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**A Statistical Representation of Results of the U.S. Geological Survey's  
1995 National Assessment of Conventional Oil and Gas Resources**

**by**

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**This report is preliminary and has not been reviewed for conformity with U.S.  
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## ABSTRACT

The 20/80 law is a heuristic law that has evolved over the years into the following rule of thumb for many populations: 20% of the population accounts for 80% of the total value. This law states quantitatively that often a relatively small portion of a population accounts for most of the total value of the population. This principle is certainly applicable in the case of conventional oil and nonassociated-gas resources in the United States where the few largest plays have an overwhelming amount of the total resources.

The general  $p100/q100$  law in statistical form is defined with the statistic  $q$  as a function of  $p$  where  $p$  is the population proportion and  $q$  is the proportion of total value. The general  $p100/q100$  law in probabilistic form is defined with the parameter  $q$  as a function of  $p$  for any probability distribution that models the population distribution. Using the lognormal distribution, the  $p100/q100$  law in lognormal form is derived with the lognormal  $q$  being a fractal, where  $q$  possesses the scale invariance property.

The  $p100/q100$  law in lognormal form was applied to data on technically recoverable resources in oil and nonassociated-gas (conventional) plays in the onshore areas and adjoining State waters of the United States. These data were generated as part of the U.S. Geological Survey's 1995 National Assessment of United States oil and gas resources. The theoretical percentages of total resources using the lognormal  $q$  are extremely close to the empirical percentages from the oil and nonassociated-gas data using the statistic  $q$ . For example, 20% of the 274 oil plays account for 73.05% of the total oil resources of the plays if we use the lognormal  $q$ , or for 75.52% if we use the statistic  $q$ ; 20% of the 239 nonassociated-gas plays account for 76.32% of the total nonassociated-gas resources of the plays if we use the lognormal  $q$ , or for 78.87% if we use the statistic  $q$ .

## INTRODUCTION

Vilfredo Pareto (1848-1923), an Italian engineer turned economist and sociologist, was a pioneer in econometrics, applying the statistical methods of the physical sciences to the social sciences. Pareto discovered that, generally, a relatively small portion of a population accounts for most of the total value of the population. That is, in any set of elements, the critical elements usually constitute a minority of the set (Douglass and Douglass, 1993). Pareto's principle is certainly applicable in the case of technically recoverable petroleum resources in oil and nonassociated-gas (conventional) plays in the United States, where the few largest plays have an overwhelming amount of the total resources (Gautier and others, 1995; U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995).

As originally proposed, Pareto's law (Pareto, 1897; Turla and Hawkins, 1983) was an empirical relation describing the distribution of income among the population of a country; viz.,

20 percent of the people in Italy owned 80 percent of the wealth.

From this principle, the heuristic "20/80 law" has evolved over the years into the following simple rule of thumb, which is applicable to many populations:

20% of the population accounts for 80% of the total value.

Numerous applications of this law exist in the management sciences. For example,

20% of the customers account for 80% of the sales;

20% of the employees account for 80% of the work; and,

20% of the components account for 80% of the cost.

The principle is observed in other disciplines as well. The objective of the present paper is to generalize "the 20/80 law" using the lognormal distribution and apply the generalization to data on oil and nonassociated-gas resources to obtain for each:  $p$ 100% of the plays account for  $q$ 100% ( $q$  is a function of  $p$ ) of the total resources of the plays. Fractal lognormal percentage theory is developed first; the oil data are analyzed next; and, finally, the theory is applied to the nonassociated-gas data.

## FRACTAL LOGNORMAL PERCENTAGE THEORY

The material of this section was first presented in Crovelli (1995) and is reproduced here for completeness.

### THE $p$ 100/ $q$ 100 LAW IN STATISTICAL FORM:

Consider a population of  $N$  values for a random variable  $X$  arranged in increasing order of magnitude:

$$x_1, x_2, \dots, x_N.$$

The total value of the population is:

$$\sum_{i=1}^N x_i.$$

Given a proportion  $p$  ( $0 \leq p \leq 1$ ) of largest values of the population, the partial sum of the  $p100\%$  of largest values of the population is:

$$\sum_{i=[(1-p)N]+1}^N x_i,$$

where  $[\cdot]$  is the greatest integer value function.

Let the proportion of the total value of the population accounted for by  $p100\%$  of the population be given by  $q$  ( $0 \leq q \leq 1$ ), where the statistic  $q$  is defined as:

$$q = \frac{\sum_{i=[(1-p)N]+1}^N x_i}{\sum_{i=1}^N x_i}.$$

An alternative form is:

$$q = \frac{p \sum_{i=[(1-p)N]+1}^N x_i / pN}{\sum_{i=1}^N x_i / N}.$$

Summarizing,  $p$  is a proportion (or fraction) of the population, and  $p100\%$  is a percentage of the population; whereas,  $q$  is a proportion (or fraction) of the total value,  $q100\%$  is a percentage of the total value, and  $p100\%$  of the population accounts for  $q100\%$  of the total value.

### THE $p100/q100$ LAW IN PROBABILISTIC FORM

Given a population of values of a random variable  $X$  having a probability distribution that models the population distribution, then  $p100\%$  of the population values account for  $q100\%$  of the total value, where the parameter  $q$  is called the proportion of total value and defined in terms of conditional expectation,  $E(X|\cdot)$ , as follows.

$$q = \frac{pE(X | X > x_p)}{E(X)}, \quad 0 \leq p \leq 1,$$

with  $p = P(X > x_p)$  and  $x_p$  is called the  $p100$ th fractile.

Various probability distributions could be used as models for the population distribution. We will consider the lognormal probability distribution. The percentage theory based on the Pareto distribution has been developed and applied to petroleum field size data (Crovelli, 1995).

### THE LOGNORMAL PROBABILITY DISTRIBUTION

The lognormal distribution is discussed in detail in Aitchison and Brown (1957), Johnson and others (1994), and Crow and Shimizu (1988). A nonnegative random variable  $X$  has a lognormal distribution if the random variable  $Y = \ln X$  has a normal distribution.

The lognormal probability density function of  $X$  is

$$f(x) = \frac{1}{\sqrt{2\pi} \sigma x} \exp \frac{-(\ln x - \mu)^2}{2\sigma^2}, \quad x > 0$$

where  $\sigma > 0$  is a shape parameter and  $\mu$  is a scale parameter (Law and Kelton, 1991, p. 337).

The expected value or mean of  $X$  is

$$E(X) = e^{\mu + \sigma^2/2}$$

The variance of  $X$  is

$$V(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$$

The scale parameter  $\mu$  and the shape parameter  $\sigma$  of the lognormal distribution are computed from the following formulas:

$$\mu = \frac{1}{N} \sum_{i=1}^N \ln x_i$$

and

$$\sigma = \left[ \frac{N \sum_{i=1}^N (\ln x_i)^2 - \left( \sum_{i=1}^N \ln x_i \right)^2}{N^2} \right]^{1/2}$$

The following notation will be used throughout the remainder of the paper:

$X \sim \text{lognormal}(\text{shape parameter } \sigma, \text{scale parameter } \mu),$

where the symbol  $\sim$  is read “is distributed as.”

### THE LOGNORMAL DISTRIBUTION HAS A FRACTAL PROPERTY

If  $X \sim \text{lognormal}$  (shape parameter  $\sigma$ , scale parameter  $\mu$ ), let

$$X' = cX, \quad c > 0.$$

The fractal property of  $X$  under a positive multiplicative constant results in:

$$X' \sim \text{lognormal} (\text{shape parameter } \sigma, \text{ scale parameter } \mu + \ln c).$$

Note that the distributions of  $X$  and  $X'$  are identical (with the same shape parameter) except for scale. A probabilistic fractal is a probability distribution that is invariant except for scale (scale invariant or self-similar) under a given transformation (Mandelbrot, 1983, p. 343). Under a positive multiplicative constant, the lognormal distribution remains a lognormal distribution with fixed  $\sigma$  and changed  $\mu$ .

### THE $p100/q100$ LAW IN LOGNORMAL FORM

Consider the  $p100/q100$  law in probabilistic form when the population distribution is modeled as a lognormal distribution; i.e.,

$$q = \frac{pE(X|X > x_p)}{E(X)}, \quad 0 \leq p \leq 1$$

when  $X \sim \text{lognormal}$  (shape parameter  $\sigma$ , scale parameter  $\mu$ ).

Recall that the expected value of  $X$  is

$$E(X) = e^{\mu + \sigma^2/2}.$$

It can be shown (Johnson and others, 1994) that the conditional expectation is given by

$$E(X|X > x_p) = e^{\mu + \sigma^2/2} \frac{R((\ln x_p - \mu - \sigma^2)/\sigma)}{R((\ln x_p - \mu)/\sigma)}$$

where  $R$  denotes the complementary cumulative distribution function of a standard normal random variable  $Z$ , i.e.,  $R(z) \equiv P(Z > z)$ .

We define the following notation:

$$R(z_p) = P(Z > z_p) = p$$

where  $z_p$  is called the  $p$ 100th fractile of  $Z$ .

Because

$$R((\ln x_p - \mu) / \sigma) = R(z_p) = p$$

and

$$R((\ln x_p - \mu - \sigma^2) / \sigma) = R(z_p - \sigma)$$

we have

$$E(X | X > x_p) = e^{\mu + \sigma^2/2} \frac{R(z_p - \sigma)}{p}$$

Substituting into the general formula for  $q$  and simplifying, we get

$$q = R(z_p - \sigma), \quad 0 < p < 1$$

or

$$q = 1 - \Phi(z_p - \sigma), \quad 0 < p < 1$$

where  $\Phi$  denotes the standard normal cumulative distribution function.

Observe that if  $p \rightarrow 0$ , then  $q \rightarrow 0$ , and if  $p \rightarrow 1$ , then  $q \rightarrow 1$ . Thus we define  $q = 0$  if  $p = 0$ , and  $q = 1$  if  $p = 1$ . The special case when  $p = 0.5$  and  $z_p = 0$  yields

$$q = R(-\sigma) = 1 - \Phi(-\sigma)$$

The parameter  $q$  has the very remarkable property of being scale-free; i.e.,  $q$  does not depend upon the scale parameter  $\mu$ . Because the parameter  $q$  is scale-free,  $q$  stays the same for all lognormal distributions with the same shape parameter  $\sigma$ , as in the case of the fractal property of the lognormal distribution under a positive multiplicative constant. Because it is scale invariant, the proportion  $q$  can be considered to be a parametric fractal.

The proportion  $q$  of the  $p$ 100/ $q$ 100 law in statistical form is called the statistic  $q$ , in probabilistic form it is called the parameter  $q$ , and in lognormal form it is called the lognormal  $q$ .

## OIL AND NONASSOCIATED-GAS DATA

The U.S. Geological Survey periodically makes appraisals of the oil and gas resources of the Nation. In its 1995 National Assessment (Gautier and others, 1995; U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995), the onshore areas and adjoining State waters of the Nation were assessed. The basic assessment unit was the play.

A conventional accumulation of the 1995 National Assessment is one in which oil or gas is trapped as a discrete deposit (field) that is usually bounded by a downdip water contact. A conventional play is a set of known and (or) postulated conventional accumulations sharing similar geologic, geographic, and temporal properties. Conventional plays were assessed as described by Gautier and Dolton (1995).

For purposes of this report, terms such as oil resources, oil data, and oil plays refer to conventional oil plays of the 1995 National Assessment; terms involving nonassociated-gas refer to conventional nonassociated-gas plays of the 1995 National Assessment. An accumulation with a gas-oil ratio in excess of 20,000 is considered a nonassociated-gas accumulation.

As part of the 1995 National Assessment, 274 conventional oil plays were assessed. Estimates of mean, undiscovered, technically recoverable resources for these plays total 30.25 billion barrels of oil (U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995). The mean, undiscovered, technically recoverable oil resources estimated for each of the 274 conventional oil plays constitute the data set used in the following section on Application of Theory to Conventional Oil Resources.

Estimates of mean, undiscovered, technically recoverable resources for the 239 conventional nonassociated-gas plays assessed as part of the 1995 National Assessment total 258.69 trillion cubic feet of gas (U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995). The mean, undiscovered, technically recoverable gas resources estimated for each of the 239 conventional nonassociated-gas plays constitute the data set used in the section on Application of Theory to Conventional Nonassociated-Gas Resources.



## APPLICATION OF THEORY TO CONVENTIONAL OIL RESOURCES

The fractal lognormal percentage theory is applied in this section to the oil data. This new probabilistic methodology for percentage assessment lends itself as an ideal spreadsheet software application.

A frequency distribution of the 274 oil plays (in million barrels mean oil) is displayed in Figure 1. Because the plotted random variable is the logarithm of mean oil,  $\text{Ln}$  (million barrels mean oil), the bell-shaped distribution suggests that the oil resources are approximately distributed as a lognormal distribution.

Spreadsheet software was used to make various calculations involving the oil data. Table 1 is a spreadsheet with 274 rows corresponding to the 274 oil plays. Each oil play of Table 1 is identified by the play number that was used in the 1995 National Assessment (Gautier and others, 1995). The data are listed and several calculations are made, including those necessary for the statistic  $q$ . The lognormal parameters were computed: scale parameter  $\mu = 3.3669$  and shape parameter  $\sigma = 1.4559$ .

Table 2 is a spreadsheet of the calculations from the oil data that are necessary for the lognormal  $q$  as a function of various values of  $p$ . The formula for the lognormal  $q$  in the case of the oil data is the following:

$$q = R(z_p - 1.4559) = 1 - \Phi(z_p - 1.4559), \quad 0 < p < 1$$

Therefore, for any specified value of  $p$ , we can obtain:  $p100\%$  of the oil plays account for  $q100\%$  of the total oil resources of the plays. The fractal lognormal percentage assessment of conventional oil resources is summarized in Table 3. The corresponding graph of the summary is given in Figure 2.

Note that in Table 3 and Figure 2, the theoretical percentages of total oil resources using the lognormal  $q$  are extremely close to the empirical percentages from the oil data using the statistic  $q$ . For example, 20% of the 274 oil plays account for 73.05% of the total oil resources of the plays if we use the lognormal  $q$ , or for 75.52% if we use the statistic  $q$ . Plays presumably vary in their degree of heterogeneity, and this can be quantitatively captured by this method. The method also forms a basis for comparing plays of different geologic styles, and similar geologic styles.

## Frequency distribution of oil data

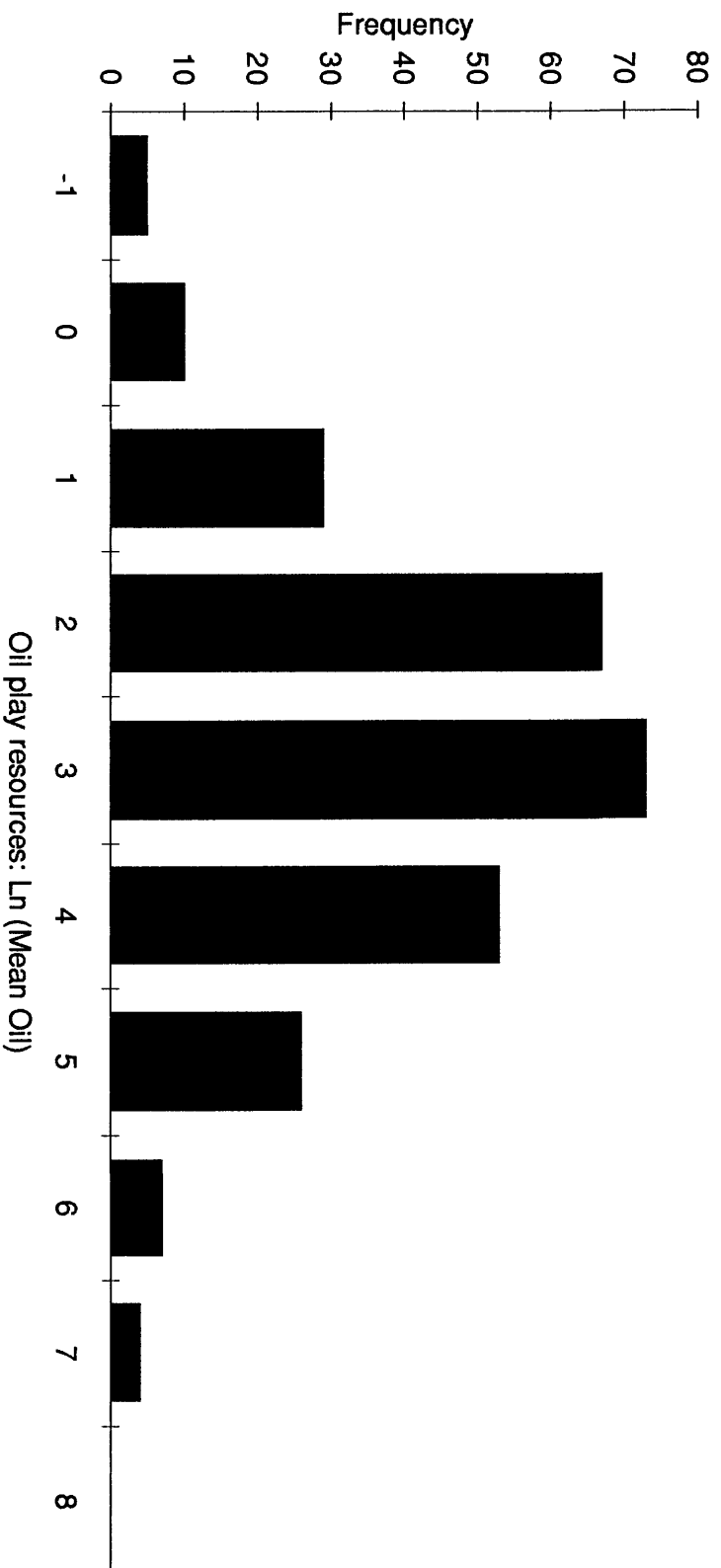


Figure 1. Frequency distribution of 274 oil plays (in million barrels mean oil). Data and bin frequencies are included with Table 1.

Table 1. Spreadsheet of oil data and calculations for the statistic  $q$ .

$n$	Play No.	Mean Oil (MMBBL)	Ln of Mean Oil	Sorted Mean Oil	Partial sum Sorted	Statistic $q$ Partial/Total	<i>Bin</i>	<i>Frequency</i>
1	101	1564.4	7.3553	1592.8	1592.800	0.06645	-1	5
2	102	1254.3	7.1343	1564.4	3157.200	0.13172	0	10
3	103	1489.2	7.3060	1489.2	4646.400	0.19385	1	29
4	104	493.7	6.2019	1254.3	5900.700	0.24619	2	67
5	109	617.3	6.4254	809.2	6709.900	0.27995	3	73
6	110	226.9	5.4245	646.8	7356.700	0.30693	4	53
7	111	1592.8	7.3732	617.3	7974.000	0.33269	5	26
8	205	61.2	4.1141	583	8557.000	0.35701	6	7
9	301	52.1	3.9532	522.3	9079.300	0.37880	7	4
10	302	9.1	2.2083	507	9586.300	0.39995	8	0
11	304	646.8	6.4720	493.7	10080.000	0.42055		
12	308	172.6	5.1510	402.3	10482.300	0.43734		
13	309	57.2	4.0466	321.2	10803.500	0.45074		
14	405	18.2	2.9014	298.9	11102.400	0.46321		
15	406	1.1	0.0953	296.1	11398.500	0.47556		
16	703	20.9	3.0397	264.9	11663.400	0.48661		
17	801	3.7	1.3083	253.7	11917.100	0.49720		
18	1002	85.2	4.4450	252.3	12169.400	0.50772		
19	1003	62	4.1271	251.1	12420.500	0.51820		
20	1004	583	6.3682	249.6	12670.100	0.52861		
21	1005	238.8	5.4756	248.9	12919.000	0.53900		
22	1006	11.7	2.4596	238.8	13157.800	0.54896		
23	1008	35.9	3.5807	230.5	13388.300	0.55858		
24	1009	9.7	2.2721	229.1	13617.400	0.56814		
25	1010	50	3.9120	226.9	13844.300	0.57760		
26	1101	8.8	2.1748	223.6	14067.900	0.58693		
27	1102	21	3.0445	214.2	14282.100	0.59587		
28	1103	6.6	1.8871	213	14495.100	0.60476		
29	1104	52.4	3.9589	203.6	14698.700	0.61325		
30	1105	14.3	2.6603	194.6	14893.300	0.62137		
31	1106	223.6	5.4099	194.6	15087.900	0.62949		
32	1107	59	4.0775	192.4	15280.300	0.63752		
33	1109	6.6	1.8871	182.1	15462.400	0.64511		
34	1201	33.8	3.5205	172.6	15635.000	0.65231		
35	1202	19.4	2.9653	168.5	15803.500	0.65934		
36	1204	16.3	2.7912	152.4	15955.900	0.66570		
37	1211	103.9	4.6434	152	16107.900	0.67204		
38	1301	126.2	4.8379	142.8	16250.700	0.67800		
39	1302	251.1	5.5259	142	16392.700	0.68393		
40	1311	296.1	5.6907	132	16524.700	0.68943		
41	1312	249.6	5.5199	127.2	16651.900	0.69474		
42	1401	214.2	5.3669	126.2	16778.100	0.70001		
43	1402	142.8	4.9614	126.2	16904.300	0.70527		
44	1403	264.9	5.5794	124.7	17029.000	0.71047		
45	1404	89.8	4.4976	109.3	17138.300	0.71503		
46	1405	109.3	4.6941	106.6	17244.900	0.71948		
47	1406	30.8	3.4275	106.4	17351.300	0.72392		
48	1407	18.3	2.9069	103.9	17455.200	0.72826		

Table 1. (cont.)

<i>n</i>	Play No.	Mean Oil (MMBBL)	Ln of Mean Oil	Sorted Mean Oil	Partial sum Sorted	Statistic q Partial/Total
49	1701	0.9	-0.1054	103.8	17559.000	0.73259
50	1803	0.6	-0.5108	103.5	17662.500	0.73690
51	1901	321.2	5.7721	95	17757.500	0.74087
52	1902	15.4	2.7344	92.6	17850.100	0.74473
53	1903	24	3.1781	90.7	17940.800	0.74852
54	1905	4.5	1.5041	89.8	18030.600	0.75226
55	1906	6.7	1.9021	87.7	18118.300	0.75592
56	1907	11	2.3979	86.9	18205.200	0.75955
57	2002	40.5	3.7013	85.2	18290.400	0.76310
58	2004	3.4	1.2238	83.7	18374.100	0.76659
59	2005	142	4.9558	83.4	18457.500	0.77007
60	2014	2.7	0.9933	81.7	18539.200	0.77348
61	2101	40.1	3.6914	81.4	18620.600	0.77688
62	2102	152	5.0239	80.2	18700.800	0.78022
63	2104	2.3	0.8329	79.2	18780.000	0.78353
64	2105	18.1	2.8959	75	18855.000	0.78666
65	2106	20.8	3.0350	75	18930.000	0.78979
66	2204	21.3	3.0587	71.3	19001.300	0.79276
67	2206	30.5	3.4177	70.6	19071.900	0.79571
68	2207	31.4	3.4468	67.8	19139.700	0.79854
69	2210	7.8	2.0541	66.7	19206.400	0.80132
70	2301	7	1.9459	66.2	19272.600	0.80408
71	2302	2.4	0.8755	63.2	19335.800	0.80672
72	2305	22.5	3.1135	63.1	19398.900	0.80935
73	2402	9	2.1972	63.1	19462.000	0.81198
74	2403	33.5	3.5115	62.4	19524.400	0.81459
75	2404	2.7	0.9933	62	19586.400	0.81717
76	2504	18.7	2.9285	61.4	19647.800	0.81973
77	2705	1.1	0.0953	61.2	19709.000	0.82229
78	2706	0.9	-0.1054	60.8	19769.800	0.82482
79	2707	1.3	0.2624	59.1	19828.900	0.82729
80	2802	1.4	0.3365	59	19887.900	0.82975
81	2803	3.7	1.3083	58.6	19946.500	0.83220
82	2805	57.8	4.0570	58.1	20004.600	0.83462
83	2806	43.7	3.7773	58.1	20062.700	0.83704
84	2807	11.4	2.4336	58	20120.700	0.83946
85	2808	40	3.6889	57.8	20178.500	0.84188
86	2903	3.1	1.1314	57.8	20236.300	0.84429
87	2904	5.1	1.6292	57.2	20293.500	0.84667
88	2906	2.7	0.9933	56.1	20349.600	0.84901
89	2907	0.8	-0.2231	55.3	20404.900	0.85132
90	2910	10.8	2.3795	55.1	20460.000	0.85362
91	3101	168.5	5.1269	53.8	20513.800	0.85586
92	3102	53.5	3.9797	53.5	20567.300	0.85810
93	3103	55.3	4.0128	52.4	20619.700	0.86028
94	3105	49.3	3.8979	52.1	20671.800	0.86246
95	3106	17.7	2.8736	50.6	20722.400	0.86457
96	3301	20.1	3.0007	50	20772.400	0.86665
97	3302	6.8	1.9169	49.5	20821.900	0.86872

Table 1. (cont.)

<i>n</i>	Play No.	Mean Oil (MMBBL)	Ln of Mean Oil	Sorted Mean Oil	Partial sum Sorted	Statistic q Partial/Total
98	3303	81.4	4.3994	49.3	20871.200	0.87078
99	3304	522.3	6.2582	48.3	20919.500	0.87279
100	3305	53.8	3.9853	48	20967.500	0.87479
101	3306	192.4	5.2596	47.7	21015.200	0.87678
102	3307	59.1	4.0792	46.9	21062.100	0.87874
103	3309	48.3	3.8774	46.5	21108.600	0.88068
104	3310	21.1	3.0493	45.4	21154.000	0.88258
105	3312	67.8	4.2166	43.7	21197.700	0.88440
106	3313	58.1	4.0622	42.6	21240.300	0.88618
107	3401	22	3.0910	41.3	21281.600	0.88790
108	3402	24.7	3.2068	40.5	21322.100	0.88959
109	3405	229.1	5.4342	40.4	21362.500	0.89127
110	3406	18.1	2.8959	40.1	21402.600	0.89295
111	3407	28.3	3.3429	40	21442.600	0.89462
112	3408	6.1	1.8083	39.3	21481.900	0.89626
113	3501	31.1	3.4372	39.2	21521.100	0.89789
114	3502	19	2.9444	38.7	21559.800	0.89951
115	3503	8.3	2.1163	38.1	21597.900	0.90110
116	3504	58.6	4.0707	35.9	21633.800	0.90259
117	3506	11.7	2.4596	34.6	21668.400	0.90404
118	3603	230.5	5.4403	34.6	21703.000	0.90548
119	3604	248.9	5.5171	34.3	21737.300	0.90691
120	3606	63.1	4.1447	33.9	21771.200	0.90833
121	3607	49.5	3.9020	33.8	21805.000	0.90974
122	3701	41.3	3.7209	33.5	21838.500	0.91113
123	3703	12.6	2.5337	32.7	21871.200	0.91250
124	3704	15.8	2.7600	32.5	21903.700	0.91385
125	3705	18.9	2.9392	31.5	21935.200	0.91517
126	3706	18.5	2.9178	31.4	21966.600	0.91648
127	3707	34.6	3.5439	31.1	21997.700	0.91778
128	3708	6.2	1.8245	30.8	22028.500	0.91906
129	3801	11.9	2.4765	30.5	22059.000	0.92033
130	3802	6.5	1.8718	30.5	22089.500	0.92161
131	3901	7.1	1.9601	30.5	22120.000	0.92288
132	3905	61.4	4.1174	30.1	22150.100	0.92413
133	3907	14.5	2.6741	29.6	22179.700	0.92537
134	3908	34.3	3.5351	29.2	22208.900	0.92659
135	4001	4.1	1.4110	29.2	22238.100	0.92781
136	4004	95	4.5539	28.8	22266.900	0.92901
137	4005	18	2.8904	28.3	22295.200	0.93019
138	4301	32.7	3.4874	28.3	22323.500	0.93137
139	4402	126.2	4.8379	27.3	22350.800	0.93251
140	4403	252.3	5.5306	27	22377.800	0.93363
141	4404	39.2	3.6687	26.6	22404.400	0.93474
142	4405	30.5	3.4177	26.4	22430.800	0.93584
143	4406	507	6.2285	26.2	22457.000	0.93694
144	4407	80.2	4.3845	26	22483.000	0.93802
145	4408	8.2	2.1041	25	22508.000	0.93907
146	4409	63.2	4.1463	24.8	22532.800	0.94010

Table 1. (cont.)

<i>n</i>	Play No.	Mean Oil (MMBBL)	Ln of Mean Oil	Sorted Mean Oil	Partial sum Sorted	Statistic q Partial/Total
147	4410	132	4.8828	24.7	22557.500	0.94113
148	4411	809.2	6.6960	24	22581.500	0.94213
149	4412	194.6	5.2709	23.4	22604.900	0.94311
150	4501	26.2	3.2658	23.4	22628.300	0.94408
151	4502	50.6	3.9240	22.5	22650.800	0.94502
152	4504	45.4	3.8155	22	22672.800	0.94594
153	4505	152.4	5.0265	21.3	22694.100	0.94683
154	4506	106.4	4.6672	21.1	22715.200	0.94771
155	4601	16.2	2.7850	21.1	22736.300	0.94859
156	4701	58.1	4.0622	21	22757.300	0.94947
157	4704	26.6	3.2809	21	22778.300	0.95034
158	4705	127.2	4.8458	20.9	22799.200	0.95122
159	4708	213	5.3613	20.8	22820.000	0.95208
160	4710	298.9	5.7001	20.8	22840.800	0.95295
161	4715	66.7	4.2002	20.6	22861.400	0.95381
162	4716	203.6	5.3162	20.3	22881.700	0.95466
163	4719	42.6	3.7519	20.1	22901.800	0.95550
164	4722	106.6	4.6691	19.7	22921.500	0.95632
165	4724	20.3	3.0106	19.4	22940.900	0.95713
166	4725	20.6	3.0253	19	22959.900	0.95792
167	4726	32.5	3.4812	18.9	22978.800	0.95871
168	4728	26.4	3.2734	18.9	22997.700	0.95950
169	4733	81.7	4.4031	18.8	23016.500	0.96028
170	4734	34.6	3.5439	18.7	23035.200	0.96106
171	4735	56.1	4.0271	18.5	23053.700	0.96183
172	4737	15.4	2.7344	18.3	23072.000	0.96260
173	4738	30.1	3.4045	18.3	23090.300	0.96336
174	4739	26	3.2581	18.2	23108.500	0.96412
175	4740	71.3	4.2669	18.1	23126.600	0.96487
176	4742	29.2	3.3742	18.1	23144.700	0.96563
177	4743	31.5	3.4500	18	23162.700	0.96638
178	4744	103.8	4.6425	17.7	23180.400	0.96712
179	4745	63.1	4.1447	16.9	23197.300	0.96782
180	4901	55.1	4.0091	16.7	23214.000	0.96852
181	4902	25	3.2189	16.3	23230.300	0.96920
182	4905	83.4	4.4236	16.3	23246.600	0.96988
183	4906	75	4.3175	16.2	23262.800	0.97056
184	4910	90.7	4.5076	15.8	23278.600	0.97122
185	4911	124.7	4.8259	15.5	23294.100	0.97186
186	4912	103.5	4.6396	15.4	23309.500	0.97251
187	4916	46.5	3.8395	15.4	23324.900	0.97315
188	4917	87.7	4.4739	15.3	23340.200	0.97379
189	4918	57.8	4.0570	15.3	23355.500	0.97442
190	4919	194.6	5.2709	15	23370.500	0.97505
191	4921	38.1	3.6402	14.7	23385.200	0.97566
192	4925	24.8	3.2108	14.5	23399.700	0.97627
193	4928	28.8	3.3604	14.3	23414.000	0.97687
194	4930	21.1	3.0493	13.5	23427.500	0.97743
195	4932	70.6	4.2570	12.8	23440.300	0.97796

Table 1. (cont.)

<i>n</i>	Play No.	Mean Oil (MMBBL)	Ln of Mean Oil	Sorted Mean Oil	Partial sum Sorted	Statistic q Partial/Total
196	4934	60.8	4.1076	12.6	23452.900	0.97849
197	4936	40.4	3.6988	12.2	23465.100	0.97900
198	4937	79.2	4.3720	11.9	23477.000	0.97949
199	4938	182.1	5.2046	11.7	23488.700	0.97998
200	4942	29.6	3.3878	11.7	23500.400	0.98047
201	4943	18.9	2.9392	11.5	23511.900	0.98095
202	4945	62.4	4.1336	11.4	23523.300	0.98143
203	5001	253.7	5.5362	11.3	23534.600	0.98190
204	5002	12.2	2.5014	11.2	23545.800	0.98236
205	5003	66.2	4.1927	11	23556.800	0.98282
206	5004	10.7	2.3702	10.8	23567.600	0.98327
207	5005	27.3	3.3069	10.8	23578.400	0.98372
208	5101	46.9	3.8480	10.8	23589.200	0.98418
209	5305	48	3.8712	10.7	23599.900	0.98462
210	5309	15	2.7081	10.1	23610.000	0.98504
211	5501	29.2	3.3742	9.9	23619.900	0.98546
212	5503	15.3	2.7279	9.7	23629.600	0.98586
213	5504	18.8	2.9339	9.6	23639.200	0.98626
214	5505	10.8	2.3795	9.3	23648.500	0.98665
215	5507	8.6	2.1518	9.3	23657.800	0.98704
216	5601	5.8	1.7579	9.1	23666.900	0.98742
217	5603	0.6	-0.5108	9	23675.900	0.98779
218	5802	6.6	1.8871	8.9	23684.800	0.98816
219	5803	11.3	2.4248	8.8	23693.600	0.98853
220	5804	20.8	3.0350	8.6	23702.200	0.98889
221	5805	6	1.7918	8.5	23710.700	0.98924
222	5807	5.5	1.7047	8.4	23719.100	0.98959
223	5809	12.8	2.5494	8.3	23727.400	0.98994
224	5810	19.7	2.9806	8.3	23735.700	0.99029
225	5813	30.5	3.4177	8.3	23744.000	0.99063
226	5814	33.9	3.5234	8.2	23752.200	0.99098
227	5815	13.5	2.6027	8	23760.200	0.99131
228	5816	75	4.3175	7.8	23768.000	0.99163
229	5818	8.9	2.1861	7.7	23775.700	0.99196
230	5819	10.1	2.3125	7.6	23783.300	0.99227
231	5820	16.9	2.8273	7.1	23790.400	0.99257
232	5821	23.4	3.1527	7	23797.400	0.99286
233	5822	6.2	1.8245	6.9	23804.300	0.99315
234	5823	8.3	2.1163	6.8	23811.100	0.99343
235	5824	5.1	1.6292	6.8	23817.900	0.99372
236	5827	15.5	2.7408	6.7	23824.600	0.99400
237	5901	8	2.0794	6.7	23831.300	0.99428
238	5902	9.3	2.2300	6.6	23837.900	0.99455
239	5903	9.3	2.2300	6.6	23844.500	0.99483
240	6001	14.7	2.6878	6.6	23851.100	0.99510
241	6003	15.3	2.7279	6.5	23857.600	0.99537
242	6004	2	0.6931	6.2	23863.800	0.99563
243	6005	28.3	3.3429	6.2	23870.000	0.99589
244	6007	6.9	1.9315	6.1	23876.100	0.99614

Table 1. (cont.)

<i>n</i>	Play No.	Mean Oil (MMBBL)	Ln of Mean Oil	Sorted Mean Oil	Partial sum Sorted	Statistic q Partial/Total
245	6102	86.9	4.4648	6	23882.100	0.99640
246	6103	47.7	3.8649	5.8	23887.900	0.99664
247	6104	16.7	2.8154	5.6	23893.500	0.99687
248	6105	5.3	1.6677	5.5	23899.000	0.99710
249	6107	9.9	2.2925	5.3	23904.300	0.99732
250	6109	5.6	1.7228	5.1	23909.400	0.99753
251	6110	8.3	2.1163	5.1	23914.500	0.99775
252	6111	10.8	2.3795	4.6	23919.100	0.99794
253	6201	3.4	1.2238	4.5	23923.600	0.99813
254	6301	16.3	2.7912	4.1	23927.700	0.99830
255	6304	38.7	3.6558	3.7	23931.400	0.99845
256	6306	39.3	3.6712	3.7	23935.100	0.99861
257	6307	83.7	4.4272	3.4	23938.500	0.99875
258	6308	23.4	3.1527	3.4	23941.900	0.99889
259	6309	402.3	5.9972	3.1	23945.000	0.99902
260	6310	18.3	2.9069	2.7	23947.700	0.99913
261	6311	92.6	4.5283	2.7	23950.400	0.99924
262	6313	21	3.0445	2.7	23953.100	0.99936
263	6401	7.7	2.0412	2.4	23955.500	0.99946
264	6402	11.2	2.4159	2.3	23957.800	0.99955
265	6403	11.5	2.4423	2	23959.800	0.99964
266	6404	4.6	1.5261	1.4	23961.200	0.99970
267	6501	6.8	1.9169	1.3	23962.500	0.99975
268	6502	7.6	2.0281	1.1	23963.600	0.99980
269	6601	8.5	2.1401	1.1	23964.700	0.99984
270	6602	8.4	2.1282	0.9	23965.600	0.99988
271	6703	58	4.0604	0.9	23966.500	0.99992
272	6706	27	3.2958	0.8	23967.300	0.99995
273	6725	9.6	2.2618	0.6	23967.900	0.99997
274	6732	6.7	1.9021	0.6	23968.500	1.00000

0.6000	-0.5108	<== Min
1592.8000	7.3732	<== Max
23968.5000	922.5384	<== Total
87.4763	3.3669	<== Mean ( $\mu$ )
202.3034	1.4559	<== Standard deviation ( $\sigma$ )



Table 2. Spreadsheet of calculations from oil data of Figure 1 for the lognormal  $q$  as a function of various values of  $p$ . NORMSINV is the spreadsheet function that computes  $z$ ; NORMSDIST is the function that computes  $1 - q$ .

$p$ Pop. Prop.	$(p)(N)$ $(p)(274)$	$1 - p$	$z$ NORMSINV	$z - \sigma$ $(z - 1.4559)$	$1 - q$ NORMSDIST	lognormal $q$ (theoretical)	statistic $q$ (empirical)
0.01	2.74	0.99	2.3263	0.8704	0.8080	0.1920	0.1777
0.025	6.85	0.975	1.9600	0.5041	0.6929	0.3071	0.3288
0.05	13.7	0.95	1.6449	0.1890	0.5749	0.4251	0.4595
0.1	27.4	0.9	1.2816	-0.1743	0.4308	0.5692	0.5994
0.2	54.8	0.8	0.8416	-0.6143	0.2695	0.7305	0.7552
0.3	82.2	0.7	0.5244	-0.9315	0.1758	0.8242	0.8351
0.4	109.6	0.6	0.2533	-1.2026	0.1146	0.8854	0.8923
0.5	137	0.5	0.0000	-1.4559	0.0727	0.9273	0.9302
0.6	164.4	0.4	-0.2533	-1.7092	0.0437	0.9563	0.9566
0.7	191.8	0.3	-0.5244	-1.9803	0.0238	0.9762	0.9761
0.8	219.2	0.2	-0.8416	-2.2975	0.0108	0.9892	0.9886
0.9	246.6	0.1	-1.2816	-2.7375	0.0031	0.9969	0.9968

Table 3. Summary of fractal lognormal percentage assessment of conventional oil resources.

Percentage of oil plays $p100\%$	Percentage of total oil resources $q100\%$	
	Theoretical (lognormal $q$ )	Empirical (statistic $q$ )
0	0.00	0.00
1	19.20	17.77
2.5	30.71	32.88
5	42.51	45.95
10	56.92	59.94
20	73.05	75.52
30	82.42	83.51
40	88.54	89.23
50	92.73	93.02
60	95.63	95.66
70	97.62	97.61
80	98.92	98.86
90	99.69	99.68
100	100.00	100.00

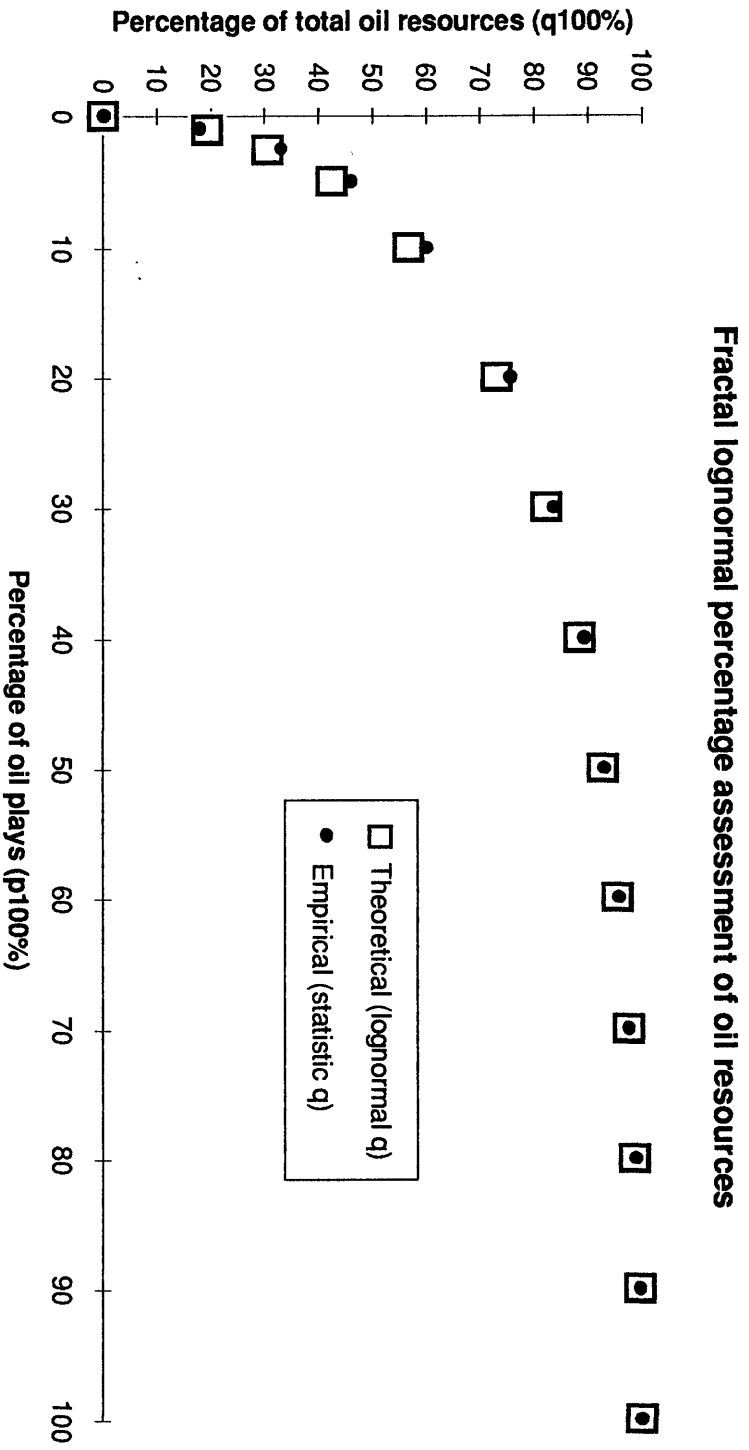


Figure 2. Theoretical percentage of total oil resources using the lognormal  $q$  as a function of the percentage of oil plays, along with the corresponding empirical values using the statistic  $q$ . Data are from Table 3.

## APPLICATION OF THEORY TO CONVENTIONAL NONASSOCIATED-GAS RESOURCES

The fractal lognormal percentage theory is applied here to the nonassociated-gas data. A frequency distribution of the 239 nonassociated-gas plays (in billion cubic feet mean gas) is displayed in Figure 3. Because the plotted random variable is the logarithm of mean gas,  $\text{Ln}$  (billion cubic feet mean gas), the bell-shaped distribution suggests that the nonassociated-gas resources are approximately distributed as a lognormal distribution.

Table 4 is a spreadsheet with 239 rows corresponding to the 239 nonassociated-gas plays. Each gas play of Table 4 is identified by the play number that was used in the 1995 National Assessment (Gautier and others, 1995). The data are listed and several calculations are made, including those necessary for the statistic  $q$ . The lognormal parameters were computed: scale parameter  $\mu = 5.3584$  and shape parameter  $\sigma = 1.5582$ .

Table 5 is a spreadsheet of the calculations from the nonassociated-gas data that are necessary for the lognormal  $q$  as a function of various values of  $p$ . The formula for the lognormal  $q$  in the case of the nonassociated-gas data is the following:

$$q = R(z_p - 1.5582) = 1 - \Phi(z_p - 1.5582), \quad 0 < p < 1$$

The fractal lognormal percentage assessment of conventional nonassociated-gas resources is summarized in Table 6. The corresponding graph of the summary is given in Figure 4.

Note that in Table 6 and Figure 4, the theoretical percentages of total nonassociated-gas resources using the lognormal  $q$  are extremely close to the empirical percentages from the nonassociated-gas data using the statistic  $q$ . For example, 20% of the 239 nonassociated-gas plays account for 76.32% of the total nonassociated-gas resources of the plays if we use the lognormal  $q$ , or for 78.87% if we use the statistic  $q$ .

### Frequency distribution of nonassociated-gas data

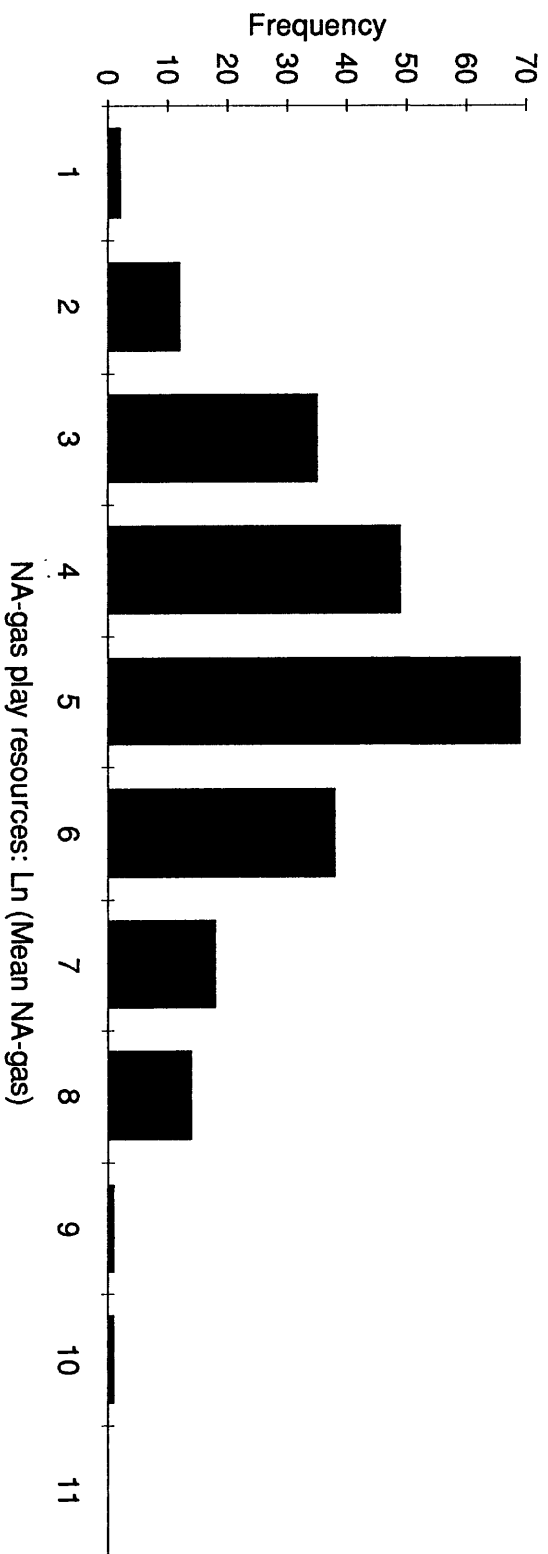


Figure 3. Frequency distribution of 239 nonassociated-gas plays (in billion cubic feet mean gas). Data and bin frequencies are included with Table 4. (NA means nonassociated.)

Table 4. Spreadsheet of nonassociated (NA)-gas data and calculations for the statistic  $q$ .

$n$	Play No.	Mean NAgas (BCF)	Ln of Mean NAgas	Sorted Mean NAgas	Partial sum Sorted	Statistic $q$ Partial/Total	<i>Bin</i>	<i>Frequency</i>
1	101	312.60	5.7449	23739.20	23739.200	0.13121		
2	102	4469.50	8.4050	13296.70	37035.900	0.20470	1	2
3	103	2042.40	7.6219	6315.30	43351.200	0.23960	2	12
4	105	5626.90	8.6353	5709.80	49061.000	0.27116	3	35
5	106	3944.10	8.2800	5626.90	54687.900	0.30226	4	49
6	109	13296.70	9.4953	5259.60	59947.500	0.33133	5	69
7	110	1886.80	7.5426	5164.20	65111.700	0.35988	6	38
8	111	23739.20	10.0749	4469.50	69581.200	0.38458	7	18
9	201	2209.80	7.7007	3944.10	73525.300	0.40638	8	14
10	205	116.40	4.7570	3935.70	77461.000	0.42813	9	1
11	302	179.20	5.1885	3795.90	81256.900	0.44911	10	1
12	303	737.90	6.6038	3203.50	84460.400	0.46682	11	0
13	401	30.00	3.4012	3187.10	87647.500	0.48443		
14	402	163.20	5.0950	3097.70	90745.200	0.50155		
15	403	26.80	3.2884	3053.70	93798.900	0.51843		
16	404	15.00	2.7081	3024.00	96822.900	0.53514		
17	407	331.10	5.8024	2940.40	99763.300	0.55140		
18	410	24.40	3.1946	2905.60	102668.900	0.56746		
19	501	235.10	5.4600	2594.60	105263.500	0.58180		
20	502	78.40	4.3618	2388.70	107652.200	0.59500		
21	701	679.60	6.5215	2307.50	109959.700	0.60775		
22	703	125.20	4.8299	2209.80	112169.500	0.61997		
23	801	22.10	3.0956	2042.40	114211.900	0.63125		
24	901	532.00	6.2766	1886.80	116098.700	0.64168		
25	902	386.70	5.9576	1830.80	117929.500	0.65180		
26	903	1209.70	7.0981	1559.40	119488.900	0.66042		
27	1001	85.30	4.4462	1462.70	120951.600	0.66851		
28	1007	174.60	5.1625	1362.20	122313.800	0.67603		
29	1301	211.90	5.3561	1280.70	123594.500	0.68311		
30	1311	487.70	6.1897	1209.70	124804.200	0.68980		
31	1701	11.10	2.4069	1173.70	125977.900	0.69629		
32	1803	4.30	1.4586	1172.10	127150.000	0.70276		
33	1902	12.00	2.4849	1125.30	128275.300	0.70898		
34	1905	30.80	3.4275	1119.00	129394.300	0.71517		
35	1907	136.80	4.9185	1080.50	130474.800	0.72114		
36	2001	1019.10	6.9267	1068.10	131542.900	0.72704		
37	2002	133.40	4.8934	1066.90	132609.800	0.73294		
38	2003	392.80	5.9733	1029.00	133638.800	0.73863		
39	2004	190.30	5.2486	1019.10	134657.900	0.74426		
40	2014	18.30	2.9069	1017.00	135674.900	0.74988		
41	2101	155.20	5.0447	994.50	136669.400	0.75538		
42	2102	102.40	4.6289	960.00	137629.400	0.76068		
43	2104	49.20	3.8959	947.10	138576.500	0.76592		
44	2105	351.10	5.8611	920.50	139497.000	0.77101		

Table 4. (cont.)

<i>n</i>	Play No.	Mean NAgas (BCF)	Ln of Mean NAgas	Sorted Mean NAgas	Partial sum Sorted	Statistic q Partial/Total
45	2107	58.30	4.0656	899.40	140396.400	0.77598
46	2206	29.60	3.3878	833.50	141229.900	0.78058
47	2207	93.10	4.5337	820.90	142050.800	0.78512
48	2212	261.60	5.5668	802.40	142853.200	0.78956
49	2301	197.50	5.2857	794.30	143647.500	0.79395
50	2403	104.60	4.6501	783.50	144431.000	0.79828
51	2504	191.40	5.2544	744.40	145175.400	0.80239
52	2701	1830.80	7.5125	737.90	145913.300	0.80647
53	2704	42.00	3.7377	724.30	146637.600	0.81047
54	2706	13.00	2.5649	679.60	147317.200	0.81423
55	2808	90.90	4.5098	651.60	147968.800	0.81783
56	2809	400.20	5.9920	639.70	148608.500	0.82137
57	2901	234.20	5.4562	573.60	149182.100	0.82454
58	2903	19.30	2.9601	563.70	149745.800	0.82765
59	2904	10.70	2.3702	548.40	150294.200	0.83068
60	2907	6.90	1.9315	539.60	150833.800	0.83367
61	3102	318.00	5.7621	532.00	151365.800	0.83661
62	3103	70.80	4.2599	530.50	151896.300	0.83954
63	3105	111.40	4.7131	517.40	152413.700	0.84240
64	3107	95.40	4.5581	511.60	152925.300	0.84523
65	3307	271.80	5.6051	487.70	153413.000	0.84792
66	3401	39.60	3.6788	473.20	153886.200	0.85054
67	3402	52.20	3.9551	459.00	154345.200	0.85307
68	3403	126.50	4.8402	447.00	154792.200	0.85554
69	3408	174.10	5.1596	444.50	155236.700	0.85800
70	3501	128.70	4.8575	424.40	155661.100	0.86035
71	3502	36.40	3.5946	409.70	156070.800	0.86261
72	3503	379.10	5.9378	407.90	156478.700	0.86487
73	3504	150.90	5.0166	402.40	156881.100	0.86709
74	3515	55.20	4.0110	400.20	157281.300	0.86930
75	3601	2905.60	7.9744	398.30	157679.600	0.87150
76	3602	365.90	5.9024	397.00	158076.600	0.87370
77	3603	3097.70	8.0384	395.80	158472.400	0.87588
78	3604	920.50	6.8249	392.80	158865.200	0.87806
79	3606	444.50	6.0970	388.60	159253.800	0.88020
80	3701	138.40	4.9301	387.40	159641.200	0.88234
81	3702	165.90	5.1114	386.70	160027.900	0.88448
82	3703	16.40	2.7973	379.10	160407.000	0.88658
83	3704	352.50	5.8651	370.20	160777.200	0.88862
84	3705	97.50	4.5799	365.90	161143.100	0.89065
85	3706	69.60	4.2428	359.50	161502.600	0.89263
86	3707	20.10	3.0007	352.50	161855.100	0.89458
87	3708	30.00	3.4012	351.10	162206.200	0.89652
88	3901	49.60	3.9040	348.20	162554.400	0.89845

Table 4. (cont.)

<i>n</i>	Play No.	Mean NAgas (BCF)	Ln of Mean NAgas	Sorted Mean NAgas	Partial sum Sorted	Statistic q Partial/Total
89	3903	359.50	5.8847	345.40	162899.800	0.90036
90	3905	21.20	3.0540	338.30	163238.100	0.90222
91	4004	388.60	5.9626	331.10	163569.200	0.90405
92	4101	34.00	3.5264	326.60	163895.800	0.90586
93	4401	3935.70	8.2778	318.00	164213.800	0.90762
94	4402	144.40	4.9726	312.60	164526.400	0.90935
95	4403	135.60	4.9097	307.70	164834.100	0.91105
96	4404	3203.50	8.0720	303.50	165137.600	0.91272
97	4406	397.00	5.9839	303.00	165440.600	0.91440
98	4407	1172.10	7.0666	287.00	165727.600	0.91598
99	4410	188.10	5.2370	271.80	165999.400	0.91749
100	4411	398.30	5.9872	270.40	166269.800	0.91898
101	4412	303.50	5.7154	268.90	166538.700	0.92047
102	4502	87.50	4.4716	261.60	166800.300	0.92191
103	4504	724.30	6.5852	259.80	167060.100	0.92335
104	4505	217.80	5.3836	257.70	167317.800	0.92477
105	4601	99.10	4.5961	253.50	167571.300	0.92617
106	4701	833.50	6.7256	245.10	167816.400	0.92753
107	4702	548.40	6.3070	242.40	168058.800	0.92887
108	4703	1029.00	6.9363	240.10	168298.900	0.93020
109	4704	326.60	5.7887	239.20	168538.100	0.93152
110	4705	2307.50	7.7439	235.10	168773.200	0.93282
111	4709	6315.30	8.7507	234.20	169007.400	0.93411
112	4710	783.50	6.6638	228.40	169235.800	0.93537
113	4717	242.40	5.4906	227.50	169463.300	0.93663
114	4718	3187.10	8.0669	224.50	169687.800	0.93787
115	4719	563.70	6.3345	219.40	169907.200	0.93909
116	4720	3024.00	8.0143	219.30	170126.500	0.94030
117	4721	103.90	4.6434	218.00	170344.500	0.94150
118	4722	3795.90	8.2417	217.80	170562.300	0.94271
119	4723	5709.80	8.6499	217.40	170779.700	0.94391
120	4724	1125.30	7.0258	215.50	170995.200	0.94510
121	4725	303.00	5.7137	213.40	171208.600	0.94628
122	4726	539.60	6.2908	211.90	171420.500	0.94745
123	4727	5164.20	8.5495	208.40	171628.900	0.94860
124	4729	3053.70	8.0241	206.20	171835.100	0.94974
125	4730	88.50	4.4830	197.50	172032.600	0.95083
126	4731	1559.40	7.3521	191.40	172224.000	0.95189
127	4732	820.90	6.7104	190.30	172414.300	0.95294
128	4733	517.40	6.2488	188.10	172602.400	0.95398
129	4734	219.30	5.3904	180.20	172782.600	0.95498
130	4735	794.30	6.6775	179.20	172961.800	0.95597
131	4736	1462.70	7.2880	177.20	173139.000	0.95695
132	4737	395.80	5.9809	174.60	173313.600	0.95791

Table 4. (cont.)

<i>n</i>	Play No.	Mean NAgas (BCF)	Ln of Mean NAgas	Sorted Mean NAgas	Partial sum Sorted	Statistic q Partial/Total
133	4738	473.20	6.1595	174.10	173487.700	0.95887
134	4739	224.50	5.4139	173.30	173661.000	0.95983
135	4740	899.40	6.8017	171.30	173832.300	0.96078
136	4741	2940.40	7.9863	165.90	173998.200	0.96170
137	4742	744.40	6.6126	163.20	174161.400	0.96260
138	4743	345.40	5.8447	155.20	174316.600	0.96346
139	4744	99.80	4.6032	152.20	174468.800	0.96430
140	4745	253.50	5.5354	151.20	174620.000	0.96513
141	4901	651.60	6.4794	150.90	174770.900	0.96597
142	4903	5259.60	8.5678	144.40	174915.300	0.96677
143	4904	287.00	5.6595	138.40	175053.700	0.96753
144	4905	240.10	5.4811	138.00	175191.700	0.96829
145	4907	111.10	4.7104	136.80	175328.500	0.96905
146	4909	530.50	6.2738	135.60	175464.100	0.96980
147	4910	639.70	6.4610	133.40	175597.500	0.97054
148	4912	960.00	6.8669	128.70	175726.200	0.97125
149	4913	259.80	5.5599	127.90	175854.100	0.97195
150	4915	245.10	5.5017	126.50	175980.600	0.97265
151	4916	424.40	6.0507	125.20	176105.800	0.97335
152	4918	994.50	6.9022	122.70	176228.500	0.97402
153	4920	1119.00	7.0202	116.40	176344.900	0.97467
154	4922	407.90	6.0110	111.80	176456.700	0.97528
155	4923	409.70	6.0154	111.40	176568.100	0.97590
156	4924	1080.50	6.9852	111.10	176679.200	0.97651
157	4926	1280.70	7.1552	107.40	176786.600	0.97711
158	4927	511.60	6.2375	104.60	176891.200	0.97769
159	4929	270.40	5.5999	104.50	176995.700	0.97826
160	4930	151.20	5.0186	103.90	177099.600	0.97884
161	4931	1017.00	6.9246	102.40	177202.000	0.97940
162	4933	1068.10	6.9736	99.80	177301.800	0.97996
163	4935	257.70	5.5518	99.70	177401.500	0.98051
164	4937	40.60	3.7038	99.10	177500.600	0.98105
165	4938	90.00	4.4998	99.10	177599.700	0.98160
166	4941	459.00	6.1291	99.10	177698.800	0.98215
167	4946	60.10	4.0960	97.50	177796.300	0.98269
168	4947	180.20	5.1941	95.90	177892.200	0.98322
169	5101	307.70	5.7291	95.40	177987.600	0.98375
170	5303	53.30	3.9759	93.10	178080.700	0.98426
171	5501	35.40	3.5667	90.90	178171.600	0.98476
172	5504	52.60	3.9627	90.00	178261.600	0.98526
173	5801	947.10	6.8534	88.50	178350.100	0.98575
174	5802	45.80	3.8243	87.50	178437.600	0.98623
175	5803	67.50	4.2121	85.30	178522.900	0.98670
176	5804	370.20	5.9140	78.40	178601.300	0.98714



Table 4. (cont.)

<i>n</i>	Play No.	Mean NAgas (BCF)	Ln of Mean NAgas	Sorted Mean NAgas	Partial sum Sorted	Statistic q Partial/Total
177	5805	21.70	3.0773	76.30	178677.600	0.98756
178	5807	20.40	3.0155	70.80	178748.400	0.98795
179	5809	173.30	5.1550	70.80	178819.200	0.98834
180	5812	2594.60	7.8612	69.60	178888.800	0.98873
181	5813	228.40	5.4311	67.50	178956.300	0.98910
182	5814	218.00	5.3845	65.70	179022.000	0.98946
183	5815	219.40	5.3909	64.10	179086.100	0.98982
184	5816	1173.70	7.0679	60.10	179146.200	0.99015
185	5817	122.70	4.8097	58.30	179204.500	0.99047
186	5818	19.90	2.9907	57.70	179262.200	0.99079
187	5819	2388.70	7.7785	57.30	179319.500	0.99111
188	5820	48.00	3.8712	56.60	179376.100	0.99142
189	5821	177.20	5.1773	55.20	179431.300	0.99173
190	5822	24.20	3.1864	55.20	179486.500	0.99203
191	5823	227.50	5.4272	53.30	179539.800	0.99233
192	5824	65.70	4.1851	52.60	179592.400	0.99262
193	5825	12.60	2.5337	52.50	179644.900	0.99291
194	5827	387.40	5.9595	52.20	179697.100	0.99319
195	5902	19.30	2.9601	50.60	179747.700	0.99347
196	5903	45.30	3.8133	49.60	179797.300	0.99375
197	6001	22.20	3.1001	49.20	179846.500	0.99402
198	6005	24.70	3.2068	48.00	179894.500	0.99429
199	6101	215.50	5.3730	46.00	179940.500	0.99454
200	6104	33.80	3.5205	45.80	179986.300	0.99479
201	6107	99.70	4.6022	45.30	180031.600	0.99504
202	6108	56.60	4.0360	44.10	180075.700	0.99529
203	6111	29.10	3.3707	42.00	180117.700	0.99552
204	6202	206.20	5.3288	40.60	180158.300	0.99574
205	6203	57.30	4.0483	39.60	180197.900	0.99596
206	6204	76.30	4.3347	36.40	180234.300	0.99616
207	6205	573.60	6.3519	35.40	180269.700	0.99636
208	6206	1066.90	6.9725	34.00	180303.700	0.99655
209	6207	11.60	2.4510	33.80	180337.500	0.99673
210	6307	171.30	5.1434	30.80	180368.300	0.99690
211	6308	99.10	4.5961	30.00	180398.300	0.99707
212	6309	802.40	6.6876	30.00	180428.300	0.99724
213	6310	46.00	3.8286	29.60	180457.900	0.99740
214	6312	1362.20	7.2169	29.10	180487.000	0.99756
215	6313	55.20	4.0110	26.80	180513.800	0.99771
216	6318	107.40	4.6766	24.70	180538.500	0.99785
217	6405	268.90	5.5943	24.40	180562.900	0.99798
218	6501	152.20	5.0252	24.20	180587.100	0.99811
219	6502	402.40	5.9974	23.20	180610.300	0.99824
220	6503	447.00	6.1026	22.20	180632.500	0.99836

Table 4. (cont.)

<i>n</i>	Play No.	Mean NAgas (BCF)	Ln of Mean NAgas	Sorted Mean NAgas	Partial sum Sorted	Statistic q Partial/Total
221	6505	52.50	3.9608	22.10	180654.600	0.99849
222	6701	95.90	4.5633	21.70	180676.300	0.99861
223	6702	213.40	5.3632	21.20	180697.500	0.99872
224	6703	208.40	5.3395	20.40	180717.900	0.99884
225	6704	217.40	5.3817	20.10	180738.000	0.99895
226	6706	111.80	4.7167	19.90	180757.900	0.99906
227	6708	138.00	4.9273	19.30	180777.200	0.99916
228	6714	64.10	4.1604	19.30	180796.500	0.99927
229	6716	44.10	3.7865	18.30	180814.800	0.99937
230	6717	99.10	4.5961	16.40	180831.200	0.99946
231	6718	70.80	4.2599	15.00	180846.200	0.99955
232	6719	104.50	4.6492	13.00	180859.200	0.99962
233	6720	338.30	5.8239	12.60	180871.800	0.99969
234	6721	50.60	3.9240	12.00	180883.800	0.99975
235	6725	57.70	4.0553	11.60	180895.400	0.99982
236	6727	127.90	4.8512	11.10	180906.500	0.99988
237	6732	239.20	5.4773	10.70	180917.200	0.99994
238	6802	23.20	3.1442	6.90	180924.100	0.99998
239	6901	348.20	5.8528	4.30	180928.400	1.00000

4.3000	1.4586	<== Min
23739.2000	10.0749	<== Max
180928.4000	1280.6593	<== Total
757.0226	5.3584	<== Mean ( $\mu$ )
2012.8771	1.5582	<== Standard deviation ( $\sigma$ )

Table 5. Spreadsheet of calculations from nonassociated-gas data of Figure 3 for the lognormal  $q$  as a function of various values of  $p$ . NORMSINV is the spreadsheet function that computes  $z$ ; NORMSDIST is the function that computes  $1 - q$ .

$p$ Pop. Prop.	$(p)(N)$ $(p)(239)$	$1 - p$	$z$ NORMSINV	$z - \sigma$ $(z - 1.5582)$	$1 - q$ NORMSDIST	lognormal $q$ (theoretical)	statistic $q$ (empirical)
0.1	23.9	0.9	1.2816	-0.2766	0.3910	0.6090	0.6406
0.2	47.8	0.8	0.8416	-0.7166	0.2368	0.7632	0.7887
0.3	71.7	0.7	0.5244	-1.0338	0.1506	0.8494	0.8642
0.4	95.6	0.6	0.2533	-1.3049	0.0960	0.9040	0.9121
0.5	119.5	0.5	0.0000	-1.5582	0.0596	0.9404	0.9445
0.6	143.4	0.4	-0.2533	-1.8115	0.0350	0.9650	0.9678
0.7	167.3	0.3	-0.5244	-2.0826	0.0186	0.9814	0.9828
0.8	191.2	0.2	-0.8416	-2.3998	0.0082	0.9918	0.9924
0.9	215.1	0.1	-1.2816	-2.8398	0.0023	0.9977	0.9977
0.05	11.95	0.95	1.6449	0.0867	0.5345	0.4655	0.4659
0.025	5.975	0.975	1.9600	0.4018	0.6561	0.3439	0.3306
0.01	2.39	0.99	2.3263	0.7681	0.7788	0.2212	0.2183

Table 6. Summary of fractal lognormal percentage assessment of conventional nonassociated-gas resources. (NA means nonassociated.)

Percentage of NA gas plays $p100\%$	Percentage of total NA gas resources $q100\%$	
	Theoretical (lognormal $q$ )	Empirical (statistic $q$ )
0	0.00	0.00
1	22.12	21.83
2.5	34.39	33.06
5	46.55	46.59
10	60.90	64.06
20	76.32	78.87
30	84.94	86.42
40	90.40	91.21
50	94.04	94.45
60	96.50	96.78
70	98.14	98.28
80	99.18	99.24
90	99.77	99.77
100	100.00	100.00

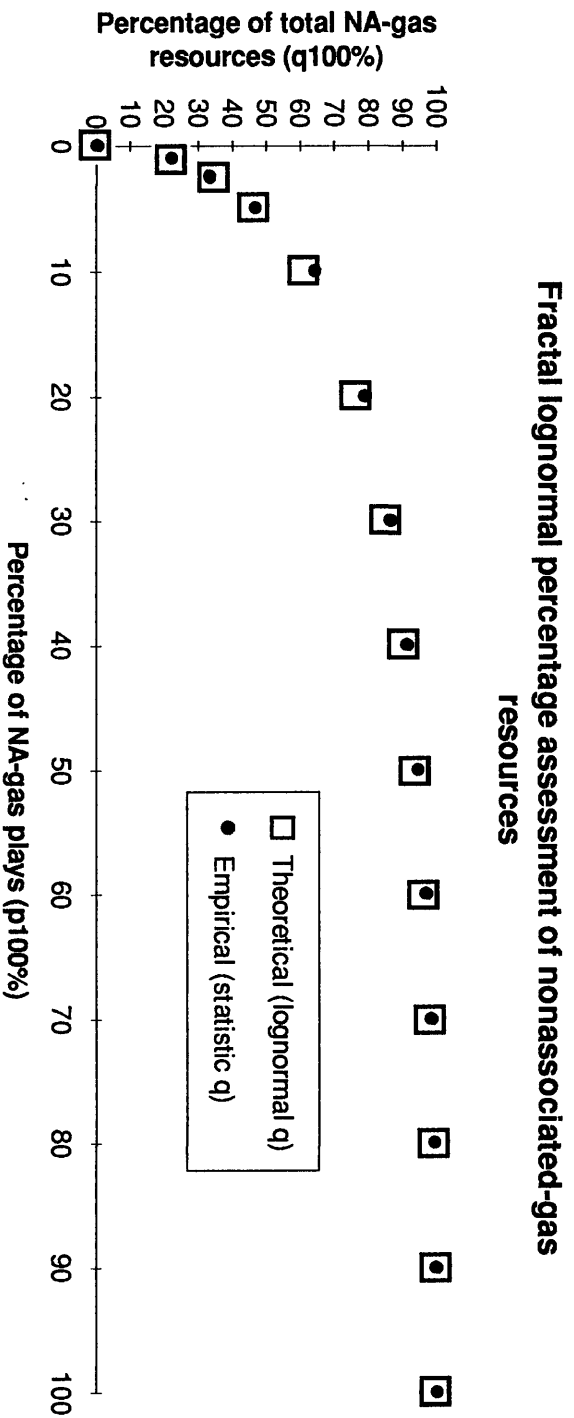


Figure 4. Theoretical percentage of total nonassociated-gas resources using the lognormal  $q$  as a function of the percentage of nonassociated-gas plays, along with the corresponding empirical values using the statistic  $q$ . Data are from Table 6. (NA means nonassociated.)

## SUMMARY AND CONCLUSIONS

1. "The 20/80 law" is generalized as the  $p100/q100$  law in probabilistic form for any probability distribution that models the population distribution. The parameter  $q$  is called the proportion of total value and defined in terms of the population proportion  $p$  and conditional expectation.
2. When the population distribution is modeled with the lognormal distribution, the  $p100/q100$  law in probabilistic form produces the  $p100/q100$  law in lognormal form. The proportion of total value is derived, and the parameter  $q$  is a function only of  $p$  and the shape parameter  $\sigma$ . The parameter  $q$  is scale-free; i.e., it does not depend upon the lognormal scale parameter  $\mu$ . Because it is scale invariant, the proportion  $q$  is a parametric fractal.
3. The  $p100/q100$  law in lognormal form can be applied to conventional petroleum resources in oil and nonassociated-gas plays in the United States. The lognormal  $q$  is a function of  $p$  for each commodity such that  $p100\%$  of the plays account for  $q100\%$  of the total resources of the plays. In both cases the theoretical percentages of total resources using the lognormal  $q$  are extremely close to the empirical percentages from the data using the statistic  $q$ . For example, 20% of the 274 oil plays account for 73.05% of the total oil resources of the plays if we use the lognormal  $q$ , or for 75.52% if we use the statistic  $q$ ; 20% of the 239 nonassociated-gas plays account for 76.32% of the total nonassociated-gas resources of the plays if we use the lognormal  $q$ , or for 78.87% if we use the statistic  $q$ .
4. This new probabilistic methodology for percentage assessment of resources lends itself as an ideal application for spreadsheet software.

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