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A statistical test for periodicity hypothesis in the crater formation rate

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SUMMARY

The hypothesis that the crater formation rate exhibits periodicity is examined by adopting a criterion proposed by Broadbent, which is more stringent than those adopted by previous authors. Data sets of Alvarez & Muller, Rampino & Stothers and of Grieve are tested. The data set of Rampino & Stothers is found to satisfy the adopted criterion for periodicity with period $P = 30$ Myr. Again, small craters ($D < 10$ km) in the data set of Grieve satisfy the criterion even better with $P = 30$ Myr and 50 Myr, but large craters do not satisfy the criterion. Removal of some of the very young craters (ages < 8 Myr) yields three significant periods, 16.5, 30 and 50 Myr. Taken at face value, the result would indicate that small impactors hit the Earth at intervals of 16.5 Myr and that this period is modulated by the galactic tide.

1 INTRODUCTION

Large craters are record of impacts of large bodies which may have brought significant strain to the terrestrial environment. Measured ages of craters provide information on the possible variation of the number of bodies in the inner solar system.

It has been claimed by a number of authors that the distribution of crater formation ages is not random but periodic. Thus, Seyfert & Sirkin (1979) divided dates of impacts into groups separated by 26 Myr, and suggested that the impacts triggered tectonic activity. Alvarez & Muller (1984) claimed to have detected a periodicity of 28.4 Myr in the records of 11 craters, while Rampino & Stothers (1986) detected a periodicity of 30 ± 1 Myr from the records of 65 craters of ages less than 600 Myr. On the other hand, Grieve *et al.* (1985) and Tremaine (1986) question the detected periodicities on various grounds. Bailey, Wilkinson & Wolfendale (1987) also reviewed the claimed periodicity and argued that such a periodicity cannot be brought about by known perturbations such as passing stars or molecular clouds. The periodicity or otherwise in the crater records has an implication regarding the structure of comet cloud which is associated with the solar system. Thus Stothers (1988), adopting the 30 Myr periodicity, argues that the active part of the cloud of comets is the observed Oort cloud with semi-major axis, a greater than 2.5×10^4 AU, while an hypothesis of a solar campaign has also been proposed (Davis, Hut & Muller 1984). Again, Clube & Napier (1982, 1984, 1986) favour capture of giant comets as the solar system periodically passes through spiral arms and/or giant molecular clouds which are concentrated near the galactic plane. Thus it is important to examine carefully whether or not the age distribution of craters exhibits periodicity.

In the present paper, a method proposed by Broadbent (1955, 1956) together with a Monte Carlo method will be applied to various data sets on the age of craters to see whether or not the periodicity hypothesis can be supported. In Section 2, a brief account will be given of the methods adopted by previous authors, and a detailed account will also be given of Broadbent's method. Then we proceed to discuss data sets provided by Alvarez & Muller (1984), Rampino & Stothers (1986) and Grieve (1987). It will be shown that the data set used by Rampino & Stothers and that of Grieve, which include small craters, support the periodicity hypothesis but that other data sets are not consistent with the periodicity hypothesis.

2 METHODS

In this section, we describe the methods adopted by the previous authors to test the periodicity hypothesis and explain the method proposed by Broadbent which will be adopted in the present work.

Let $t_1, \ldots, t_N$ be the ages ascribed to craters, and define the function

$$f(t) = \Delta(t - t_1) + \Delta(t - t_2) + \ldots \Delta(t - t_N), \tag{2.1}$$

where $\Delta(t - t_i)$ is either a rectangular function or a Gaussian function centred at t_i with width or variance appropriate to uncertainties in the age determination. Alvarez & Muller (1984) calculated the periodogram (or Fourier transform) of the function and found a peak at frequency 0.035 Myr^{-1} which corresponds to the period, 28.4 Myr; their calculation is based on 11 craters whose ages range from 14.8 to 210 Myr. This method, however, suffers from the lack of a criterion to judge whether the obtained peak is statistically significant or not.

The method adopted by Rampino & Stothers (1986) is based on Stothers (1979). If there is an exact periodicity in the series $T_1, T_2, \ldots, T_N$, there should be constants α and P such that

$$T_i = \alpha + r_i P \qquad i = 1, 2, \ldots, N, \tag{2.2}$$

where r_i is zero or an integer; in this expression P is the period and α corresponds to the latest epoch of crater formation. For a given set of α and P, one can obtain the series $T_1, T_2, \ldots, T_N$ such that $|T_i - t_i| \le P/2$. Define the quantity

$$\frac{s^2}{P^2} = \frac{1}{N} \sum_{i=1}^{N} q_i^2, \qquad q_i = (T_i - t_i)/P. \tag{2.3}$$

Clearly, s^2/P^2 is a measure of deviation of the series $t_1, \ldots, t_N$ from an exact periodicity. Stothers (1979) proposed to search a set of values of α and P such that the quantity s^2/P^2 is minimized. Adopting this method, Rampino & Stothers (1986) obtained $\alpha = 5 \pm 5$ Myr and $P = 30 \pm 1$ Myr from the 65 craters with age <600 Myr. As will be shown later in the present section, this is equivalent to the method proposed earlier by Broadbent (1955, 1956). In their work, however, a criterion different from the one proposed by Broadbent was adopted. On the other hand, Grieve *et al.* (1985) adopted a similar method but in their work, the quantity q_i in equation (2.3) is not the difference $(T_i - t_i)/P$ but the difference between two successive ages divided by an adopted value of P or, in case it exceeds 1/2, the difference minus an integer which is closest to it. They adopted ages for 26 craters and concluded that one would need evidence beyond arguments based on time-series analysis in order to establish periodicities in the rate of impacts.

Finally, we refer to the method of Broadbent, which is the one we adopt in the following. As mentioned, this method consists of calculating the quantity s^2/P^2 defined by equation (2.3). Broadbent uses $d = P/2$ in place of the assumed period, P. For a random sequence $t_1, t_2, \ldots, t_N$, the quantity s^2/d^2 has the mean 1/3 and variance $4N/45$. In other words, s^2/P^2 has the mean 1/12 and variance $N/45$. Hence, a criterion for periodicity is that s^2/P^2 be significantly less than 1/12. Broadbent, however, shows that this criterion is not very significant. When s^2/P^2 (or s^2/d^2) is plotted against d, for a series of random numbers $t_1, t_2, \ldots, t_N$, s^2/P^2 is a violently oscillating function and s^2/P^2 takes on values far less than 1/12 for several values of $1/d$. Thus, he proposes the criterion

$$\sqrt{N}\left(\frac{1}{3} - \frac{s^2}{d^2}\right) > 1 \tag{2.4}$$

as the condition to be satisfied for the periodicity hypothesis to be accepted as statistically significant.

For randomly distributed $t_1, t_2, t_3, \ldots, t_N$, the inequality (2.4) is satisfied once in more than 1000 trials, and it is more stringent than in ordinary tests. However, because the period, P and the epoch α are derived from the data, one would need to have such a stringent condition. The 5, 1 and 0.1 per cent points of the left-hand side of the inequality are 0.49, 0.69 and 0.92, respectively. This criterion was derived theoretically but it is based on a large number of numerical experiments and in this sense, it is somewhat empirical. He applied the method to judge whether the energy levels of some atomic nuclei are quantized (multiples of a certain quantum). This criterion holds good for $N > 10$. For smaller N, it is not possible to substantiate a periodicity by statistical argument, as might be expected. In the present paper, we will adopt the above inequality to test the periodicity.

3 DATA SETS

In order to test the claimed periodicity in the rate of crater formation, it is necessary to define data sets on crater ages, since the results of the tests may depend on which set of data are used. In this work, we adopt three sets of data. First, we consider the data set of Alvarez & Muller (1984), since the periodicity was first claimed quantitatively by them. The data consists of 11 craters whose ages were supposedly accurately known. Then we consider the data set given, as in Table 1, by Rampino & Stothers (1986). The data consist of 65 craters of ages less than 600 Myr. Finally, we adopt the data set of Grieve (1987), which consists of 102 craters. The data set is larger than those of Alvarez & Muller and of Rampino & Stothers and contains revised age determinations of those craters adopted by the earlier authors.

4 RESULTS

4.1 Alvarez & Muller data set

The Alvarez–Muller data set of 11 craters has been tested by calculating the quantity s^2/P^2 as defined by equation (2.3). As Grieve *et al.* (1985) and Broadbent (1956) state, this quantity can be made as small as one wishes by taking small values of P, because the data set largely consists of ages which are represented by integral multiples of Myr. Hence, in minimizing s^2/P^2 when plotted against P, a lower bound on P ought to be specified. Again, a period greater than say, 100 Myr is not to be expected, because the data set is apparently not consistent with such a period. By calculating s^2/P^2 for various values of $P > 1$ Myr, a minimum value of s^2/P^2 has been found at $s^2/P^2 = 0.0222$ (or $s/P = 0.149$) for $P = 28$ Myr and $\alpha = 12$ Myr. On the criterion adopted by Grieve *et al.* (1985) that s/P be less than $0.29 - 3 \times$ standard deviation $= 0.29 - 3 \times 0.13\, N^{-1/2}$ ($= 0.17$) for the periodicity hypothesis to be statistically significant, the calculated value of s/P satisfies the criterion.

However, if one applies the criterion proposed by Broadbent (1956), one obtains an opposite conclusion: the left-hand side of the inequality (2.4) equals

$$\sqrt{N}\left(\frac{1}{3} - \frac{s^2}{d^2}\right) = 0.811$$

and is less than unity, contrary to the criterion.

In order to see whether the obtained value $s^2/P^2 = 0.0222$ is significant or otherwise by another means, a Monte Carlo

Table 1. Data sets to be used for testing periodicity hypothesis.

Data set	No. of craters	Age
Alvarez&Muller (1984)	11	0∼ 240*Myr*
Rampino&Stothers (1986)	65	0∼600
Grieve (1987)	102	0∼1970

calculation similar to that described by Tremaine (1986) has been carried out. For a random set of values $t_1, \ldots, t_{11}$, the quantity s^2/P^2 is calculated for various values of the epoch α and period P, and a minimum value of s^2/P^2 is obtained. For another set of random values $t_1, t_2, \ldots, t_{11}$, the corresponding minimum of s^2/P^2 is calculated. One hundred samples have been done to see how frequently the quantity s^2/P^2 takes on a value less than 0.0222. Of the 100 cases, 11 cases have been found where $s^2/P^2 < 0.0222$. In other words, the s^2/P^2 takes on as small as 0.0222 or less in 11 out of 100 cases where the sequence $t_1, t_2, \ldots, t_{11}$ is randomly generated. So, the data set used by Alvarez & Muller to claim a periodicity is not statistically significant, as indeed it appears at a glance.

4.2 Rampino & Stothers data set

Rampino & Stothers (1986) adopted 65 craters with ages less than 600 Myr and obtained a periodicity with $P = 32 \pm 1$ Myr and $\alpha = 5 \pm 4$ Myr. We repeated the analysis and obtained the same result. The minimum value of s^2/P^2 is found to be 0.0493 (or $s/P = 0.222$). If one applies the criterion of Broadbent (1956), the left-hand side of the inequality (2.4) is found to be

$$\sqrt{N}\left(\frac{1}{3} - \frac{s^2}{d^2}\right) = 1.098 \text{ for } \alpha = 4.5 \text{ Myr}$$

which is greater than unity. The criterion is satisfied for $\alpha = 4.5 \pm 2$ Myr. The data set of Rampino & Stothers can thus be used as an argument for periodicity in the crater ages. Some comments are necessary regarding the choice of α. s^2/P^2 is not very sensitive to changes in the value of α and this gives rise to some uncertainty in the value of α which satisfies the adopted criterion. For this data set, another minimum has been found at $P = 50$ Myr and $\alpha = 4 \pm 4$ Myr, which satisfies the Broadbent criterion. This is ascribed by Rampino & Stothers (1986) to the fact that many of the ages are expressed in multiples of 50 Myr. If so, the derived period, $P = 50$ Myr would be an artefact. The same period is, however, derived from the data set of Grieve (1987), and we postpone the discussion thereof until Section 5.

In the present discussion, the 65 craters have been treated with equal weight. If, however, one divides the craters according to their ages, 44 have ages ≤250 Myr. This means that young craters are better preserved than older ones. It seemed worthwhile, therefore, to consider the young craters separately. In Fig. 1 s/P is plotted against P for the 44 craters. The minimum value of s^2/P^2 ($s/P = 0.191$) for the 44 craters is found to be 0.0365 for $P = 31$ Myr and $\alpha = 5$ Myr. When one applies the Broadbent criterion it is found that

$$\sqrt{N}\left(\frac{1}{3} - \frac{s^2}{d^2}\right) = 1.242$$

so that the criterion is amply satisfied. The adopted criterion is satisfied for $P = 31$ Myr and $\alpha = 5 \pm 3$ Myr. Thus the data set adopted by Rampino & Stothers exhibits periodicity whether the craters are young or old.

4.3 Grieve data set

As stated, the data set of Grieve (1987) contains 102 craters with ages ≤1970 Myr. For some craters, only upper limits are given to their ages. In the present analysis, we adopt the upper limit as the age of a crater, where it is given.

Of the 102 craters, 61 have diameters less than 10 km. Bearing in mind that large craters have higher probability of preservation, statistical tests for periodicity have first been carried out for the craters with diameters ≥ 10 km.

For the 41 craters with $D \geq 10$ km, minima of s^2/P^2 have been found for which

$$P = 31 \text{ Myr} \quad \alpha = 3 \pm 1 \text{ Myr} \quad s^2/P^2 = 0.0676$$

$$P = 50 \text{ Myr} \quad \alpha = 5 \pm 1 \text{ Myr} \quad s^2/P^2 = 0.0562.$$

It is interesting to note that an assumed period $P = 50$ Myr gives a lower value of s^2/P^2 than the period $P = 31$ Myr. Further,

$$\sqrt{N}\left(\frac{1}{3} - \frac{s^2}{d^2}\right) = 0.403 \quad \text{for } P = 31 \text{ Myr},$$

$$= 0.695 \quad \text{for } P = 50 \text{ Myr}.$$

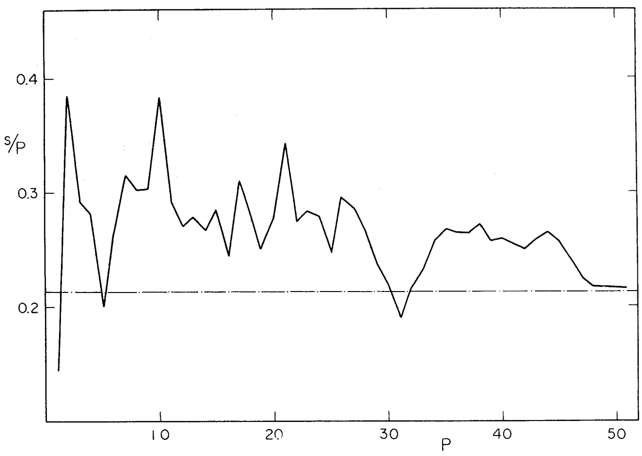

Figure 1. s/P is plotted against period, P in Myr. The horizontal line gives the level below which the Broadbent criterion is satisfied. s/P is calculated from the data set of Rampino & Stothers (1986). Note that the criterion is satisfied at $P = 31$ Myr.

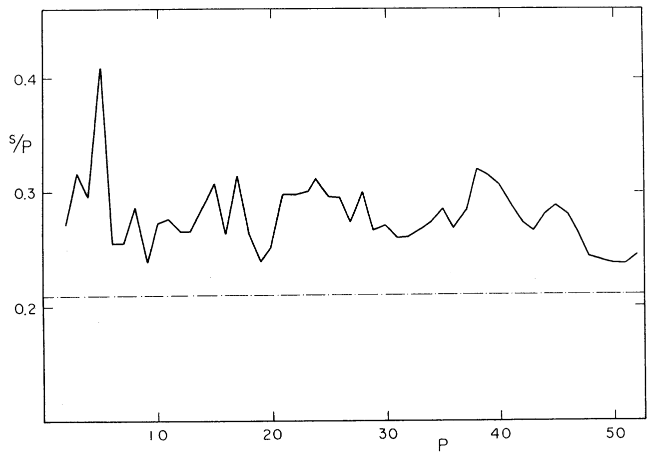

Figure 2. s/P is plotted against P for craters in the data set of Grieve (1987). Large craters ($D \geq 10$ km and ages less than 600 Myr) are used to calculate s/P. Note that the criterion for periodicity is not satisfied for conceivable values of P.

In other words, the Broadbent criterion is not satisfied for $P=31$ and 50 Myr periodicities (see Fig. 2).

Next, we consider craters with ages less than 240 Myr. There are 30 such craters with $D \geq 10$ km. Three interesting minima of s^2/P^2 have been found;

$P=16$ Myr $\alpha=4$ Myr $s^2/P^2=0.0581$,

$P=31$ Myr $\alpha=4$ Myr $s^2/P^2=0.0620$,

$P=53$ Myr $\alpha=4 \pm 2$ Myr $s^2/P^2=0.0502$.

Clearly, the Broadbent criterion is not satisfied for these periods, because the minimal values of s^2/P^2 are too large. Finally, we consider 98 craters with ages ≤ 600 Myr regardless of diameters. The following minimum has been obtained;

$P=30$ Myr $\alpha=2$ Myr $s^2/P^2=0.0552$.

For this minimum, we have

$$\sqrt{N}\left(\frac{1}{3}-\frac{s^2}{d^2}\right)=1.114$$

so that the Broadbent criterion is amply satisfied. The result is rather surprising, because Stothers (1988) and others argued that large craters show periodicity in ages. Thus, one is led to carry out a similar calculation for craters such that are ≤ 600 Myr and $D<10$ km. There are 57 such craters. For these craters, a minimum of s^2/P^2 has been found such that

$P=30$ Myr $\alpha=1.5 \sim 2$ Myr $s^2/P^2=0.0424$

and that

$$\sqrt{N}\left(\frac{1}{3}-\frac{s^2}{d^2}\right)=1.23.$$

There is a similar minimum at $P=50$ Myr (see Fig. 3). Finally, the data set such that ages ≤ 240 Myr and $D<10$ km has been investigated. For such craters, it has been found that $N=33$,

$P=29$ Myr $\alpha=3.5 \sim 4.5$ Myr $s^2/P^2=0.0380$

$P=52$ Myr $\alpha=2.0 \sim 3.0$ Myr $s^2/P^2=0.0361$

and that

$$\sqrt{N}\left(\frac{1}{3}-\frac{s^2}{d^2}\right)=1.04 \text{ and } 1.08.$$

A less conspicuous minimum has also been found for $P=17$ Myr and $\alpha=1$ Myr.

To summarize, small craters satisfy the adopted criterion for periodicity better than the larger ones ($D \geq 10$ km). However, there is an additional period $P=50$ Myr which satisfies the adopted criterion equally as well as period $P=30$ Myr. A similar test has been done by excluding such craters that their age determinations have uncertainties greater than 20 Myr. The result obtained is very similar to the ones obtained above. For those with $D \leq 10$ km, two significant periods have been obtained at 30 and 50 Myr, while for $D>10$ km, statistically significant periods have not been obtained.

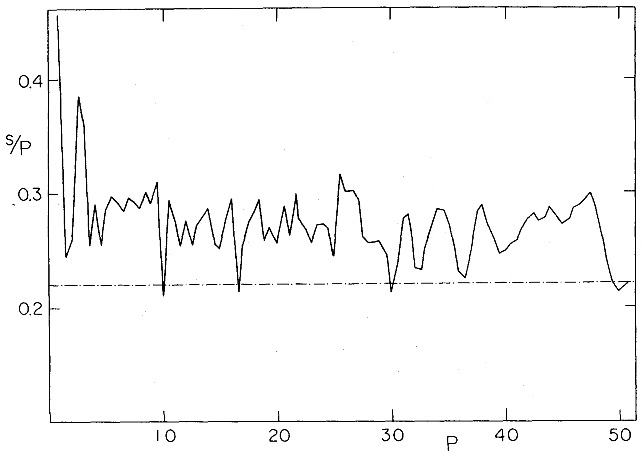

Figure 3. s/P is plotted against P for craters in the data set of Grieve (1987). Small craters ($D<10$ km and ages less than 600 Myr) are taken into account and four craters with ages ≤ 8 Myr are removed to suppress the effect of very young craters. Note that the Broadbent criterion is satisfied at three periods, $P=16.5$, 30 and 50 Myr.

It is conceivable that the values of α obtained (the latest epoch of impacts) may be due to the fact that there are many young craters. In the data set of Grieve (1987), there are 13 craters with ages ≤ 8 Myr. In order to remove any possible effect due to this situation, we have removed four craters with $D<10$ km and ages ≤ 8 Myr. For this data set, s/P is plotted against P in Fig. 3. Four significant periods are seen. These are 10, 16.5, 30 and 50 Myr for $\alpha=2 \pm 2$ Myr. The period $P=10$ Myr may be ascribed to the artefact of many ages being expressed as multiples of 10 Myr. Thus, not only is the value of α not much changed, but a period $P=16.5$ Myr has been detected. A similar calculation has been done by removing four more craters with ages ≤ 8 Myr but the overall result remains unaltered. Thus, the suppression of the effect of very young craters not only leaves the value of α unaltered, but brings about a significant period, $P=16.5$ Myr.

To summarize, small craters ($D<10$ km) in the data set of Grieve (1987) satisfy the Broadbent criterion for periodicity. Significant periods are $P=30$ and 50 Myr. When the effect of very young craters is suppressed, another period, $P=16.5$ Myr is detected.

5 CONCLUSIONS AND DISCUSSION

A criterion proposed by Broadbent (1986) has been adopted to test the hypothesis of periodicity in the age distributions of craters. Data sets of Alvarez & Muller (1984), Rampino & Stothers (1986) and of Grieve (1987) have been tested. The data sets of Rampino & Stothers as well as the craters compiled by Grieve with ages $t \leq 600$ Myr have been found to satisfy the criterion of Broadbent. The epoch of the latest bombardment has been found at 2 ± 2 Myr BP and this does not seem to depend much on the existence or otherwise of many craters with ages ≤ 8 Myr. The present investigation, thus, confirms the claim of Rampino & Stothers (1986), although, periodicity for craters with $D \geq 10$ km has not been

Table 2. Data sets and the results of tests for periodicity.

Data set		No. of craters	Derived period	Broadbent criterion
Alvarez&Muller(1984)		11	28Myr	not satisfied
Rampino&Stothers(1986)	all	65	32±2, 50	satisfied
	ages≤ 250Myr	44	31	satisfied
Grieve(1987)	D≥10km, age≤ 600Myr	41	50	not satisfied
	D≥10km, age≤ 240Myr	30	53	not satisfied
	age≤ 600Myr	98	30, 52	satisfied
	D<10km, age≤ 600Myr	57	30, 50	satisfied
	D<10km, ages≤ 600Myr, 4 craters with ages ≤ 8Myr removed	53	16.5, 30, 50	satisfied
	D<10km, age≤ 240Myr	33	29, 52	satisfied

confirmed. The results are summarized in Table 2. Keeping in mind that the adopted criterion is a rather stringent one, the results obtained here appear impressive. The set of data for 25 craters compiled by Shoemaker & Wolfe (1986) has also been tested, with negative result. For the data set of Grieve, three periods have been obtained for small craters which equally satisfy the adopted criterion for the periodicity hypothesis.

It seems therefore difficult to draw a definite conclusion regarding the periodicity hypothesis for the crater formation rate, because, whether the hypothesis is substantiated by the adopted criterion or not depends on which data set is used. If the data set of Rampino & Stothers (1986) is accepted, one could argue that $P=30$ Myr is the significant period. If only one period $P=30$ Myr had been derived from the data set of Grieve (1987), it would have been reasonable to regard the hypothesis as statistically significant. Unfortunately, three periods have been detected and a unique interpretation appears difficult. Therefore, it seems still premature to draw a definite conclusion on the hypothesis. In order to draw a definite conclusion, it is required to compile a data set which represents unbiased records of craters.

Nevertheless, if one accepts at face value the result obtained here, it would seem possible to argue that the periods $P=30$ and 50 Myr are the multiples of the fundamental period, $P=16.5$ Myr. Clube (1986) and Napier (1987) have noted this period ($P=15$ Myr) in the records of geomagnetic reversals and in the crater data of Shoemaker & Wolfe (1986), although no significant period has been detected here for the latter. The result indicates that small impactors hit the Earth at intervals of 16 Myr as might be expected through the disintegration of giant comets (Clube & Napier 1984), and that this process itself is modulated by the tidal force of the galactic disc in which matter is concentrated towards the galactic mid-plane, whereas large impactors hit randomly.

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