the closure rule for the Savings–Investment balance. For the basic model version, *DMPS* is flexible, permitting *MPS* to be adjusted by a uniform rate for selected (one or more) nongovernment institutions.

Savings–Investment Balance

$$\sum_{i \in INSDNG} MPS_i \cdot \left(1 - TINS_i\right) \cdot YI_i + GSAV + EXR \cdot \overline{FSAV} = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c \qquad (45)$$

$$\begin{bmatrix} \text{non-govern-} \\ \text{ment savings} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix} = \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix}$$

Equation (45) states that total savings and total investment have to be equal. Total savings is the sum of savings from domestic nongovernment institutions, the government, and the rest of the world, with the last item converted into domestic currency. Total investment is the sum of the values of fixed investment (gross fixed capital formation) and stock changes.

In the basic model version, the flexible variable, *DMPS*, performs the task of clearing this balance (compare with equation 44). None of the other items in the Savings–Investment balance is free to vary to assure that the balance holds. Given that the balancing role is performed by the savings side, this closure represents a case of "investment-driven" savings. In the GAMS code, additional Savings–Investment closures have also been programmed. Under closure 2 (see Table 3), *DMPS* is fixed and *MPSADJ* is flexible. For closure 3, in which investment is savings-driven, *IADJ* is flexible whereas both *MPSADJ* and *DMPS* are fixed.

Up to this point, the model as stated is not square; the number of equations exceeds the number of variables by one. However, the model satisfies Walras' law: one equation is functionally dependent on the others and can be dropped. The Savings–Investment balance or the current-account balance is commonly eliminated.) After eliminating one equation, the model is square and, in the absence of errors in formulation, a unique solution typically exists. Instead of dropping one equation, it is also possible to add one variable to the macroeconomic balance equations. The solution value of this variable should be zero. If not, one or more equations are not satisfied and a general equilibrium solution has not been found. This is the approach followed in the GAMS version of the model. A variable called *WALRAS* is added to the Savings–Investment balance. No equation is dropped.

After this adjustment, the model presented is complete and self-contained. In the basic model version, three more equations (and three new variables that appear in them) are added. The reason for including these is that they permit the formulation of the "balanced" Savings–Investment closures 4 and 5. We will return to the closure issue later, after presenting the new equations and their notation.

Total Absorption

$$TABS = \sum_{h\in H}\sum_{c\in C} PQ_c \cdot QH_{c\,h} + \sum_{a\in A}\sum_{c\in C}\sum_{h\in H} PXAC_{a\,c} \cdot QHA_{a\,c\,h} + \sum_{c\in C} PQ_c \cdot QG_c + \sum_{c\in C} PQ_c \cdot QINV_c + \sum_{c\in C} PQ_c \cdot qdst_c \tag{46}$$

$$\begin{bmatrix} \textit{total} \\ \textit{absorption} \end{bmatrix} = \begin{bmatrix} \textit{household} \\ \textit{market} \\ \textit{consumption} \end{bmatrix} + \begin{bmatrix} \textit{household} \\ \textit{home} \\ \textit{consumption} \end{bmatrix} + \begin{bmatrix} \textit{government} \\ \textit{consumption} \end{bmatrix} + \begin{bmatrix} \textit{fixed} \\ \textit{investment} \end{bmatrix} + \begin{bmatrix} \textit{stock} \\ \textit{change} \end{bmatrix}$$

where

TABS = total nominal absorption.

Total absorption is measured as the total value of domestic final demands, which equals GDP at market prices plus imports minus exports. The new variable, *TABS*, records this value.

Ratio of Investment to Absorption

$$INVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$$

$$\begin{bmatrix} \text{investment-} \\ \text{absorption} \\ \text{ratio} \end{bmatrix} \cdot \begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} = \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix} \tag{47}$$

where

INVSHR = investment share in nominal absorption.

The right-hand side of this equation defines the total investment value (compare with equation 45). On the left-hand side, total absorption is multiplied by a new free variable, *INVSHR*. At equilibrium, this variable measures the ratio between investment and absorption.

Ratio of Government Consumption to Absorption

$$GOVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QG_c$$

$$\begin{bmatrix} \text{government} \\ \text{consumption-} \\ \text{absorption} \\ \text{ratio} \end{bmatrix} \cdot \begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} \tag{48}$$

where

GOVSHR = government consumption share in nominal absorption.

This final equation is similar to equation (47) except that investment is replaced by government consumption. The right-hand side defines the value of government consumption (compare with equation 38). On the left-hand side, total absorption is multiplied by a new free variable, *GOVSHR* , which measures the ratio between government consumption and absorption.

The presence of equations (46), (47), and (48) and the three new variables makes it possible to specify Savings–Investment closures 4 and 5, which represent versions of "balanced" macroeconomic adjustment that may be preferable for model simulations aimed at generating plausible, real-world responses to shocks (compare with discussion of *Macroeconomic Balances* in Chapter 3). For the investment-driven, Savings–investment closures 1 and 2, the burden of adjusting to absorption shocks is assumed in full by household consumption.[29] Under closure 3, with savings-driven investment, the adjustment burden falls on investment.

Savings–Investment closures 4 and 5 in Table 3, which are also programmed in the GAMS version of the model, impose a balanced adjustment in the aggregate components of absorption. Under both, the shares of nominal absorption for investment and government consumption (*INVSHR* and *GOVSHR*) are fixed at base levels while the quantity adjustment factors for fixed investment demand and government consumption (*IADJ* and *GADJ*) are endogenized. The two closures differ as to whether *DMPS* or *MPSADJ* is the flexible variable that generates the Savings–Investment equilibrium.

Under these two closures, any change in total absorption would, in nominal terms, be spread evenly across all three components of absorption; given the shares for investment and government consumption, the share for household consumption is implicitly defined. Adjustments in the nongovernment savings value clear the savings- investment balance. The magnitude of the savings adjustment, which is influenced by changes in investment and government consumption (for the latter via changes in government savings), determines the availability of resources for household consumption.

[29]However, for simulations with single-period models (like the current model) aimed at exploring welfare impacts of exogenous shocks, the Savings–Investment closure 1 or 2 is often preferable since the model is unable to capture future welfare changes associated with current changes in investment (compare with the *Macroeconomic Balances* discussion in Chapter 3).

5. THE STANDARD MODEL IN GAMS

The GAMS input files contained in the CD-ROM that accompanies this manual include country data files that enable the user to conduct simulations with the standard model using data for a selected country. It is also straightforward to apply this modeling system to alternative country data sets generated by the user. This chapter provides a brief guide to the GAMS files and suggestions on how to use this modeling system. The files themselves include additional explanatory comments.

Table 5 summarizes the contents of the different files and Figure 3 provides a schematic representation of the structure of the GAMS model and data files.[30] The modeling system is segmented into two main files, *mod.gms* and *sim.gms*. This segmentation corresponds to the two main steps in a typical CGE modeling project. In the first main file, mod.gms, the model, which is identical to that detailed in Chapter 4, is set up and calibrated to a country data set that is read in the form of an "include" file (*<name>.dat*). The sample data sets illustrate how data sets should be defined.[31] The SAMs may be included directly in the *<name>.dat* file or be read into this file using GAMS GDX file command that comes with recent GAMS release (or via a link to a spreadsheet using the program XLLINK, which has to be installed separately for older releases of GAMS. The former approach—direct inclusion of the SAM and the rest of the data—is often preferable because it is less error-prone, and it facilitates model documentation and transportability between different users and computers. If the account imbalances in the SAM exceed a low cutoff point, a simple SAM balancing program in the file *sambal.inc* is activated. The file *varinit.inc* is used to initialize all variables at base levels. In the optional

[30]The CD-ROM also includes an example file for a SAM aggregation program (samagg.gms). It may be used independently of the other GAMS files, in which case the appropriately aggregated SAM should be inserted in the country data file. Alternatively, after some adjustments, samagg.gms may be used as an include file in the country data set, immediately before the inclusion of sambal.inc. If so, the set AC in the country data file should be expanded to include all SAM accounts (both of the initial SAM and the aggregated SAM). The rest of the sets should be defined on the basis of the accounts in the aggregated SAM.

[31]Three sample data sets are included: test.dat, which is based on data from Mozambique, and is designed to test many of the model features; swazilan.dat, which includes macrodata for Swaziland that has only one element in each account set; and zimbabwe.dat, a Zimbabwe data set. When applied to test.dat or swazilan.dat, the standard model can be solved using the student version of GAMS; the Zimbabwe data set is larger and requires a full version of GAMS (including solvers for NLP and MCP problems). Data sets for a number of other countries are also available.

Table 5—File structure in GAMS standard CGE modeling system

File name	Description
mod.gms	All items (sets, parameters, variables) that appear in the standard model equations as well as the equations themselves and the CGE model are declared. Except for the sets, these items are also defined. The model is solved for the base.
<name>.dat	Include files for mod.gms with country-specific data sets (named after the country they represent), one of which should be included. The data consists of set elements (used to define model sets), a SAM, elasticities, selected physical factor quantities, commodity value shares for home consumption (if needed), and a parameter transforming SAM tax data.
sambal.inc	Include file for <name>.dat. A simple program that balances the SAM if its account imbalances exceed a cut-off point.
varinit.inc	Include file for mod.gms (and, optionally, for sim.gms). All model variables are initialized.
varlow.inc	Optional include file for mod.gms. Imposes lower limits on selected model variables.
repbase.inc	Include file for mod.gms. Using data from the base solution, defines an economic structure table, a GDP table, and a macro-SAM.
sim.gms	Restarted from mod.gms. The file includes (a) declarations and definitions of sets for simulations, experiment parameters, closures for macrosystem constraints, and closures for factor markets; (b) a loop over the set of current simulations that contains definitions of simulation-specific parameters and variables, a solve statement, and an include file defining report parameters; (c) preparation and processing of report parameters (in include files), checks for errors in report parameters, and a display of report parameters.
repsetup.inc	Include file for sim.gms that includes (a) declarations and definitions for sets used in reports; and (b) declarations of report parameters.
reploop.inc	Include file for sim.gms. For each simulation, the file defines report parameters for (a) the levels of each model variable;[a] (b) the value of parameters that are subject to change in simulations; (c) the incomes and expenditures of each SAM account; (d) national accounts data; (e) macro- and factor-market closure; (f) consistency checks for data in (c) and (d).
repperc.inc	Include file for sim.gms. For all relevant parameters under (a) through (d) in reploop.inc, computation of percentage change from base for nonbase simulations.[b]
repsum.inc	Include file for sim.gms. Summary results tables based on report parameters defined in reploop.inc and repperc.inc.

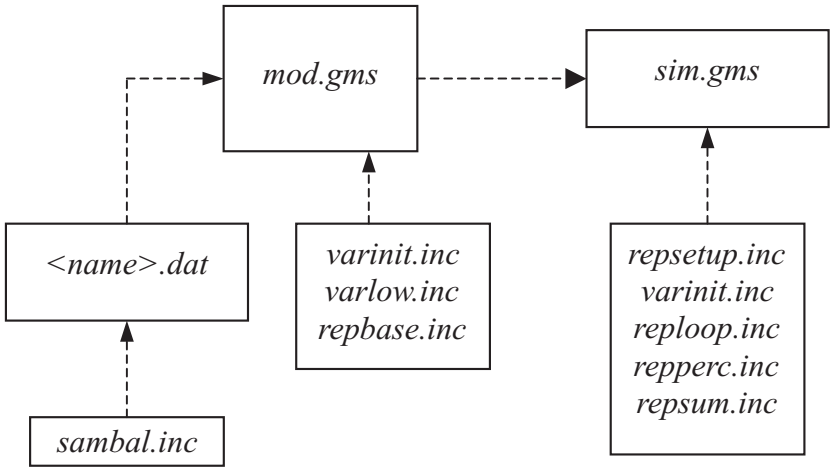
[a]These parameters have the same name as the corresponding variable with X added at the end.
[b]These parameters have the same name as the corresponding parameter in reploop.inc with P added at the end.

file varlow.inc, lower limits close to zero are imposed for selected variables as this may improve solver performance.

Two models are defined inside *mod.gms*, one for MCP (mixed-complementarity programming) and one for NLP (nonlinear programming) solvers.[32] The MCP model is identical to the model presented above. The NLP model differs in that it also includes an objective function. The objective function is needed given that this is an optimization problem, but it has no influence on the solution since there is only one feasible solution that satisfies all constraints. After having solved the model for the base, the program calls up the file *repbase.inc*, which generates a report on the base solution.

In *sim.gms*, which restarts from the save files of *mod.gms*, simulations are defined and carried out.[33] A note at the beginning of the file specifies the steps required when additional simulations are introduced. For each simulation, the user can choose between alternative closures for macroeconomic constraints (compare with Table 3) and factor markets (three alternatives for each factor and simulation; see summary in Chapter 3). The user has the option of selecting the base levels of the model variables as the solver's starting point for selected simulations (by including the file *varinit.inc*); this may facilitate the solver's task of finding a solution relative to the default, according to which it uses the variable levels from the preceding model solution. Report parameters are declared in the include file repsetup.inc and defined in the include files reploop.inc, repperc.inc, and repsum.inc. The parameters are designed to contain most of the information that an analyst may be interested in; Table 5 provides details. Repsum.inc may be used as a starting point for user-defined reports that highlight information of interest in a specific application.

Figure 3—The structure of GAMS model and data files



[32]For information on solvers, visit the GAMS Development Corporation website (www.gams.com).
[33]For save and restart facilities in GAMS, see Brooke et al. (1998, 199).

The modeling system presented can be used in a variety of ways. The first and most straightforward approach is to carry out simulations with one of the existing data sets without making any changes in the modeling structure. Here the user is required only to define new simulations. The file *sim.gms* includes a note that summarizes the core steps to take when carrying out additional simulations.

In a second approach, users may wish to take the additional step of applying the model to their own data set. If so, it is preferable to structure the data set in the same way as the sample data files. The most critical additional step is to generate a properly formatted SAM. If an available SAM has a different format (for example, exports from activity accounts instead of commodity accounts or a different treatment of taxes), we strongly recommend that the user reformat the SAM (a task that can be done inside the GAMS include file). The alternative of adjusting the model code to a differently formatted SAM is likely to be more time-consuming and error-prone. Once the model properly calibrates to the new data set, the user can proceed with simulations.

The third approach is also the most involved. Here, in combination with 1 and 2, more advanced users may wish to change the model, a step that involves changing the files *mod.gms* and, quite likely, *<name>.dat*, as existing model elements (sets, parameters, variables, and equations) are modified or new ones are declared and defined. If the user is also applying the model to a new data set (as in the second approach above), it is probably easier to divide the process into two steps, first generating a data set to which the original model calibrates and second modifying the model. Changes in the model structure will also require the user to modify and/or add to the report system, for example, adding new parameters to account for new model variables and modifying the parameters that define the incomes and expenditures of SAM accounts.[34]

After having read this manual, we recommend that users familiarize themselves with the contents of the different files. For users who limit themselves to the first approach, the most important task is to become familiar with the file *sim.gms* and its include files. For users who also add their own database, as in the second approach described above, it is also crucial to be aware of the detailed structure of the standard SAM (described in Chapter 2 and exemplified in the country data files) and how it may differ from the original format of any new SAM that the user wants to apply. A thorough study of the modeling system is required for users who, in addition, wish to modify the model, using it as a tool to develop further in different directions.

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[34]The modeling system includes consistency checks on the report parameters that will generate error messages if, for example, the reports show imbalances between the income and spending of SAM accounts.

APPENDIX A: MATHEMATICAL SUMMARY STATEMENT FOR THE STANDARD CGE MODEL

SETS

$\alpha \in A$	activities
$\alpha \in ACES(\subset A)$	activities with a CES function at the top of the technology nest
$\alpha \in ALEO(\subset A)$	activities with a Leontief function at the top of the technology nest
$c \in C$	commodities
$c \in CD(\subset C)$	commodities with domestic sales of domestic output
$c \in CDN(\subset C)$	commodities not in CD
$c \in CE(\subset C)$	exported commodities
$c \in CEN(\subset C)$	commodities not in CE
$c \in CM(\subset C)$	imported commodities
$c \in CMN(\subset C)$	commodities not in CM
$c \in CT(\subset C)$	transactions service commodities
$c \in CX(\subset C)$	commodities with domestic production
$f \in F$	factors
$i \in INS$	institutions (domestic and rest of the world)
$i \in INSD(\subset INS)$	domestic institutions
$i \in INSDNG$ $(\subset INSD)$	domestic nongovernment institutions
$h \in H(\subset INSDNG)$	households

PARAMETERS

Latin Letters

$cwts_c$	weight of commodity c in the CPI
$dwts_c$	weight of commodity c in the producer price index
$ica_{c\,a}$	quantity of c as intermediate input per unit of activity a
$icd_{c\,c'}$	quantity of commodity c as trade input per unit of c' produced and sold domestically
$ice_{c\,c'}$	quantity of commodity c as trade input per exported unit of c'
$icm_{c\,c'}$	quantity of commodity c as trade input per imported unit of c'
$inta_a$	quantity of aggregate intermediate input per activity unit

iva_a	quantity of value-added per activity unit
$\overline{mps_i}$	base savings rate for domestic institution i
$mps01_c$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
pwe_c	export price (foreign currency)
pwm_c	import price (foreign currency)
$qdst_c$	quantity of stock change
$\overline{qg_c}$	base-year quantity of government demand
$\overline{qinv_c}$	base-year quantity of private investment demand
$shif_{if}$	share for domestic institution i in income of factor f
$shii_{ii'}$	share of net income of i' to i ($i' \in INSDNG'$; $i \in INSDNG$)
$t\alpha_\alpha$	tax rate for activity a
te_c	export tax rate
tf_f	direct tax rate for factor f
$\overline{tins_i}$	exogenous direct tax rate for domestic institution i
$tins01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
tm_c	import tariff rate
tq_c	rate of sales tax
$trnsfr_{if}$	transfer from factor f to institution i
tva_a	rate of value-added tax for activity a

Greek Letters

α_a^a	efficiency parameter in the CES activity function
α_a^{va}	efficiency parameter in the CES value-added function
α_a^{ac}	shift parameter for domestic commodity aggregation function
α_c^q	Armington function shift parameter
α_c^t	CET function shift parameter
β_{ach}^h	marginal share of consumption spending on home commodity c from activity a for household h
β_{ch}^m	marginal share of consumption spending on marketed commodity c for household h
δ_a^a	CES activity function share parameter
δ_{ac}^{ac}	share parameter for domestic commodity aggregation function
δ_c^q	Armington function share parameter
δ_c^t	CET function share parameter
δ_{fa}^{va}	CES value-added function share parameter for factor f in activity a
γ_{ch}^m	subsistence consumption of marketed commodity c for household h
γ_{ach}^h	subsistence consumption of home commodity c from activity a for household h

	θ_{ac}	yield of output c per unit of activity a
	ρ_a^a	CES production function exponent
	ρ_a^{va}	CES value-added function exponent
	ρ_c^{ac}	domestic commodity aggregation function exponent
	ρ_c^q	Armington function exponent
	ρ_c^t	CET function exponent
EXOGENOUS VARIABLES	$\overline{CPI}$	consumer price index
	$\overline{DTINS}$	change in domestic institution tax share (= 0 for base; exogenous variable)
	$\overline{FSAV}$	foreign savings (FCU)
	$\overline{GADJ}$	government consumption adjustment factor
	$\overline{IADJ}$	investment adjustment factor
	$\overline{MPSADJ}$	savings rate scaling factor (= 0 for base)
	$\overline{QFS}_f$	quantity supplied of factor
	$\overline{TINSADJ}$	direct tax scaling factor (= 0 for base; exogenous variable)
	$\overline{WFDIST}_{fa}$	wage distortion factor for factor f in activity a
ENDOGENOUS VARIABLES	$DMPS$	change in domestic institution savings rates (= 0 for base; exogenous variable)
	DPI	producer price index for domestically marketed output
	EG	government expenditures
	EH_h	consumption spending for household
	EXR	exchange rate (*LCU* per unit of *FCU*)
	$GOVSHR$	government consumption share in nominal absorption
	$GSAV$	government savings
	$INVSHR$	investment share in nominal absorption
	MPS_i	marginal propensity to save for domestic non-government institution (exogenous variable)
	PA_a	activity price (unit gross revenue)
	PDD_c	demand price for commodity produced and sold domestically
	PDS_c	supply price for commodity produced and sold domestically
	PE_c	export price (domestic currency)
	$PINTA_a$	aggregate intermediate input price for activity a
	PM_c	import price (domestic currency)
	PQ_c	composite commodity price
	PVA_a	value-added price (factor income per unit of activity)
	PX_c	aggregate producer price for commodity
	$PXAC_{ac}$	producer price of commodity c for activity a
	QA_a	quantity (level) of activity
	QD_c	quantity sold domestically of domestic output

QE_c	quantity of exports
$QF_{f\,a}$	quantity demanded of factor *f* from activity *a*
QG_c	government consumption demand for commodity
$QH_{c\,h}$	quantity consumed of commodity c by household *h*
$QHA_{a\,c\,h}$	quantity of household home consumption of commodity c from activity *a* for household *h*
$QINTA_a$	quantity of aggregate intermediate input
$QINT_{c\,a}$	quantity of commodity *c* as intermediate input to activity *a*
$QINV_c$	quantity of investment demand for commodity
QM_c	quantity of imports of commodity
QQ_c	quantity of goods supplied to domestic market (composite supply)
QT_c	quantity of commodity demanded as trade input
QVA_a	quantity of (aggregate) value-added
QX_c	aggregated marketed quantity of domestic output of commodity
$QXAC_{a\,c}$	quantity of marketed output of commodity *c* from activity *a*
$TABS$	total nominal absorption
$TINS_i$	direct tax rate for institution *i* ($i \in INSDNG$)
$TRII_{i\,i'}$	transfers from institution *i'* to *i* (both in the set INSDNG)
WF_f	average price of factor *f*
YF_f	income of factor *f*
YG	government revenue
YI_i	income of domestic nongovernment institution
$YIF_{i\,f}$	income to domestic institution *i* from factor *f*

EQUATIONS

Price Block

Import price

$$PM_c = pwm_c \cdot (1 + tm_c) \cdot EXR + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c'c} \qquad c \in CM \qquad (1)$$

$$\begin{bmatrix} \text{import} \\ \text{price} \\ (LCU) \end{bmatrix} = \begin{bmatrix} \text{import} \\ \text{price} \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} \text{tariff} \\ \text{adjust-} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ (LCU \text{ per} \\ FCU) \end{bmatrix} + \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{import unit} \end{bmatrix}$$

Export price

$$PE_c = pwe_c \cdot (1 - te_c) \cdot EXR - \sum_{c' \in CT} PQ_{c'} \cdot ice_{c'c} \qquad c \in CE \qquad (2)$$

$$\begin{bmatrix} \text{export} \\ \text{price} \\ (LCU) \end{bmatrix} = \begin{bmatrix} \text{export} \\ \text{price} \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} \text{tariff} \\ \text{adjust-} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ (LCU \text{ per} \\ FCU) \end{bmatrix} - \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{export unit} \end{bmatrix}$$

Demand price of domestic nontraded goods

$$PDD_c = PDS_c + \sum_{c' \in CT} PQ_{c'} \cdot icd_{c'c} \qquad c \in CD \qquad (3)$$

$$\begin{bmatrix} \text{domestic} \\ \text{demand} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{domestic} \\ \text{supply} \\ \text{price} \end{bmatrix} + \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{unit of} \\ \text{domestic sales} \end{bmatrix}$$

Absorption

$$PQ_c \cdot (1 - tq_c) \cdot QQ_c = PDD_c \cdot QD_c + PM_c \cdot QM_c \qquad c \in (CD \cup CM) \qquad (4)$$

$$\begin{bmatrix} \text{absorption} \\ \text{(at demand} \\ \text{prices net of} \\ \text{sales tax)} \end{bmatrix} = \begin{bmatrix} \text{domestic demand price} \\ \text{times} \\ \text{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \text{import price} \\ \text{times} \\ \text{import quantity} \end{bmatrix}$$

Marketed output value

$$PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c \qquad c \in CX \qquad (5)$$

$$\begin{bmatrix} \text{producer price} \\ \text{times marketed} \\ \text{output quantity} \end{bmatrix} = \begin{bmatrix} \text{domestic supply price} \\ \text{times} \\ \text{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \text{export price} \\ \text{times} \\ \text{export quantity} \end{bmatrix}$$

Activity price

$$PA_a = \sum_{c \in C} PXAC_{a\,c} \cdot \theta_{a\,c} \qquad a \in A \qquad (6)$$

$$\begin{bmatrix} \text{activity} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{producer prices} \\ \text{times yields} \end{bmatrix}$$

Aggregate intermediate input price

$$PINTA_a = \sum_{c \in C} PQ_c \cdot ica_{c\,a} \qquad a \in A \qquad (7)$$

$$\begin{bmatrix} \text{aggregate} \\ \text{intermediate} \\ \text{input price} \end{bmatrix} = \begin{bmatrix} \text{intermediate input cost} \\ \text{per unit of aggregate} \\ \text{intermediate input} \end{bmatrix}$$

Activity revenue and costs

$$PA_a \cdot (1 - ta_a) \cdot QA_a = PVA_a \cdot QVA_a + PINTA_a \cdot QINTA_a \qquad a \in A \qquad (8)$$

$$\begin{bmatrix} \text{activity price} \\ \text{(net of taxes)} \\ \text{times activity level} \end{bmatrix} = \begin{bmatrix} \text{value-added} \\ \text{price times} \\ \text{quantity} \end{bmatrix} + \begin{bmatrix} \text{aggregate} \\ \text{intermediate} \\ \text{input price times} \\ \text{quantity} \end{bmatrix}$$

Consumer price index

$$\overline{CPI} = \sum_{c \in C} PQ_c \cdot cwts_c \qquad (9)$$

$$\begin{bmatrix} \text{consumer} \\ \text{price index} \end{bmatrix} = \begin{bmatrix} \text{prices times} \\ \text{weights} \end{bmatrix}$$

Producer price index for nontraded market output

$$DPI = \sum_{c \in C} PDS_c \cdot dwts_c \qquad (10)$$

$$\begin{bmatrix} \text{producer price index} \\ \text{for non-traded outputs} \end{bmatrix} = \begin{bmatrix} \text{prices times} \\ \text{weights} \end{bmatrix}$$

Production and Trade Block

CES technology: Activity production function

$$QA_a = \alpha_a^a \cdot \left(\delta_a^a \cdot QVA_a^{-\rho_a^a} + (1-\delta_a^a) \cdot QINTA_a^{-\rho_a^a} \right)^{-\frac{1}{\rho_a^a}} \qquad a \in ACES \qquad (11)$$

$$\begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix} = CES \begin{bmatrix} \text{quantity of aggregate value added,} \\ \text{quantity of aggregate intermediate input} \end{bmatrix}$$

CES technology: Value-added intermediate-input quantity ratio

$$\frac{QVA_a}{QINTA_a} = \left(\frac{PINTA_a}{PVA_a} \cdot \frac{\delta_a^a}{1-\delta_a^a} \right)^{\frac{1}{1+\rho_a^a}} \qquad a \in ACES \qquad (12)$$

$$\begin{bmatrix} \text{value-added} - \\ \text{intermediate-} \\ \text{input quantity} \\ \text{ratio} \end{bmatrix} = f \begin{bmatrix} \text{intermediate-input} : \\ \text{value-added} \\ \text{price ratio} \end{bmatrix}$$

Leontief technology: Demand for aggregate value-added

$$QVA_a = iva_a \cdot QA_a \qquad a \in ALEO \qquad (13)$$

$$\begin{bmatrix} \text{demand for} \\ \text{value added} \end{bmatrix} = f \begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix}$$

Leontief technology: Demand for aggregate intermediate input

$$QINTA_a = inta_a \cdot QA_a \qquad a \in ALEO \qquad (14)$$

$$\begin{bmatrix} \text{demand for aggregate} \\ \text{intermediate input} \end{bmatrix} = f \begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix}$$

Value-added and factor demands

$$QVA_a = \alpha_a^{va} \cdot \left(\sum_{f \in F} \delta_{f\,a}^{va} \cdot QF_{f\,a}^{-\rho_a^{va}} \right)^{-\frac{1}{\rho_a^{va}}} \qquad a \in A \qquad (15)$$

$$\begin{bmatrix} \text{quantity of aggregate} \\ \text{value added} \end{bmatrix} = CES \begin{bmatrix} \text{factor} \\ \text{inputs} \end{bmatrix}$$

Factor demand

$$WF_f \cdot \overline{WFDIST}_{f\,a} = PVA_a \cdot (1 - tva_a) \cdot QVA_a \cdot \left(\sum_{f \in F'} \delta^{va}_{f\,a} \cdot QF_{f\,a}{}^{-\rho^{va}_a} \right)^{-1} \cdot \delta^{va}_{f\,a} \cdot QF_{f\,a}{}^{-\rho^{va}_a - 1} \qquad a \in A,\ f \in F \qquad (16)$$

$$\begin{bmatrix} \text{marginal cost of} \\ \text{factor } f \text{ in activity } a \end{bmatrix} = \begin{bmatrix} \text{marginal revenue product} \\ \text{of factor } f \text{ in activity } a \end{bmatrix}$$

Disaggregated intermediate input demand

$$QINT_{c\,a} = ica_{c\,a} \cdot QINTA_a \qquad a \in A,\ c \in C \qquad (17)$$

$$\begin{bmatrix} \text{intermediate demand} \\ \text{for commodity } c \\ \text{from activity } a \end{bmatrix} = f \begin{bmatrix} \text{aggregate intermediate} \\ \text{input quantity} \\ \text{for activity } a \end{bmatrix}$$

Commodity production and allocation

$$QXAC_{a\,c} + \sum_{h \in H} QHA_{a\,c\,h} = \theta_{a\,c} \cdot QA_a \qquad a \in A,\ a \in CX \qquad (18)$$

$$\begin{bmatrix} \text{marketed quantity} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix} + \begin{bmatrix} \text{household home} \\ \text{consumption} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix} = \begin{bmatrix} \text{production} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix}$$

Output aggregation function

$$QX_c = \alpha^{ac}_c \cdot \left(\sum_{a \in A} \delta^{ac}_{a\,c} \cdot QXAC_{a\,c}{}^{-\rho^{ac}_c} \right)^{-\frac{1}{\rho^{ac}_c - 1}} \qquad c \in CX \qquad (19)$$

$$\begin{bmatrix} \text{aggregate} \\ \text{marketed} \\ \text{production of} \\ \text{commodity } c \end{bmatrix} = CES \begin{bmatrix} \text{activity-specific} \\ \text{marketed} \\ \text{production of} \\ \text{commodity } c \end{bmatrix}$$

First-order condition for output aggregation function

$$PXAC_{a\,c} = PX_c \cdot QX_c \left(\sum_{a \in A'} \delta^{ac}_{a\,c} \cdot QXAC^{-\rho^{ac}_c}_{a\,c} \right)^{-1} \cdot \delta^{ac}_{a\,c} \cdot QXAC^{-\rho^{ac}_c - 1}_{a\,c} \qquad a \in A,\ c \in CX \qquad (20)$$

$$\begin{bmatrix} \text{marginal cost of com-} \\ \text{modity } c \text{ from activity } a \end{bmatrix} = \begin{bmatrix} \text{marginal revenue product of} \\ \text{commodity } c \text{ from activity } a \end{bmatrix}$$

Production and Trade Block (continued)

Output transformation (CET) function

$$QX_c = \alpha_c^t \cdot \left(\delta_c^t \cdot QE_c^{\rho_c^t} + (1-\delta_c^t)\cdot QD_c^{\rho_c^t}\right)^{\frac{1}{\rho_c^t}} \qquad c \in (CE \cap CD) \qquad (21)$$

$$\begin{bmatrix}\text{aggregate marketed}\\ \text{domestic output}\end{bmatrix} = CET\begin{bmatrix}\text{export quantity, domestic}\\ \text{sales of domestic output}\end{bmatrix}$$

Export-domestic supply ratio

$$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c}\cdot\frac{1-\delta_c^t}{\delta_c^t}\right)^{\frac{1}{\rho_c^t-1}} \qquad c \in (CE \cap CD) \qquad (22)$$

$$\begin{bmatrix}\textit{export-domestic}\\ \textit{supply ratio}\end{bmatrix} = f\begin{bmatrix}\textit{export-domestic}\\ \textit{price ratio}\end{bmatrix}$$

Output transformation for non-exported commodities

$$QX_c = QD_c + QE_c$$

$$\begin{bmatrix}\textit{aggregate}\\ \textit{marketed}\\ \textit{domestic output}\end{bmatrix} = \begin{bmatrix}\textit{domestic market}\\ \textit{sales of domestic}\\ \textit{output [for}\\ c \in (CD \cap CEN)]\end{bmatrix} + \begin{bmatrix}\textit{exports [for}\\ c \in (CE \cap CDN)]\end{bmatrix} \qquad \begin{matrix} c \in \\ (CD \cap CEN) \\ \cup \\ (CE \cup CDN)\end{matrix} \qquad (23)$$

Composite supply (Armington) function

$$QQ_c = \alpha_c^q \cdot \left(\delta_c^q \cdot QM_c^{-\rho_c^q} + (1-\delta_c^q)\cdot QD_c^{-\rho_c^q}\right)^{-\frac{1}{\rho_c^q}} \qquad c \in (CM \cap CD) \qquad (24)$$

$$\begin{bmatrix}\textit{composite}\\ \textit{supply}\end{bmatrix} = f\begin{bmatrix}\textit{import quantity, domestic}\\ \textit{use of domestic output}\end{bmatrix}$$

Import-domestic demand ratio

$$\frac{QM_c}{QD_c} = \left(\frac{PDD_c}{PM_c}\cdot\frac{\delta_c^q}{1-\delta_c^q}\right)^{\frac{1}{1+\rho_c^q}} \qquad c \in (CM \cap CD) \qquad (25)$$

$$\begin{bmatrix}\textit{import-domestic}\\ \textit{demand ratio}\end{bmatrix} = f\begin{bmatrix}\textit{domestic-import}\\ \textit{price ratio}\end{bmatrix}$$

Composite supply for non-imported outputs and nonproduced imports

$$QQ_c = QD_c + QM_c \qquad c \in (CD \cap CMN) \cup (CM \cup CDN) \qquad (26)$$

$$\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = \begin{bmatrix} \text{domestic use of} \\ \text{marketed domestic} \\ \text{output [for} \\ c \in (CD \cap CMN)] \end{bmatrix} + \begin{bmatrix} \text{imports [for} \\ c \in (CM \cap CDN)] \end{bmatrix}$$

Demand for transactions services

$$QT_c = \sum_{c' \in C'} \left(icm_{c\,c'} \cdot QM_{c'} + ice_{c\,c'} \cdot QE_{c'} + icd_{c\,c'} \cdot QD_{c'} \right) \qquad c \in CT \qquad (27)$$

$$\begin{bmatrix} \text{demand for} \\ \text{transactions} \\ \text{services} \end{bmatrix} = \begin{bmatrix} \text{sum of demands} \\ \text{for imports, exports,} \\ \text{and domestic sales} \end{bmatrix}$$

Institution Block

Factor income

$$YF_f = \sum_{a \in A} WF_f \cdot \overline{WFDIST}_{f\,a} \cdot QF_{f\,a} \qquad f \in F \qquad (28)$$

$$\begin{bmatrix} \text{income of} \\ \text{factor } f \end{bmatrix} = \begin{bmatrix} \text{sum of activity payments} \\ \text{(activity-specific wages} \\ \text{times employment levels)} \end{bmatrix}$$

Institutional factor incomes

$$YIF_{i\,f} = shif_{i\,f} \cdot \left[\left(1 - tf_f\right) \cdot YF_f - trnsfr_{row\,f} \cdot EXR \right] \qquad i \in INSD,\ f \in F \qquad (29)$$

$$\begin{bmatrix} \text{income of} \\ \text{institution } i \\ \text{from factor } f \end{bmatrix} = \begin{bmatrix} \text{share of income} \\ \text{of factor } f \text{ to} \\ \text{institution } i \end{bmatrix} \cdot \begin{bmatrix} \text{income of factor } f \\ \text{(net of tax and} \\ \text{transfer to RoW)} \end{bmatrix}$$

Income of domestic, nongovernment institutions

$$YI_i = \sum_{f \in F} YIF_{i\,f} + \sum_{i' \in INSDNG'} TRII_{i\,i'} + trnsfr_{i\,gov} \cdot \overline{CPI} + trnsfr_{i\,row} \cdot EXR \qquad i \in INSDNG \qquad (30)$$

$$\begin{bmatrix} \text{income of} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{factor} \\ \text{income} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from other domestic} \\ \text{non-government} \\ \text{institutions} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from} \\ \text{government} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from} \\ \text{RoW} \end{bmatrix}$$

Institution Block (continued)

Intra-institutional transfers

$$TRII_{i\,i'} = shii_{i\,i'} \cdot \left(1 - MPS_{i'}\right) \cdot \left(1 - TINS_{i'}\right) \cdot YI_{i'} \qquad i \in INSDNG,\ i' \in INSDNG' \qquad (31)$$

$$\begin{bmatrix} \text{transfer from} \\ \text{institution } i' \text{ to } i \end{bmatrix} = \begin{bmatrix} \text{share of net income} \\ \text{of institution } i' \\ \text{transfered to } i \end{bmatrix} \cdot \begin{bmatrix} \text{income of institution} \\ i', \text{ net of savings and} \\ \text{direct taxes} \end{bmatrix}$$

Household consumption expenditure

$$EH_h = \left(1 - \sum_{i \in INSDNG} shii_{i\,h}\right) \cdot \left(1 - MPS_h\right) \cdot \left(1 - TINS_h\right) \cdot YI_h \qquad h \in H \qquad (32)$$

$$\begin{bmatrix} \text{household income} \\ \text{disposable for} \\ \text{consumption} \end{bmatrix} = \begin{bmatrix} \text{household income, net of direct} \\ \text{taxes, savings, and transfers to} \\ \text{other non-government institutions} \end{bmatrix}$$

Household consumption demand for marketed commodities

$$PQ_c \cdot QH_{c\,h} = PQ_c \cdot \gamma^m_{c\,h} + \beta^m_{c\,h} \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot \gamma^m_{c'\,h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a\,c'} \cdot \gamma^h_{a\,c'\,h}\right) \qquad c \in C,\ h \in H \qquad (33)$$

$$\begin{bmatrix} \text{household consumption} \\ \text{spending on market} \\ \text{commodity } c \end{bmatrix} = f \begin{bmatrix} \text{total household consumption} \\ \text{spending, market price of } c\text{, and other} \\ \text{commodity prices (market and home)} \end{bmatrix}$$

Household consumption demand for home commodities

$$PXAC_{a\,c} \cdot QHA_{a\,c\,h} = PXAC_{a\,c} \cdot \gamma^h_{a\,c\,h} + \beta^h_{a\,c\,h} \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot \gamma^m_{c'\,h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a\,c'} \cdot \gamma^h_{a\,c'\,h}\right) \qquad a \in A,\ c \in C,\ h \in H \qquad (34)$$

$$\begin{bmatrix} \text{household consumption} \\ \text{spending on home commodity} \\ c \text{ from activity } a \end{bmatrix} = f \begin{bmatrix} \text{total household consumption spending,} \\ \text{producer price, and other} \\ \text{commodity prices (market and home)} \end{bmatrix}$$

Investment demand

$$QINV_c = \overline{IADJ} \cdot \overline{qinv}_c \qquad c \in CINV \qquad (35)$$

$$\begin{bmatrix} \text{fixed investment} \\ \text{demand for} \\ \text{commodity } c \end{bmatrix} = \begin{bmatrix} \text{adjustment factor} \\ \text{times} \\ \text{base-year fixed} \\ \text{investment} \end{bmatrix}$$

Government consumption demand

$$QG_c = \overline{GADJ} \cdot \overline{qg}_c \qquad c \in C \qquad (36)$$

$$\begin{bmatrix} \text{government} \\ \text{consumption} \\ \text{demand for} \\ \text{commodity } c \end{bmatrix} = \begin{bmatrix} \text{adjustment factor} \\ \text{times} \\ \text{base-year government} \\ \text{consumption} \end{bmatrix}$$

Government revenue

$$\begin{aligned} YG = & \sum_{i \in INSDNG} TINS_i \cdot YI_i + \sum_{f \in F} tf_f \cdot YF_f + \sum_{a \in A} tva_a \cdot PVA_a \cdot QVA_a \\ & + \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a + \sum_{c \in CM} tm_c \cdot pwm_c \cdot QM_c \cdot EXR + \sum_{c \in CE} te_c \cdot pwe_c \cdot QE_c \cdot EXR \\ & + \sum_{c \in C} tq_c \cdot PQ_c \cdot QQ_c + \sum_{f \in F} YIF_{gov\,f} + trnsfr_{gov\,row} \cdot EXR \end{aligned} \qquad (37)$$

$$\begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = \begin{bmatrix} \text{direct taxes} \\ \text{from} \\ \text{institutions} \end{bmatrix} + \begin{bmatrix} \text{direct taxes} \\ \text{from} \\ \text{factors} \end{bmatrix} + \begin{bmatrix} \text{value-} \\ \text{added} \\ \text{tax} \end{bmatrix}$$

$$+ \begin{bmatrix} \text{activity} \\ \text{tax} \end{bmatrix} + \begin{bmatrix} \text{import} \\ \text{tariffs} \end{bmatrix} + \begin{bmatrix} \text{export} \\ \text{taxes} \end{bmatrix} + \begin{bmatrix} \text{sales} \\ \text{tax} \end{bmatrix} + \begin{bmatrix} \text{factor} \\ \text{income} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from} \\ \text{RoW} \end{bmatrix}$$

Government expenditures

$$EG = \sum_{c \in C} PQ_c \cdot QG_c + \sum_{i \in INSDNG} trnsfr_{i\,gov} \cdot \overline{CPI} \qquad (38)$$

$$\begin{bmatrix} \text{government} \\ \text{spending} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{transfers to domestic} \\ \text{non-government} \\ \text{institutions} \end{bmatrix}$$

System Constraint Block

Factor market

$$\sum_{a \in A} QF_{f\,a} = \overline{QFS}_f \qquad f \in F \qquad (39)$$

$$\begin{bmatrix} \text{demand for} \\ \text{factor } f \end{bmatrix} = \begin{bmatrix} \text{supply of} \\ \text{factor } f \end{bmatrix}$$

System Constraint Block (continued)

Composite commodity markets

$$QQ_c = \sum_{a \in A} QINT_{c\,a} + \sum_{h \in H} QH_{c\,h} + QG_c + QINV_c + qdst_c + QT_c \qquad c \in C \qquad (40)$$

$$\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = \begin{bmatrix} \text{intermediate} \\ \text{use} \end{bmatrix} + \begin{bmatrix} \text{household} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix} + \begin{bmatrix} \text{trade} \\ \text{input use} \end{bmatrix}$$

Current account balance for rest of the world (in foreign currency)

$$\sum_{c \in CM} pwm_c \cdot QM_c + \sum_{f \in F} trnsfr_{row\,f} = \sum_{c \in CE} pwe_c \cdot QE_c + \sum_{i \in INSD} trnsfr_{i\,row} + \overline{FSAV} \qquad (41)$$

$$\begin{bmatrix} \text{import} \\ \text{spending} \end{bmatrix} + \begin{bmatrix} \text{factor} \\ \text{transfers} \\ \text{to RoW} \end{bmatrix} = \begin{bmatrix} \text{export} \\ \text{revenue} \end{bmatrix} + \begin{bmatrix} \text{institutional} \\ \text{transfers} \\ \text{from RoW} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix}$$

Government balance

$$YG = EG + GSAV \qquad (42)$$

$$\begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{expenditures} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix}$$

Direct institutional tax rates

$$TINS_i = \overline{tins}_i \cdot \left(1 + \overline{TINSADJ} \cdot tins01_i\right) + \overline{DTINS} \cdot tins01_i \qquad i \in INSDNG \qquad (43)$$

$$\begin{bmatrix} \text{direct tax} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix}$$

Institutional savings rates

$$MPS_i = \overline{mps}_i \cdot \left(1 + \overline{MPSADJ} \cdot mps01_i\right) + DMPS \cdot mps01_i \qquad i \in INSDNG \qquad (44)$$

$$\begin{bmatrix} \text{savings} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix}$$

Savings–Investment Balance

$$\sum_{i \in INSDNG} MPS_i \cdot (1 - TINS_i) \cdot YI_i + GSAV + EXR \cdot \overline{FSAV} = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c \tag{45}$$

$$\begin{bmatrix} \text{non-govern-} \\ \text{ment savings} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix} = \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix}$$

Total absorption

$$TABS = \sum_{h \in H} \sum_{c \in C} PQ_c \cdot QH_{c\,h} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{a\,c} \cdot QHA_{a\,c\,h} + \sum_{c \in C} PQ_c \cdot QG_c + \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c \tag{46}$$

$$\begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} = \begin{bmatrix} \text{household} \\ \text{market} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{household} \\ \text{home} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix}$$

Ratio of investment to absorption

$$INVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c \tag{47}$$

$$\begin{bmatrix} \text{investment-} \\ \text{absorption} \\ \text{ratio} \end{bmatrix} \cdot \begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} = \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix}$$

Ratio of government consumption to absorption

$$GOVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QG_c \tag{48}$$

$$\begin{bmatrix} \text{government} \\ \text{consumption-} \\ \text{absorption} \\ \text{ratio} \end{bmatrix} \cdot \begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix}$$

APPENDIX B: CORE GAMS CODE FOR STANDARD CGE MODEL

```
*SETS ==========================================================

AC                  global set for model accounts-aggregated microsam accounts
A(AC)               activities
ACES(A)             activities with CES fn at top of technology nest
ALEO(A)             activities with Leontief fn at top of technology nest
C(AC)               commodities
CD(C)               commodities with domestic sales of output
CDN(C)              commodities without domestic sales of output
CE(C)               exported commodities
CEN(C)              non-exported commodities
CM(C)               imported commodities
CMN(C)              non-imported commodities
CX(C)               commodities with output
F(AC)               factors
INS(AC)             institutions
INSD(INS)           domestic institutions
INSDNG(INSD)        domestic non-government institutions
H(INSDNG)           households


*PARAMETERS ====================================================

 Parameters         other than tax rates
 alphaa(A)          shift parameter for top level CES function
 alphaac(C)         shift parameter for domestic commodity aggregation fn
 alphaq(C)          shift parameter for Armington function
 alphat(C)          shift parameter for CET function
 alphava(A)         shift parameter for CES activity production function
 betah(A,C,H)       marg shr of hhd cons on home com c from act a
 betam(C,H)         marg share of hhd cons on marketed commodity c
 cwts(C)            consumer price index weights
 deltaa(A)          share parameter for top level CES function
 deltaac(A,C)       share parameter for domestic commodity aggregation fn
 deltaq(C)          share parameter for Armington function
 deltat(C)          share parameter for CET function
 deltava(F,A)       share parameter for CES activity production function
 dwts(C)            domestic sales price weights
 gammah(A,C,H)      per-cap subsist cons for hhd h on home com c fr act a
 gammam(C,H)        per-cap subsist cons of market com c for hhd h
 ica(C,A)           intermediate input c per unit of aggregate intermediate
```

```
 inta(A)          aggregate intermediate input coefficient
 iva(A)           aggregate value added coefficient
 icd(C,CP)        trade input of c per unit of com cp produced & sold dom'ly
 ice(C,CP)        trade input of c per unit of com cp exported
 icm(C,CP)        trade input of c per unit of com cp imported
 mps01(INS)       0-1 par for potential flexing of savings rates
 mpsbar(INS)      marg prop to save for dom non-gov inst ins (exog part)
 qdst(C)          inventory investment by sector of origin
 qbarg(C)         exogenous (unscaled) government demand
 qbarinv(C)       exogenous (unscaled) investment demand
 rhoa(A)          CES top level function exponent
 rhoac(C)         domestic commodity aggregation function exponent
 rhoq(C)          Armington function exponent
 rhot(C)          CET function exponent
 rhova(A)         CES activity production function exponent
 shif(INS,F)      share of dom. inst i in income of factor f
 shii(INS,INSP)   share of inst i in post-tax post-sav income of inst ip
 supernum(H)      LES supernumerary income
 theta(A,C)       yield of commodity c per unit of activity a
 tins01(INS)      0-1 par for potential flexing of dir tax rates
 trnsfr(INS,AC)   transfers fr inst. or factor ac to institution ins
*Tax rates
 ta(A)            rate of tax on producer gross output value
 te(C)            rate of tax on exports
 tf(F)            rate of direct tax on factors (soc sec tax)
 tinsbar(INS)     rate of (exog part of) direct tax on dom inst ins
 tm(C)            rate of import tariff
 tq(C)            rate of sales tax
 tva(A)           rate of value-added tax

*VARIABLES ==========================================================

CPI               consumer price index (PQ-based)
DPI               index for domestic producer prices (PDS-based)
DMPS              change in marginal propensity to save for selected inst
DTINS             change in domestic institution tax share
EG                total current government expenditure
EH(H)             household consumption expenditure
EXR               exchange rate
FSAV              foreign savings
GADJ              government demand scaling factor
GOVSHR            govt consumption share of absorption
GSAV              government savings
IADJ              investment scaling factor (for fixed capital formation)
INVSHR            investment share of absorption
MPS(INS)          marginal propensity to save for dom non-gov inst ins
MPSADJ            savings rate scaling factor
PA(A)             output price of activity a
```

```
PDD(C)            demand price for com c produced & sold domestically
PDS(C)            supply price for com c produced & sold domestically
PE(C)             price of exports
PINTA(A)          price of intermediate aggregate
PM(C)             price of imports
PQ(C)             price of composite good c
PVA(A)            value added price
PWE(C)            world price of exports
PWM(C)            world price of imports
PX(C)             average output price
PXAC(A,C)         price of commodity c from activity a
QA(A)             level of domestic activity
QD(C)             quantity of domestic sales
QE(C)             quantity of exports
QF(F,A)           quantity demanded of factor f from activity a
QFS(F)            quantity of factor supply
QG(C)             quantity of government consumption
QH(C,H)           quantity consumed of marketed commodity c by household h
QHA(A,C,H)        quantity consumed of home commodity c fr act a by hhd h
QINT(C,A)         quantity of intermediate demand for c from activity a
QINTA(A)          quantity of aggregate intermediate input
QINV(C)           quantity of fixed investment demand
QM(C)             quantity of imports
QQ(C)             quantity of composite goods supply
QT(C)             quantity of trade and transport demand for commodity c
QVA(A)            quantity of aggregate value added
QX(C)             quantity of aggregate marketed commodity output
QXAC(A,C)         quantity of ouput of commodity c from activity a
TABS              total absorption
TINS(INS)         rate of direct tax on domestic institutions ins
TINSADJ           direct tax scaling factor
TRII(INS,INSP)    transfers to dom inst insdng from insdngp
WALRAS            Savings-Investment imbalance (should be zero)
WF(F)             economy-wide wage (rent) for factor f
WFDIST(F,A)       factor wage distortion variable
YF(F)             factor income
YG                total current government income
YIF(INS,F)        income of institution ins from factor f
YI(INS)           income of (domestic non-governmental) institution ins

*EQUATIONS ==========================================================

*Price block  =======================================================
 PMDEF(C)         domestic import price
 PEDEF(C)         domestic export price
 PDDDEF(C)        demand price for com c produced and sold domestically
 PQDEF(C)         value of sales in domestic market
 PXDEF(C)         value of marketed domestic output
```

```
 PADEF(A)          output price for activity a
 PINTADEF(A)       price of aggregate intermediate input
 PVADEF(A)         value-added price
 CPIDEF            consumer price index
 DPIDEF            domestic producer price index

*Production and trade block ==========================================
 CESAGGPRD(A)      CES aggregate prod fn (if CES top nest)
 CESAGGFOC(A)      CES aggregate first-order condition (if CES top nest)
 LEOAGGINT(A)      Leontief aggreg intermed demand (if Leontief top nest)
 LEOAGGVA(A)       Leontief aggreg value-added demand (if Leontief top nest)
 CESVAPRD(A)       CES value-added production function
 CESVAFOC(F,A)     CES value-added first-order condition
 INTDEM(C,A)       intermediate demand for commodity c from activity a
 COMPRDFN(A,C)     production function for commodity c and activity a
 OUTAGGFN(C)       output aggregation function
 OUTAGGFOC(A,C)    first-order condition for output aggregation function
 CET(C)            CET function
 CET2(C)           domestic sales and exports for outputs without both
 ESUPPLY(C)        export supply
 ARMINGTON(C)      composite commodity aggregation function
 COSTMIN(C)        first-order condition for composite commodity cost min
 ARMINGTON2(C)     comp supply for com without both dom sales and imports
 QTDEM(C)          demand for transactions (trade and transport) services

*Institution block ===============================================
 YFDEF(F)          factor incomes
 YIFDEF(INS,F)     factor incomes to domestic institutions
 YIDEF(INS)        total incomes of domest non-gov't institutions
 EHDEF(H)          household consumption expenditures
 TRIIDEF(INS,INSP) transfers to inst ins from inst insp
 HMDEM(C,H)        LES cons demand by hhd h for marketed commodity c
 HADEM(A,C,H)      LES cons demand by hhd h for home commodity c fr act a
 INVDEM(C)         fixed investment demand
 GOVDEM(C)         government consumption demand
 EGDEF             total government expenditures
 YGDEF             total government income

*System constraint block ==========================================
 COMEQUIL(C)       composite commodity market equilibrium
 FACEQUIL(F)       factor market equilibrium
 CURACCBAL         current-account balance (of RoW)
 GOVBAL            government balance
 TINSDEF(INS)      direct tax rate for inst ins
 MPSDEF(INS)       marg prop to save for inst ins
 SAVINVBAL         Savings-Investment balance
 TABSEQ            total absorption
 INVABEQ           investment share in absorption
 GDABEQ            government consumption share in absorption
```

```
*Notational convention inside equations:
*Parameters and "invariably" fixed variables are in lower case.
*Potentially "variable" variables are in upper case.

*Price block ========================================================
 PMDEF(C)$CM(C)..
  PM(C) =E= pwm(C)*(1 + tm(C))*EXR + SUM(CT, PQ(CT)*icm(CT,C));

 PEDEF(C)$CE(C)..
  PE(C) =E= pwe(C)*(1 - te(C))*EXR - SUM(CT, PQ(CT)*ice(CT,C));

 PDDDEF(C)$CD(C).. PDD(C) =E= PDS(C) + SUM(CT, PQ(CT)*icd(CT,C));

 PQDEF(C)$(CD(C) OR CM(C))..
       PQ(C)*(1 - tq(c))*QQ(C) =E= PDD(C)*QD(C) + PM(C)*QM(C);

 PXDEF(C)$CX(C)..  PX(C)*QX(C) =E= PDS(C)*QD(C) + PE(C)*QE(C);

 PADEF(A)..  PA(A) =E= SUM(C, PXAC(A,C)*theta(A,C));

 PINTADEF(A).. PINTA(A) =E= SUM(C, PQ(C)*ica(C,A)) ;

 PVADEF(A)..  PA(A)*(1-ta(A))*QA(A) =E= PVA(A)*QVA(A) + PINTA(A)*QINTA(A) ;

 CPIDEF..  CPI =E= SUM(C, cwts(C)*PQ(C)) ;

 DPIDEF..  DPI =E= SUM(CD, dwts(CD)*PDS(CD)) ;

*Production and trade block ==========================================

*CESAGGPRD and CESAGGFOC apply to activities with CES function at
*top of technology nest.

 CESAGGPRD(A)$ACES(A)..
   QA(A) =E= alphaa(A)*(deltaa(A)*QVA(A)**(-rhoa(A))
                + (1-deltaa(A))*QINTA(A)**(-rhoa(A)))**(-1/rhoa(A)) ;

 CESAGGFOC(A)$ACES(A)..
   QVA(A) =E= QINTA(A)*((PINTA(A)/PVA(A))*(deltaa(A)/
                                  (1 - deltaa(A))))**(1/(1+rhoa(A))) ;

*LEOAGGINT and LEOAGGVA apply to activities with Leontief function at
*top of technology nest.

 LEOAGGINT(A)$ALEO(A)..  QINTA(A) =E= inta(A)*QA(A) ;

 LEOAGGVA(A)$ALEO(A)..  QVA(A) =E= iva(A)*QA(A) ;
```

```
*CESVAPRD, CESVAFOC, INTDEM apply at the bottom of the technology nest
*(for all activities).

 CESVAPRD(A)..
   QVA(A) =E= alphava(A)*(SUM(F,
                        deltava(F,A)*QF(F,A)**(-rhova(A))) )**(-1/rhova(A)) ;

 CESVAFOC(F,A)$deltava(F,A)..
   WF(F)*wfdist(F,A) =E=
   PVA(A)*(1-tva(A))
   * QVA(A) * SUM(FP, deltava(FP,A)*QF(FP,A)**(-rhova(A)) )**(-1)
   *deltava(F,A)*QF(F,A)**(-rhova(A)-1);

 INTDEM(C,A)$ica(C,A).. QINT(C,A) =E= ica(C,A)*QINTA(A);

 COMPRDFN(A,C)$theta(A,C)..
    QXAC(A,C) + SUM(H, QHA(A,C,H)) =E= theta(A,C)*QA(A) ;

 OUTAGGFN(C)$CX(C)..
   QX(C) =E= alphaac(C)*SUM(A, deltaac(A,C)*QXAC(A,C)
                                            **(-rhoac(C)))**(-1/rhoac(C));

 OUTAGGFOC(A,C)$deltaac(A,C)..
   PXAC(A,C) =E=
   PX(C)
   * QX(C) * SUM(AP, deltaac(AP,C)*QXAC(AP,C)**(-rhoac(C)) )**(-1)
   *deltaac(A,C)*QXAC(A,C)**(-rhoac(C)-1);

 CET(C)$(CE(C) AND CD(C))..
    QX(C) =E= alphat(C)*(deltat(C)*QE(C)**rhot(C) +
                           (1 - deltat(C))*QD(C)**rhot(C))**(1/rhot(C)) ;

 ESUPPLY(C)$(CE(C) AND CD(C))..
   QE(C) =E=  QD(C)*((PE(C)/PDS(C))*
                 ((1 - deltat(C))/deltat(C)))**(1/(rhot(C)-1)) ;

 CET2(C)$( (CD(C) AND CEN(C)) OR (CE(C) AND CDN(C)) )..
   QX(C) =E= QD(C) + QE(C);

 ARMINGTON(C)$(CM(C) AND CD(C))..
   QQ(C) =E= alphaq(C)*(deltaq(C)*QM(C)**(-rhoq(C)) +
                        (1 -deltaq(C))*QD(C)**(-rhoq(C)))**(-1/rhoq(C)) ;

 COSTMIN(C)$(CM(C) AND CD(C))..
   QM(C) =E= QD(C)*((PDD(C)/PM(C))*(deltaq(C)/(1 - deltaq(C))))
                                                        **(1/(1 + rhoq(C)));

 ARMINGTON2(C)$( (CD(C) AND CMN(C)) OR (CM(C) AND CDN(C)) )..
   QQ(C) =E= QD(C) + QM(C);
```

```
 QTDEM(C)$CT(C)..
   QT(C) =E= SUM(CP, icm(C,CP)*QM(CP) + ice(C,CP)*QE(CP) + icd(C,CP)*QD(CP));

*Institution block ==================================================

 YFDEF(F)..  YF(F) =E= SUM(A, WF(F)*wfdist(F,A)*QF(F,A));

 YIFDEF(INSD,F)$shif(INSD,F)..
  YIF(INSD,F) =E= shif(INSD,F)*((1-tf(f))*YF(F) - trnsfr('ROW',F)*EXR);

 YIDEF(INSDNG)..
  YI(INSDNG) =E=
   SUM(F, YIF(INSDNG,F))  + SUM(INSDNGP, TRII(INSDNG,INSDNGP))
   + trnsfr(INSDNG,'GOV')*CPI + trnsfr(INSDNG,'ROW')*EXR;

 TRIIDEF(INSDNG,INSDNGP)$(shii(INSDNG,INSDNGP))..
  TRII(INSDNG,INSDNGP) =E= shii(INSDNG,INSDNGP)
            * (1 - MPS(INSDNGP)) * (1 - TINS(INSDNGP))* YI(INSDNGP);

 EHDEF(H)..
  EH(H) =E= (1 - SUM(INSDNG, shii(INSDNG,H))) * (1 - MPS(H))
                                              * (1 - TINS(H)) * YI(H);

 HMDEM(C,H)$betam(C,H)..
   PQ(C)*QH(C,H) =E=
    PQ(C)*gammam(C,H)
         + betam(C,H)*( EH(H) - SUM(CP, PQ(CP)*gammam(CP,H))
                             - SUM((A,CP), PXAC(A,CP)*gammah(A,CP,H))) ;

 HADEM(A,C,H)$betah(A,C,H)..
   PXAC(A,C)*QHA(A,C,H) =E=
     PXAC(A,C)*gammah(A,C,H)
                  + betah(A,C,H)*(EH(H) - SUM(CP, PQ(CP)*gammam(CP,H))
                          - SUM((AP,CP), PXAC(AP,CP)*gammah(AP,CP,H))) ;

 INVDEM(C)..  QINV(C) =E= IADJ*qbarinv(C);

 GOVDEM(C)..  QG(C) =E= GADJ*qbarg(C);

 YGDEF..
   YG =E= SUM(INSDNG, TINS(INSDNG)*YI(INSDNG))
           + SUM(f, tf(F)*YF(F))
           + SUM(A, tva(A)*PVA(A)*QVA(A))
           + SUM(A, ta(A)*PA(A)*QA(A))
           + SUM(C, tm(C)*pwm(C)*QM(C))*EXR
           + SUM(C, te(C)*pwe(C)*QE(C))*EXR
           + SUM(C, tq(C)*PQ(C)*QQ(C))
           + SUM(F, YIF('GOV',F))
           + trnsfr('GOV','ROW')*EXR;
```

```
 EGDEF..  EG =E= SUM(C, PQ(C)*QG(C)) + SUM(INSD, trnsfr(INSD,'GOV'))*CPI;

*System constraint block ==============================================

 FACEQUIL(F)..  SUM(A, QF(F,A)) =E= QFS(F);

 COMEQUIL(C)..
   QQ(C) =E= SUM(A, QINT(C,A)) + SUM(H, QH(C,H)) + QG(C)
                 + QINV(C) + qdst(C) + QT(C);

 CURACCBAL..
  SUM(C, pwm(C)*QM(C)) + SUM(F, trnsfr('ROW',F)) =E=
  SUM(C, pwe(C)*QE(C)) + SUM(INSD, trnsfr(INSD,'ROW')) + FSAV;

 GOVBAL.. YG =E= EG + GSAV;

 TINSDEF(INSDNG)..
  TINS(INSDNG) =E= tinsbar(INSDNG)*(1 + TINSADJ*tins01(INSDNG)) +
DTINS*tins01(INSDNG);

 MPSDEF(INSDNG)..
  MPS(INSDNG) =E= mpsbar(INSDNG)*(1 + MPSADJ*mps01(INSDNG)) + DMPS*mps01(INSDNG);

 SAVINVBAL..
   SUM(INSDNG, MPS(INSDNG) * (1 - TINS(INSDNG)) * YI(INSDNG))
    + GSAV + FSAV*EXR =E=
   SUM(C, PQ(C)*QINV(C)) + SUM(C, PQ(C)*qdst(C)) + WALRAS;

 TABSEQ..
  TABS =E=
   SUM((C,H), PQ(C)*QH(C,H)) + SUM((A,C,H), PXAC(A,C)*QHA(A,C,H))
  + SUM(C, PQ(C)*QG(C)) + SUM(C, PQ(C)*QINV(C)) + SUM(C, PQ(C)*qdst(C));

 INVABEQ.. INVSHR*TABS =E= SUM(C, PQ(C)*QINV(C)) + SUM(C, PQ(C)*qdst(C));

 GDABEQ..  GOVSHR*TABS =E= SUM(C, PQ(C)*QG(C));
```

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About this Manual

The purpose of this manual is to contribute to and facilitate the use of computable general equilibrium (CGE) models in the analysis of issues related to food policy in developing countries. The volume includes a detailed presentation of a static "standard" CGE model and its required database and incorporates features of particular importance in developing countries. The manual discusses the implementation of the model in GAMS and is accompanied by a CD-ROM that includes the GAMS files for the model, sample databases, simulations, solution reports, and a social accounting matrix (SAM) aggregation program. Although the volume provides a standardized framework for analysis, the analyst is not forced to make "one-size-fits-all" assumptions. The GAMS code is written to give the analyst considerable flexibility in model specification.

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