A STATIONARY APPROXIMATION IN MATCHED-FIELD PROCESSING FOR A MOVING UNDERWATER ACOUSTIC SOURCE

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1. INTRODUCTION

A normal-mode model for the acoustic field due to a moving source, derived by Hawker [1], was recently used in the development of procedures for applying matched-field processing (MFP) techniques to a moving source [2]. This approach involved matching "measured" array covariance matrices at multiple frequencies using replica matrices computed with a phase-expanded form of the Hawker model. Excellent results were obtained, but the technique was compute-intensive. In this paper, a procedure is described for using snapshot vectors at a single frequency to generate the replica matrices. By comparing the performance of the MFP techniques applied to simulated data, it is shown that this new stationary approximation procedure allows the position of a moving source to be estimated as acurately as, but much more efficiently than, the full multiple-frequency procedure.

2. SIMULATION OF MEASURED DATA

The simulated environment consisted of a 500-m channel with an Arctic upward refracting sound-speed profile, and a bottom consisting of a 60-m sediment layer with sound speed 2000 m/s and density 1.5 g/ml, and an underlying layer with sound speed 2600 m/s and density 2.5 g/ml. When these layers were modelled as fluids in the generation of a normal mode model at 20 Hz, 12 propagating modes were obtained.

Data were simulated for a vertical array spanning the water column. The array consisted of 20 sensors evenly spaced from 25 m to 500 m depth, with increments of 25 m. For each simulation a single source was present at a depth of 100 m and a velocity of 10 m/s outward along a radial from the array. The initial range of the source was set to either 20,000, 25,000, 30,000, 35,000 or 40,000 m. The time domain model of Hawker [1] was used in conjunction with the above 12-mode model to compute discrete noise-free time series for each sensor at a sampling rate of 51.2 Hz. Data segments of 10 s were Fourier transformed, giving a frequency resolution of 0.1 Hz, and array covariance matrices were computed for each of the seven discrete frequencies between 19.7 Hz and 20.3 Hz. These matrices were averaged over time for integration times of 10, 20, 50, 100, 200, 500, or 1000 s.

For the MFP procedure involving full matching of source motion [2], the set of seven time-averaged measured matrices $M(f_j)$ (one for each frequency f_j) for each simulation were stored and used directly. For the MFP procedures involving the conventional snapshot and our new stationary approximation, these matrices were averaged across frequency and the resulting time-and-frequency-averaged matrix $\overline{M}$ for that simulation was used.

3. MATCHED-FIELD PROCESSING

The 12-mode model was used to generate replica data for matching the simulated measured data. Depending on the input data and the type of matching to be performed, one of three forms of the generalized Bartlett beamformer (GBF) was used for the matching. In each case the GBF power was normalized with respect to both the measured and replica data so that it corresponded to the correlation coefficient for the match.

For the full matching of source position and velocity, including Doppler shifts, the following form was used:

$$P_{\text{full}} = \frac{\sum_j \operatorname{Tr}\,[M(f_j)R(f_j)]}{\sum_j \lambda_N^M(f_j)\lambda_N^R(f_j)}, \tag{1}$$

where P_{full} is the GBF power for the full source motion matching procedure, $M(f_j)$ is the measured covariance matrix for frequency f_j, $R(f_j)$ is the replica matrix for frequency f_j, and λ_N^M and λ_N^R are the largest eigenvalues of M and R, respectively. $R(f_j)$ was generated by summation of outer product matrices formed from vectors computed using the phase-expanded form of the Hawker model (Eq. 2 of [2]). The summation was performed at 10-s intervals over the corresponding integration time. Here $R(f_j)$ is a function of source position, velocity, and frequency f_j.

For the stationary approximation matching of source position and velocity, in which,Doppler shifts are not matched, the following form was used:

$$P_{\text{stat\_approx}} = \frac{\operatorname{Tr}\,[\overline{M}\,\overline{R}]}{\lambda_N^{\overline{M}}\lambda_N^{\overline{R}}}, \tag{2}$$

where $P_{\text{stat\_approx}}$ is the GBF power for the stationary approximation matching procedure, and $(\overline{\cdot})$ denotes the time-and-frequency-averaged matrix. In this form, $\overline{R} = \sum_k \mathbf{r}(t_k)\mathbf{r}^*(t_k)$, where $\mathbf{r}(t_k)$ is the stationary snapshot vector at time t_k, and the summation is over the integration time as above. The snapshot vectors were generated using the standard form of the normal mode model. Here, $\overline{R}$ is a function of source position and velocity but is defined at only a single frequency.

Table 1: Normalized GBF power (correlation coefficient) for MFP of data simulated for a source moving at 10 m/s. The data were matched using the full matching procedure, the stationary approximation and the snapshot method.

Integration Time (s)	Number of Averages	Source Motion (m)	Normalized GBF Power (Correlation Coefficient)		
			Full Moving	Stationary Approximation	Snapshot
10	1	100	1.0000 ± 0.0000	0.9970 ± 0.0017	0.9970 ± 0.0017
20	2	200	1.0000 ± 0.0000	0.9970 ± 0.0017	0.9943 ± 0.0021
50	5	500	1.0000 ± 0.0000	0.9966 ± 0.0019	0.8787 ± 0.0065
100	10	1000	1.0000 ± 0.0000	0.9966 ± 0.0017	0.7562 ± 0.0121
200	20	2000	0.9999 ± 0.0001	0.9970 ± 0.0015	0.6952 ± 0.0209
500	50	5000	0.9995 ± 0.0003	0.9989 ± 0.0005	0.6368 ± 0.0034
1000	100	10000	0.9994 ± 0.0003	0.9998 ± 0.0002	0.6879 ± 0.0031

For the conventional snapshot matching of source position, the following form was used:

$$P_{\text{snapshot}} = \frac{\mathbf{r}^* \overline{M} \mathbf{r}}{\lambda_N^{\overline{M}} |\mathbf{r}|^2}, \tag{3}$$

where P_{snapshot} is the power for the conventional snapshot form of the GBF, and $\mathbf{r}$ is the stationary snapshot replica vector. Here, $\mathbf{r}$ is a function of source position only.

MFP was performed using the grid-search-optimization (GSO) technique. GSO involves the generation of a search grid of power values for various source positions, followed by identification of local maxima in this grid, which are then used as initial estimates in the optimization of source position. The result is a set of discrete estimates of the source position. Since the array was vertical and the environment was range-independent, the power only depended on source range and depth (not bearing). For the full and stationary approximation forms of the GBF (Eqs. 1 and 2), the source speed and heading were set to their known values.

The depth search region extended from 50 to 150 m, with increments of 25 m. The specification of the range search region depended on the form of the matching. For the full and stationary approximation forms, the region was set to ±1000 m from the known initial range of the source, with increments of 100 m. (Here, source position and velocity are matched for a given initial position.) For the snapshot form, which does not match velocity, the region was defined to be from 100 m before the initial range to 100 m beyond the final range for that simulation. The increments for the range search grid were 100 m for integration times from 1 to 200 s, 200 m for 500 s, and 250 m for 1000 s.

MFP was performed for three processors, seven integration times, and five initial source ranges. The maximum correlation in each set of GSO outputs was taken, and these were averaged across the five ranges to give means and standard deviations of the GBF power for each processor and integration time.

4. RESULTS

Table 1 shows the results of these analyses. As expected for the noise-free conditions here, the full matching procedure gave essentially perfect matches for all integration times.

The stationary approximation procedure also gave very high correlations for all conditions here; these correlations were only slightly less than unity. This was the case even though the data matrices were averaged in both time and frequency, and the matching did not take Doppler shifts into account. The high correlations indicate that simply matching the source track at the reference frequency provides an excellent means of estimating the position though not the direction of motion of the moving source, and that it is not necessary to model the Doppler effects. Using the stationary approximation requires an order of magnitude less computations than the full multiple frequency procedure.

The snapshot matching procedure gave multiple peaks of approximately equal correlation values at various positions along the source track. These correlation values rapidly decreased as the integration time, and corresponding extent of source motion, increased. For integration times of the order of a few minutes, the correlations decreased to less than 0.7, compared to values of greater than 0.99 for the stationary approximation procedure.

5. CONCLUSIONS

The use of the stationary approximation procedure provides a highly effective approach to the estimation of the position of a moving source. It is almost as effective as the use of the multi-frequency full matching method, and much more effective than matching with simple snapshot vectors. The stationary approximation procedure does not involve Doppler shifts and is substantially less compute-intensive than the full procedure, since it involves matching replicas generated at only the single reference frequency. It may also be adapted for use with precomputed grids of field values to yield further increases of speed.

REFERENCES

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