

U. S. AIR FORCE
PROJECT RAND
RESEARCH MEMORANDUM

A STATISTICAL THEORY OF TARGET DETECTION
BY PULSED RADAR

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RM-754

ASTIA Document Number AD 101287

1 December 1947
Reissued 25 April 1952

Assigned to _____

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SUMMARY

This report presents data from which one may obtain the probability that a pulsed-type radar system will detect a given target at any range. This is in contrast to the usual method of obtaining radar range as a single number, which can be taken mathematically to imply that the probability of detection is zero at any range greater than this number, and certainty at any range less than this number.

Three variables, which have so far received little quantitative attention in the subject of radar range, are introduced in the theory:

1. The time taken to detect the target.
2. The average time interval between false alarms (false indications of targets).
3. The number of pulses integrated.

It is shown briefly how the results for pulsed-type systems may be applied in the case of continuous-wave systems.

Those concerned with systems analysis problems including radar performance may profitably use this work as one link in a chain involving several probabilities. For instance, the probability that a given aircraft will be detected at least once while flying any given path through a specified model radar network may be calculated using the data presented here as a basis, provided that additional probability data on such things as outage time etc., are available.

The theory developed here does not take account of interference such as clutter or man-made static, but assumes only random noise at the receiver input. Also, an ideal type of electronic integrator and detector are assumed. Thus the results are the best that can be obtained under ideal conditions. It is not too difficult, however, to make reasonable assumptions which will permit application of the results to the currently available types of radar.

The first part of this report is a restatement of known radar fundamentals and supplies continuity and background for what follows.

The mathematical part of the theory is not contained herein, but will be issued subsequently as a separate report⁽²³⁾.

SYMBOLS

- A_e = effective area of antenna for receiving.
 B = beamwidth of antenna.
 c = velocity of light.
 C = total shunt capacity of input circuit.
 δ = factor which accounts for losses in transmission lines, T R switches, atmospheric absorption, etc.
 e = rms noise voltage.
 E_p = transmitted energy per pulse.
 E_R = received energy per pulse.
 f_r = pulse repetition frequency.
 f_{sc} = scanning frequency.
 Δf = bandwidth for noise purposes.
 $\Delta f'$ = input circuit bandwidth.
 Δf_{cw} = combined R F and I F bandwidth of continuous-wave-system receiver.
 F = bandwidth multiplying factor = 1 for simple L C circuit.
 γ = number of pulses received during detection time.
 γ' = γ/N
 g_m = mutual conductance of first receiver tube.
 G = gain of transmitting antenna.
 h_r = height of radar antenna.
 h_t = target height.
 $I_0(z)$ = modified Bessel function of the first kind.
 k = Boltzmann's constant.
 λ = wave length of transmitter.
 L = sweep length in miles.
 n = $\tau_{fa} \cdot f_r \cdot \eta$
 n' = n/N
 η = number of pulse intervals per sweep.
 η' = number of separate velocity channels in continuous-wave-system receiver.
 N = number of pulses integrated, or, in cw system, the number of independent variates (of length $1/\Delta f_{cw}$) integrated.
 N_{sc} = number of pulses per scan.
 NF = overall noise figure of the receiver.

P_N = probability that N pulses of noise will exceed a given level.
 P = probability that N pulses of signal plus noise will exceed the bias level.
 P' = probability that at least one group of N integrated pulses will exceed the bias level within the detection time.
 P_{av} = average power.
 P_t = transmitted power.
 P_{min} = minimum detectable power at receiver.
 p = effective input noise power to receiver.
 r = resistance.
 R = radar range.
 R_{max} = maximum radar range.
 R_0 = idealized radar range.
 R_{eq} = equivalent noise resistance of first receiver stage.
 R_1 = total shunt resistance of first receiver input circuit.
 ΔR = range interval for integration with a moving target.
 σ = scattering cross-sectional area of target.
 τ_p = pulse length.
 τ_{fa} = false alarm interval.
 τ_d = detection time.
 τ_i = maximum integration time for moving target.
 T = absolute temperature.
 T_a = absolute temperature of space radiation received by antenna.
 T_R = absolute temperature of room.
 v = velocity of the target.
 v_g = velocity of traveling gate.
 V = visibility factor of receiver.
 ω = angular velocity of antenna.
 x = received signal pulse energy in units of $k T_R \overline{NF}$.
 y = noise level in units of the rms value of noise — the bias level.

A STATISTICAL THEORY OF TARGET DETECTION BY PULSED RADAR

PART I - INTRODUCTION

THE USUAL RADAR RANGE EQUATION

Most radar engineers are now well acquainted with the following equation used to determine the maximum range of a pulsed radar system:

$$R_{\max} = \left[\frac{P_t}{P_{\min}} \frac{GA_e \sigma \delta}{16 \pi^2} \right]^{\frac{1}{4}} \quad (1)$$

where

- P_t = peak transmitted power in watts.
- P_{\min} = minimum peak detectable signal in watts.
- σ = scattering cross section of target in units consistent with range.
- G = gain of transmitting antenna.
- A_e = effective area of antenna for receiving in units consistent with range (usually about 2/3 of the physical aperture, $A_e = G\lambda^2/4\pi$).
- δ = a dimensionless loss factor which accounts for atmospheric absorption, losses in antenna and transmission lines, etc.

The number of pitfalls that may be encountered in the use of the above equation are almost without limit, and many of these difficulties have been recognized in the past^{(3), (18)}. Three of the most troublesome are:

1. The Scattering Cross Section

In the case of moving targets, the wide variation of this quantity with aspect, and hence with time, is a matter of vital concern. The variation of cross section as a function of frequency may also be critical.

2. The Minimum Detectable Signal, P_{\min}

The statistical nature of the noise with which P_{\min} must compete makes this an ill-defined quantity.

3. The Maximum Range, R_{\max}

The statistical nature of P_{\min} in turn makes R_{\max} a statistical quantity.

There are also lesser troubles, such as the dependence of δ , the loss constant on the range, and the contribution of reflections from the ground, sea, or other objects to the incident and received powers. (One must also remember that a target cannot ordinarily be detected at ranges (in miles) much greater than $\sqrt{2h_r} + \sqrt{2h_t}$, where h_r is the height of the radar antenna and h_t the height of the target in feet, except in the case of superrefraction, or "ducts." See pp. 55-58, Ref.(18). This is the familiar "line of sight" limitation due to the earth's curvature.

THE SCATTERING CROSS SECTION OF THE TARGET

For a stationary radar observing a stationary target, the scattering cross section is a constant. Although it may not be calculated for any but the most simple target shapes, it is not too difficult to measure. On the other hand, if either the radar or the target is in motion, the cross section becomes a function of time causing the return signal strength to fluctuate. In general, the plot of cross section as a function of angle for a complex target such as an aircraft shows two interesting features. There is a nearly continuous rapid fluctuation having an angular period in the neighborhood of a degree or so (for λ in the microwave region), and a slow variation with a period in the order of 20° or more. Both of these variations may be as great as 30 db. The question at once arises: In lieu of using the complete polar diagram of cross section vs. angle, what kind of average figure can be used, and under what conditions? The answer to this question involves such things as angular rates of the aircraft with respect to the radar, correlation times, repetition rate of the radar, and number of pulses integrated. It is almost obvious that the only general way to treat this complex problem is to consider the cross section as a statistical variable. This approach seems mathematically feasible. However, in the present report the cross section will be considered to be a constant. An attempt to justify this assumption is the following: The rapidly fluctuating correlation angle at half-power points is perhaps 0.1° . The normal variation in attitude angle of an aircraft may be about 30° per second. (This variation may be caused by small rapid changes in pitch or roll due to normal turbulence of the air as well as by systematic changes in position.) Thus, the corresponding correlation time for σ is around 1/300 second. If the observation time is essentially greater than this period, it may be assumed, as a first approximation, that the rapid fluctuations in the cross section "average out."

The slow variations (period around 20°) may or may not average out. However, if the average over all *likely* attitudes is used for σ , or to be more exact, if a weighted average is taken for σ according to the probability for any attitude, then the probability of detection may not be changed very much. Henceforth, in this report σ will be assumed to be a constant, on the basis of the above statements. It may be mentioned in passing that σ loses its meaning if the target is not uniformly illuminated. Such can be the case, for example, if waves reaching the target via two or more paths combine to produce an interference pattern at the target. This effect exists in the detection of ships by surface radar.

THE MINIMUM DETECTABLE SIGNAL

As is well known,^(1,2,3) the minimum detectable signal power in a radar receiver is fundamentally limited by three main factors; i.e., Johnson noise in circuit elements of the input circuits, shot effect and other noise in the first tube (and to some small extent succeeding tubes), and cosmic noise picked up by the antenna. There may also be man-made interference such as engine noise, radiations from other radars and radio transmitters, etc. Clutter caused by sea return, rain, clouds, land masses, etc., may reduce the minimum detectable signal by a considerable amount. The effects of clutter and man-made interference are complete subjects in themselves,⁽¹⁹⁾ and will not be treated further in this paper. A study will be made here of radar range in the absence of such interference. It is not too optimistic to suppose that circuits will eventually be designed which will largely eliminate man-made interference, and most types of clutter.

The mean squared noise voltage across a resistor of resistance r is given by

$$e^2 = 4kTr \Delta f \quad (2)$$

where

- k = Boltzmann's constant = 1.38×10^{-23} joules/degree
- T = absolute temperature of the resistor
- Δf = the frequency interval under consideration.

Though the noise at the input circuit of a receiver is usually several times this value, it provides a convenient scale for measuring the input noise. The effective input noise power is defined to be

$$p = kT_R \Delta f \overline{NF} \quad (3)$$

where \overline{NF} is the so-called noise figure of the receiver, and T_R is the absolute room temperature.* If a signal power of the same value as p were incident on the antenna and the receiver were noiseless, then the output would be the same as in the case when noise only was present.

At this point, one important result concerning the noise figure due to Herold⁽¹⁾ is pertinent:

$$\overline{NF} = \frac{T_a}{T_R} + \frac{2\pi \Delta f' R_{eq} C}{F} + \frac{f(R_1)}{-0 \text{ as } R_1 \rightarrow \infty} \quad (4)$$

where

- T_a = absolute temperature of space radiation received by the antenna.
- T_R = room temperature.

* Complete discussions and derivations will be found in the Mathematical Appendix (a separate report).⁽²³⁾

- $\Delta f'$ = bandwidth of input circuit.
 C = total shunt capacity of input circuit.
 R_{eq} = equivalent noise resistance at input (due mainly to shot noise in first tube) $\approx 2.5/g_m$ for triodes.
 F = a factor depending on the exact type of input circuit coupling (≈ 1 for simple tuned circuit).
 R_1 = input shunt resistance including effect of finite input resistance of tube.
 $f(R_1)$ = a function of R_1 , R_{eq} , C and $\Delta f'$.

This formula assumes a more or less conventional type of input tubes, such as the V H F triodes and pentodes. However, it seems reasonable to believe that the general conclusions which are reached from Eq.(4) will apply to velocity-modulated input tubes as well.

The main points to be noted about Eq.(4) are these:

1. $f(R_1)$ approaches zero as R_1 approaches infinity. R_1 may be increased by better tube design.
2. C/g_m should be made as small as possible in a tube used as the first amplifier.
3. Long pulses tend to allow smaller bandwidths for the input circuit, and hence lower noise figures.
4. If $R_{eq} C \Delta f'$ is made small enough, and R_1 large enough, the noise figure will approach T_a/T_R .

Point 4 is of the *greatest importance*. It sets a limit on the noise figure when there are no sources of noise in the receiver itself. Though such a receiver will never be built in practice, it may be possible some day to approach closely this ideal state. Then the input noise will be almost *entirely* dependent on the temperature of space;* or, in other words, on the noise received by the antenna from without the radar set. That this state of affairs is not yet at hand is evidenced by the fact that at present the noise figure for microwave receivers is around 10, and for longer-wave receivers perhaps as low as 3 or 4.

The concept, often stated, that the ideal noise figure of a receiver is 1.0 is erroneous.** This would be true only if the temperature of space were the same as room temperature. Actually the temperature of space decreases rapidly with decreasing wave length. (3)

* Though the noise figure can be decreased by increasing T_R , this would increase the actual input noise, as is apparent from Eq. (3).

** The noise figure of a receiver may be defined in such a way that the antenna must be replaced by a resistor at room temperature equal to the radiation resistance of the antenna. In this case the ideal noise figure of the receiver would be 1.0.

The average space temperature* is around room temperature at 180 megacycles and drops to around 30° absolute at 450 megacycles.⁽¹²⁾ No good measurements are available in the microwave region, but there is reason to believe that values of 10° or lower may be found. If this proves to be true, then it is conceivable that the noise figure of future microwave receivers may be improved by a factor of 100, which would mean that the range of radar sets could be more than tripled as a consequence of this one factor. It is certainly a field where research should be pushed to the utmost.

It has often been the practice to calculate the maximum range of a radar set from (1) by assuming that $P_{\min} = kT_R \Delta f \overline{NF}$, or that the minimum detectable signal power is just equal to the average noise power.

This gives

$$R_{\max} = \left[\frac{P_t G A_e \sigma \delta}{16\pi^2 k T_R \Delta f \overline{NF}} \right]^{\frac{1}{4}} \quad (5)$$

Now the energy per pulse is represented by

$$E_p = P_t \tau_p \quad (6)$$

where τ_p is the pulse length. Making this substitution in (5) gives

$$R_{\max} = \left[\frac{E_p G A_e \sigma \delta}{16\pi^2 k T_R \overline{NF}} \right]^{\frac{1}{4}} \cdot \frac{1}{(\tau_p \Delta f)^{\frac{1}{4}}} \quad (7)$$

It is usually said that if $\tau_p \Delta f$ is made equal to 1, the amplitude of the pulse after passing through the amplifier will not differ much from the amplitude which would result if the pulse were infinitely long. Without further ado, $\tau_p \Delta f$ is put equal to 1, and the resultant equation

$$R_{\max} = \left[\frac{E_p G A_e \sigma \delta}{16\pi^2 k T_R \overline{NF}} \right]^{\frac{1}{4}} \quad (8)$$

emerges as the radar range equation. Now the unfortunate fact (in some respects) is that the range of a radar set calculated by means of this formula often turns out to be rather close to the experimental range. Naturally, under these circumstances great effort has not been expended in investigating the validity of radar-range equations.

* There is a variation of the space temperature with direction⁽¹²⁾. When the antenna points near the horizon, the temperature may be higher than when it is pointed at the zenith. In particular, if any appreciable part of the radiation strikes the ground, the thermal radiation received from those directions will have a temperature nearly equal to the actual temperature of the surroundings.

The reasons for the agreement of equation (8) with experiment are many. First of all, the cross section has been, in most cases, determined by observing the maximum range of a particular target and solving equation (8) for σ . This one fact alone accounts in no small way for the agreement. Secondly, the fourth power law makes R_{\max} rather insensitive to changes in the various parameters concerned in equation (8). A much fairer test is to compare respective values of R_{\max}^4 rather than R_{\max} .

Equation (8) is in no sense perfect with regard to its agreement with experiment. Errors of as much as $\pm 30\%$ are common, and factors of 2 can often be found. However, considering all the unknown factors present in an experimental determination of maximum range with an operational radar set, this agreement is considered to be quite good.

In any field of science, theoretical equations are deduced to explain observed data. However, one is very cautious in using these equations to predict results for other experiments where the values of many of the variables differ greatly from those used in the particular experiments already performed. Most of the radar sets built to date have operated within essentially narrow limits as far as some of the parameters are concerned. Particular examples are pulse repetition frequency, and, *most important*, the number of pulses integrated. This latter quantity is not even mentioned in equation (8); but, as will be seen in the next section, it is of vital importance.

The task is now two-fold:

1. To make a satisfactory statistical definition of the range of a radar system.
2. To determine the dependence of this quantity on the parameters of a (pulsed) radar system.

PART II

THE STATISTICAL PROBLEM OF THE MINIMUM DETECTABLE SIGNAL AND THE MAXIMUM RANGE

GENERAL BACKGROUND

It has been realized by many workers in the field* that the range of a radar set is a statistical variable and must be stated in terms of probabilities rather than in the exact terms of an equation such as (8). However, the evolution of a practical working theory does not seem to have been accomplished so far. The following work is a first step in that direction.

Before beginning the explanation of equations and derivations, it will be well to glance at some of the new ideas which will be included.

The random noise, which limits the range, can at intervals assume large values due to its statistical nature. This means that there will inevitably be times when a random fluctuation of the noise will be mistaken for a signal. The average interval at which such undesirable events take place will be called the false alarm time, and it will be found that the probability of detecting a target will be a function of this time. Let the reader at once be cautioned against thinking, "If it were a noise flash, I can easily tell by looking a little later. If it were a signal, it will still be there; if it were noise, it will be gone."

The second new parameter which will be introduced is the detection time. It is apparent that if an observer can spend sufficient time in deciding whether or not a target is present on an oscilloscope screen, the probability of a correct decision being reached will be increased. It is also obvious that in any practical situation in which radar is used one cannot take unlimited time to decide whether or not a target is present. To put things on a quantitative basis, the time in which a decision shall be rendered must be specified. In this event, there will not always be time for the "second look" just mentioned; but should time permit, then the probability of detecting a target will be increased at the expense of a longer detection time. Even so, there will still be a *certain lesser probability* that the noise flashes will occur on *both occasions*. Further, it will be found that the velocity of a moving target has an appreciable effect on the detection probability, due to the fact that the signal from such a target does not "remain stationary" (see page 16).

* For an excellent qualitative statement of the problem, see Radiation Laboratory Series No.1. pp.35-47, Ref.(18).

PRELIMINARY STEPS

It is desirable to present data in the most compact form, and the first step in this direction is the elimination of the necessity for the appearance of such parameters as E_p , G , A_e , σ , δ , and \overline{NF} in the final results. To this end, a parameter R_0 is defined which is given by a slight modification of Eq. (7), as follows:

$$R = \left[\frac{E_p G A_e \sigma \delta V}{16\pi^2 E_R} \right]^{1/4}, \quad R_0 = \left[\frac{E_p G A_e \sigma \delta V}{16\pi^2 k T_R \overline{NF}} \right]^{1/4} \quad (9)$$

Here, the factor $1/\tau_p \Delta f$ has been replaced by V , the so-called visibility factor.* This factor will always be less than 1 but usually not less than 0.8, except when the Doppler effect is very large. R_0 will be called the "idealized range" for lack of a better term.

Now let the received energy per pulse at any range R be E_R . Then it is clear from the equations (9) that

$$\frac{R}{R_0} = \left[\frac{k T_R \overline{NF}}{E_R} \right]^{1/4} \quad (10)$$

and defining

$$x = \frac{E_R}{k T_R \overline{NF}} \quad (11)$$

gives from (10)

$$\frac{R}{R_0} = \frac{1}{x^{1/4}} \quad (12)$$

* The derivation of exact formulas and numerous curves of visibility factors as a function of pulse width, bandwidth, type of amplifier, and off-resonance of carrier frequency will be found in the Mathematical Appendix (a separate report)⁽²³⁾. The visibility factor is actually given by

$$V = \frac{\left(\frac{E_{max}}{E_{ss}} \right)^2}{\tau_p \Delta f}$$

where E_{max} is the maximum voltage to which the pulse rises at the receiver output, and E_{ss}

is the steady state voltage at the same point. The quantity $\left(\frac{E_{max}}{E_{ss}} \right)^2$ should be contained in (7) and (8) but is usually omitted because it is so near to unity when $\tau \Delta f = 1$. In the case where the bandpass characteristic of the amplifier is the conjugate transform of the pulse, the visibility factor is exactly unity⁽⁴⁾.

where x is now the signal pulse energy in units of the average receiver noise pulse energy. As an example, suppose $x=4$, which means that the signal power equals four times the average noise power. Suppose the probability is calculated to be 0.5 that in this case the signal will be detectable. There is then a point $P=0.5$ at $R=0.7R_0$. When a series of such points are calculated for various values of x , a curve for P as a function of R/R_0 may be drawn, assuming fixed false alarm time, etc.

INTEGRATION OF PULSES

Before proceeding further, the meaning of pulse integration must be defined in detail. In its simplest form, it merely consists of adding N successive signal pulses together and attempting to detect the sum rather than an individual pulse. Now, whatever the integrating device may be, it will not know in advance whether there is a signal or not, and hence in the absence of a signal it will add up N successive noise pulses. Therefore, the comparison is between N signal plus noise pulses and N noise pulses as contrasted to a single signal pulse to a single noise pulse. One might be tempted to say that the signal to noise ratio would be unchanged, and that integration, or addition, of pulses therefore offered no advantage. This argument neglects the fact that the noise voltage fluctuates about its average value. The mean or average value of the noise voltage is not of too much concern, for it can always be "biased out."* If we add N signal pulses of voltage V , the total signal voltage is NV . If we add N noise pulses of average voltage V_N , the average sum will be NV_N . However, the average sum can be balanced out. The question is, whether or not the fluctuation in the sum voltage is now N times the fluctuation voltage of single pulse. If the answer were yes, then integration would be futile. However, due to the random nature of the fluctuation of any single pulse, the fluctuation voltage of the sum is only about \sqrt{N} times the fluctuation voltage of a single pulse. *It is the signal to noise-fluctuation** ratio, not the signal to average noise ratio that is of paramount importance.* The greater the number of pulses integrated, the greater is the signal to fluctuation ratio, and the greater is the probability of detecting the signal, but at the expense of longer detection times.

DEFINITION OF DETECTION AND THE BIAS LEVEL

Before the false alarm time can be calculated, a definition of "detection of a signal" must be given. Detection of a signal is said to occur whenever the output of the receiver exceeds a certain predetermined value hereafter called the bias level. In the absence of any signal, this bias level will on occasion be exceeded by the noise alone. The higher the bias level is set, the more infrequently this happens. The first problem is to calculate the required bias level, given the false alarm time. Knowing this bias level, the rest of the problem is to calculate the probability that any given value of signal (plus noise) will exceed this level.

* Practically, the bias level should not be too large, or the fluctuations in the bias will become of concern. See page 11 where a method of reducing the necessary bias level by a considerable factor is discussed.

** The mathematical term for the fluctuation is the "standard deviation," usually denoted by σ .

This is well and good, one says, but is this the best means of detection? What about the operator watching a cathode ray tube - what are his criteria for calling "signal"? Of course, it is impossible to say exactly, as is evidenced by the wide variation among radar operators. One can see, though, how an operator is affected by the false alarm time. If he is told that he will be subject to severe penalties if he calls a false alarm (calls a signal when it subsequently turns out that there was none), then he will be very cautious. If a doubtful pip appears on the screen, he will use discretion and say nothing. This means that under these conditions the false alarm time is increased, and at the same time the probability of detecting a target at a given range is decreased.

The operator may use the shape of a signal pulse contrasted to that of the noise as a criteria for detection as well as amplitude differences. This is thought to be a second order effect. The operator, on the other hand, is limited to some extent by the minimum brightness ratio which the eye can detect.

It seems that the method of electronic detection proposed above will be practically as good as any other possible method, electronic, human, or otherwise, if identical false alarm times and detection times are assumed. This statement is certainly not to be considered obvious. It should be possible to make some experiments to verify this theory.

METHODS OF PULSE INTEGRATION

As stated before, to integrate pulses it is merely necessary to add them together. There are many different practical ways in which this is done. One of the simplest is the use of a cathode ray tube screen⁽⁹⁾. Due to the screen persistence time, a certain number of pulses will be effectively integrated. In this case it will not be a simple addition, being more in the nature of a weighted average. The effect of weighting is always bad. In other words, the effect of equal samples in the integrated result should be as nearly the same as possible. P P I type of presentations which use intensity modulated displays usually have much longer integration times than an A scope.

One must not overlook the human operator,* who goes along with the cathode ray tube, as a vital part of the detection mechanism. The combination of the eye and the brain makes a very good integrator. In fact, the maximum integration time for a skilled operator may easily be several seconds. The best electronic integrators for pulsed radar built to date will not better this figure to any great extent. Henceforth, a model electronic integrator which linearly adds N pulses will be assumed.

* There are a large number of factors involving observers and oscilloscopes which are quite complicated and are more or less outside the intended scope of this report. Lawson and his group have done a great deal of work on this subject, the results of which will appear in Chap.VIII of Ref.(19). Most of these experimental results are also available in Ref.(24).

Now, pulses can be integrated in the R F stages, in the I F stages, or in the video stages (13), (14), (16), (18). Furthermore, there can be one or more linear or square law detectors present*, and the integration can be done in one or more steps and in at least two different ways. Many of these possibilities are reserved for detailed treatment in a separate mathematical report⁽²³⁾.

Fortunately, the results for the various cases show little difference, with one marked exception. R F and I F integration are better than video integration for small signals (compared to the noise). However, there is no practical way known at present to take full advantage of R F or I F integration with moving targets because of the requirement that the successive received pulses must be completely coherent^{(13), (14)}. Coherent integration would be possible in the case in which both the radar and the target were stationary, but this case is not of much practical value. The difference between various types of both detectors and video integrating circuits** is small, as far as results of this kind of study are concerned. There are, of course many reasons why a choice is made in practice, such as sensitivity to small changes in amplifier gains, vulnerability to countermeasures, etc.

It is worth describing one scheme for integrating in which a pulse known to be only noise is subtracted from each possible signal plus noise pulse. N of these composite pulses are then integrated. With no signal, the average value of any number of such composite pulses is nearly zero, so that the required bias level is considerably reduced. Such a method is much less sensitive to a small change in bias level, and would usually be preferred in practice. This case is much more difficult to calculate than the straight addition case; and since sample calculations show the results to be nearly identical, the latter method has been used to obtain the curves of Figures 1 thru 50.

Figures 51 and 52 show the difference in sensitivity to bias level for this method. Figures 53 and 54 show the comparison of straight integration to the case in which a noise pulse is subtracted from each signal-plus-noise pulse.

Practical types of electronic pulse integrators often take the form of very narrow band audio filters having their center frequency at the pulse repetition frequency⁽²¹⁾ or some harmonic thereof. The action of such a filter can be understood roughly by consideration of the frequency spectrum of a finite group of N pulses. The

* It is assumed throughout this report that the video bandwidth is large compared with the I F bandwidth. Actually, the results will be affected only if the video bandwidth is small compared with the I F bandwidth⁽²⁴⁾, a condition not often found in practice.

** One might ask if there would be any advantage in having an integrator which adds the sum of the squares of the N pulses or perhaps the sum of some other function of the amplitude. Actually, it can be easily shown that this just corresponds to changing the shape of the detector curve, and what is being asked is, "Is any shape of detector curve much superior to the linear or square law form?" Apparently the answer is no. There is a "best" detector curve for every different signal strength, x , given by $\log I_0(v\sqrt{2x})$ where I_0 is a modified Bessel function. No results have been obtained for this detector function, but it is thought that the maximum difference in range between this and the square law or linear detector will not exceed five percent.

envelope of such a spectrum is simply the familiar $\sin x/x$ curve of a single pulse, while the actual curve has appreciable values only in the neighborhood of the harmonics of the repetition frequency (including dc)*. The greater N is, the more closely the spectrum clusters around these harmonics. Thus, the filter may be made narrower, excluding more and more noise, but retaining most of the signal energy. With such a narrow band filter-type of integrator it is very simple to subtract a noise pulse from each signal-plus-noise pulse by gating the receiver at a frequency *double* the center frequency of the filter. To prevent the possibility of a signal on every other gate, the sweep length would ordinarily be held at less than one-half of the pulse repetition period. The simple electronic type of integrator has the disadvantage of a fixed integration time. If the number of pulses returned from a target is greater or less than the number of pulses for which the integrator is set, the operation suffers. With the human operator, the story is different. He can adjust his integration time rapidly to fit changing situations. This procedure could be approximated electronically by the use of two or more successive integrators in series,** or by the use of so-called "weighting circuits." Such a complicated procedure does not come within the scope of this report.

METHOD OF OBTAINING THE BIAS LEVEL

By means covered in detail⁽²³⁾, in a separate mathematical report***, the probability that the sum of N pulses of noise voltage alone will be greater than an arbitrary level y is obtained. This relation may be symbolically represented by

$$P_N = f(y) \quad (13)$$

where y is measured in units of the rms value of the noise. The number of groups of noise pulses which are observed in a fixed false alarm time, τ_{fa} , is then found.

When speaking of noise pulses, it is convenient to assume mentally a range gate equal to the pulse length at a fixed range. If the range sweep is continuous, such as with an A scope, the effective number of independent noise pulses observed in one

* There is a close resemblance between such a spectrum and the diffraction pattern of an N slit grating (see any standard text book on physical optics).

** The advantage of a multistage integrator is that if a signal which is large enough so that the number of pulses which need to be integrated in order to produce a detectable signal occur in a time appreciably less than the total integration time, one of the sub-stages will detect the signal much sooner than will the final stage.

*** It turns out that the functions which describe the probability that the noise alone, or a given strength signal plus noise will have any arbitrary amplitude, are quite complicated and hence only some of the results and general procedures are given in this report. Furthermore, it should be mentioned in passing that the use of the central limit theorem, or the so-called "normal approximation," is not valid until the number of pulses integrated is of the order of 1000. This is because the values of the distribution functions far out on the tails play a major role in the calculations. Several investigators in the past have made the mistake of assuming that the normal approximation was satisfactory if N were only of the order of 10.

repetition period is given by the length of the sweep divided by the pulse length*, hereafter called η . It is apparent that $\eta = 2L/c\tau_p = 10.8L/\tau_p$ where L is the sweep length in miles, c is the velocity of light, and τ_p is the pulse length in microseconds. In the special case in which the sweep occupies the total time between pulses, $\eta = 1/\tau_p f_r$, which is merely the reciprocal of the duty cycle. The time for N pulses to occur is N/f_r . Therefore $\frac{\tau_{fa}}{N/f_r} = \frac{\tau_{fa} f_r}{N}$ groups are observed in the time τ_{fa} , assuming only one gate per sweep. Since the effective number of gates per sweep is η , the total number of independent chances for obtaining a false alarm in τ_{fa} is**

$$n' = \frac{n}{N} = \frac{\tau_{fa} f_r \eta}{N} \quad (14)$$

The false alarm time is defined as the time in which the probability is $\frac{1}{2}$ that the noise will not exceed the bias level.*** From (13) and (14),

$$(1 - P_n)^{n'} = \frac{1}{2} \quad (15)$$

from which γ , the bias level, is obtained.

PROBABILITY OF DETECTING A SIGNAL

Having established the value of the bias level, the probability that a signal will exceed this level in a given time, namely the detection time τ_d , must be calculated. The signal is assumed to consist of N integrated pulses. The time of such a pulse group is N/f_r . The number of such groups which occur in τ_d is given by

$$\gamma' = \frac{\gamma}{N} = \frac{\tau_d f_r}{N} \quad (16)$$

As a corollary to the previous definition of detection, it is now assumed that the signal is detected if any one of the γ' groups of pulses exceeds the bias level. One will ask, at this stage, 'Why not count exactly how many times the signal exceeds the

* If the range gate is much wider than the pulse length, the operation of the integrator will suffer more or less, depending on the exact type of integrator used. This corresponds somewhat to the case of an oscilloscope where the spot does not move by at least its diameter within a pulse length.

** This derivation assumes that the antenna is not scanning. With a scanning antenna, integrating channels must be disposed in angular position as well as in time. In order for (14) to hold, the number of pulses per channel per scan must be equal to or greater than N , the number of pulses which each channel integrates.

*** This very nearly, though not exactly, corresponds to the earlier definition given on p. 7.

bias level?" This would in effect correspond to a two-stage integrator. Such a device is not considered here, though it is easy to make an extension of the present theory to cover this case.

At any range R , the normalized signal strength x is obtained from Eq.(12). The probability that the signal plus noise will exceed any value y for a single group of N integrated pulses is known⁽²³⁾, and may be represented symbolically as

$$P = f(y, x). \quad (17)$$

The probability that at least one of the γ' groups will exceed the bias level y is then

$$P' = 1 - (1-P)^{\gamma'}. \quad (18)$$

Notice that γ' must be an integer for the analysis to be strictly correct. It will be satisfactory, however, if one always requires $\gamma' \geq 1$ *

EFFECT OF ANTENNA SCANNING

If the antenna is scanning, some modifications of Eq.(16) for γ' , the number of groups of pulses integrated, will be necessary⁽²⁴⁾. If, with a P P I type of presentation, the antenna moves at an angular velocity ω , and the beam width is B , then the number of pulses per target per scan will be

$$N_{sc} = \frac{Bf_r}{\omega} \quad (19)$$

and (16) is replaced by

$$\gamma' = \frac{\gamma}{N} = \frac{\tau_d f_{sc} N_{sc}}{N} \quad (20)$$

where f_{sc} is the number of scans per second. With a simple type of electronic integrator, N_{sc} must be equal to or greater than N for Eq.(20) to be valid, assuming that the integrator does not hold over from scan to scan. If the integrator does hold over from scan to scan, as an operator partially does, then it is only necessary to have $\gamma' \geq 1$ as before. In any case (20) only holds if $\tau_d f_{sc} \geq 1$.

If $\tau_d f_{sc} < 1$, then $\gamma' = N_{sc} / N$, which must be equal to or greater than 1.*

* It is always best for γ' to equal 1. In this case the integrator effectively integrates pulses during the whole of the detection time. $\gamma' > 1$ is the case in which the detection time is longer than the integration time. Here the probability for detection is greater than if the detection time were reduced to the integration time, but less than it would be if the integration time were increased to equal the detection time. The case for $\gamma' < 1$ is that one in which the number of signal pulses occurring are fewer than the number for which the integrator is set. In this case the probability of detection is reduced from the value it would have if the integrator were set for exactly the number of signal pulses which occur. To calculate this latter case would require using N to calculate the bias level as in (15), but the use of some lesser value N' in obtaining (17). This will be done, but results have not been obtained as yet.

PRESENTATION OF THE RESULTS

The results are presented in the form of a set of curves. This is necessary because of the complicated form of the analytical solutions. The parameters involved in the curves are:

P = the probability of detecting a target at range R .
 R/R_0 = the ratio of the range to the idealized range.

$n = \tau_{fa} \cdot f_r \cdot \eta^*$
 τ_{fa} = the false alarm time
 f_r = the pulse repetition rate
 η = the number of pulse intervals per sweep.

$\gamma = \tau_d \cdot f_{sc} \cdot N_{sc}^{**}$
 τ_d = the detection time
 f_{sc} = the scan frequency
 N_{sc} = the number of pulses per scan
 N = the number of pulses integrated.

A summary of the range of the variables for the curves presented will be found on page 21.

AN EXAMPLE WITH A QUASI-STATIONARY TARGET

A simple example is now solved assuming a stationary target. The radar set will also be assumed to be stationary. The following data are taken as given:

ω = angular rate of antenna = $30^\circ/\text{sec}$, $f_{sc} = 1/12$
 B = beam width of antenna = 3.0°
 f_r = pulse repetition rate = 500 per second
 τ_p = pulse length = 1 microsecond
 R_0 = idealized range for given target and average aspect = 40 miles
 τ_{fa} = required false alarm time = 5 minutes
 τ_d = required detection time = 25 seconds.

* $n = \tau_{fa}/\tau_p$ if the sweep occupies the total time between pulses.

** See also (16), and the conditions on (20). The notation used on Figs. 1-54 is $\gamma = \tau_d \cdot f_r$, which represents the special case in which there is no scanning. In general this should be replaced by $\gamma = \tau_d \cdot f_{sc} \cdot N_{sc}$.

Type of detector - electronic; integrator, $N = 50$; sweep length = 20 to 80 miles.

Step 1. Calculate N_{sc} from (19)

$$N_{sc} = \frac{Bf_r}{\omega} = \frac{3 \times 500}{30} = 50$$

Step 2. Calculate γ from (20)

$$\gamma = \tau_d f_{sc} N_{sc} = 25 \times \frac{1}{12} \times 50 = 104$$

Step 3. Calculate η from $\eta = 10.8L/\tau_p$ where L is the sweep length in miles and τ_p is the pulse length in microseconds.

$$\eta = \frac{10.8 \times (80 - 20)}{1} = 648$$

Step 4. Calculate n from (14)

$$n = \tau_{fa} \cdot f_r \cdot \eta = (5 \times 60) \times 500 \times 648 = 0.98 \times 10^8$$

Step 5. Refer to Fig. 23; $n = 10^8$ and $\gamma = 100$. Mentally interpolate a curve for $N = 50$ between $N = 30$ and $N = 100$. This curve gives probability of detection at any R/R_0 . R_0 is given as 40 miles. For instance, $P = 0.50$ at $R/R_0 = 1.07$ or at $R = 43$ miles.

MOVING TARGETS AND/OR RADAR

If there is an appreciable change of range with time between the radar and the target, a limit will ordinarily be set on the number of pulses which can be integrated. This is because the returned pulses will just fail to overlap when the target has moved through a distance $d = \tau_p c/2$ where c is the velocity of light. The effective distance over which the pulses can be assumed to contribute their full amplitude is about $\frac{1}{2}$ this value. If the rate of change of range is v , the time available for integration is

$$\tau_i = \frac{\tau_p c}{4v} \quad (21)$$

The maximum number of pulses which can be integrated in this time is

$$N_{\max} = \tau_i f_r = \frac{\tau_p f_r c}{4v} \quad (22)$$

This quantity N_{\max} is the maximum number of pulses that can be integrated,* provided that it is not greater than N_{sc} , the number of pulses per scan. In the case where $N_{\max} > N_{sc}$, then N_{sc} is the maximum number of pulses which can be integrated.

In the case of approaching targets, one may be concerned with the probability that a target will be detected by the time it has reached a certain range. Assuming the target to have started its approach at range R_1 , the probability that it will have been detected at least once by the time it reaches range R is

$$P = 1 - \prod_{R_1}^R [1 - P(R)] \quad (23)$$

where R progresses from R_1 to R in units of ΔR . The length of the ΔR intervals and the number of pulses integrated per interval are determined from the considerations given above.

An example follows in which (23) can be reduced to a particularly simple form: Assume a continuously directed beam (no scan) and the target moving toward the radar with a constant range rate v . The finite product in (23) may be approximated by

$$\log_e \prod_{R_1}^R [1 - P(R)] \approx \frac{1}{\Delta R} \int_{R_1}^R \log_e (1 - P) dR \quad (24)$$

and using $\Delta R = d = \frac{\tau_p c}{4}$ equation (23) becomes

$$P = 1 - e^{-\frac{4}{\tau_p c} \int_{R_1}^R \log_e (1 - P) dR} \quad (25)$$

The integrations necessary in the solution of this type of problem must be performed numerically, using the graphical data of figures 1 to 50.

In problems where the antenna is scanning, equation (23) may be approximated in different ways depending on the exact values of the parameters involved. These are rather simple to work out in any specific case.

* A system could presumably be built incorporating one or more velocity gates. Such a velocity gate would travel with a given preset velocity. In this case, the relative velocity of the target to that of the gate, $v - v_g$, can be used in Eq.(22) in place of the target velocity, v . The greater the number of velocity gates used, the greater will the probability be that the difference between the target velocity and some one of the gates will be very small. Therefore, in this gate the allowable value of N_{\max} will be large, and the probability of detection in this gate will be increased.

In any multi-channel receiver, such as this, the number of pulse intervals per sweep, η , must be multiplied by the number of channels in calculating n .

EXACT EFFECT OF THE NUMBER OF PULSES INTEGRATED ON THE RANGE

One might expect that for a given n and a given probability of detection, the range to the fourth power would vary as N , as was stated on page 9. This would be true with coherent integration, but with video integration the variation is between $N^{1/2}$ and N (assuming a threshold signal). This effect is due to the so-called "modulation suppression" of the weak signal by the stronger noise in the process of detection.

Fig. 55 shows the exact variation of the exponent of N , here called θ , as a function of N , and of n , for P fixed at 0.50. The effect of n is seen to be quite small.

Fig. 56 shows the variation of the exponent of N for an incremental change of N as a function of N and n . P is again fixed at 0.50. In both cases, θ approaches 0.5 as N approaches infinity; though much more slowly, in the first case.

APPLICATION OF RESULTS TO CONTINUOUS-WAVE SYSTEM

Though this report is concerned primarily with pulsed systems, the results are directly applicable to continuous-wave systems. To accomplish this, the following new notation is introduced:

P_{av} = the average cw transmitter power.

Δf_{cw} = the combined R F and I F bandwidth of the cw receiver.

η' = the number of separate velocity channels incorporated in the receiver.*

The change-over is then made by means of these substitutions:

Replace E_p by $P_{av} / \Delta f_{cw}$ in R_0

Put $\gamma = \tau_d \cdot \Delta f_{cw}$

Put $n = \tau_{fa} \cdot \Delta f_{cw} \cdot \eta'$

N is now to be taken as the number of variates (of duration $1/\Delta f_{cw}$) which are integrated after detection.**

* In both the pulse and cw analysis it has been assumed that the range or velocity gates or channels are fixed in position. In the case where such gates sweep as a function of time in order to conserve apparatus (or for any other reason), the analysis is not strictly valid. A good rule-of-thumb is that the gate should move through the amplifier pass band in a time equal to the reciprocal of the amplifier pass band. In this case the effective visibility factor is about 0.8. Curves of the visibility factor for other sweep speeds are given in the Mathematical Appendix⁽²³⁾ (a separate report).

** Integration of N variates before detection merely corresponds to narrowing the R F (or I F) bandwidth by a factor of $\frac{1}{N}$.

N must be less than γ for the theory to hold. An optimum cw system is one in which $\gamma = 1$ ($\tau_d = 1/\Delta f_{cw}$), and $N = 1$. This gives the greatest range for a given energy expended during the detection time, τ_d . This corresponds exactly to the case $N = 1$ and $\gamma = 1$ in a pulsed system. If the number of range channels η in the pulsed system is equal to the number of velocity channels η' in the cw system, then the two systems, with $N = 1$ and $\gamma = 1$, will have identical ranges for the same average power.

In either case, if $N > 1$, a larger amount of average power is required, everything else remaining equal. In the pulsed case, reducing N necessitates higher peak powers, which may be impracticable; or it necessitates longer pulse lengths, which reduces possible range-resolution and at the same time aggravates the effect of a fixed Doppler shift due to the narrowing of the receiver pass band. In the cw case, reducing N necessitates a target with reasonably constant velocity so that the signal will not wander in and out of the pass band of the receiver, and also a sufficiently slow scan so that each target "pulse" is at least as long as the reciprocal of the receiver pass band.

RANGE OF VARIABLES FOR FIGURES 1 THRU 50

Fig.	P	R/R ₀	n	γ	N
1	variable	variable	10 ⁴	1	1
2	"	"	"	10	1,3,10
3	"	"	"	100	1,3,10,30,100
4	"	"	"	1000	1,3,10,30,100,300,1000
5	"	"	"	1,10,100,1000	1
6	"	"	"	10,100,1000	10
7	"	"	"	100,1000	100
8	"	"	"	N	1,10,100,1000
9	"	"	10 ⁸	1	1
10	"	"	"	10	1,3,10
11	"	"	"	30	1,3,10,30
12	"	"	"	100	1,3,10,30,100
13	"	"	"	300	1,3,10,30,100,300
14	"	"	"	1000	1,3,10,30,100,300,1000
15	"	"	"	1,3,10,30,100,300,1000	1
16	"	"	"	10,30,100,300,1000	10
17	"	"	"	30,100,300,1000	30
18	"	"	"	100,300,1000	100
19	"	"	"	300,1000	300
20	"	"	"	N	1,10,30,100,300,1000
21	"	"	10 ⁸	1	1
22	"	"	"	10	1,3,10
23	"	"	"	100	1,3,10,30,100
24	"	"	"	1000	1,3,10,30,100,300,1000
25	"	"	"	1,10,100,1000	1
26	"	"	"	10,100,1000	10
27	"	"	"	100,1000	100
28	"	"	"	N	1,10,100,1000
29	"	"	10 ¹⁰	1	1
30	"	"	"	10	1,3,10
31	"	"	"	100	1,3,10,30,100
32	"	"	"	1000	1,3,10,30,100,300,1000
33	"	"	"	1,10,100,1000	1
34	"	"	"	10,100,1000	10
35	"	"	"	100,1000	100
36	"	"	"	N	1,10,100,1000
37	"	"	10 ¹²	1	1
38	"	"	"	10	1,3,10
39	"	"	"	100	1,3,10,30,100
40	"	"	"	1000	1,3,10,30,100,300,1000
41	"	"	"	1,10,100,1000	1
42	"	"	"	10,100,1000	10
43	"	"	"	100,1000	100
44	"	"	"	N	1,10,100,1000
45	0.50	"	variable	N	1,10,100,1000
46	0.90	"	"	N	1,10,100,1000
47	0.99	"	"	N	1,10,100,1000
48	0.50	"	10 ⁴ 10 ⁸ 10 ⁸ 10 ¹⁰	N - variable	variable
49	0.90	"	10 ⁴ 10 ⁸ 10 ⁸ 10 ¹⁰	N - variable	variable
50	0.99	"	10 ⁴ 10 ⁸ 10 ⁸ 10 ¹⁰	N - variable	variable

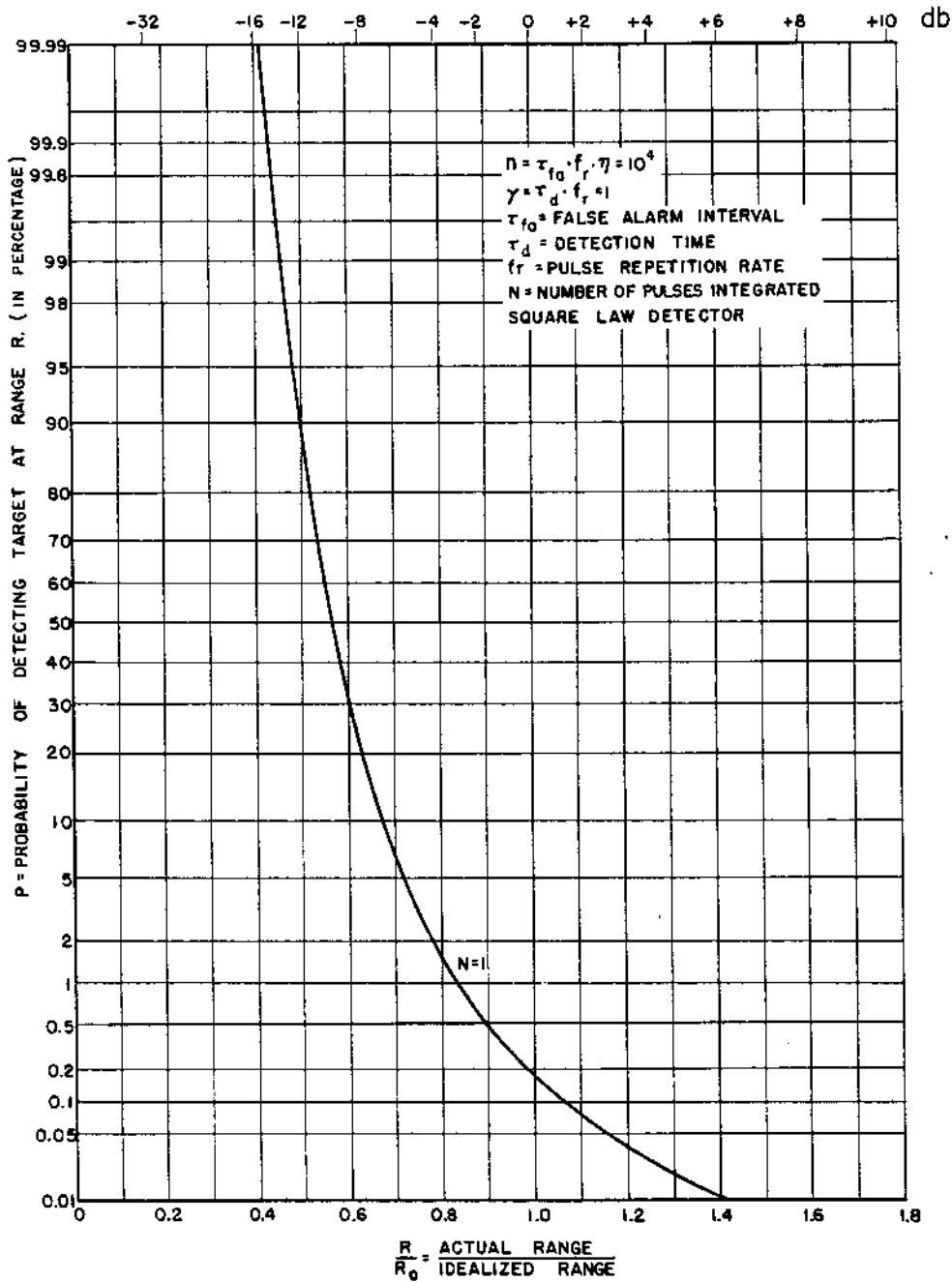


FIG. 1

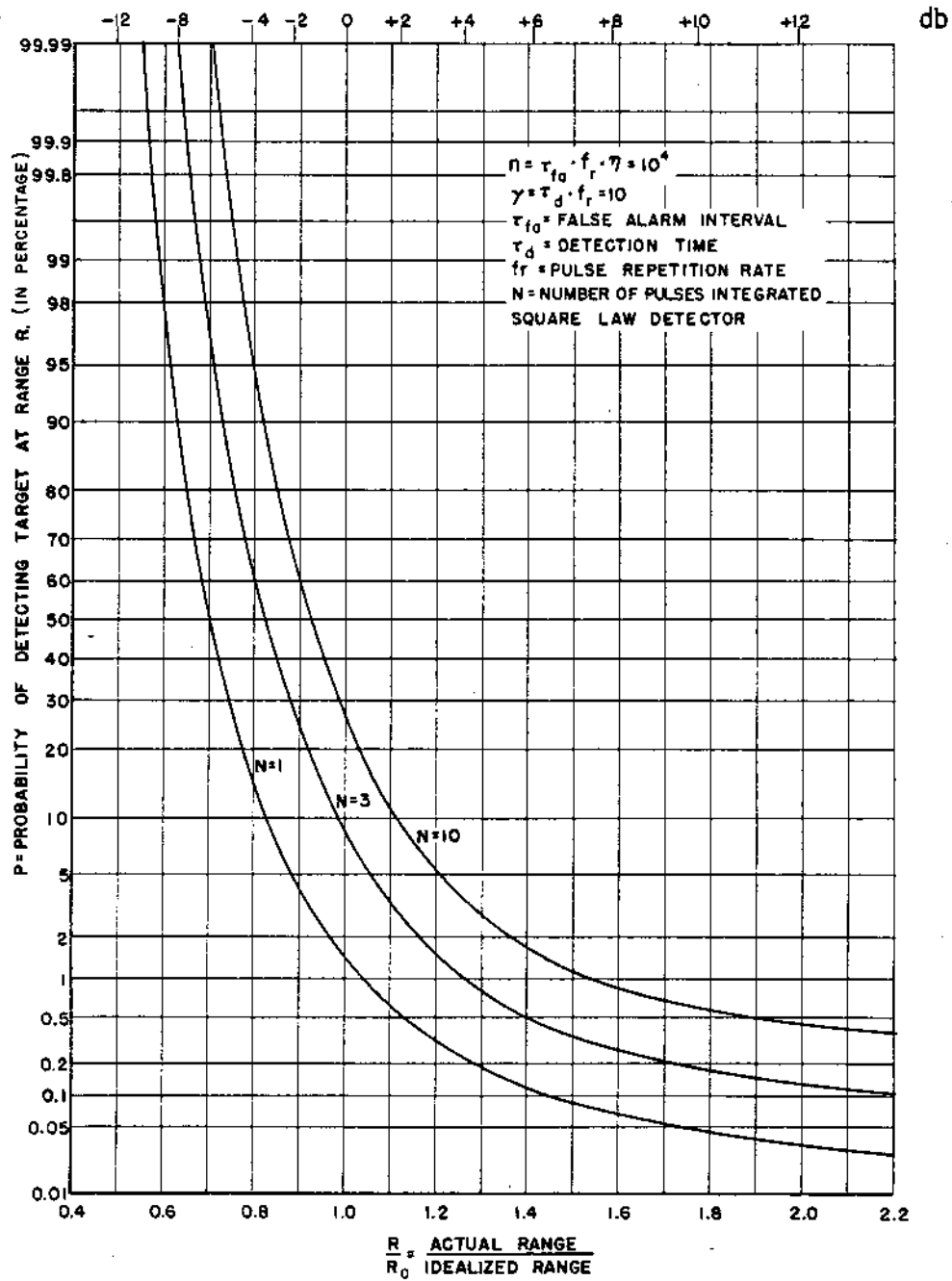
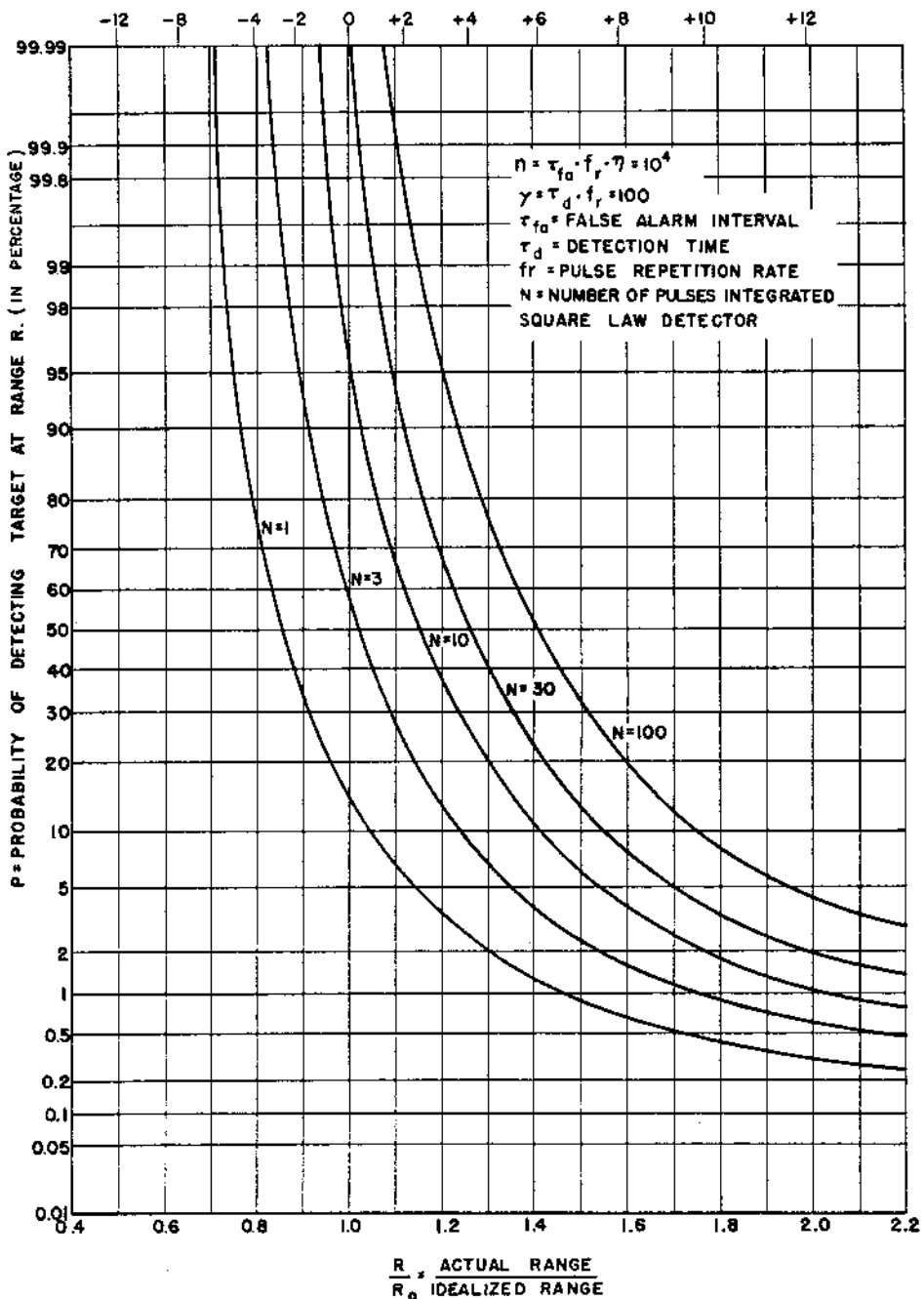


FIG. 2



db

FIG. 3

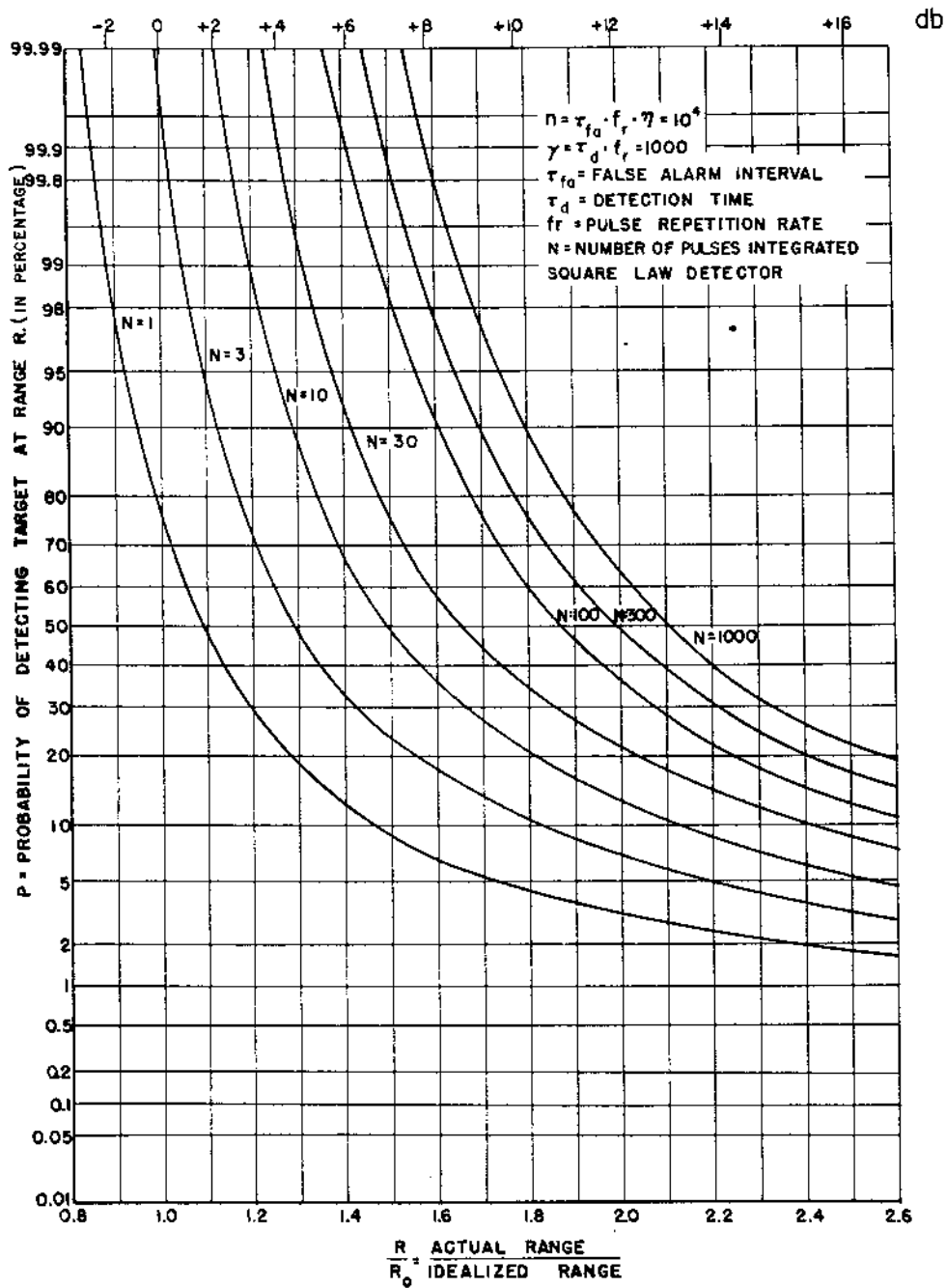


FIG. 4

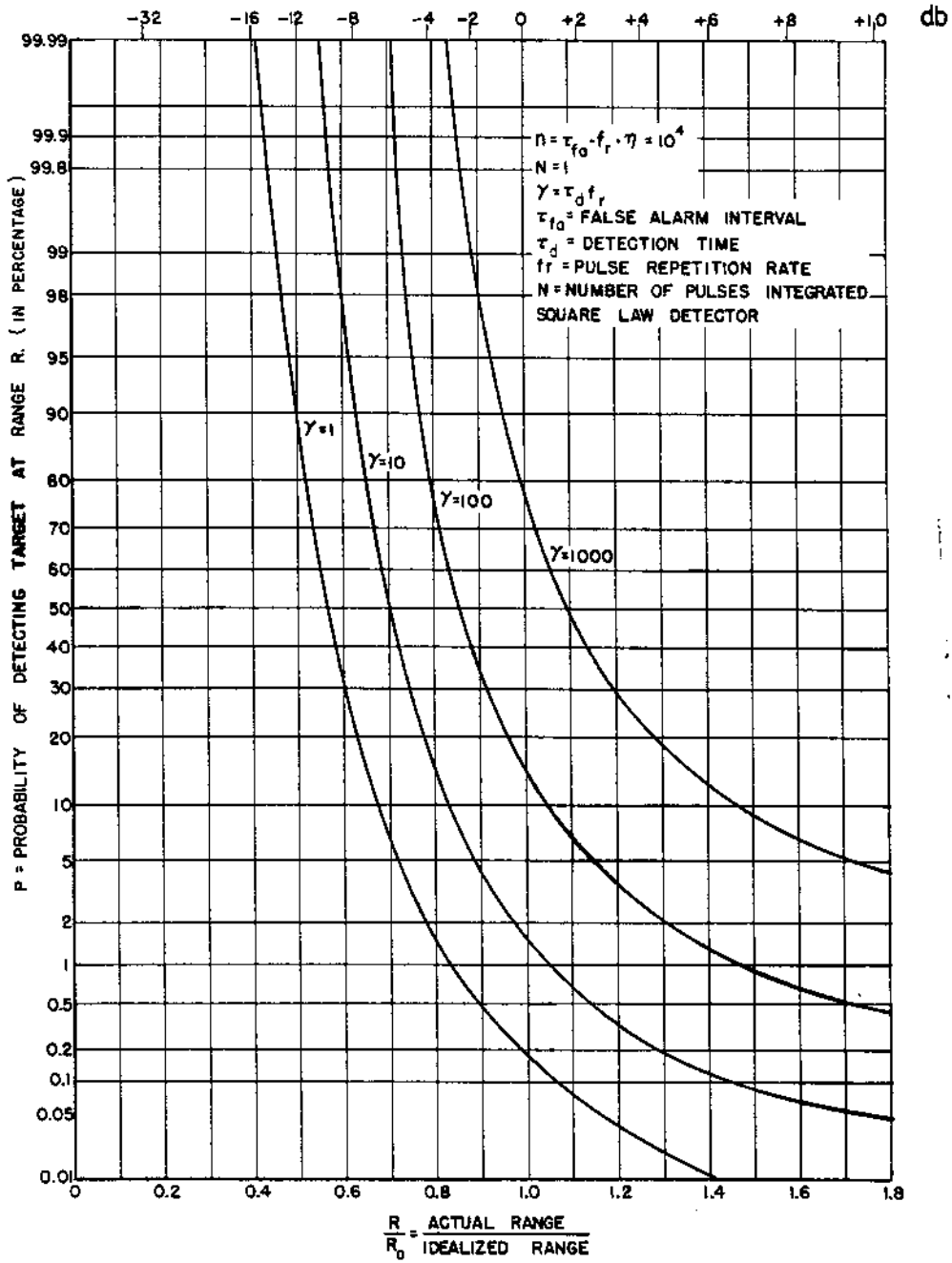


FIG. 5

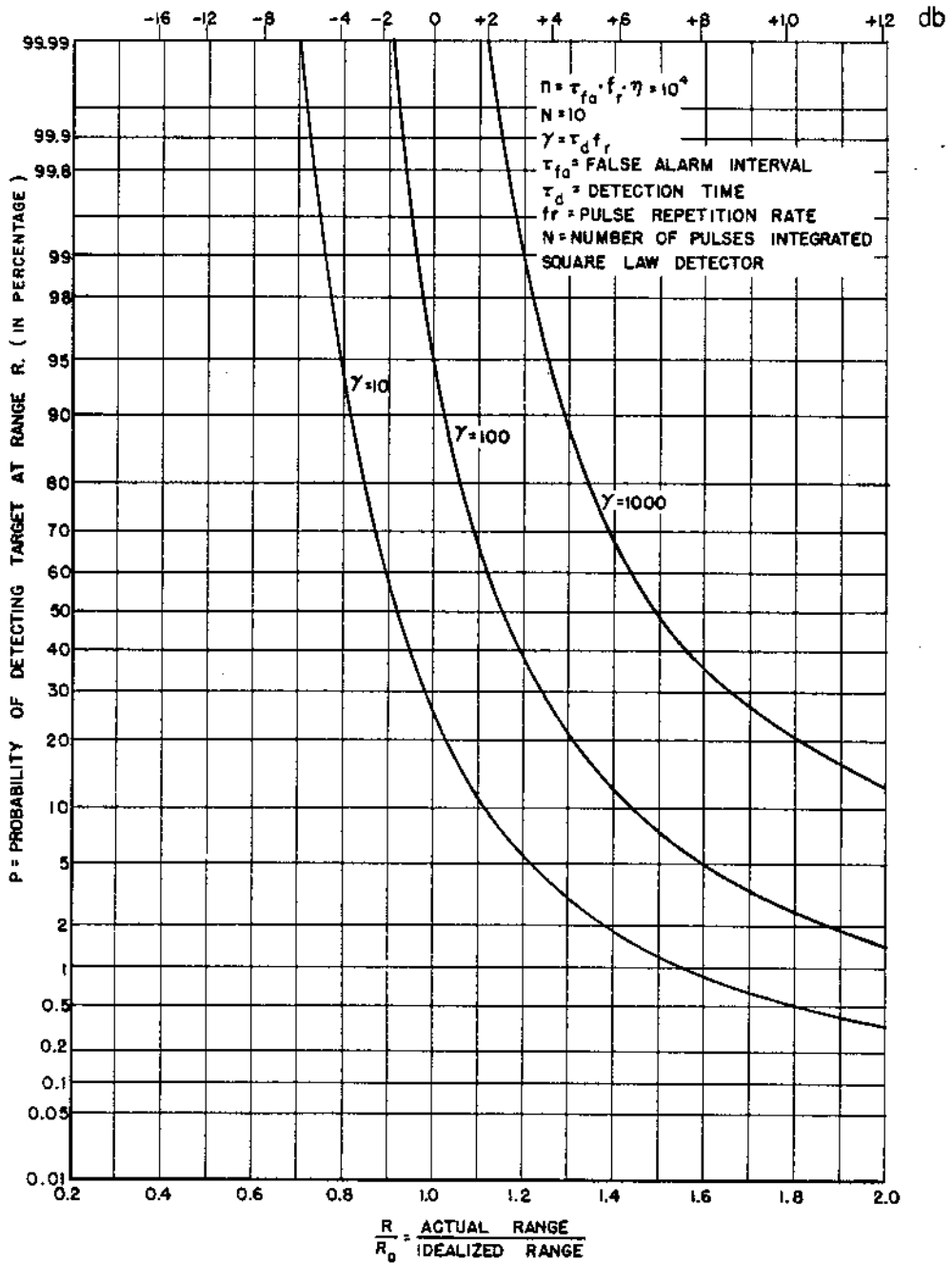


FIG. 6

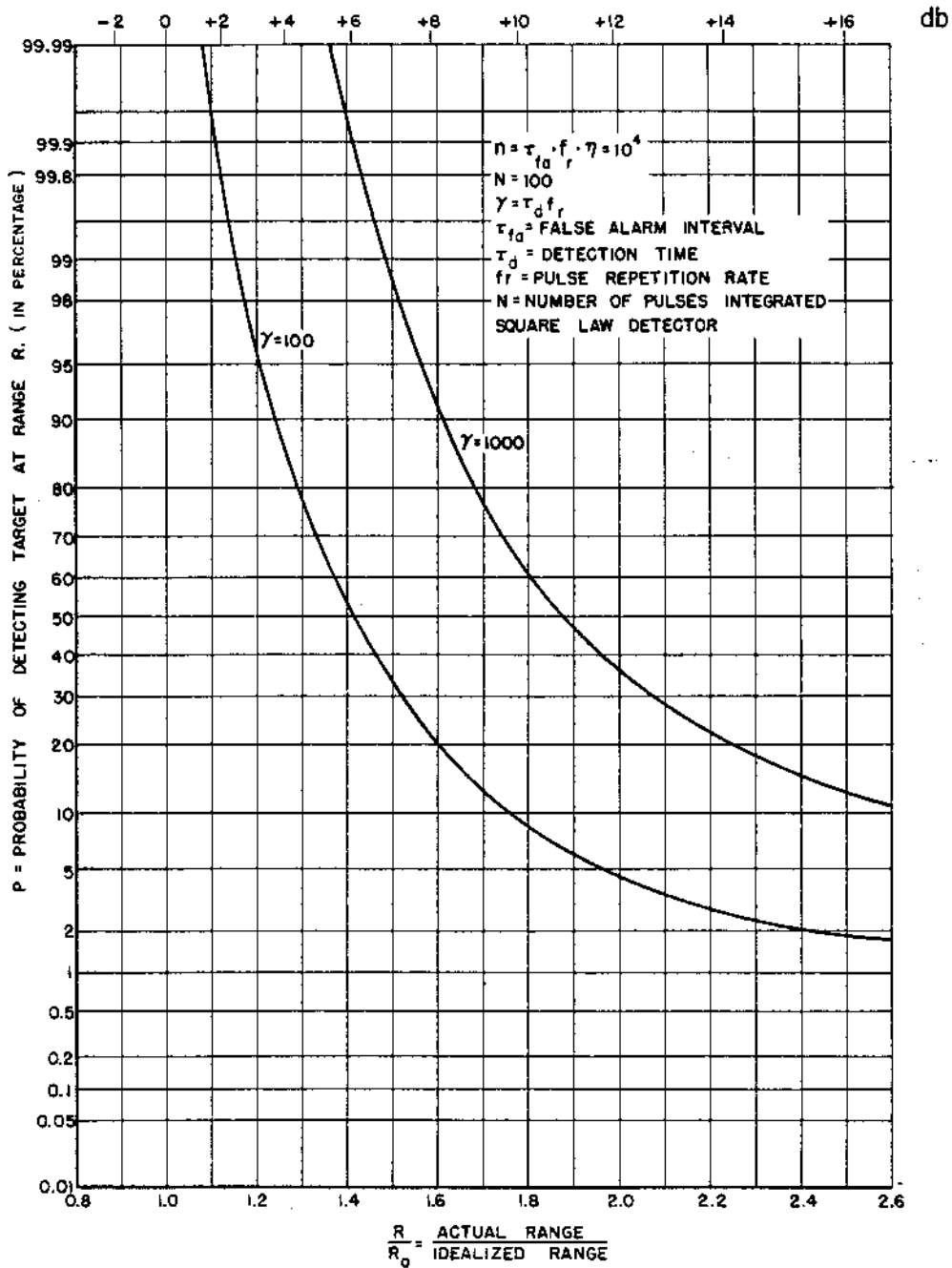


FIG. 7

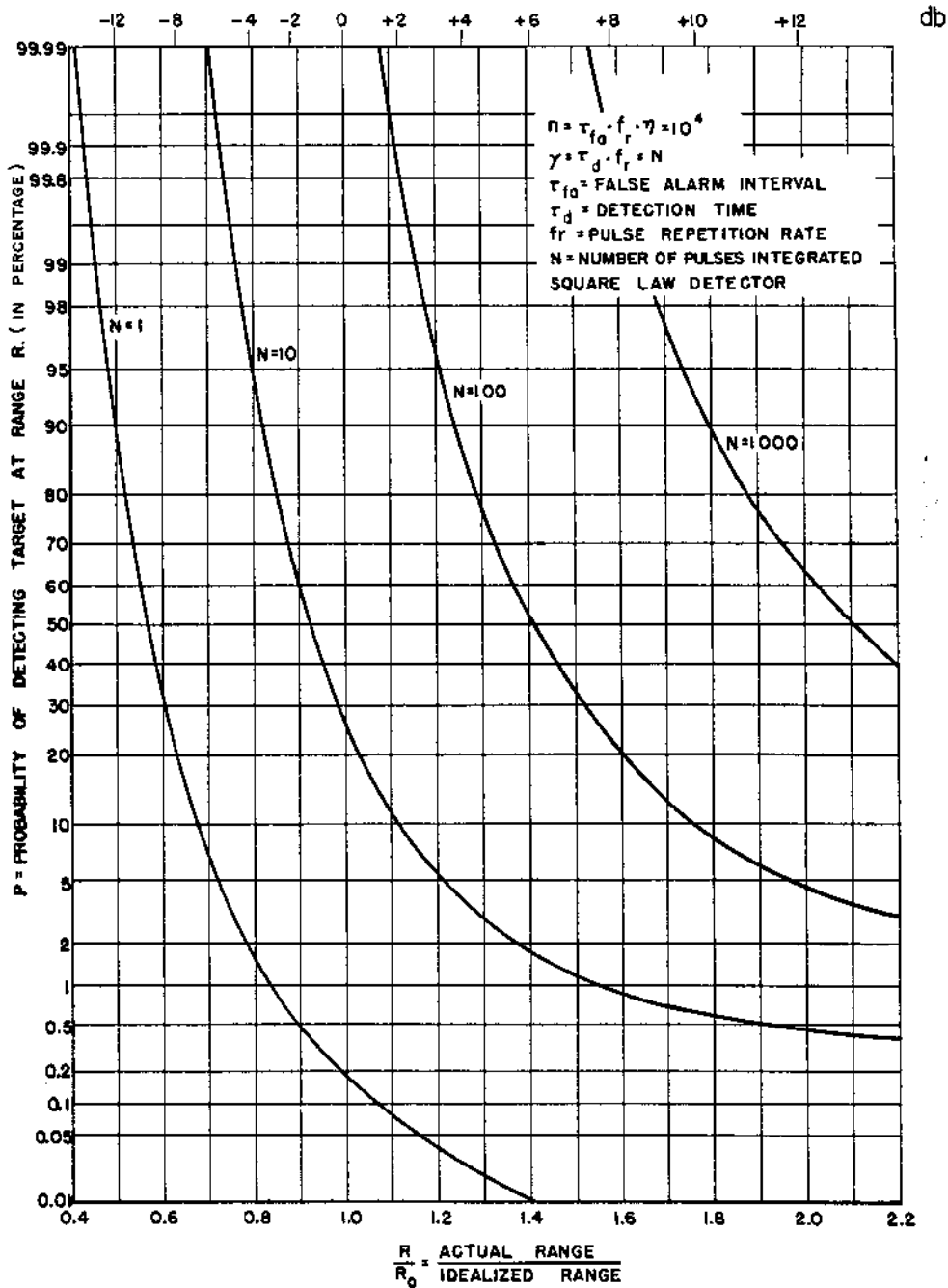


FIG. 8

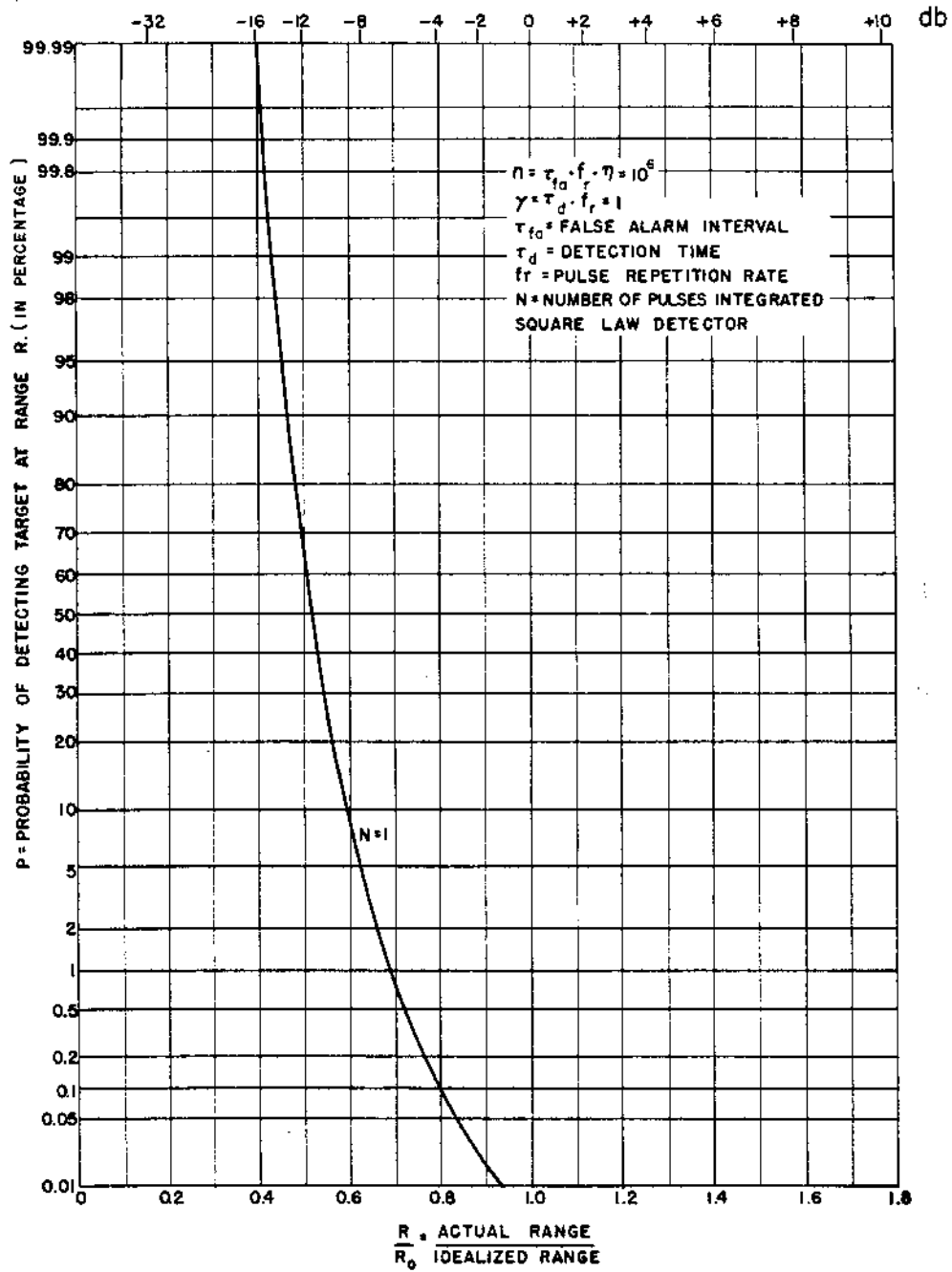


FIG. 9

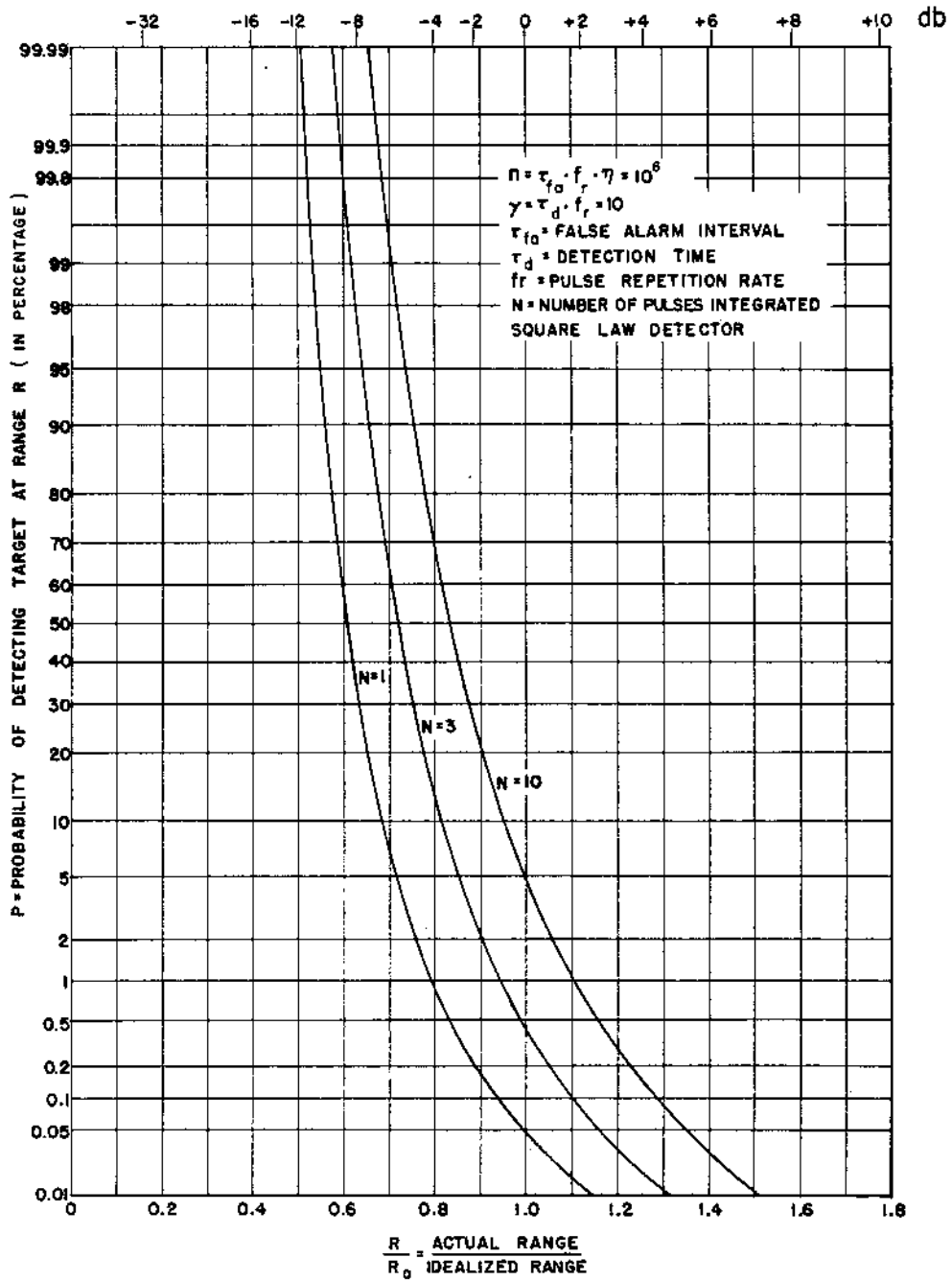


FIG. 10

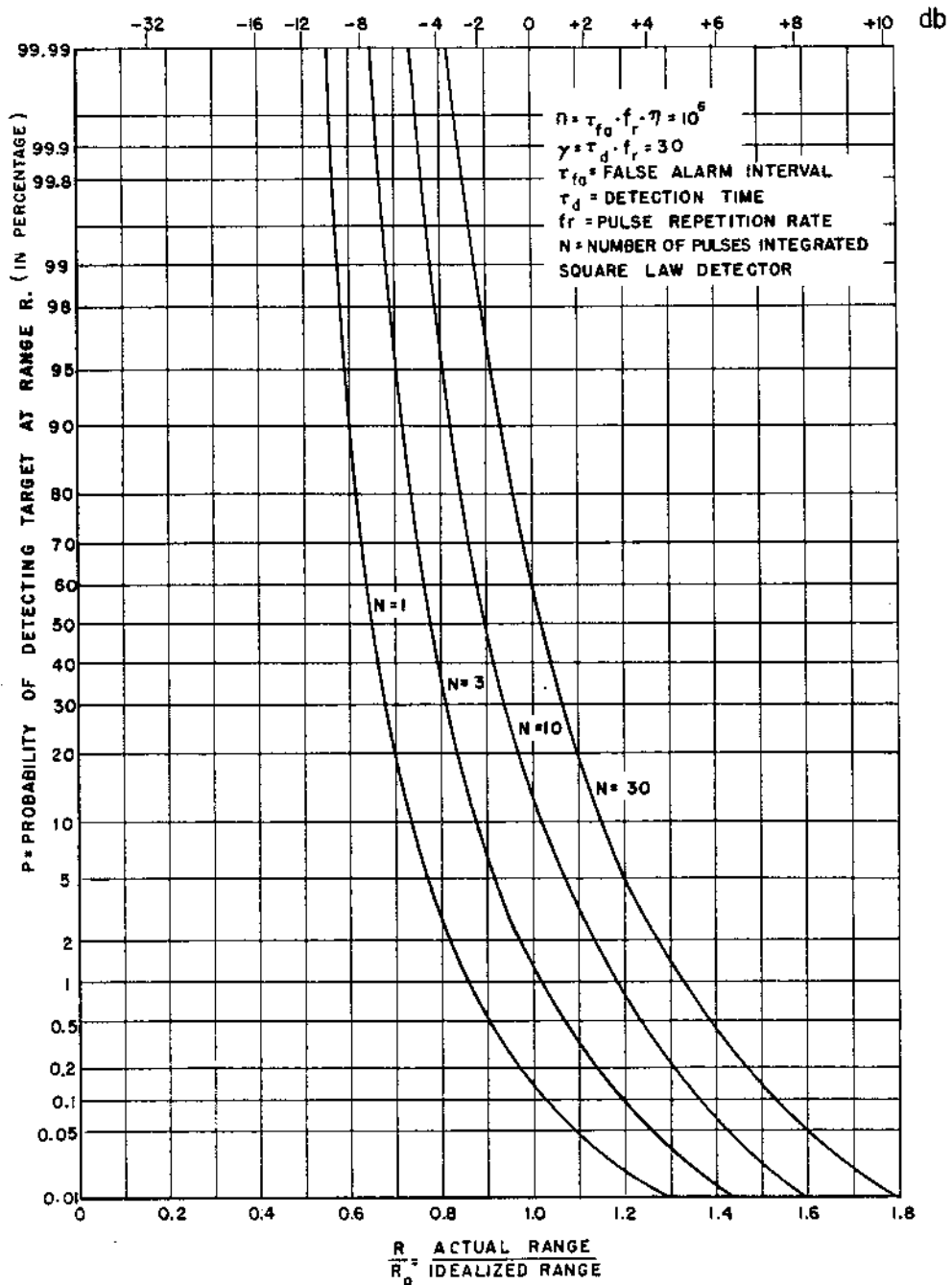


FIG. 11

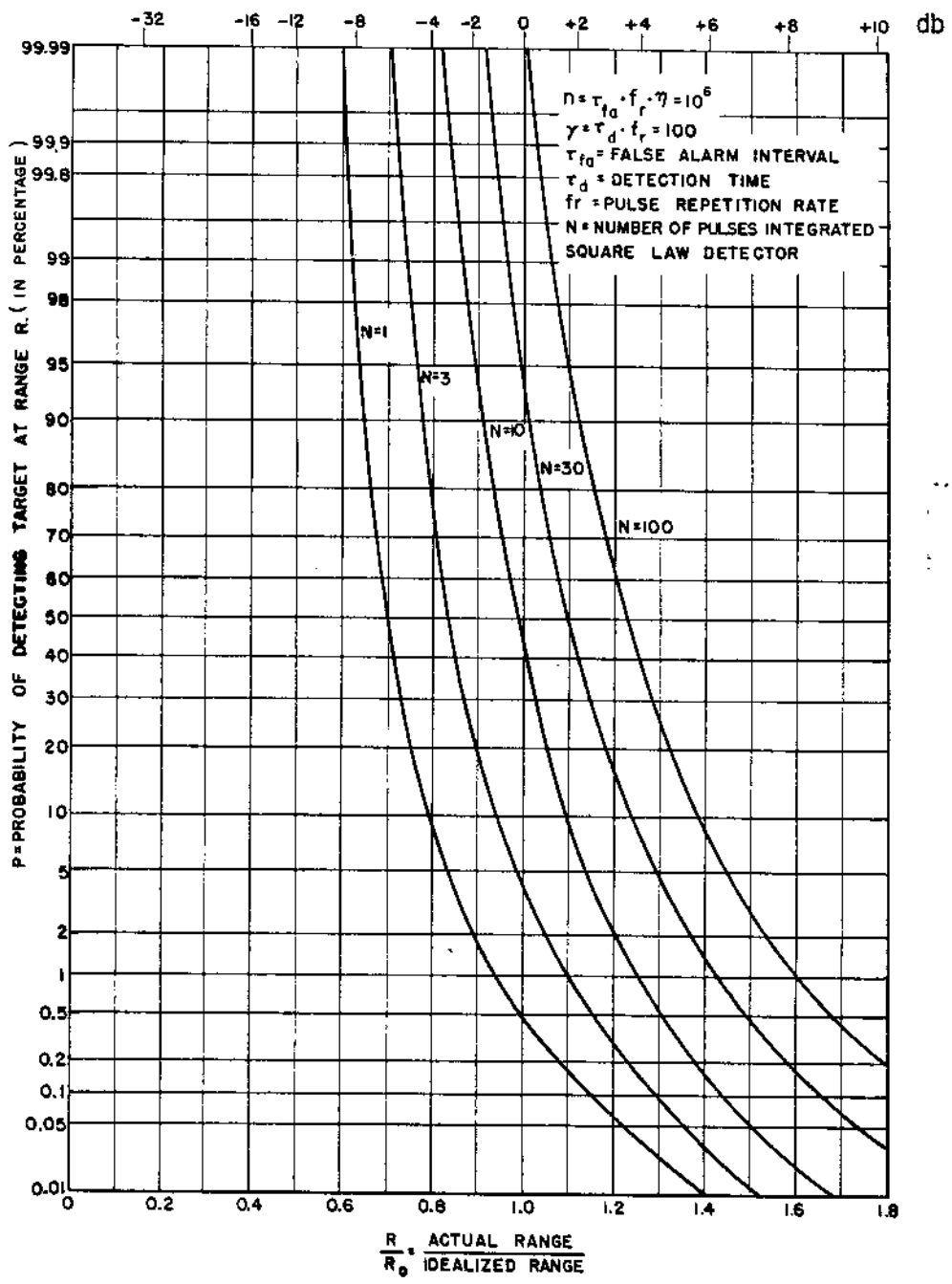


FIG. 12

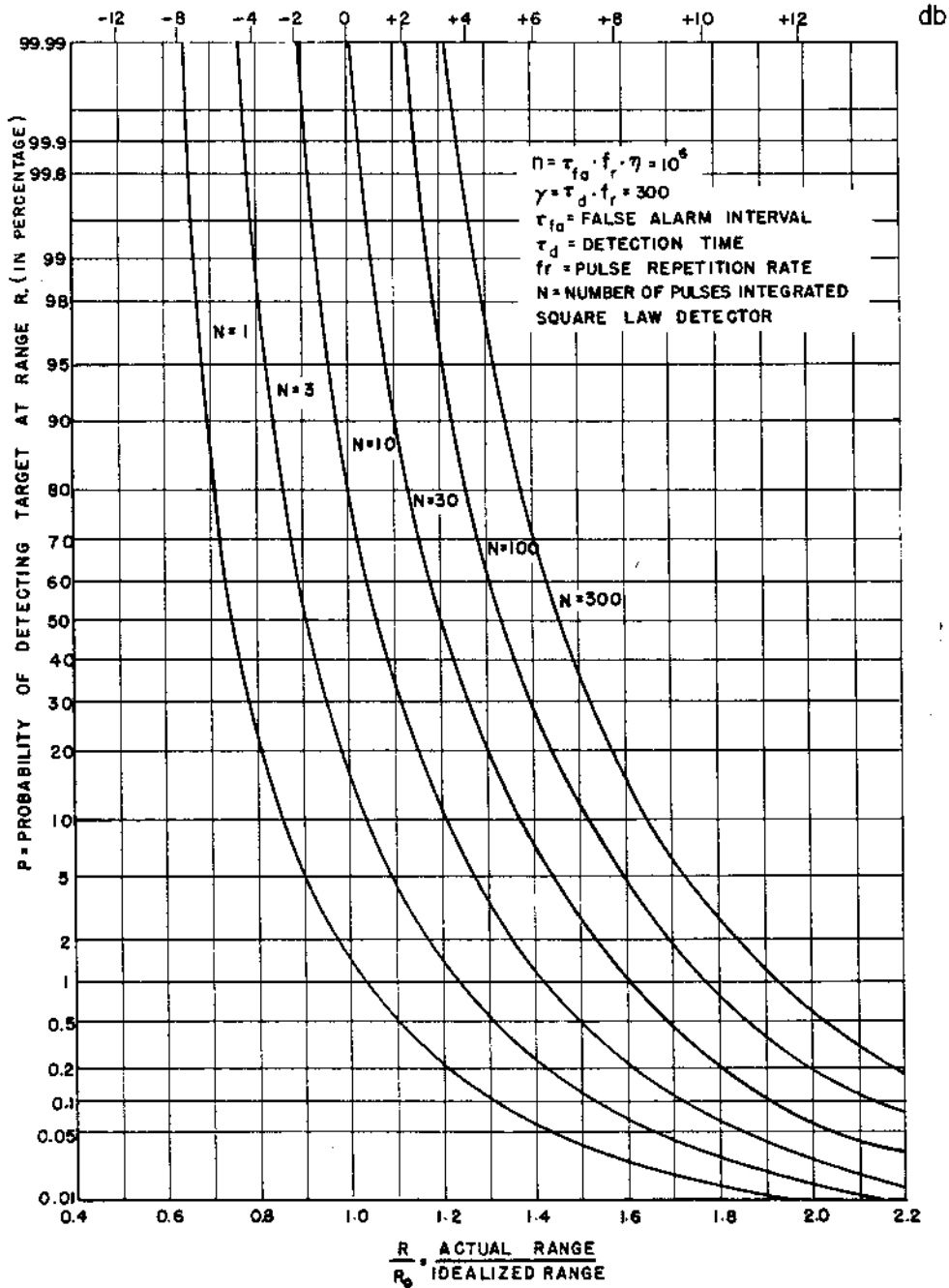
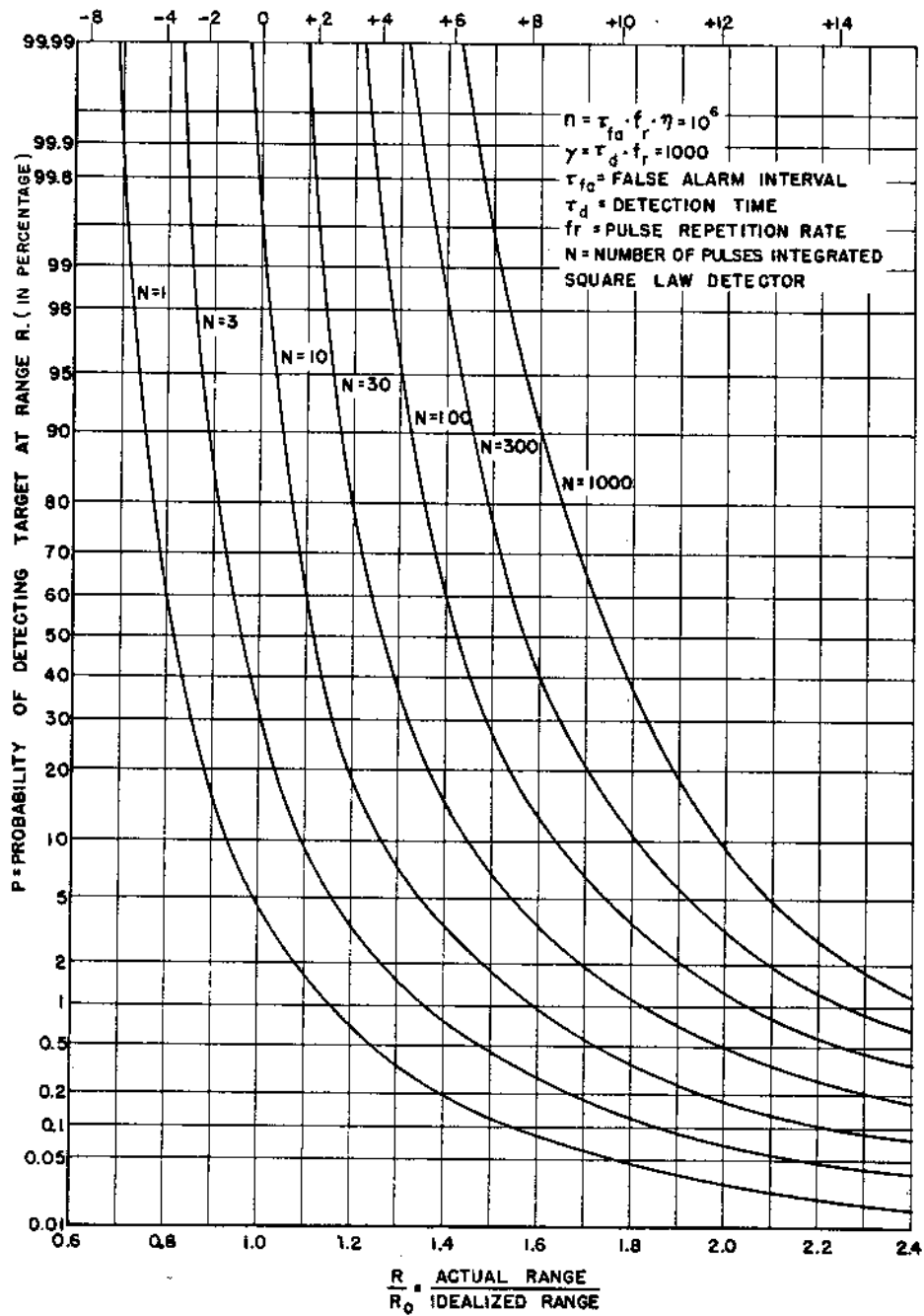


FIG. 13



db

FIG. 14

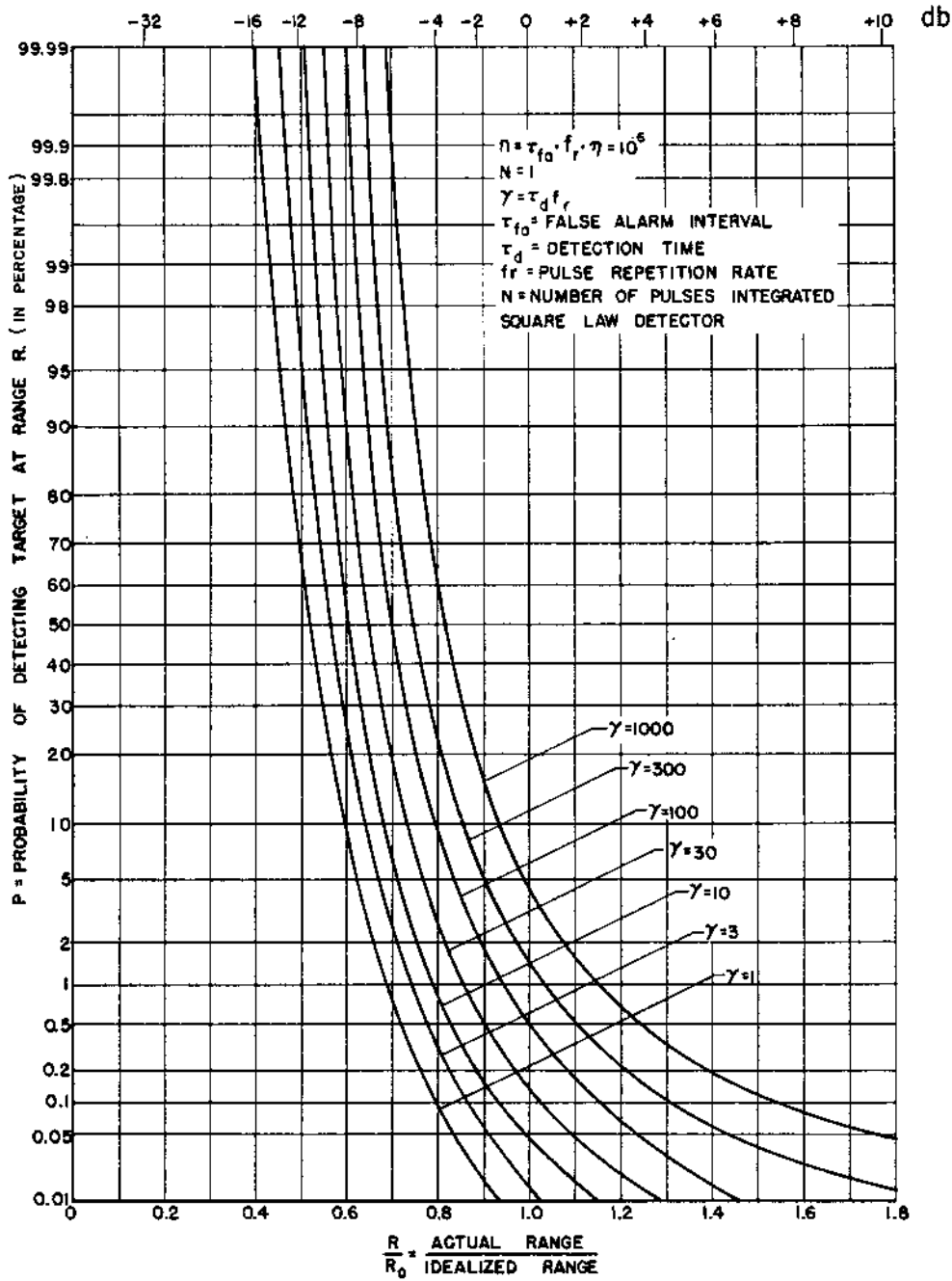
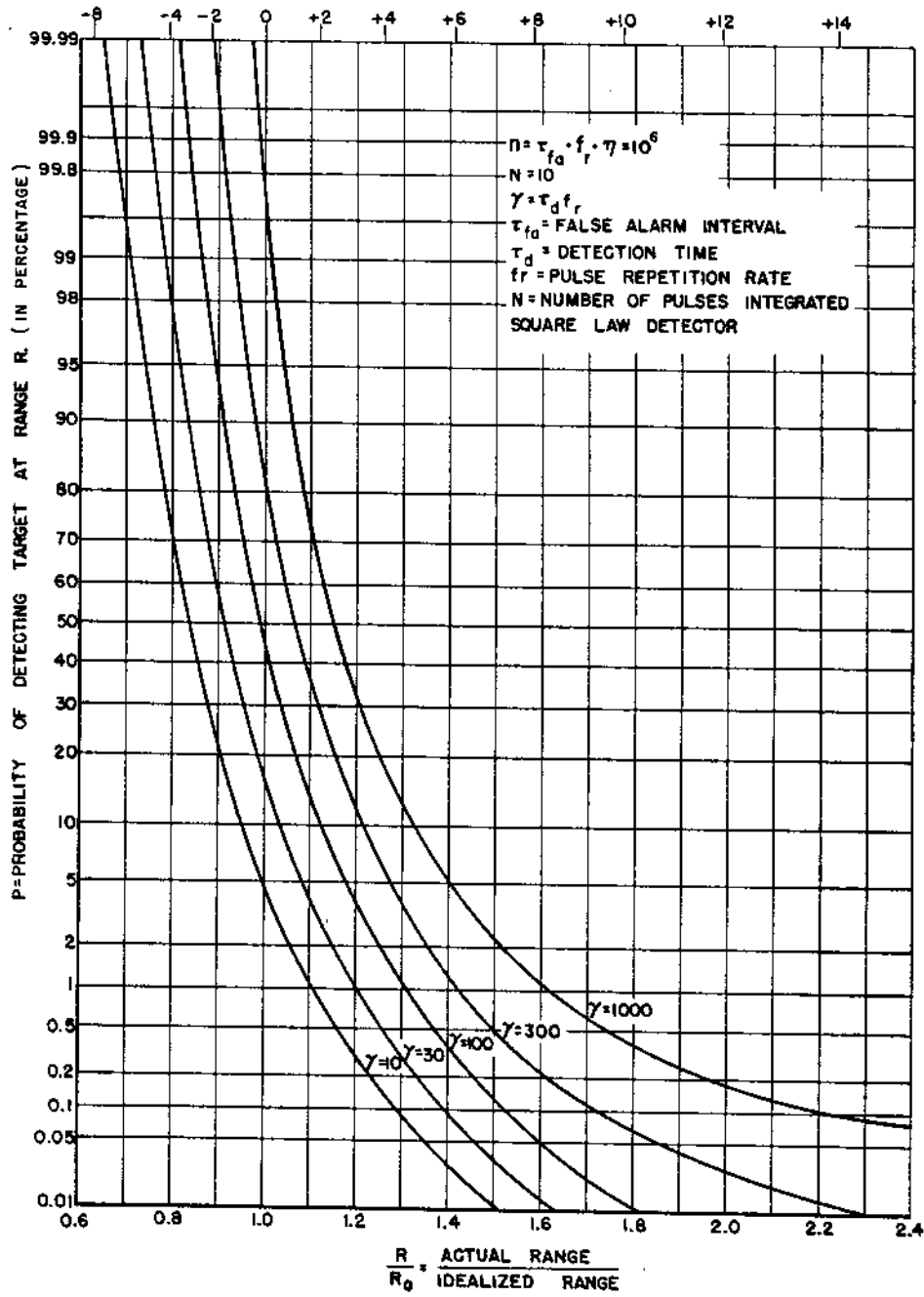


FIG. 15



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FIG. 16

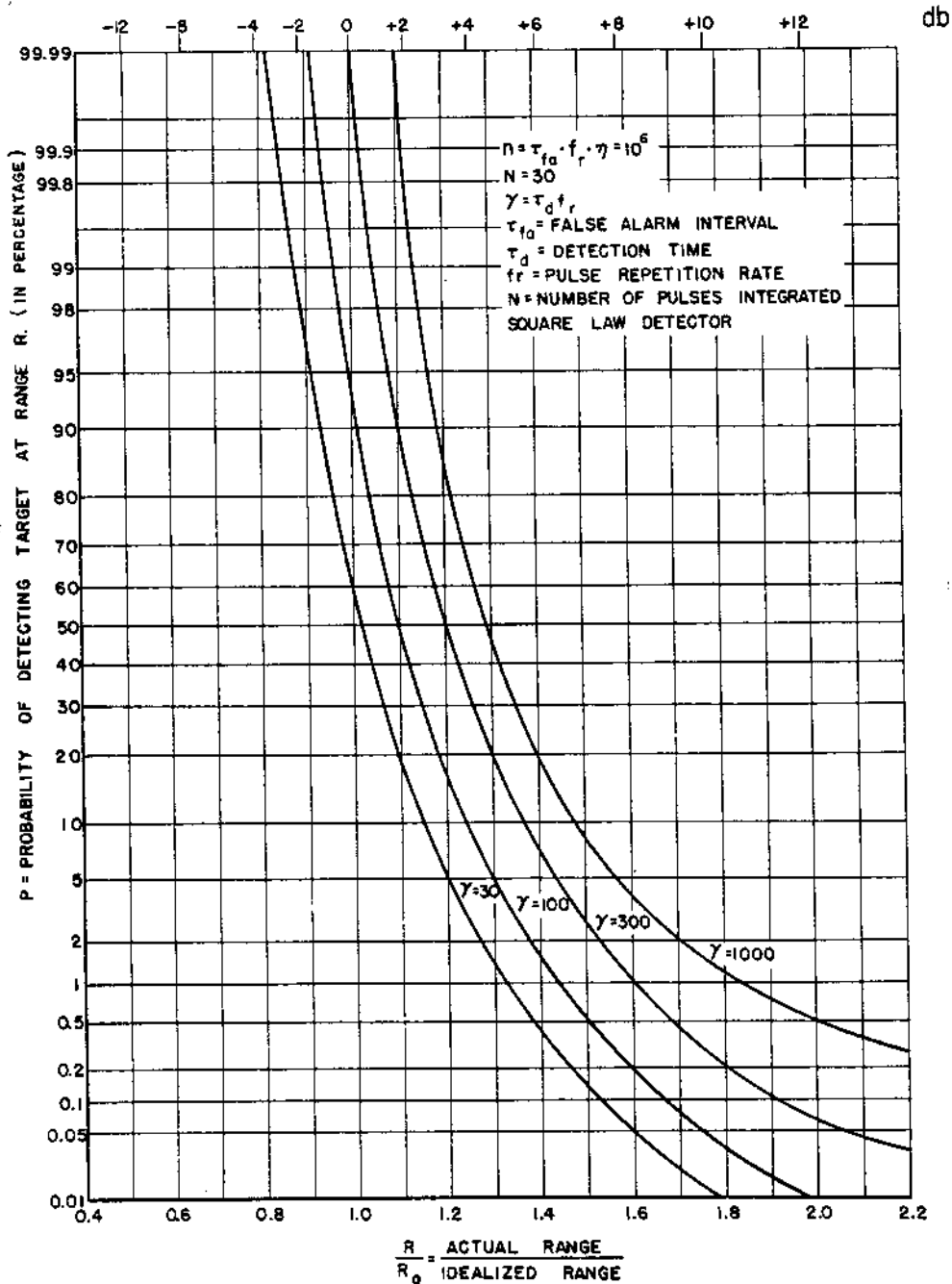


FIG. 17

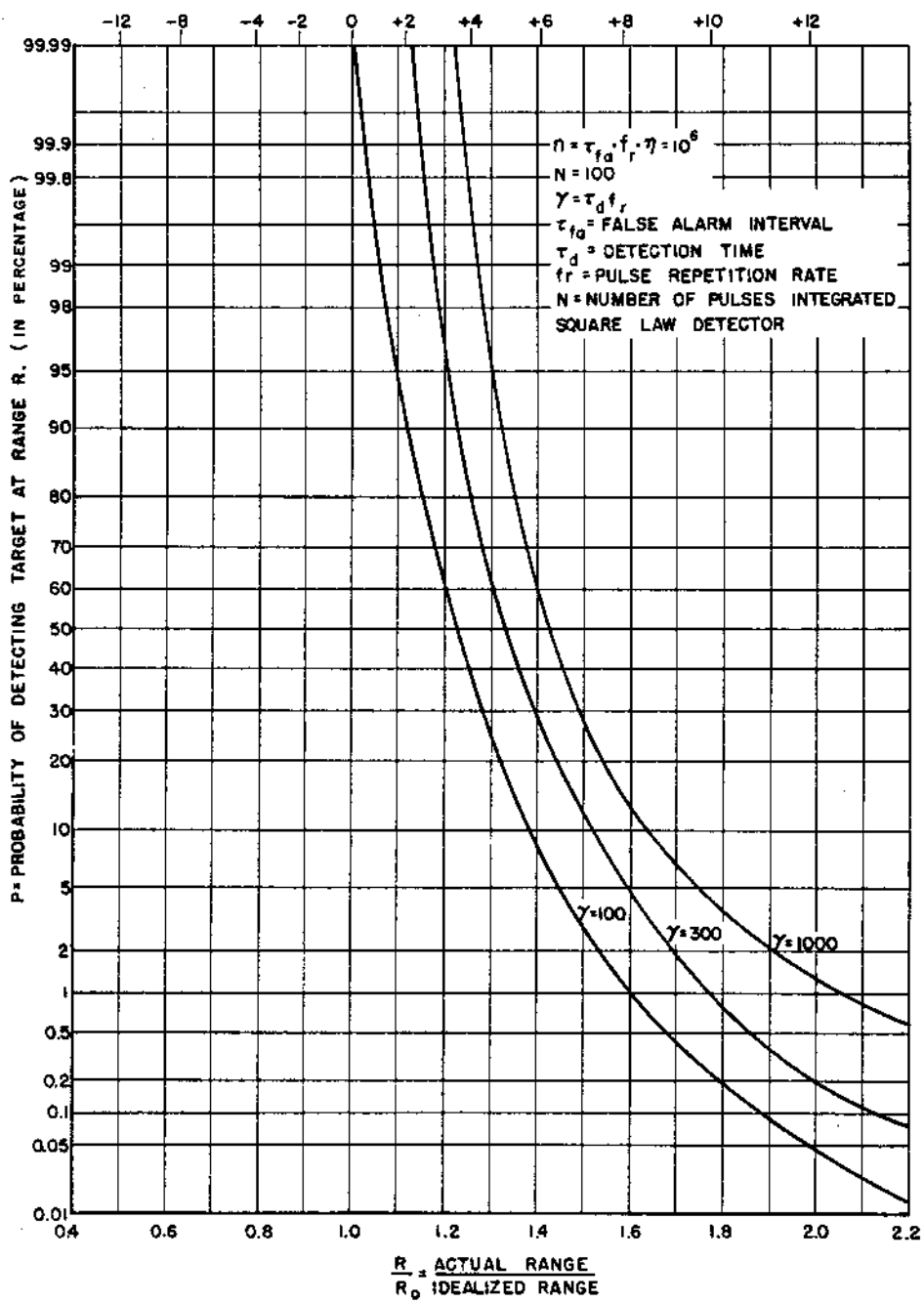


FIG. 18

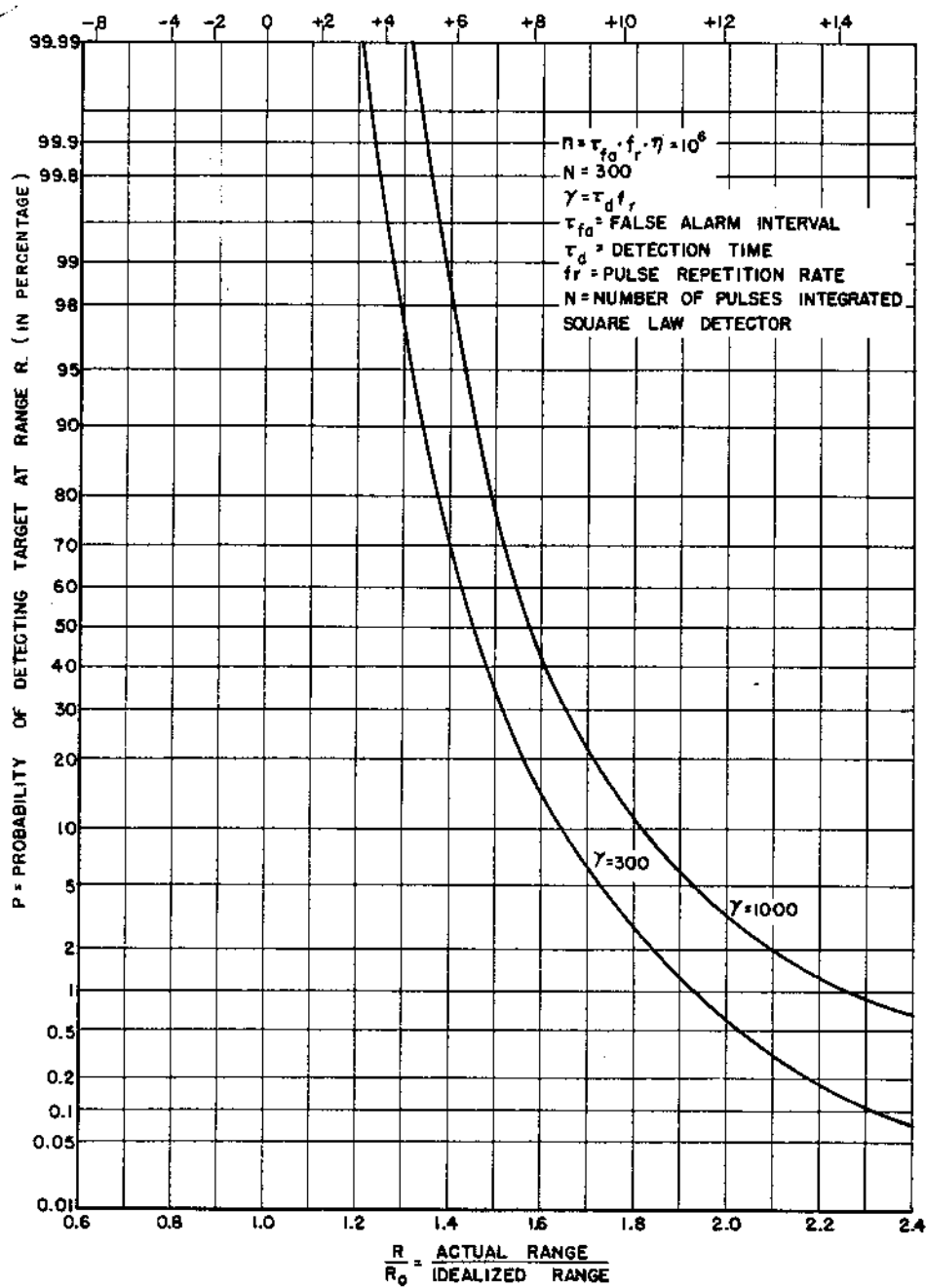


FIG. 19

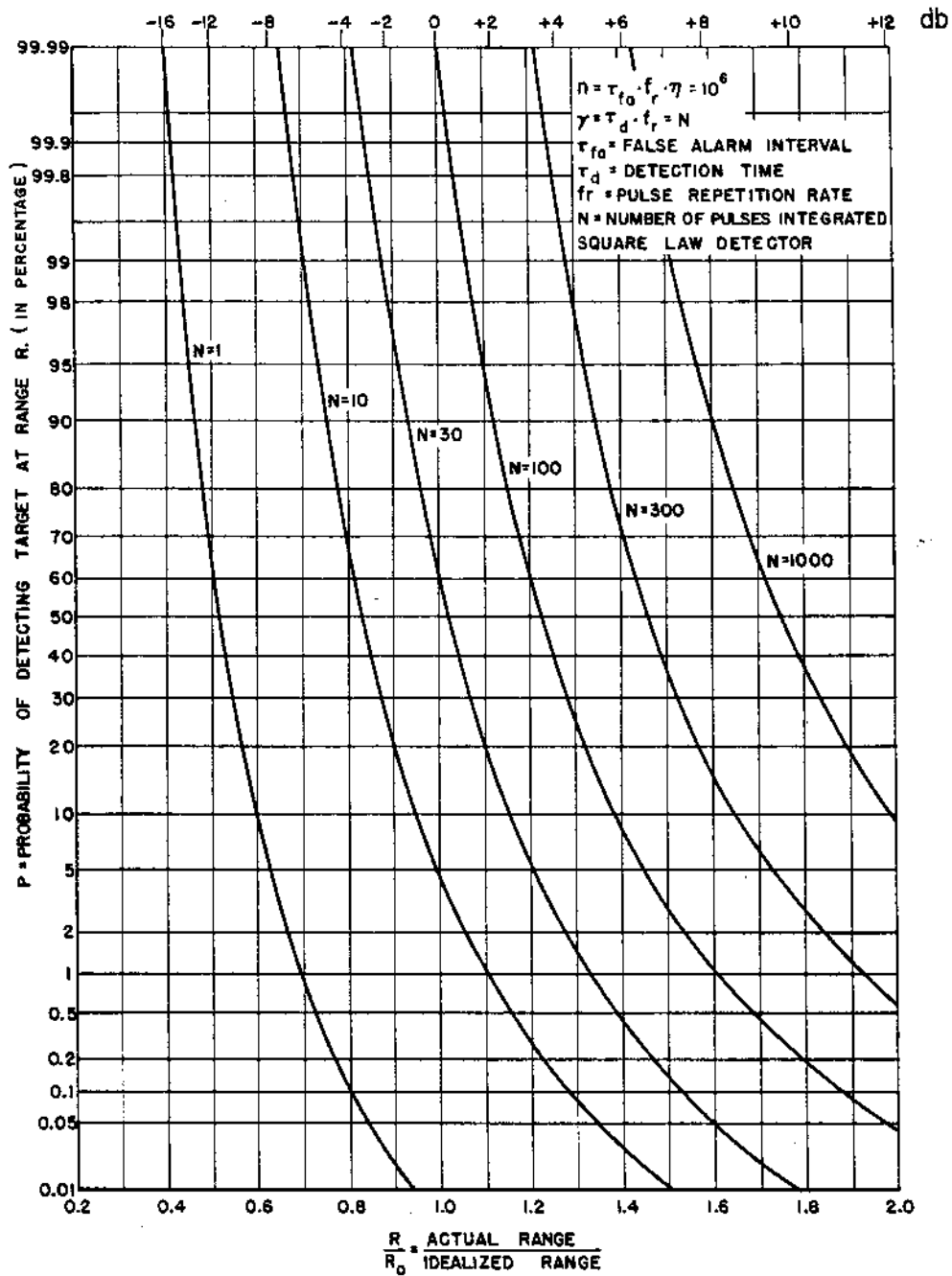


FIG. 20

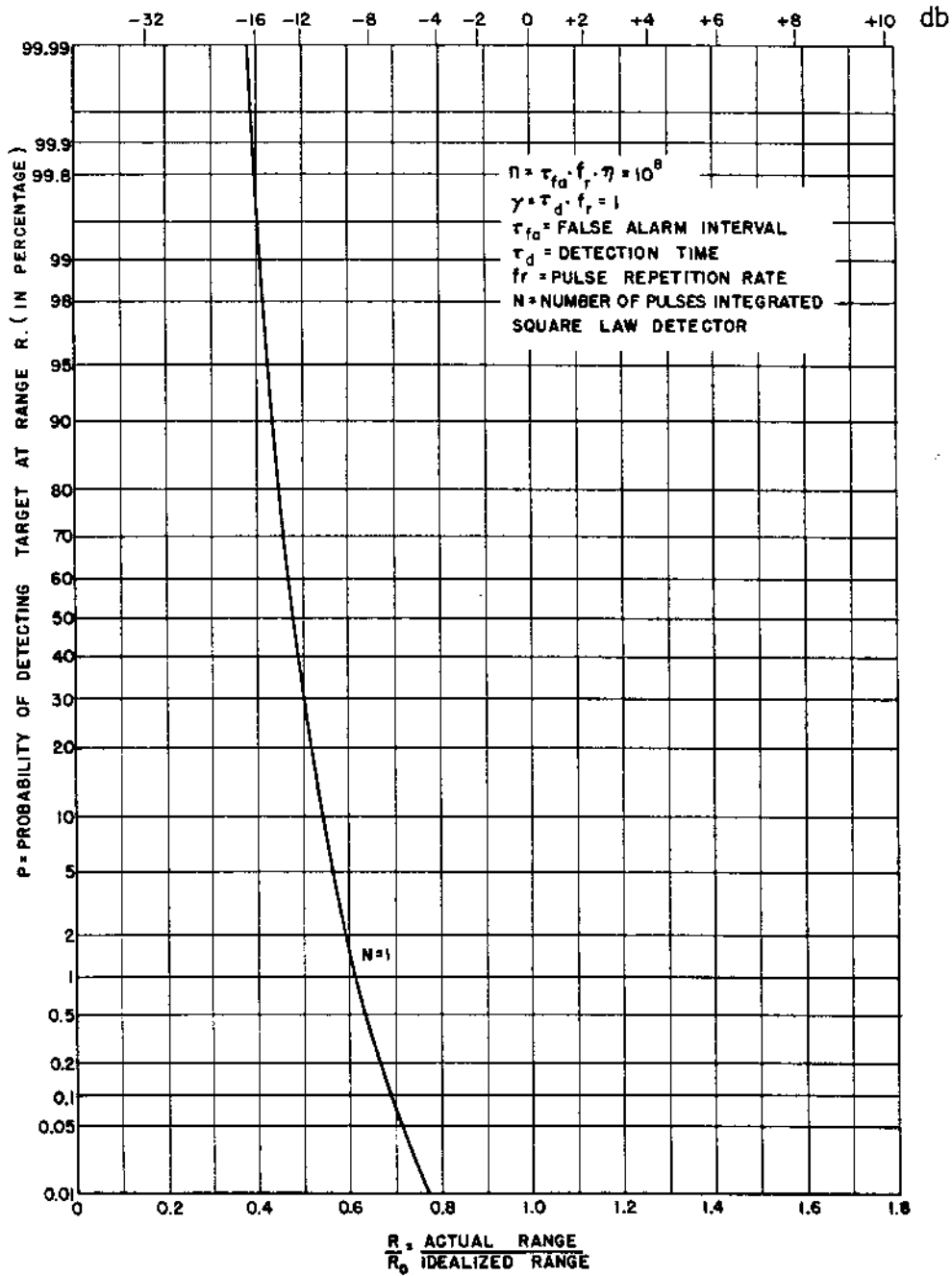


FIG. 21

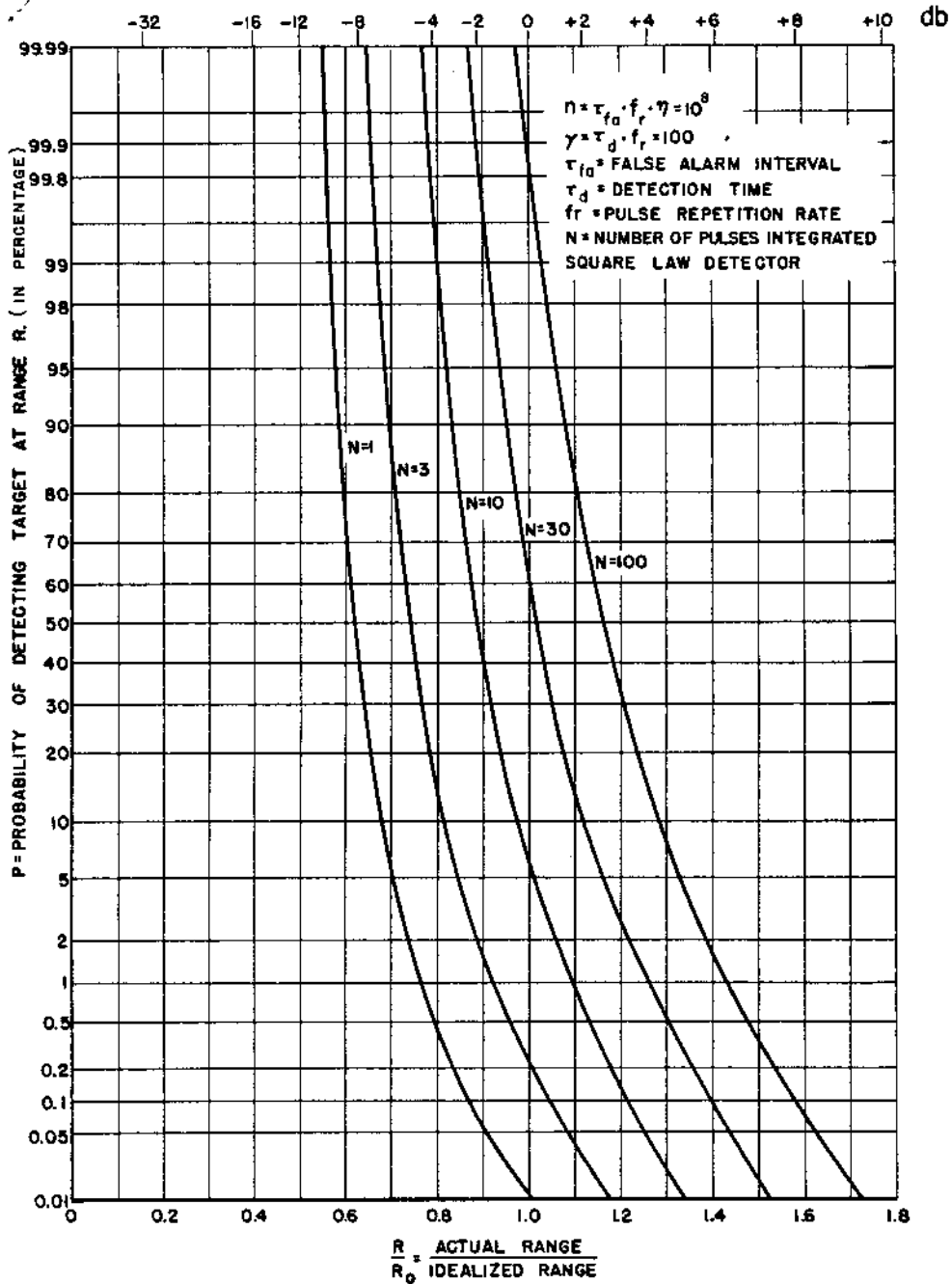


FIG. 23

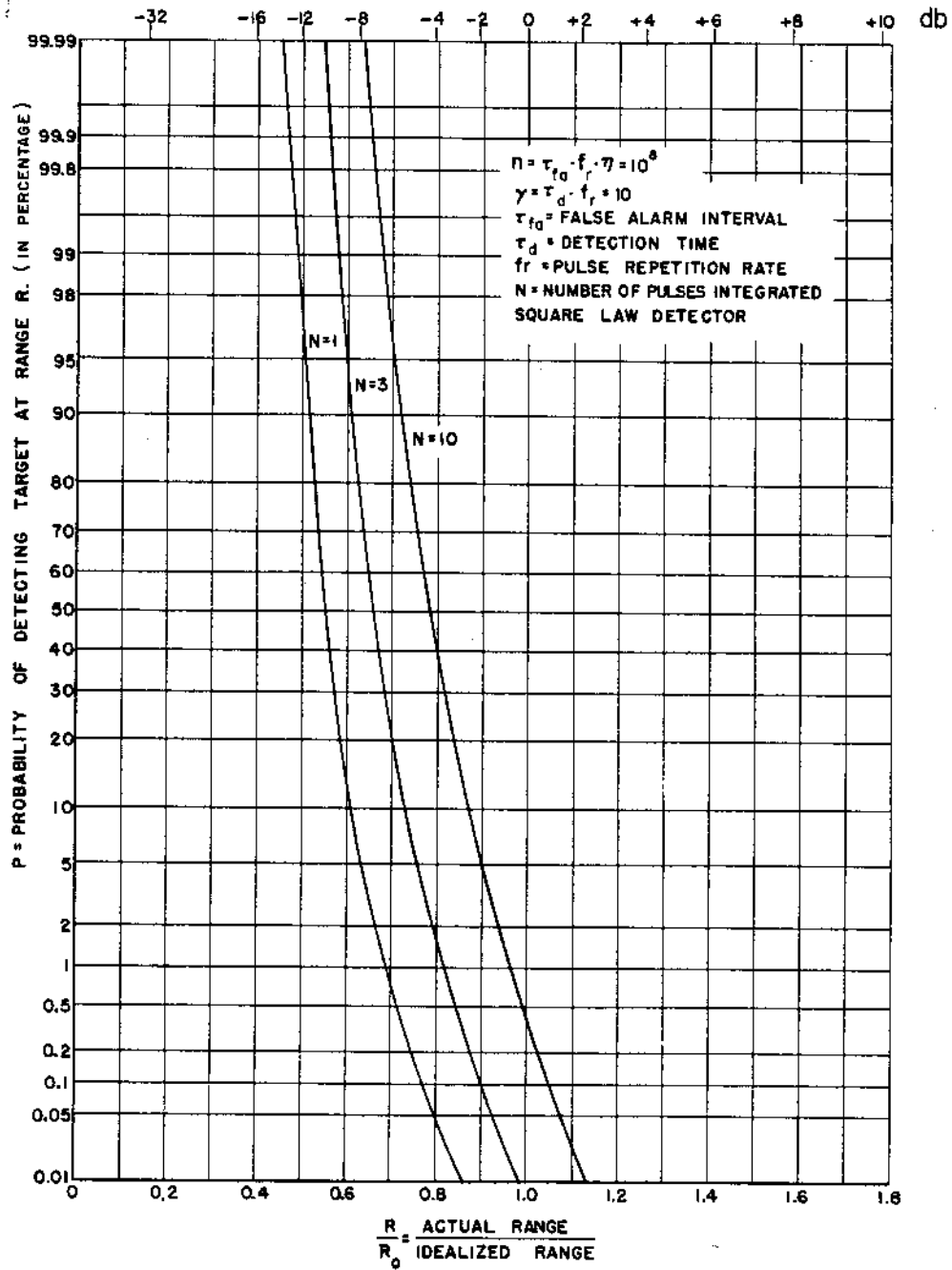


FIG. 22

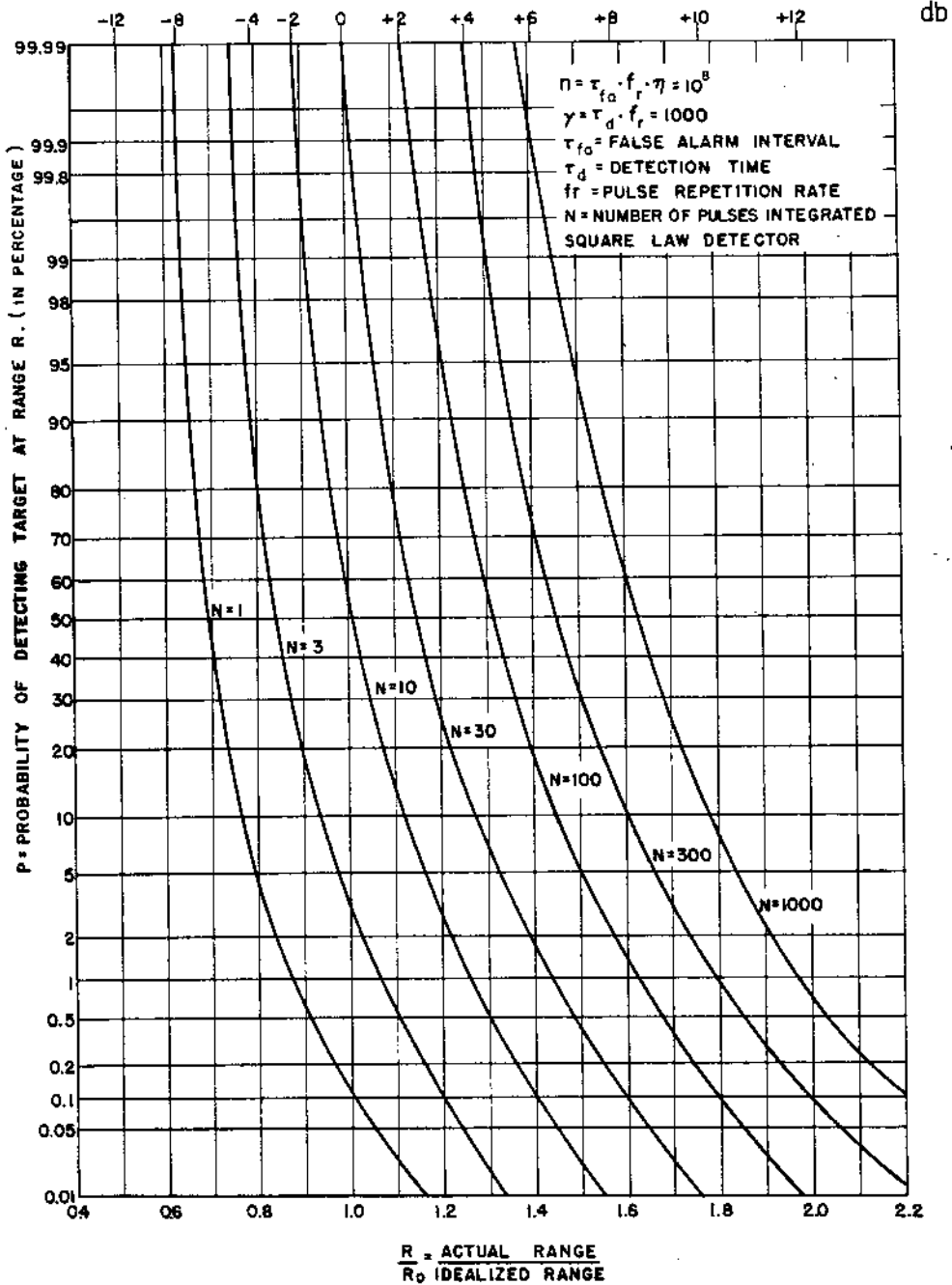


FIG. 24

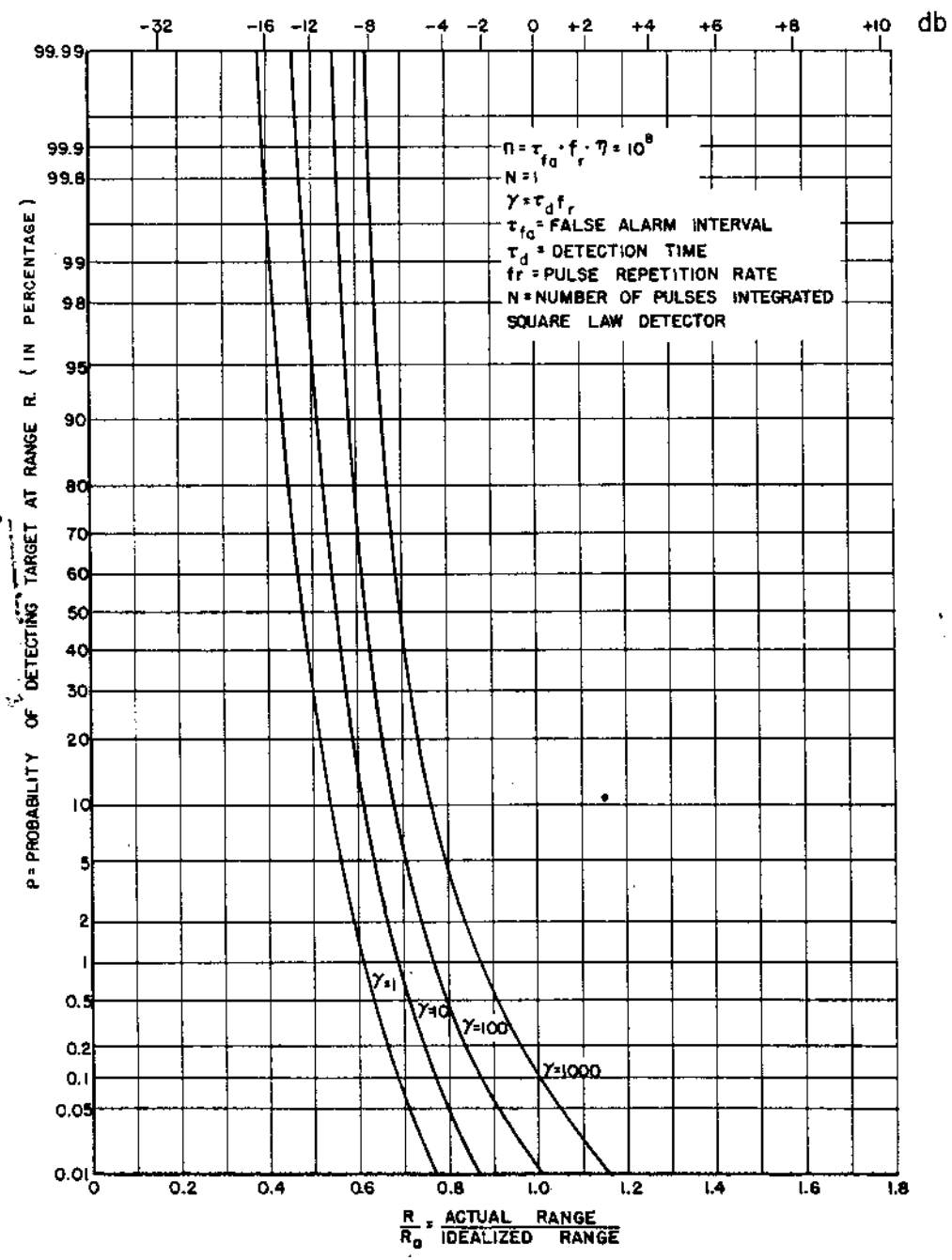


FIG. 25

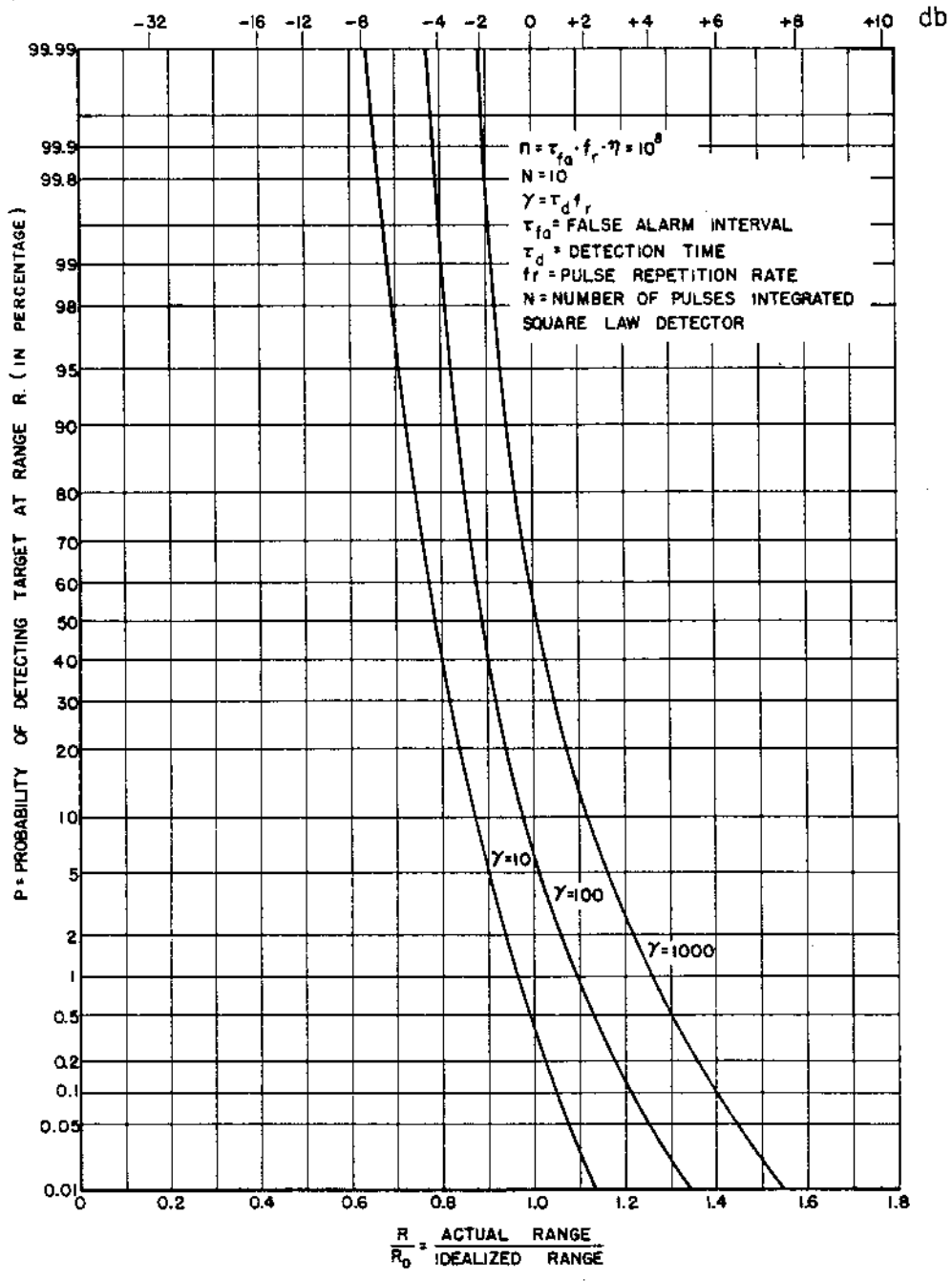


FIG. 26

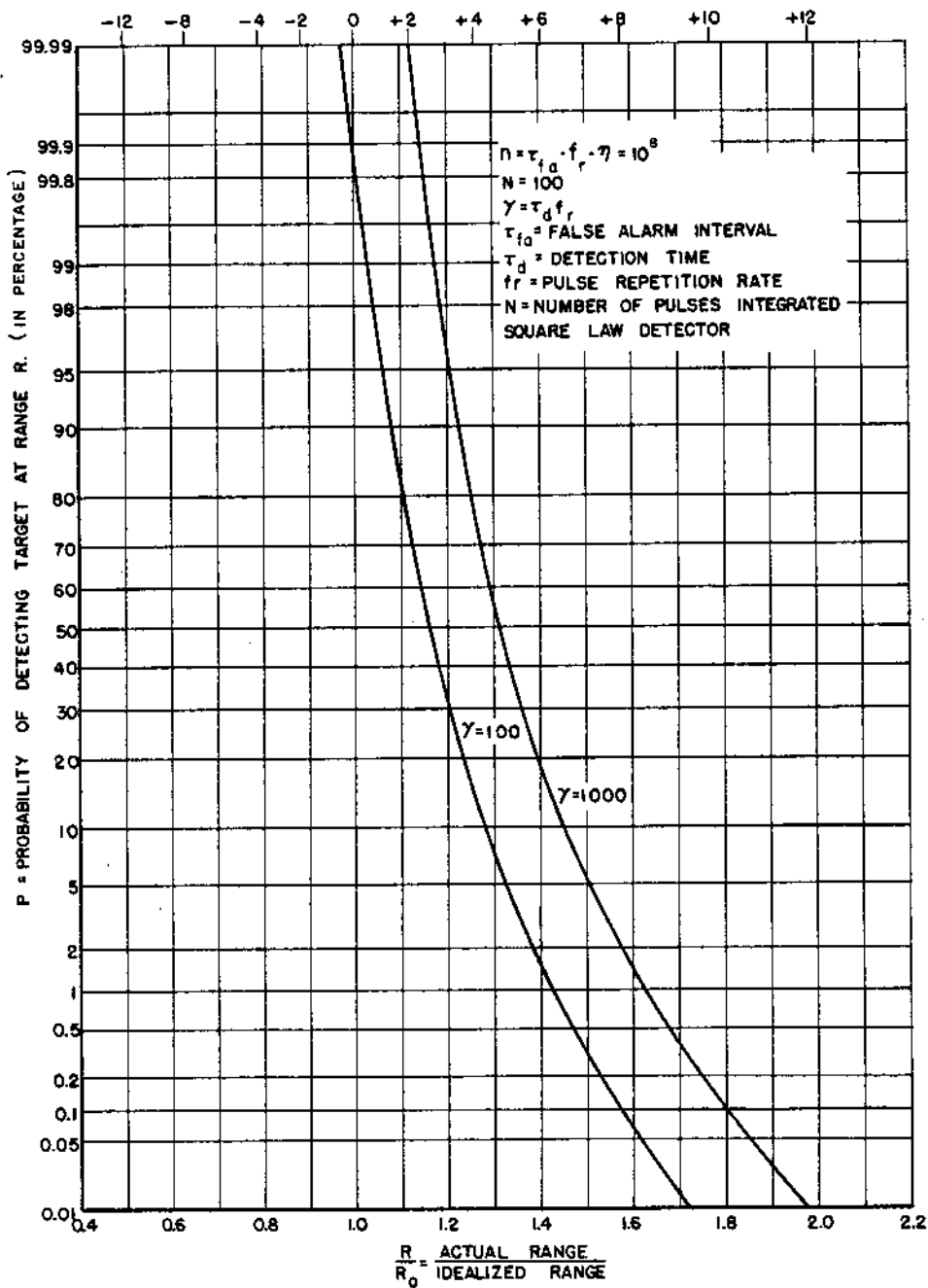


FIG. 27

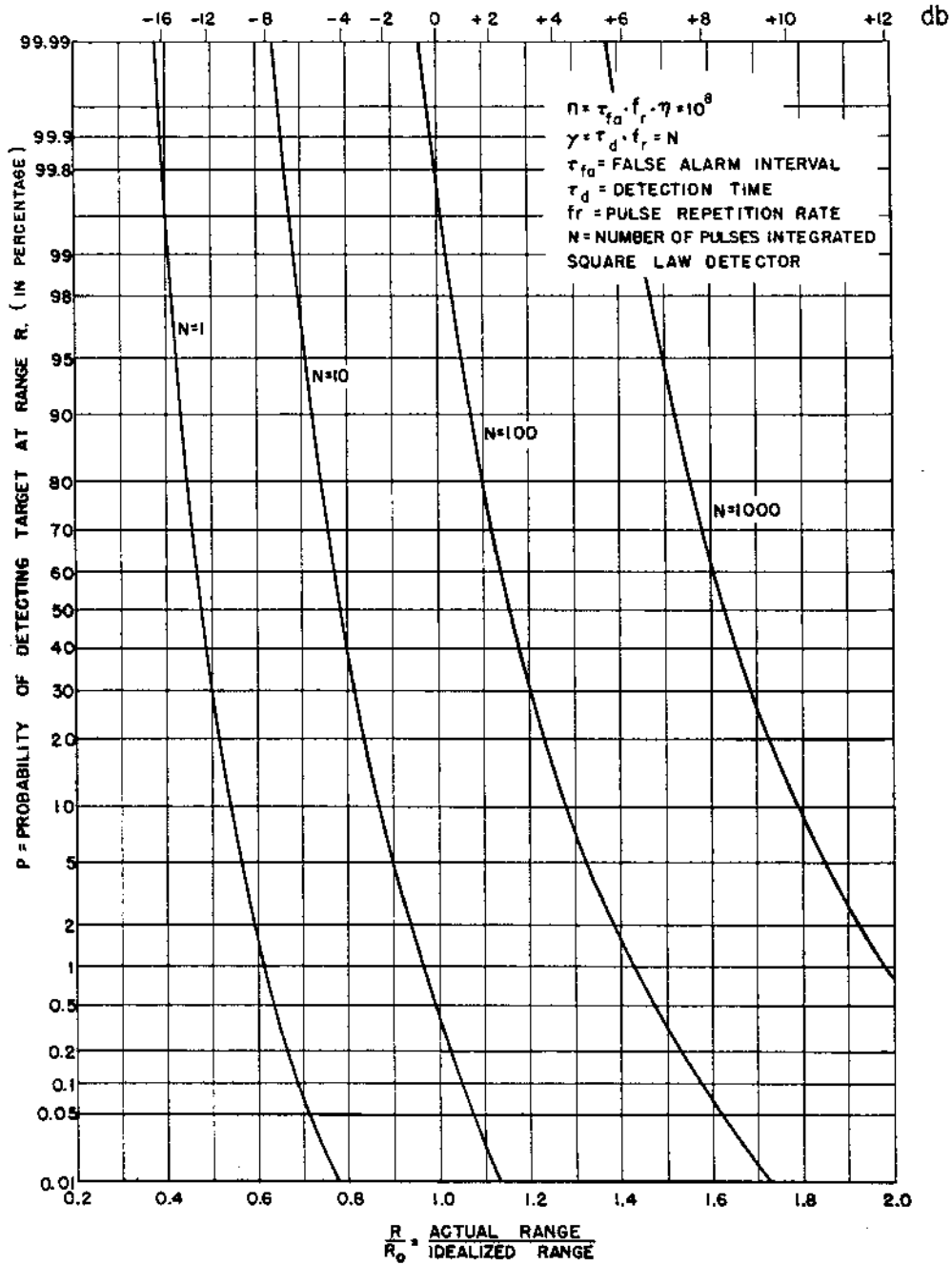


FIG. 28

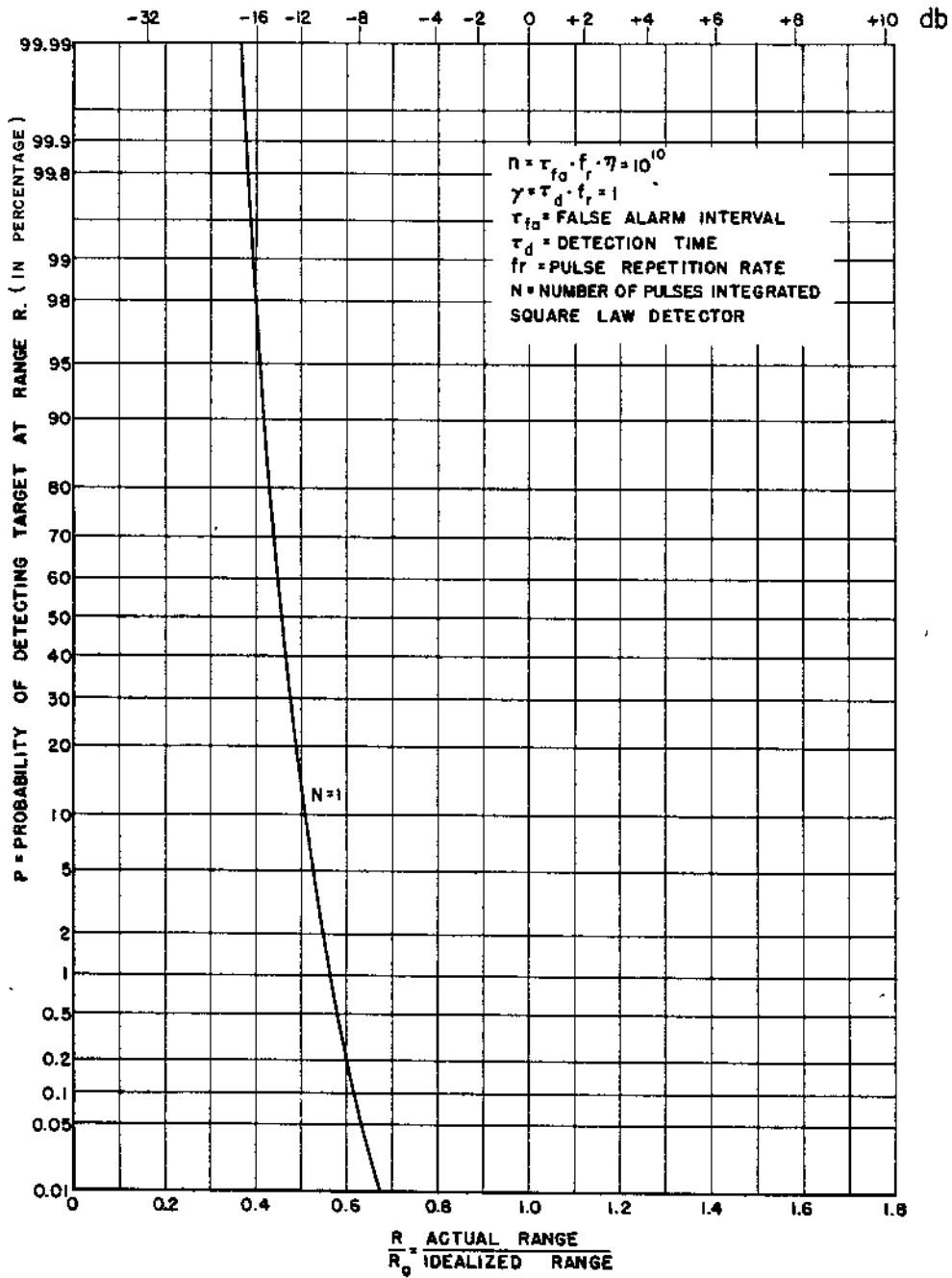


FIG. 29

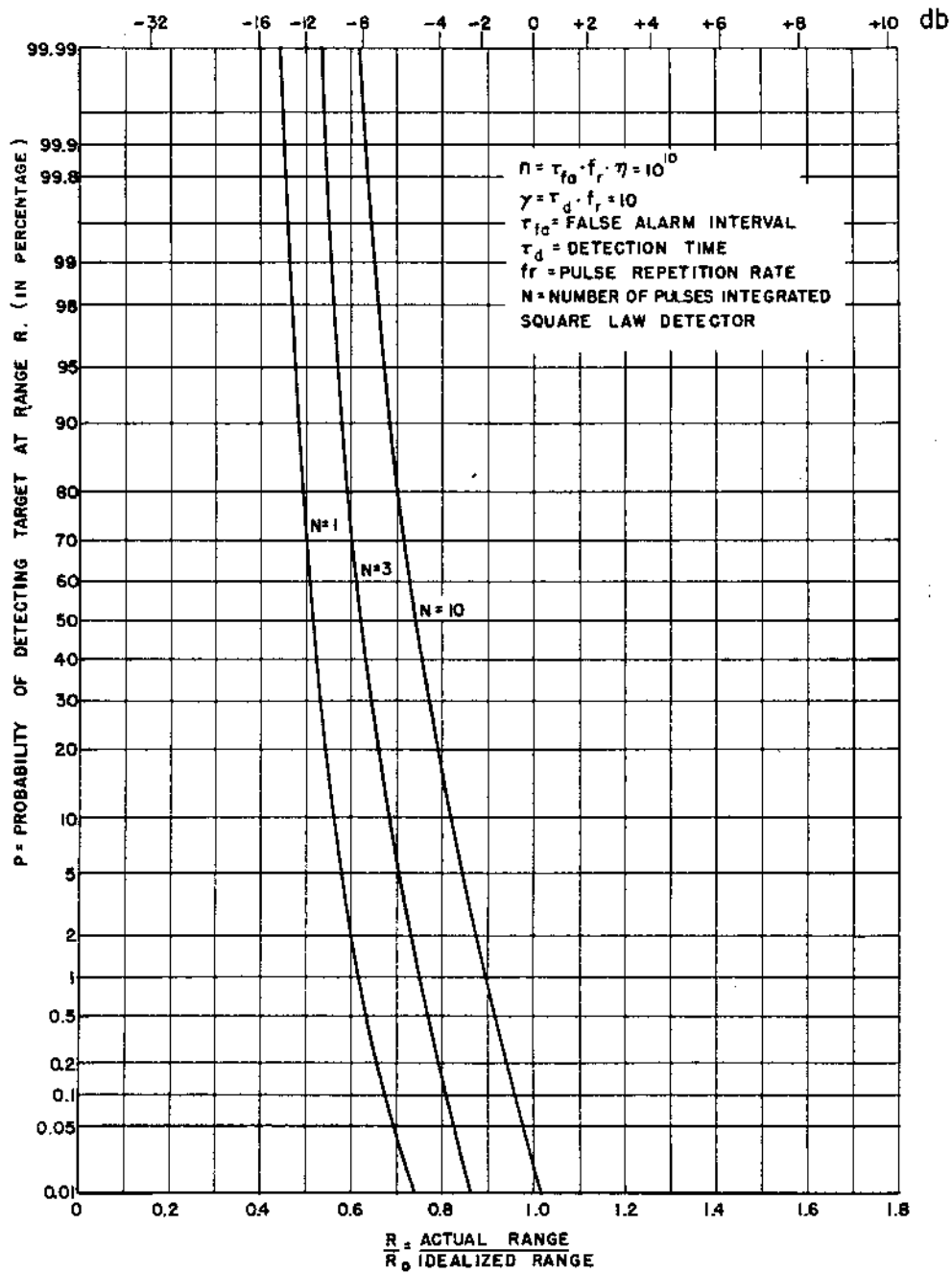


FIG. 30

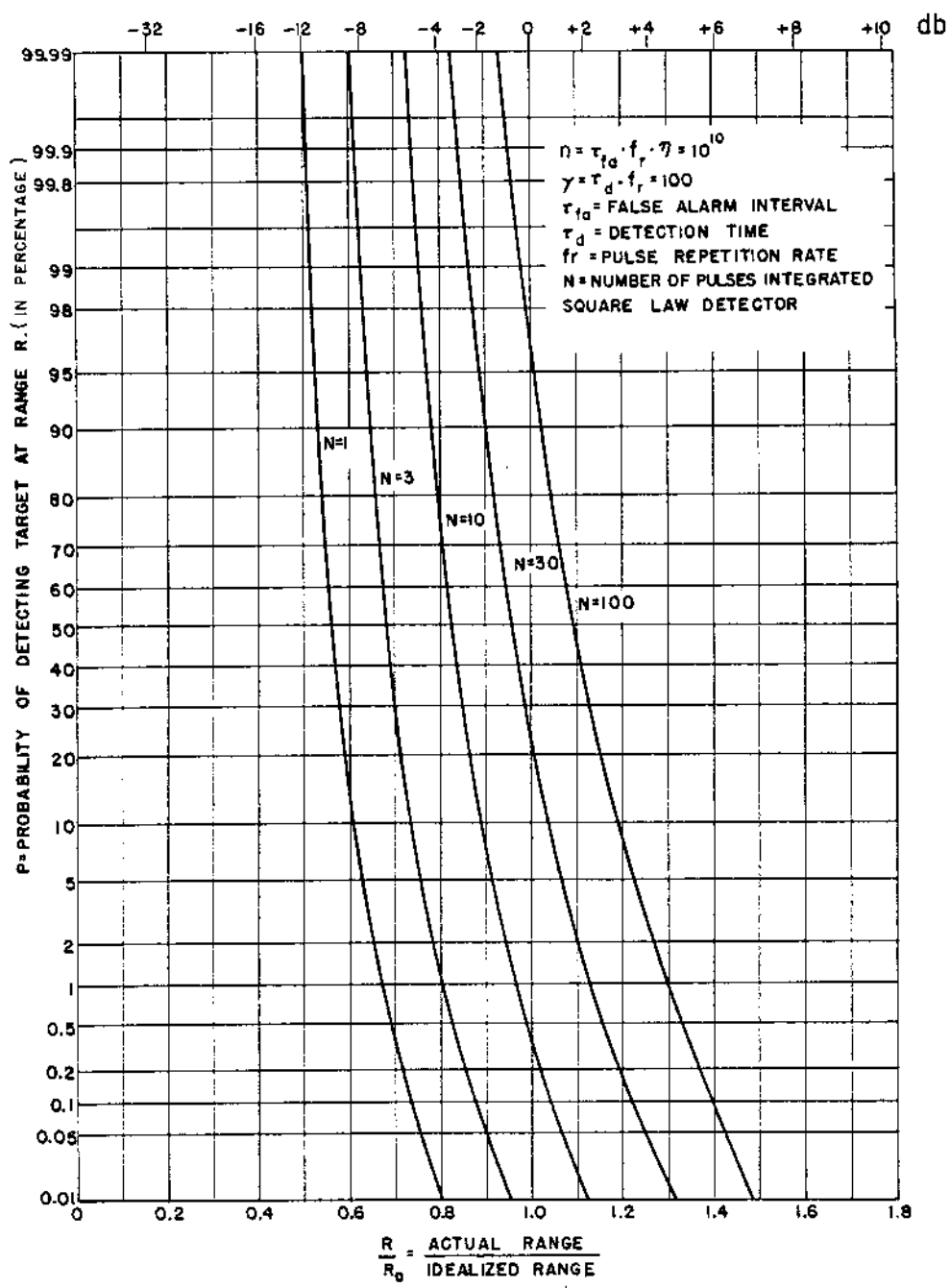


FIG. 31

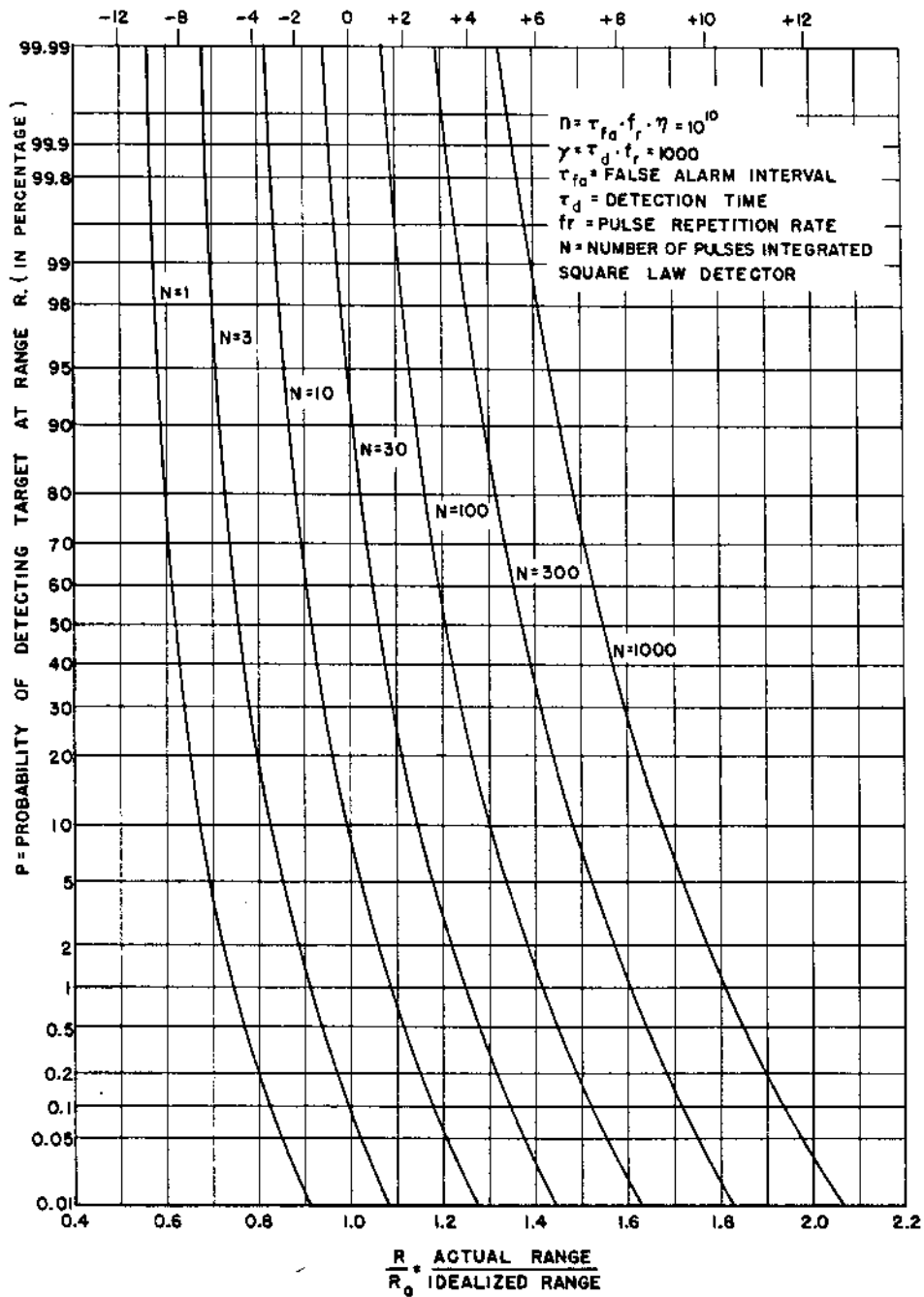


FIG. 32

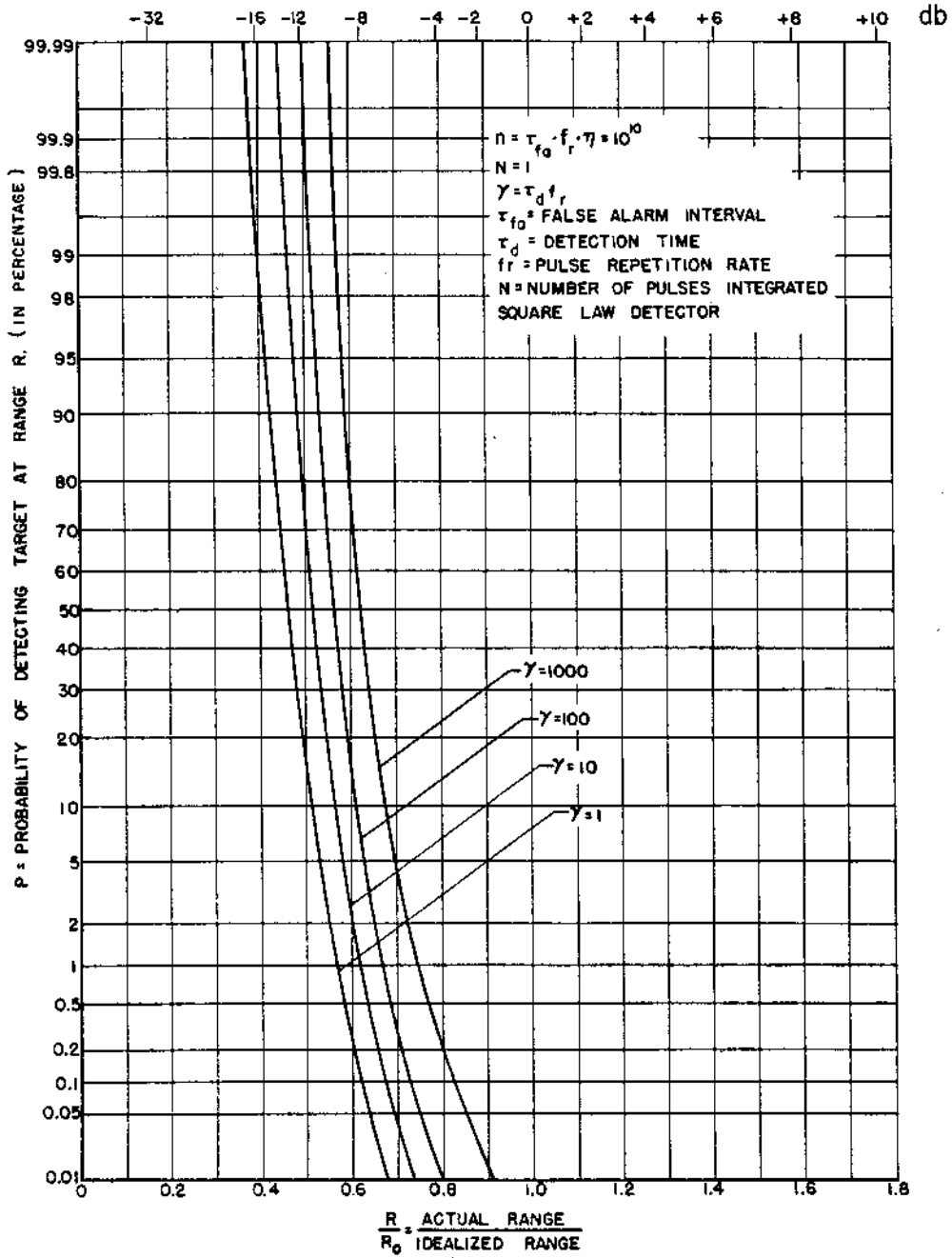


FIG. 33

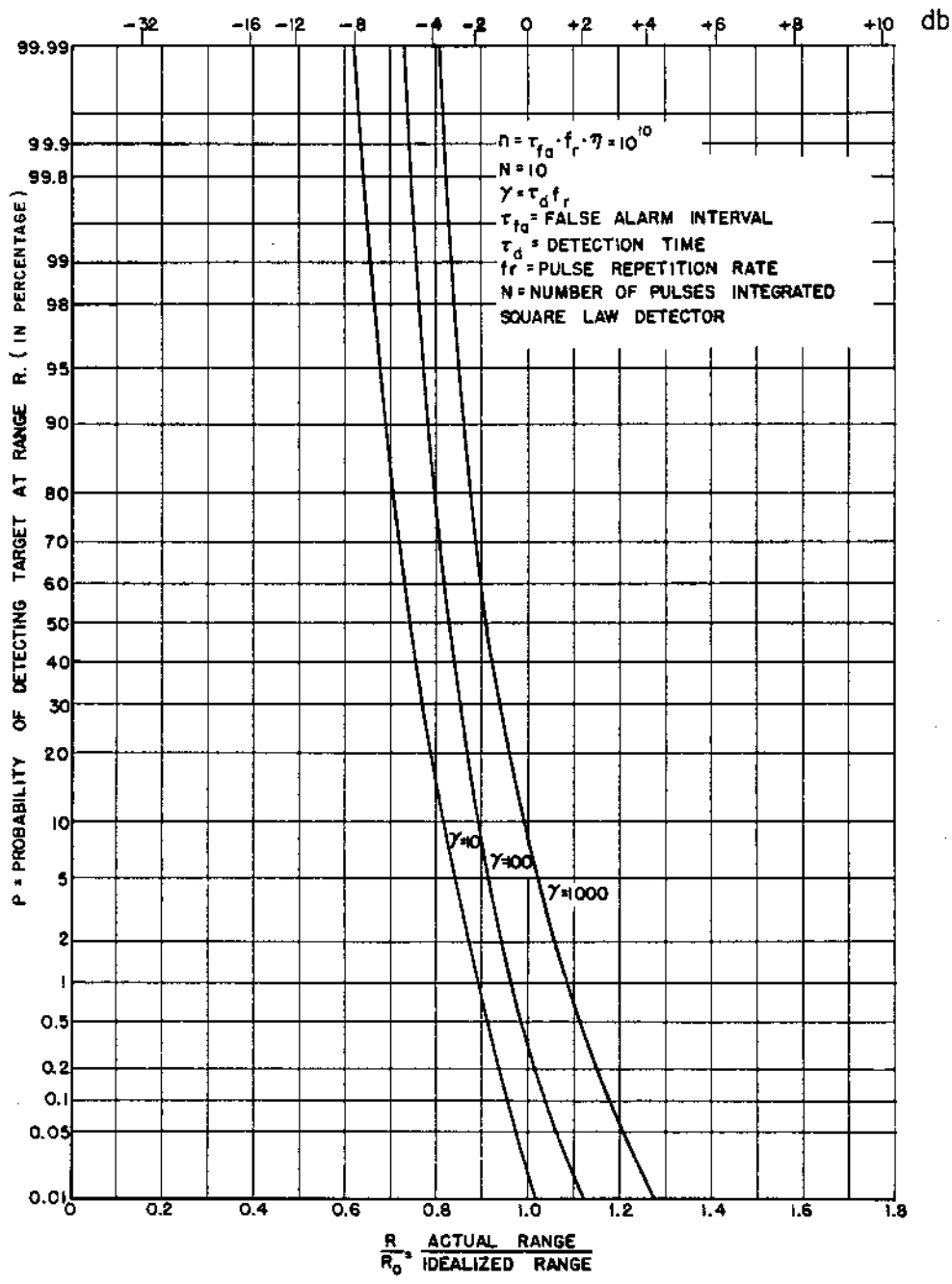


FIG. 34

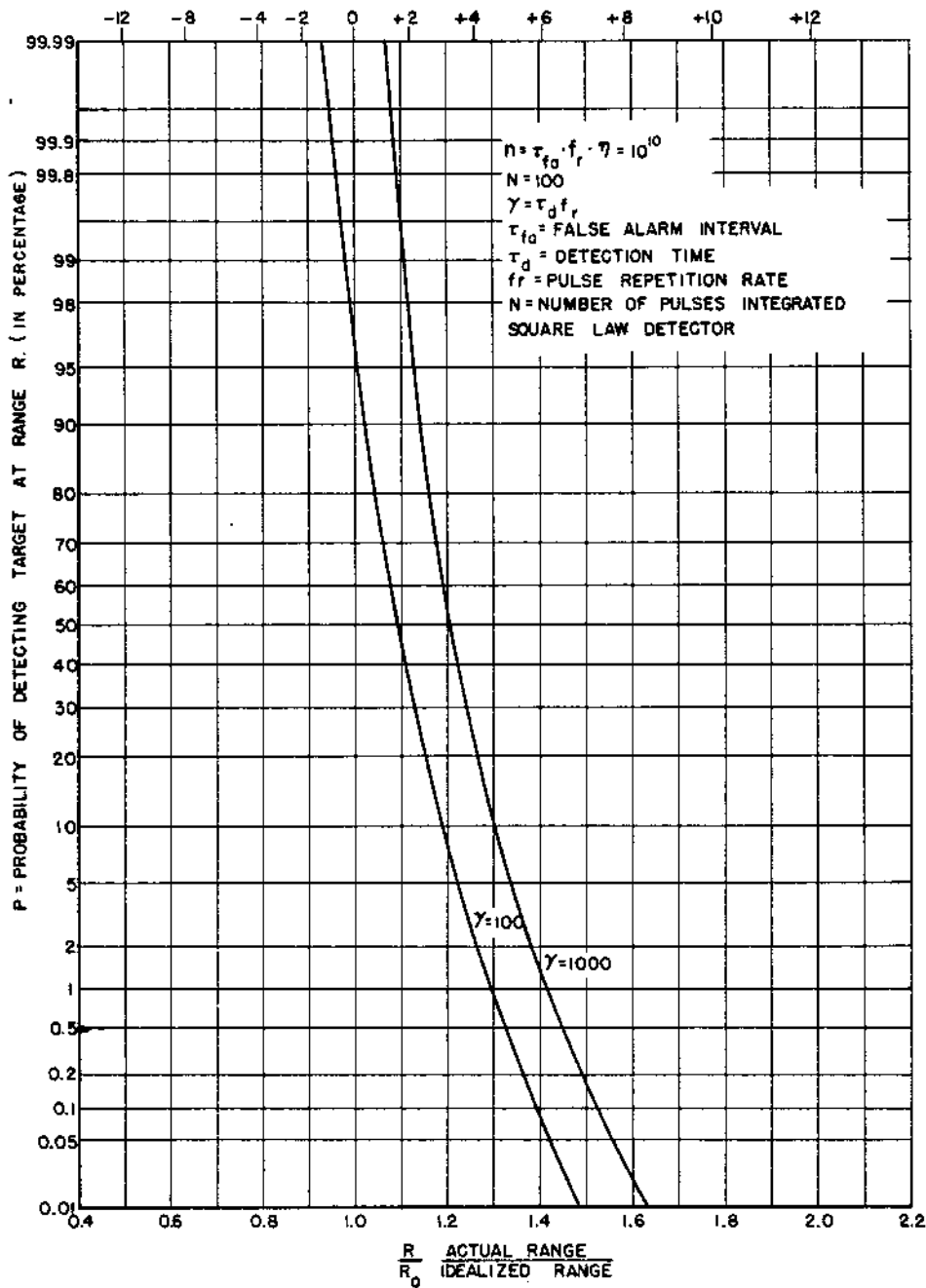


FIG. 35

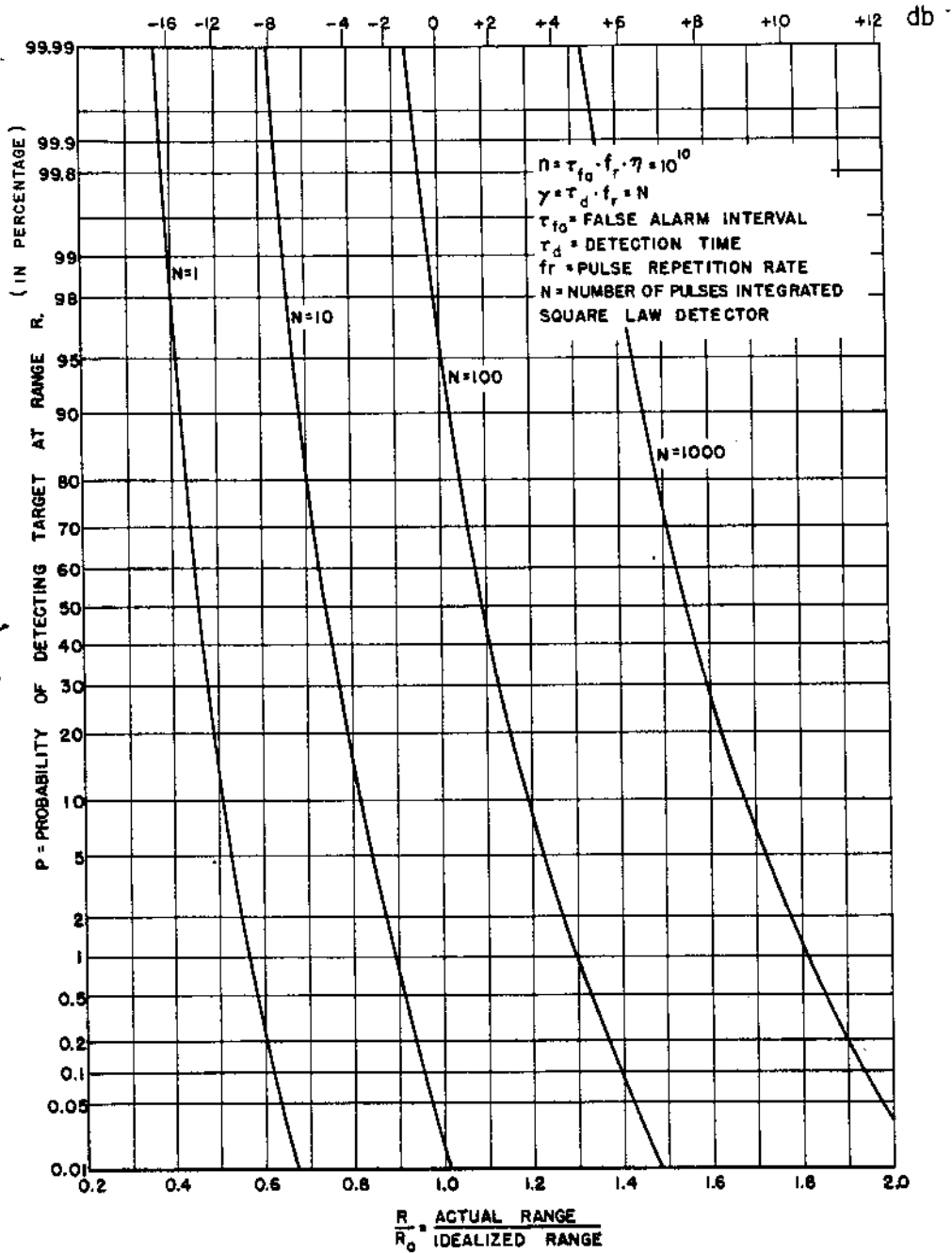


FIG. 36

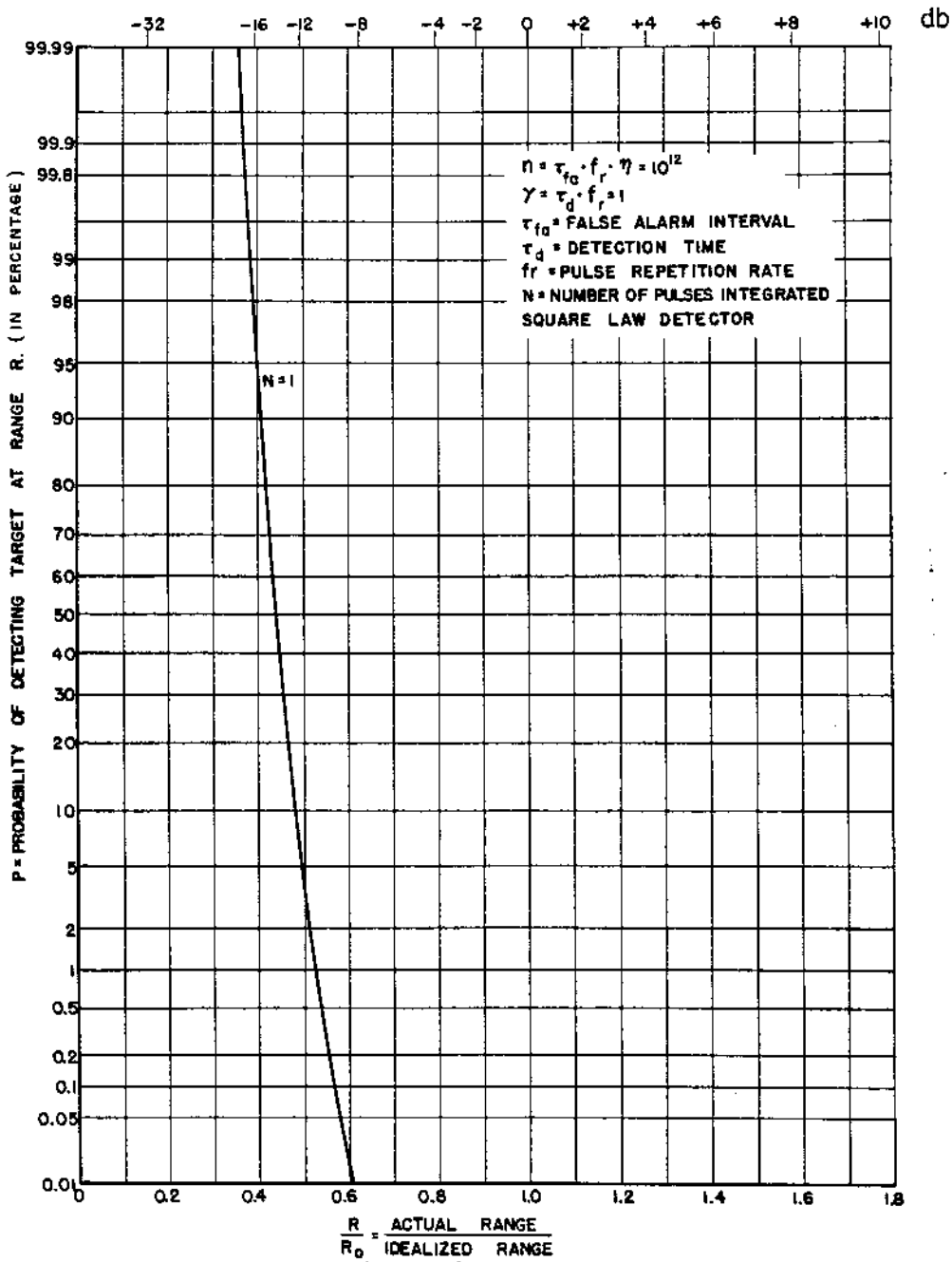


FIG. 37

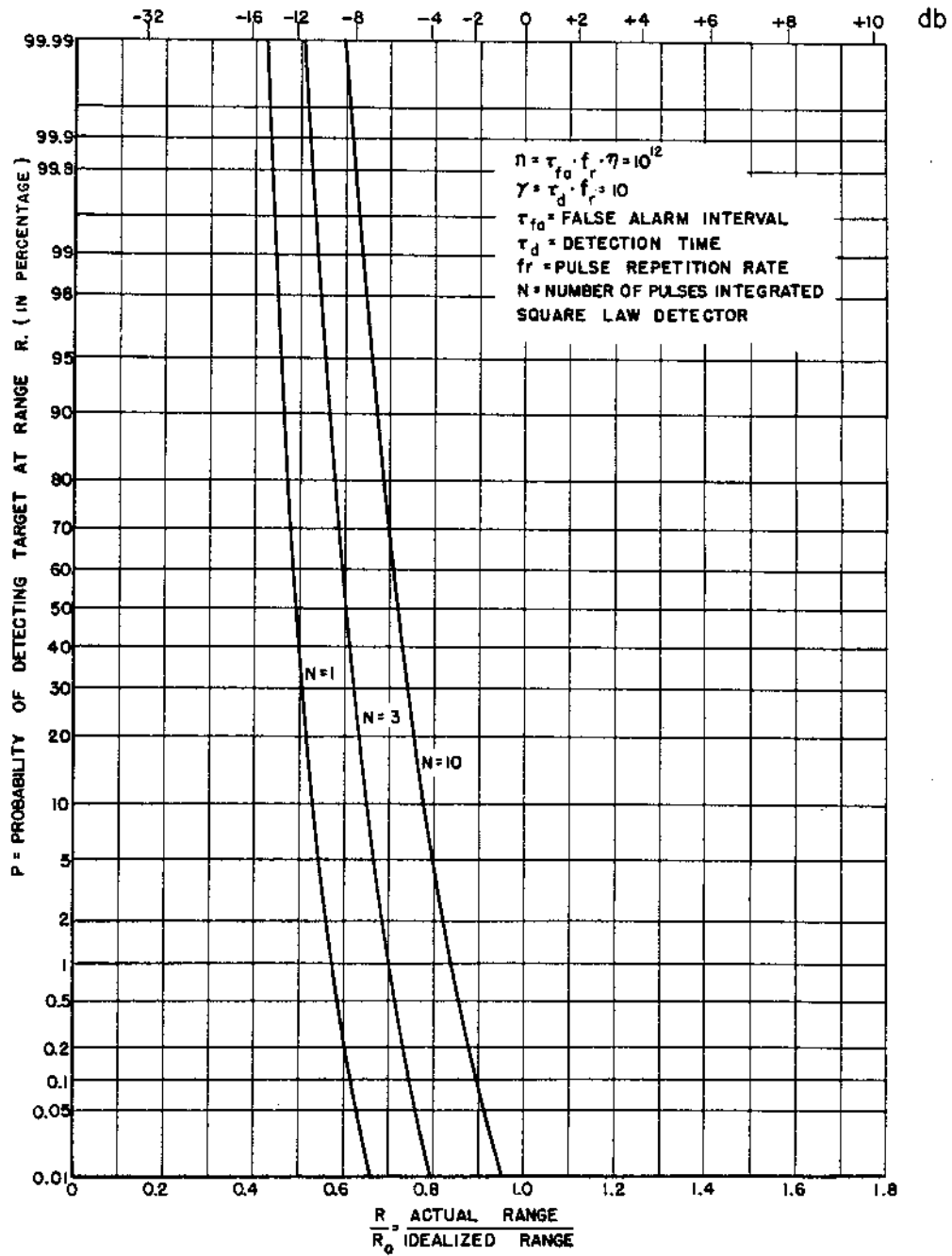


FIG. 38

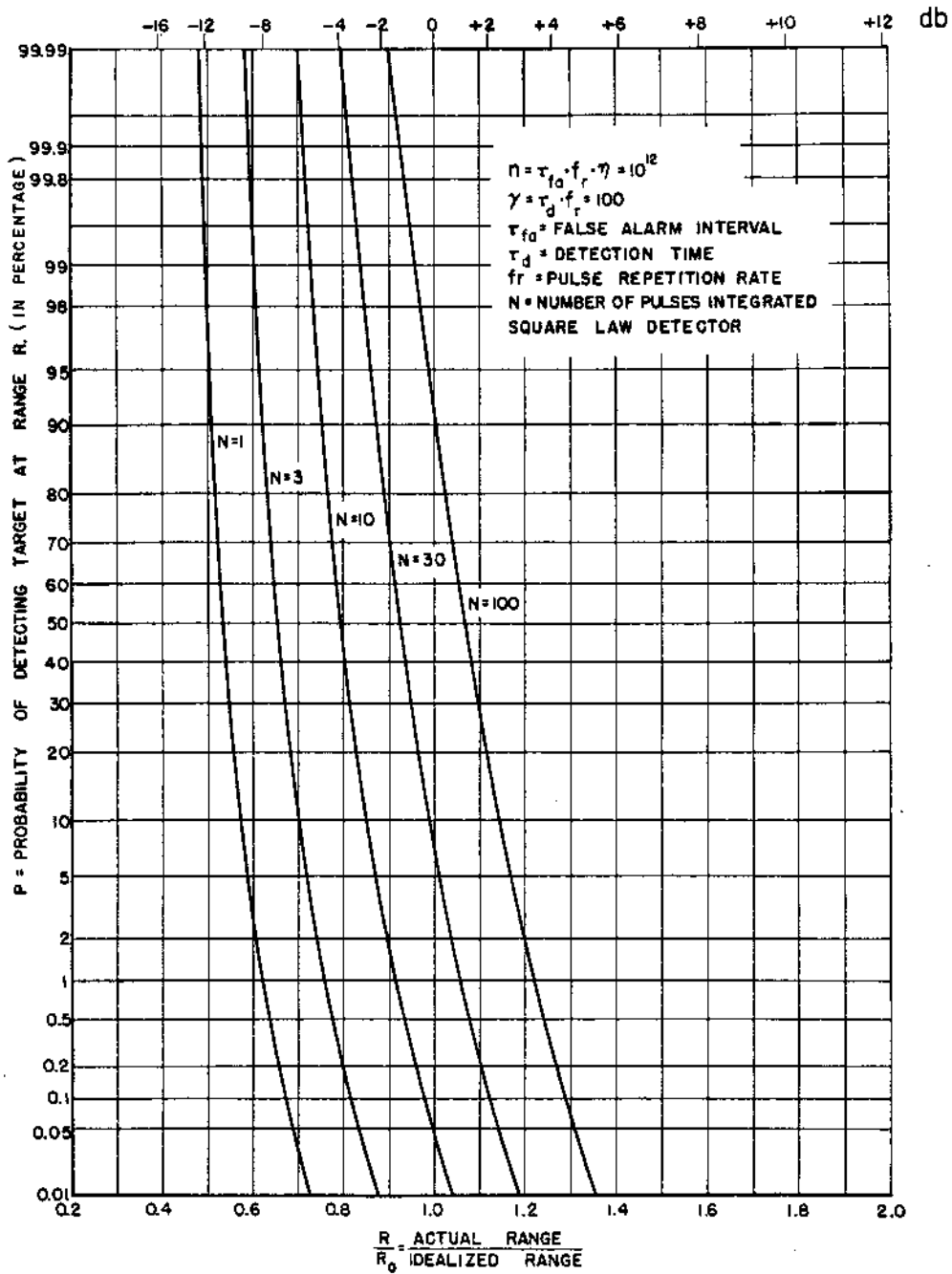


FIG. 39

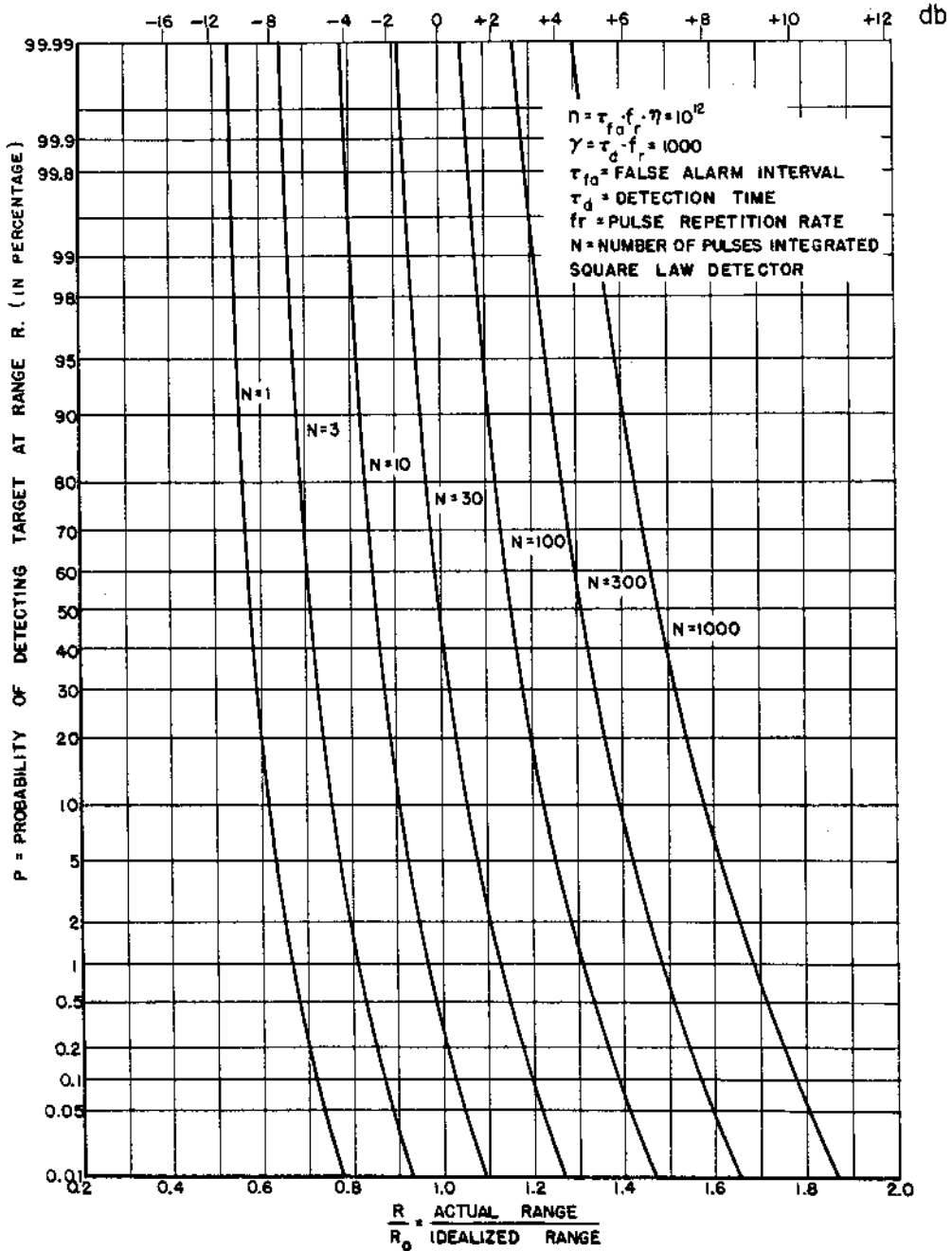


FIG. 40

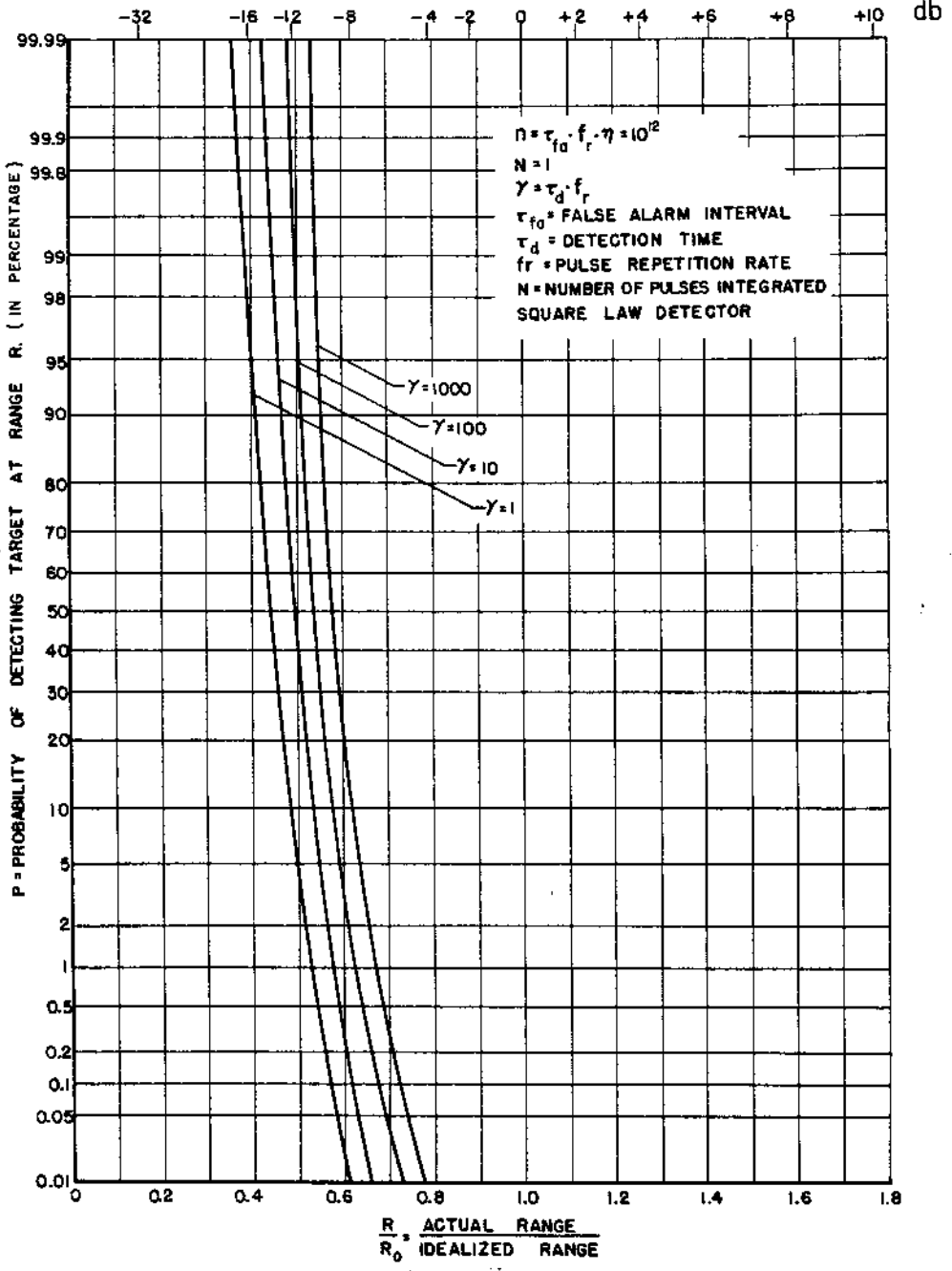


FIG. 41

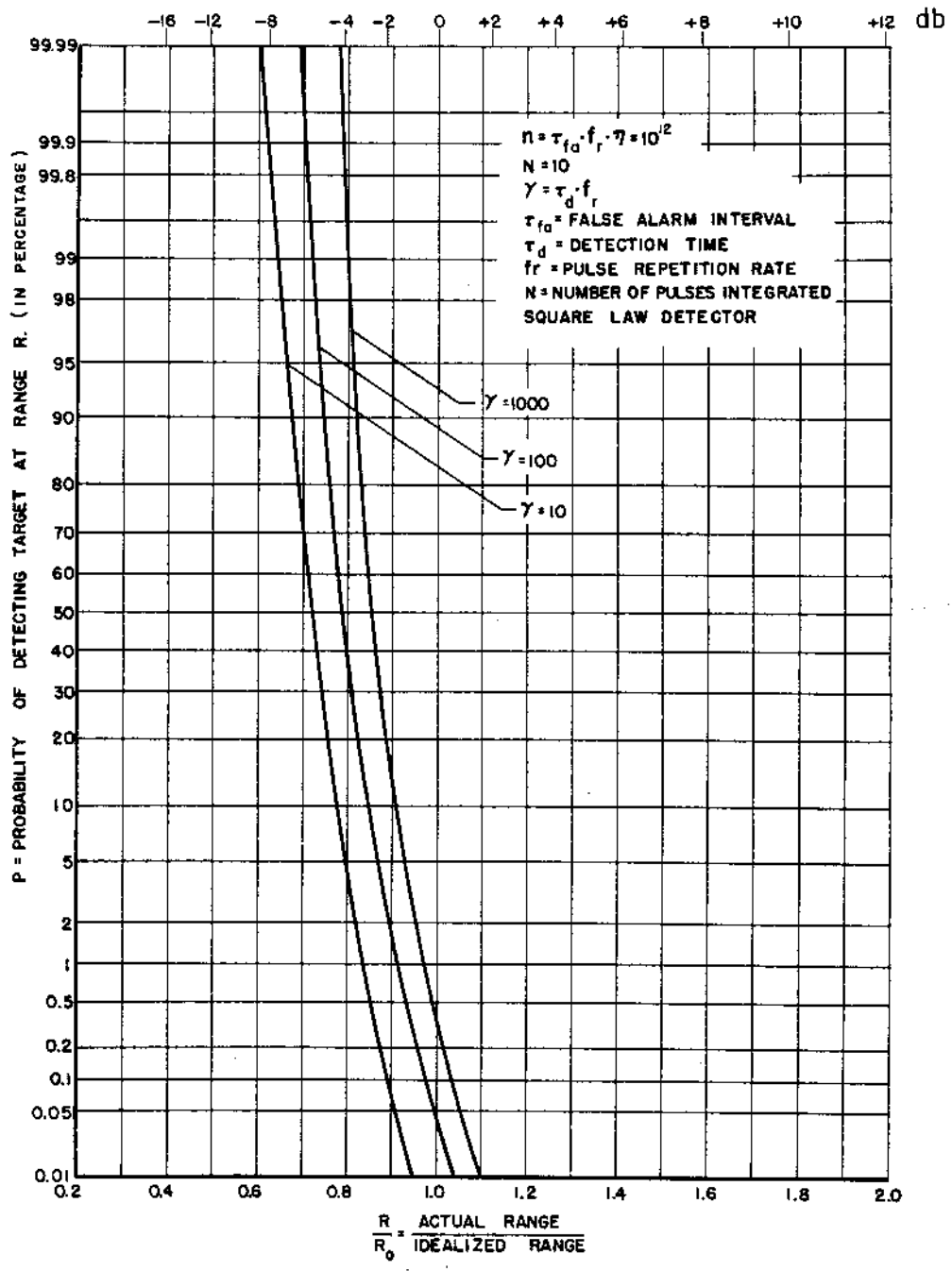


FIG. 42

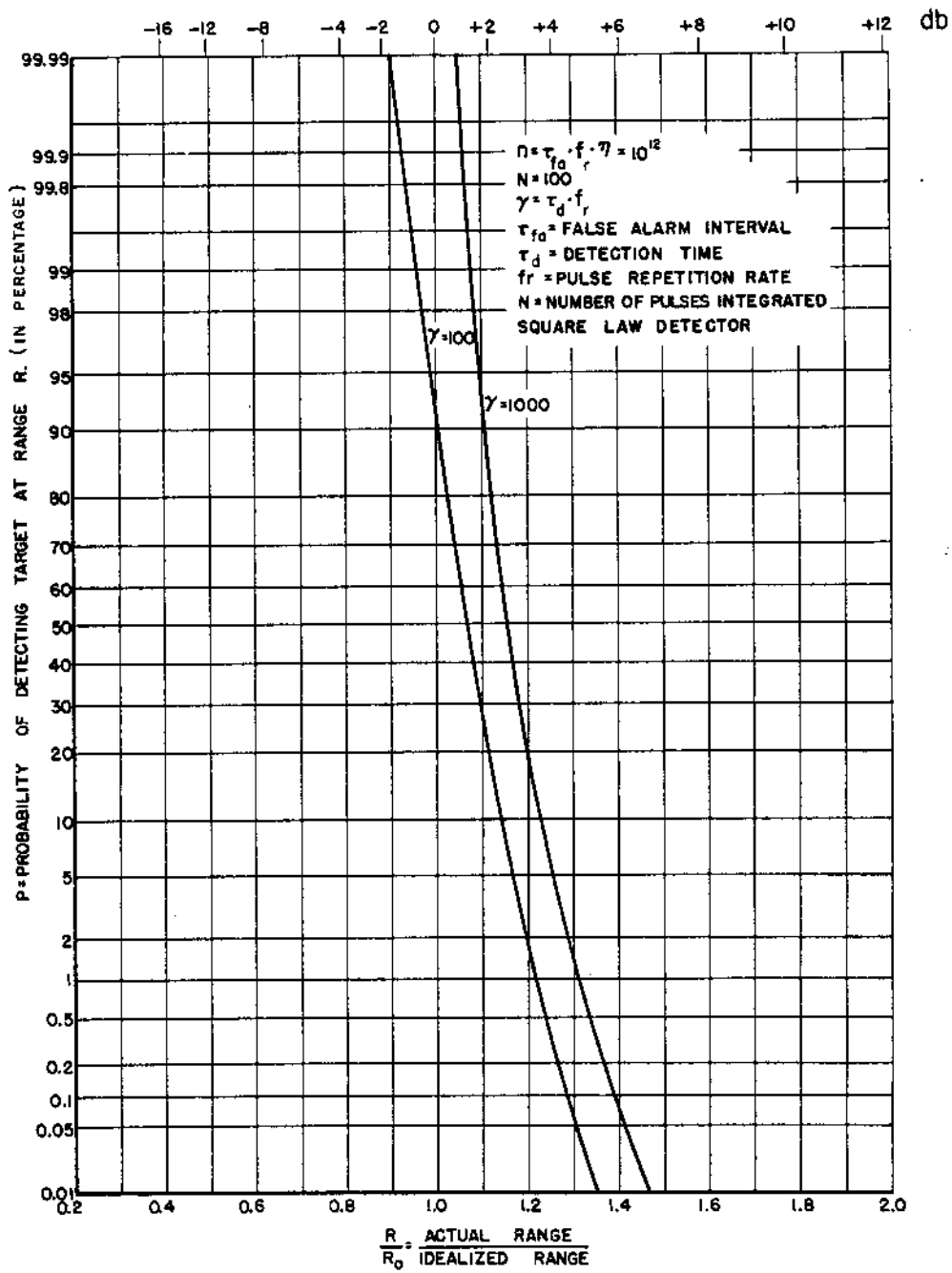


FIG. 43

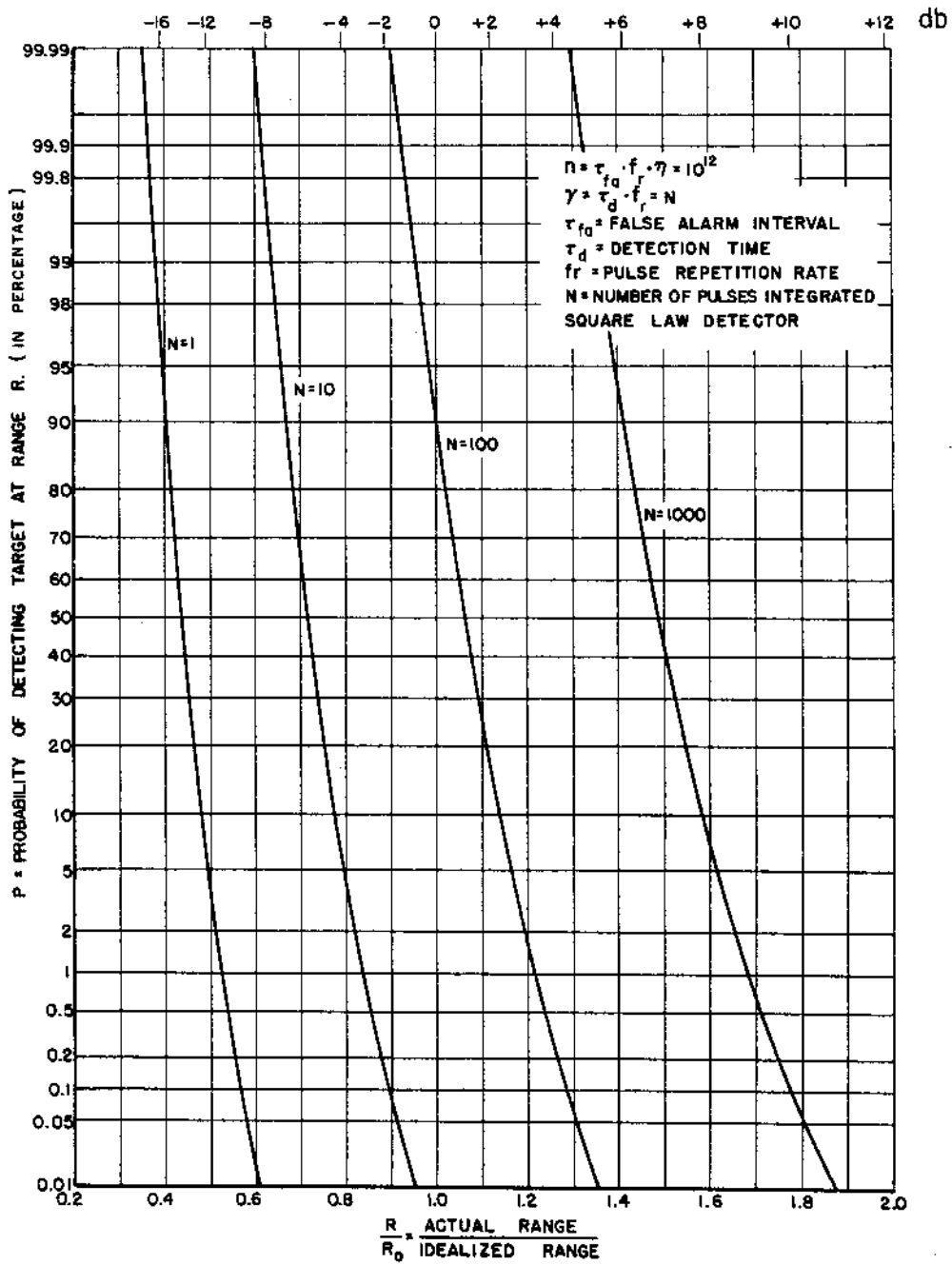


FIG. 44

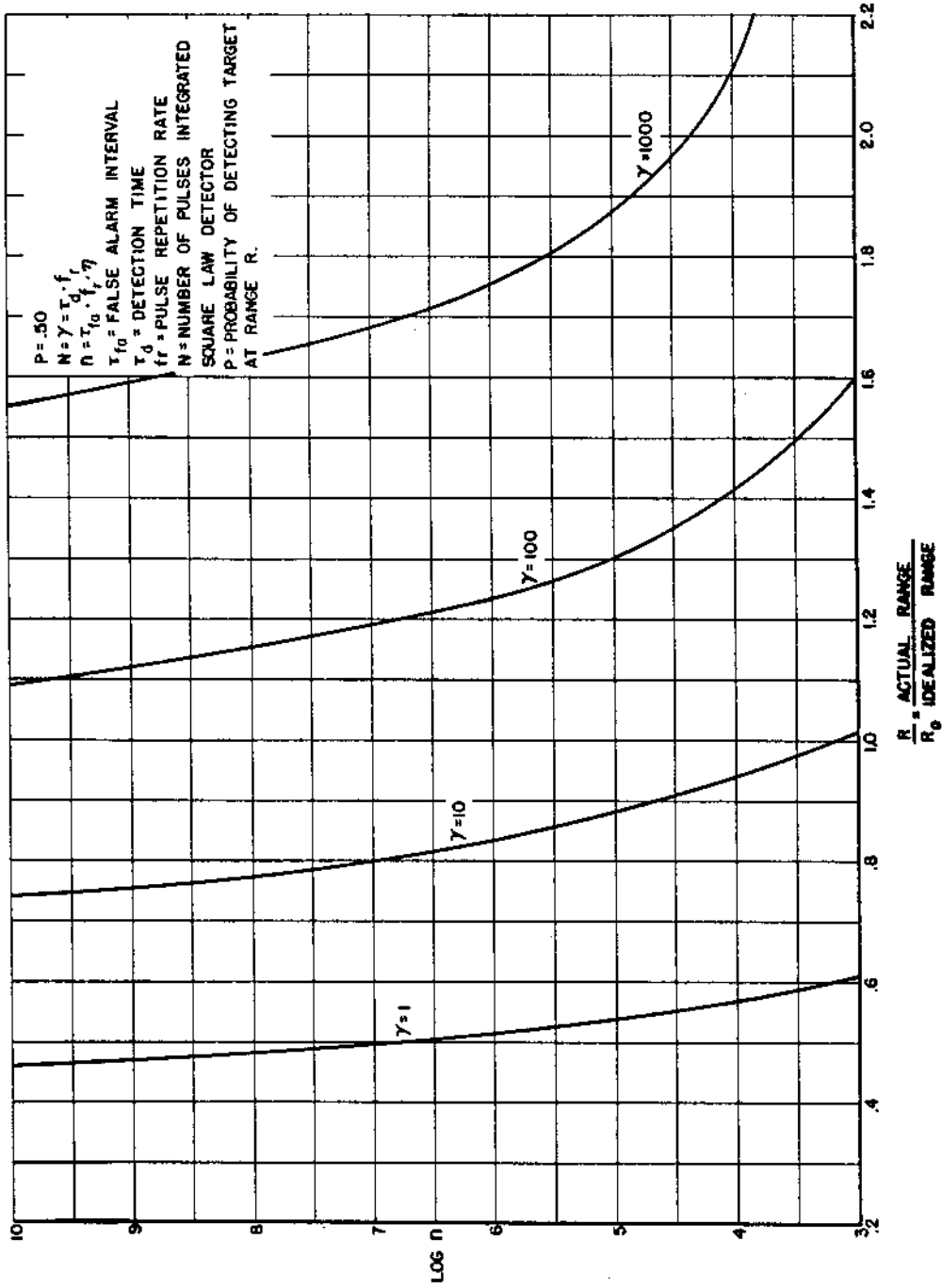


FIG. 45

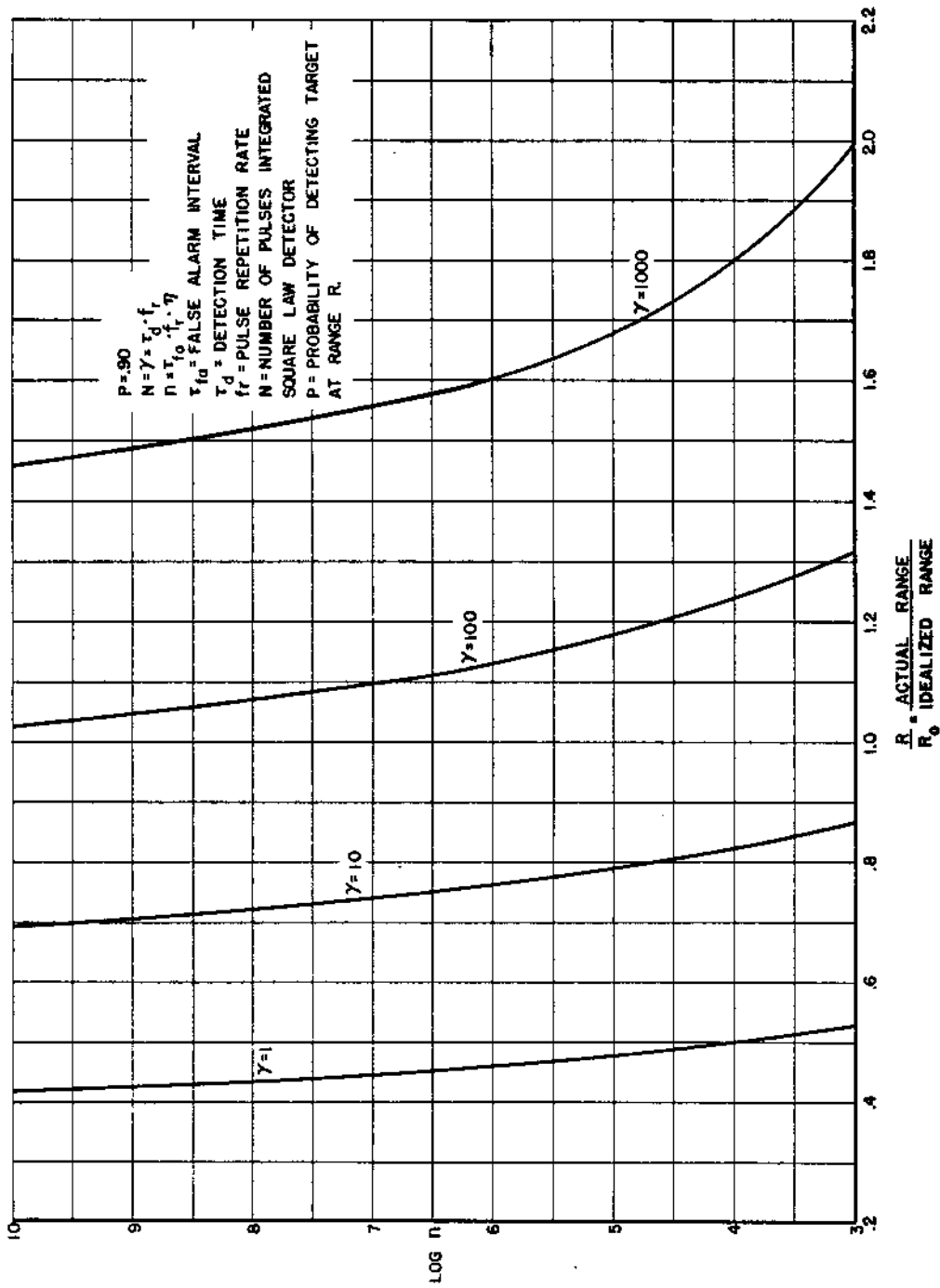


FIG. 46

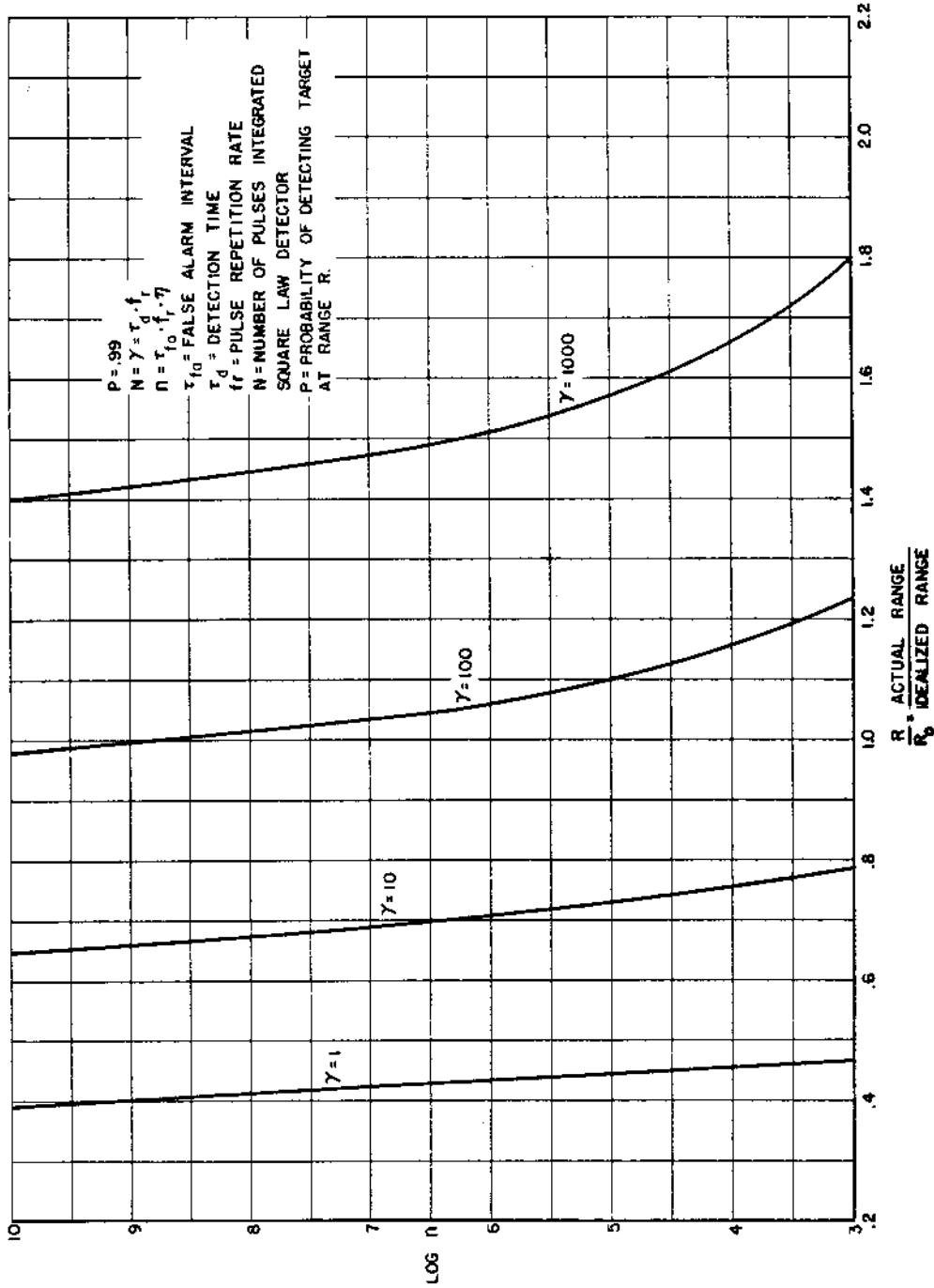


FIG. 47

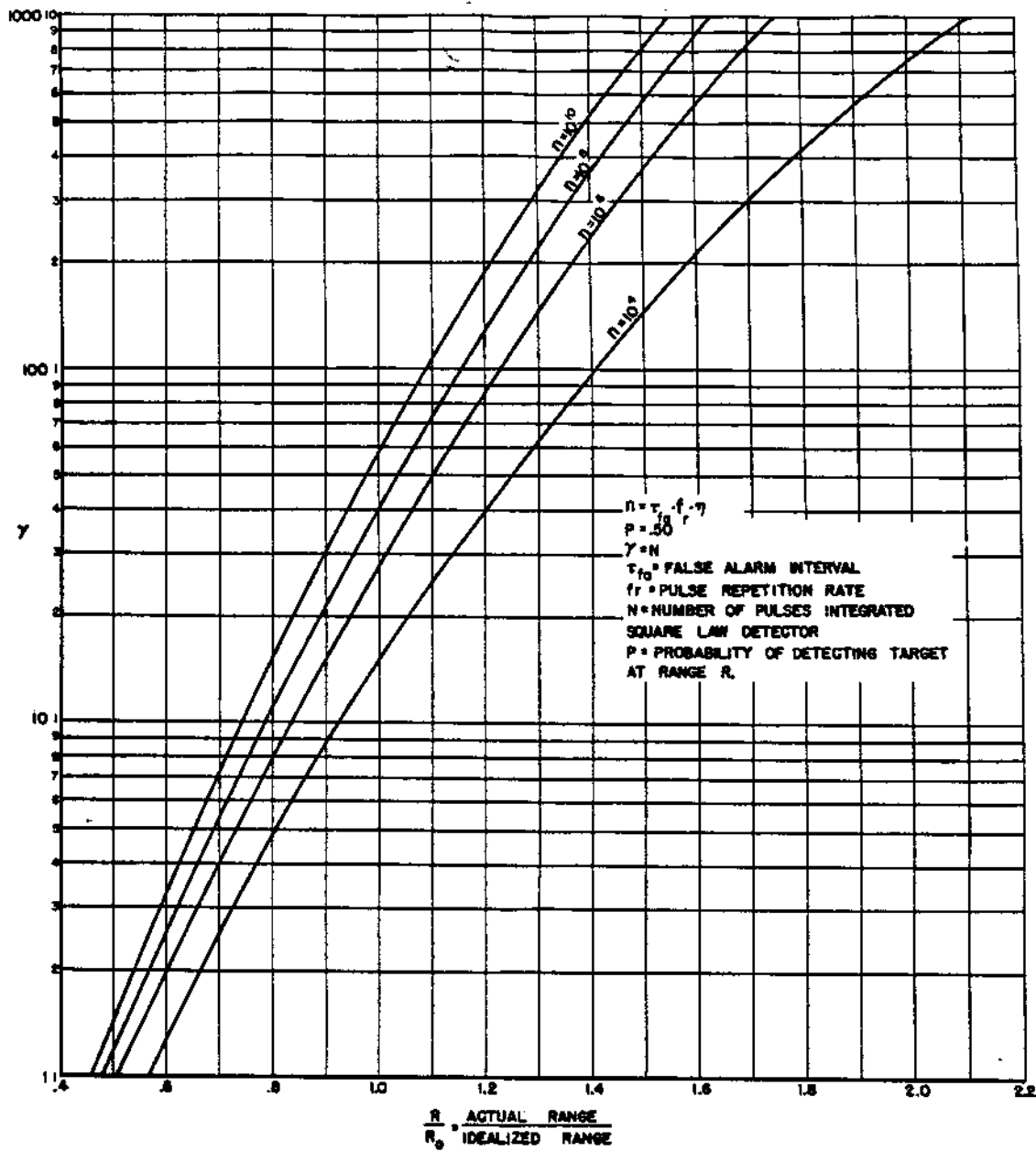


FIG. 48

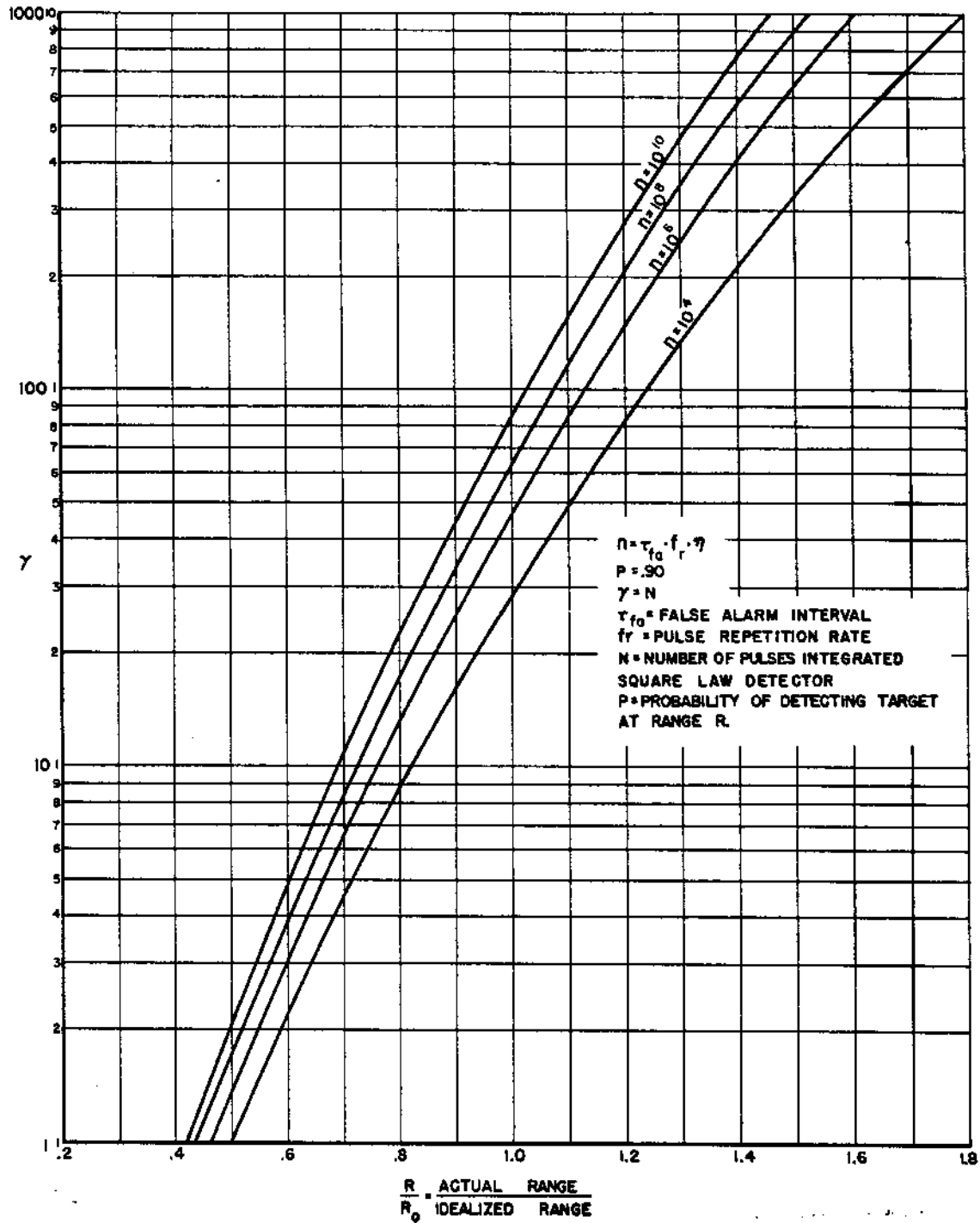


FIG. 49

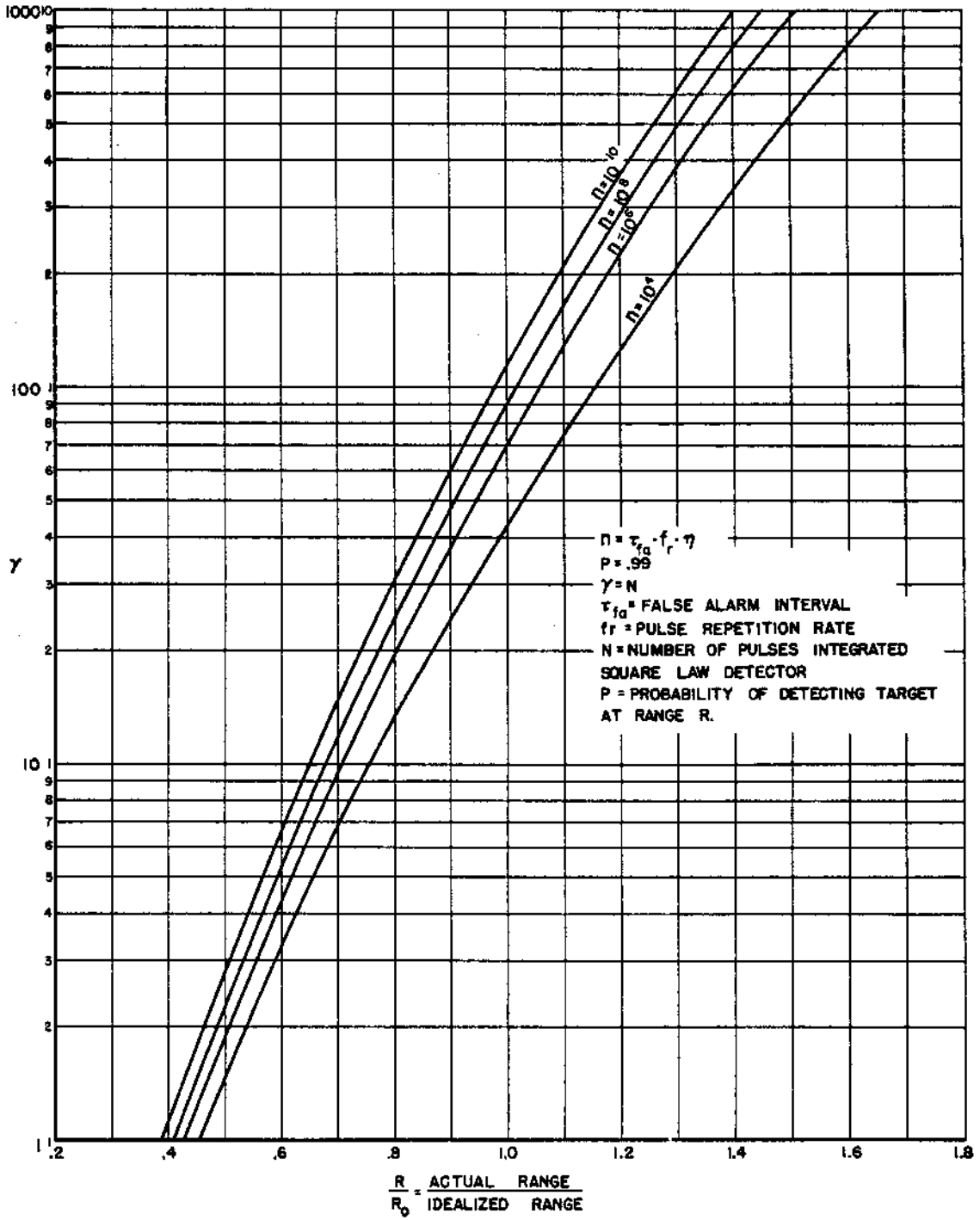


FIG. 50

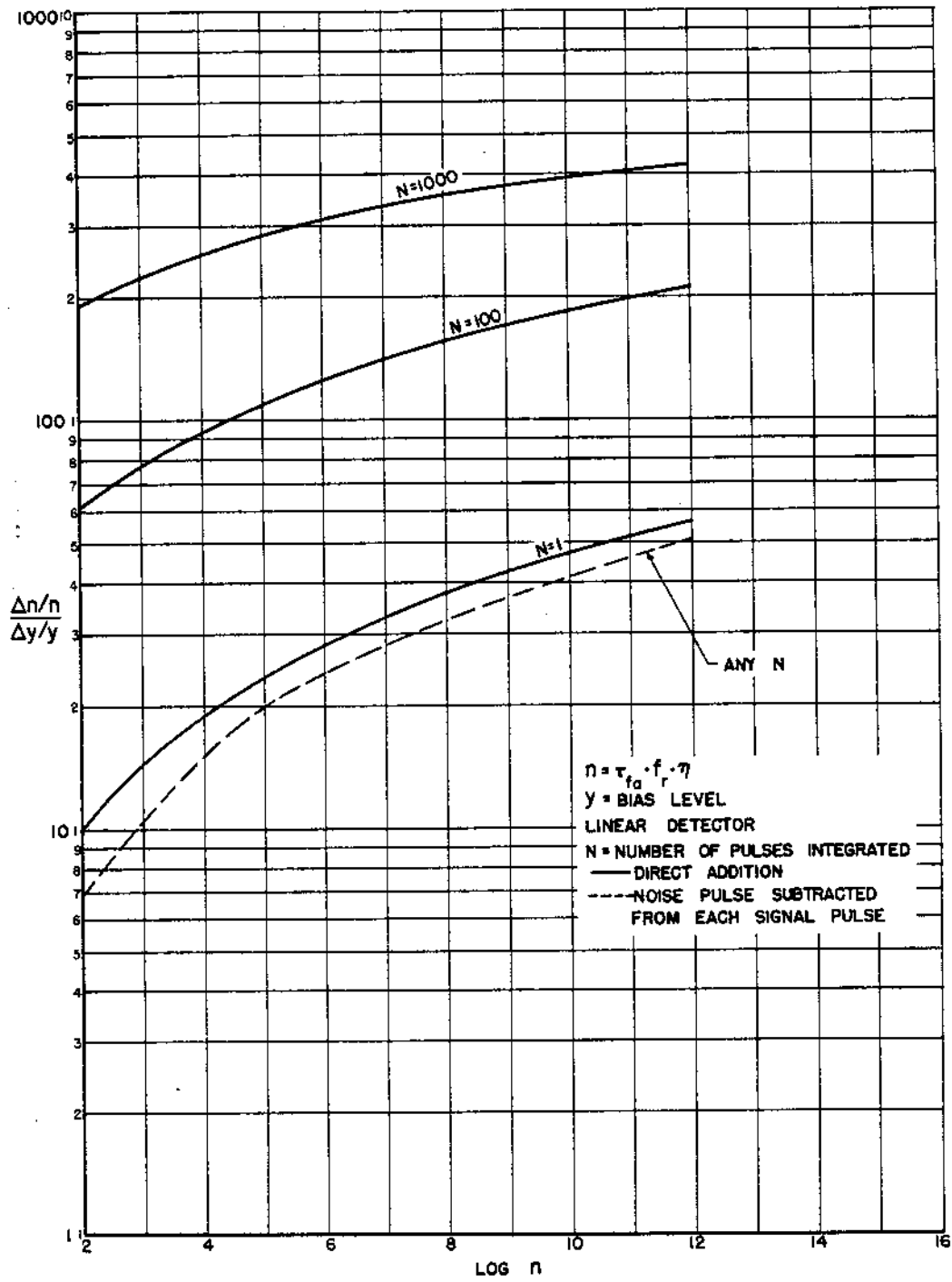


FIG. 51

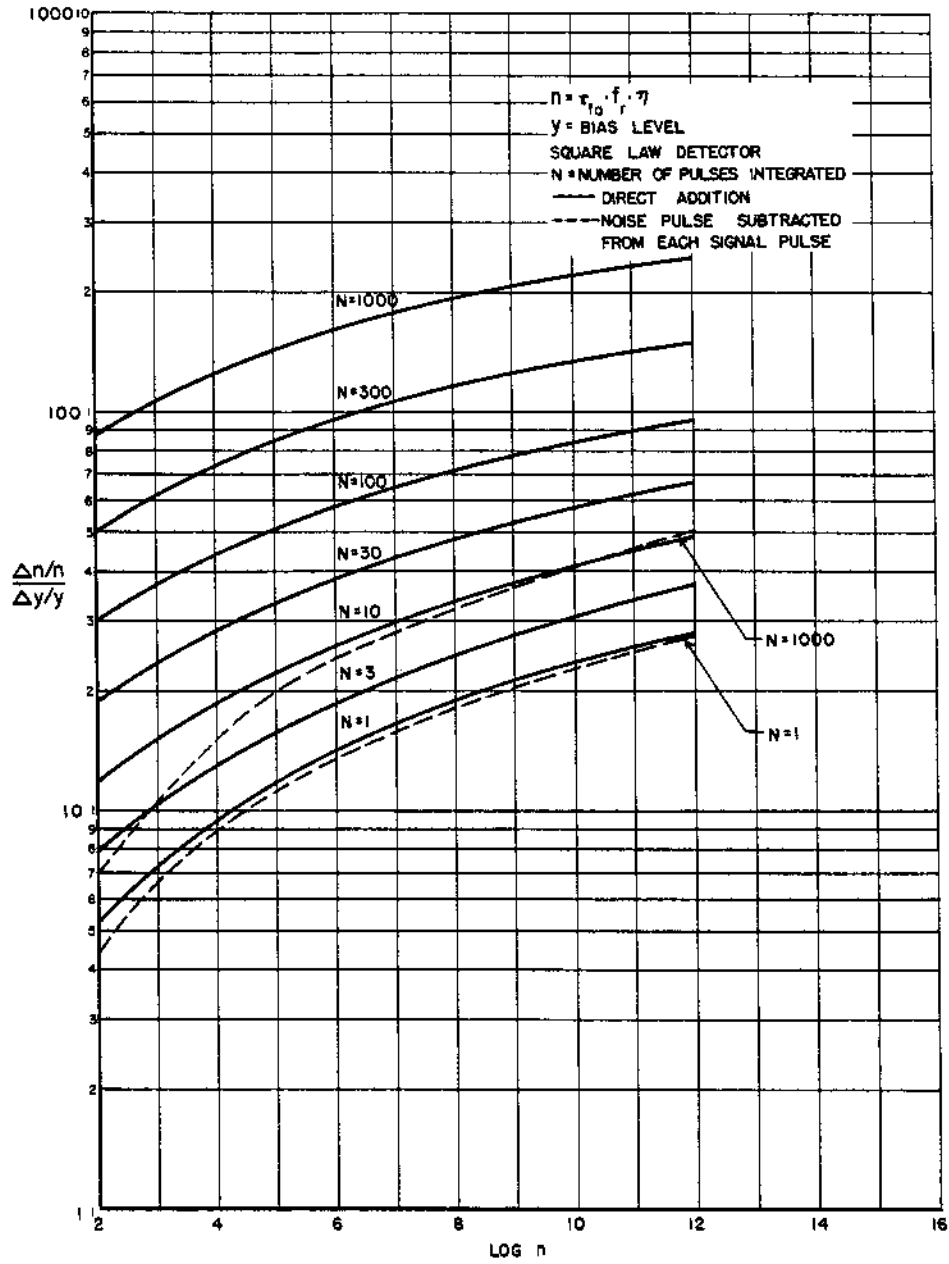


FIG. 52

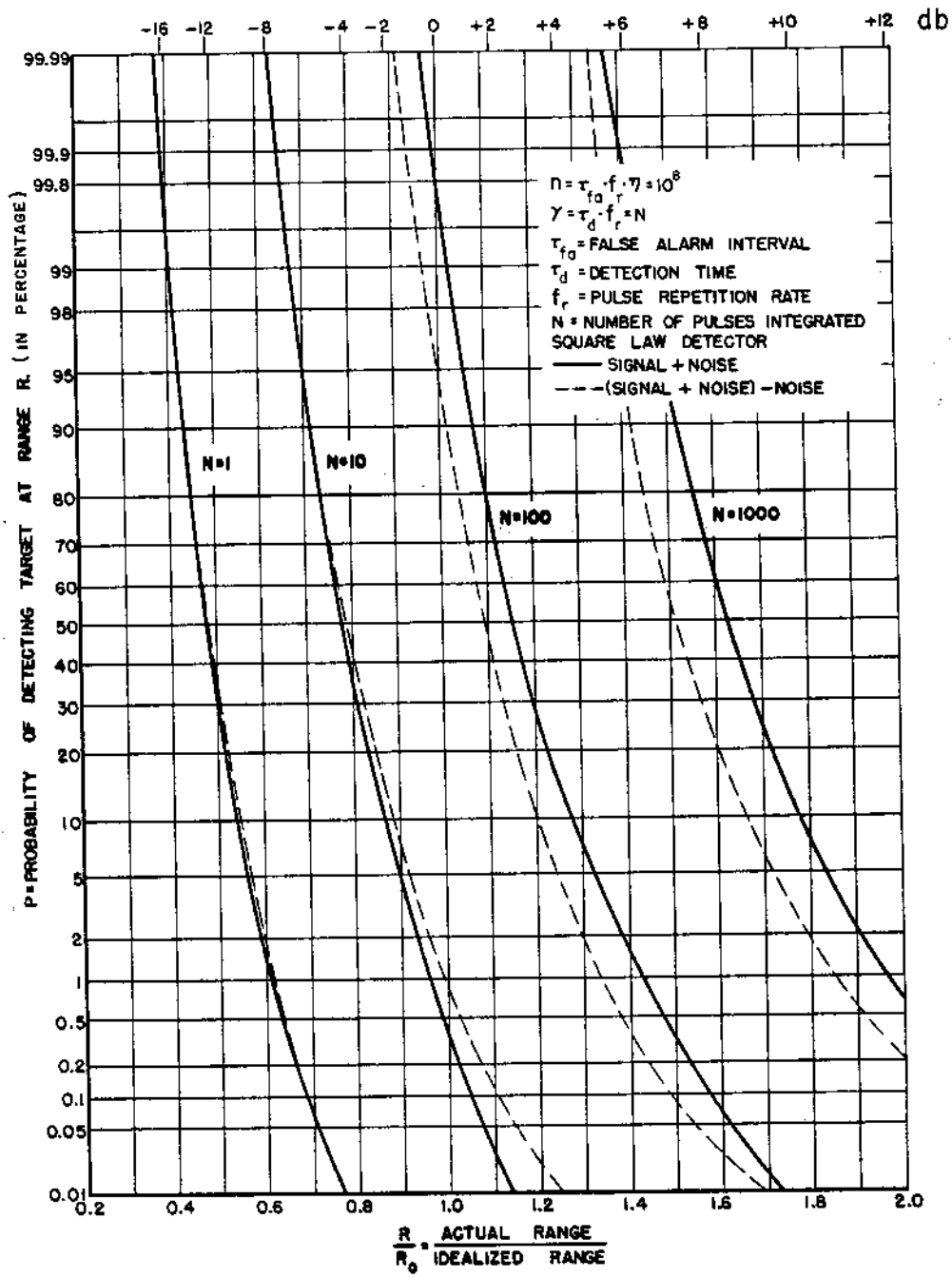


FIG. 53

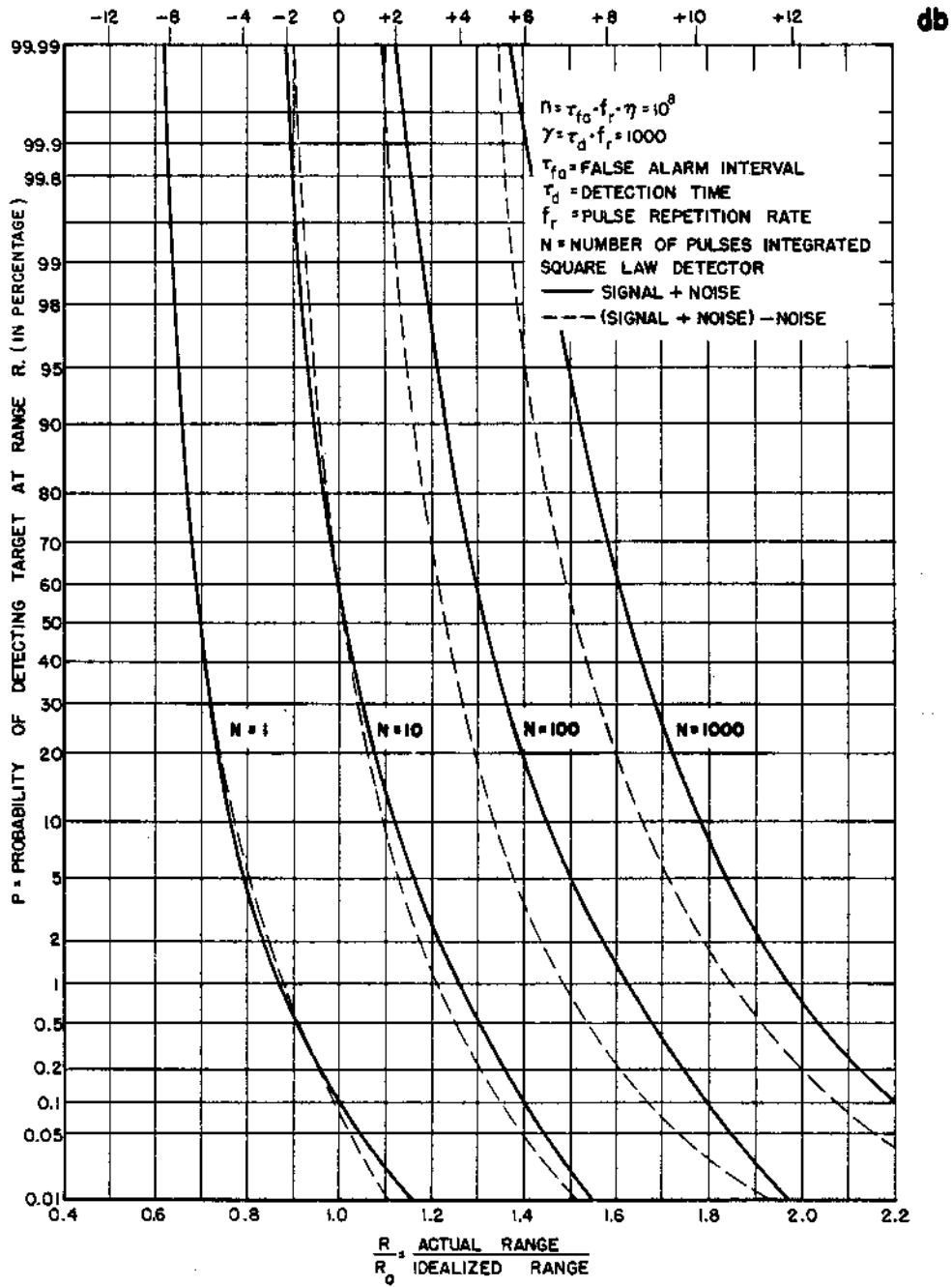


FIG. 54

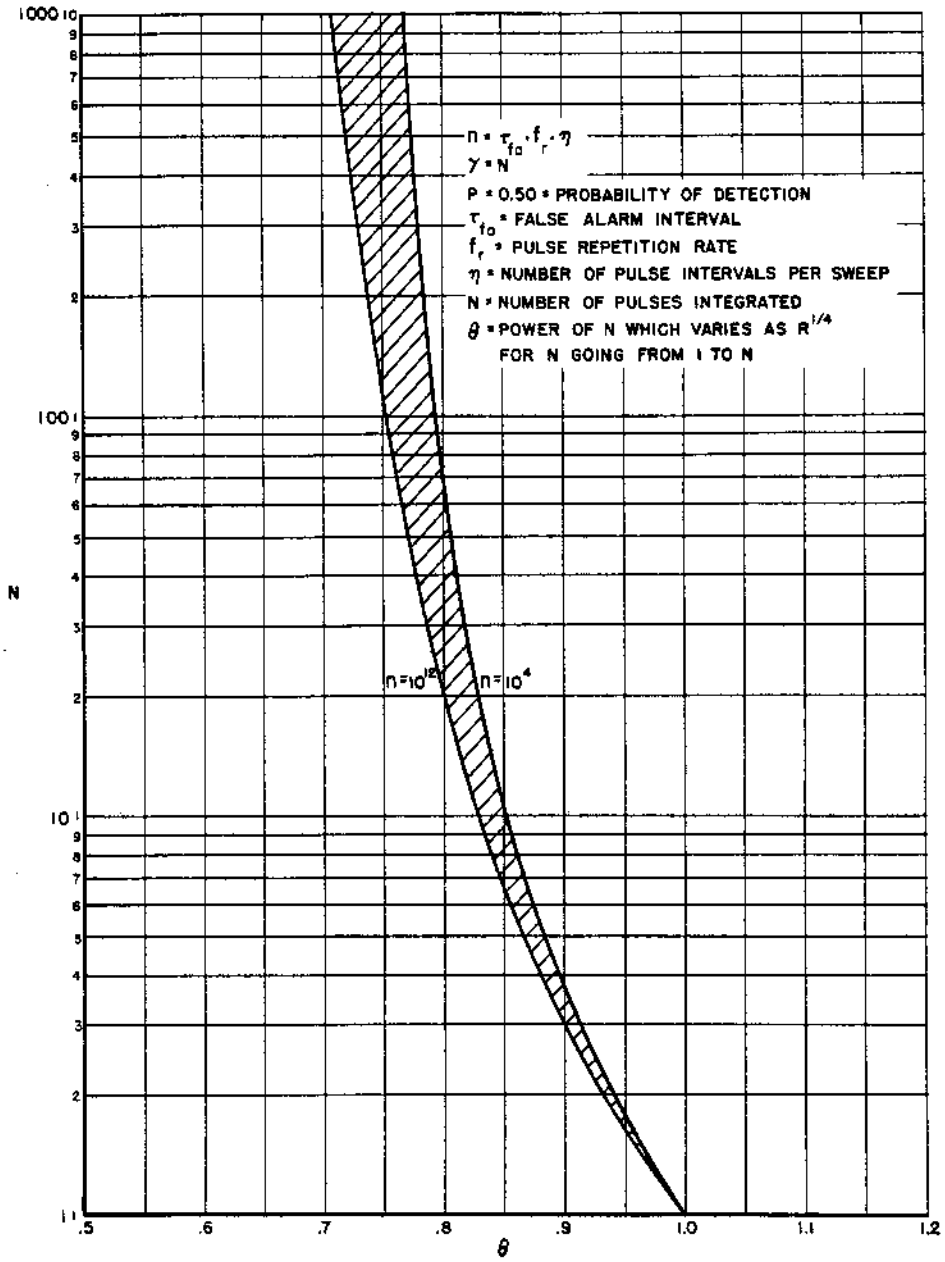


FIG. 55

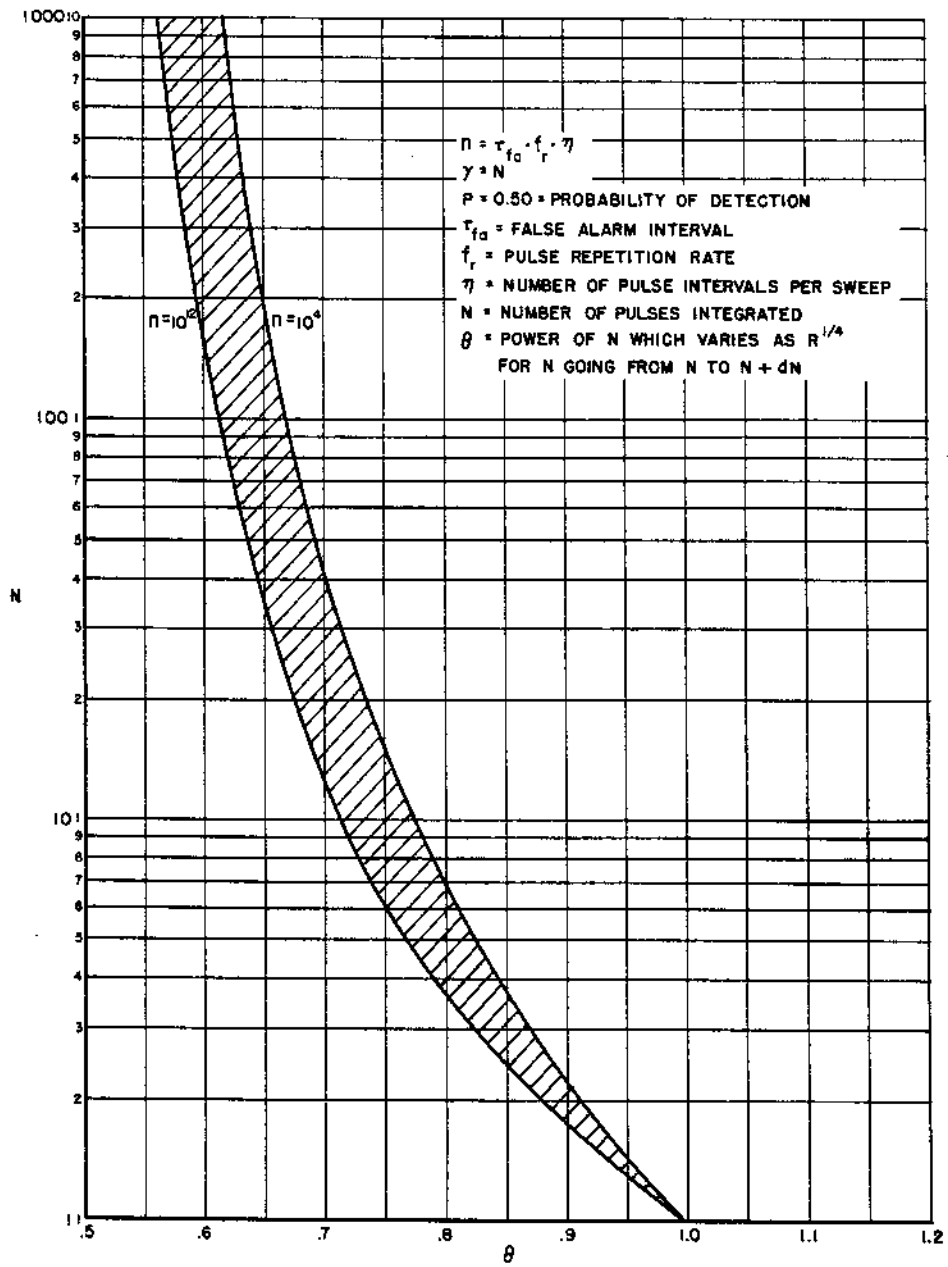


FIG. 56

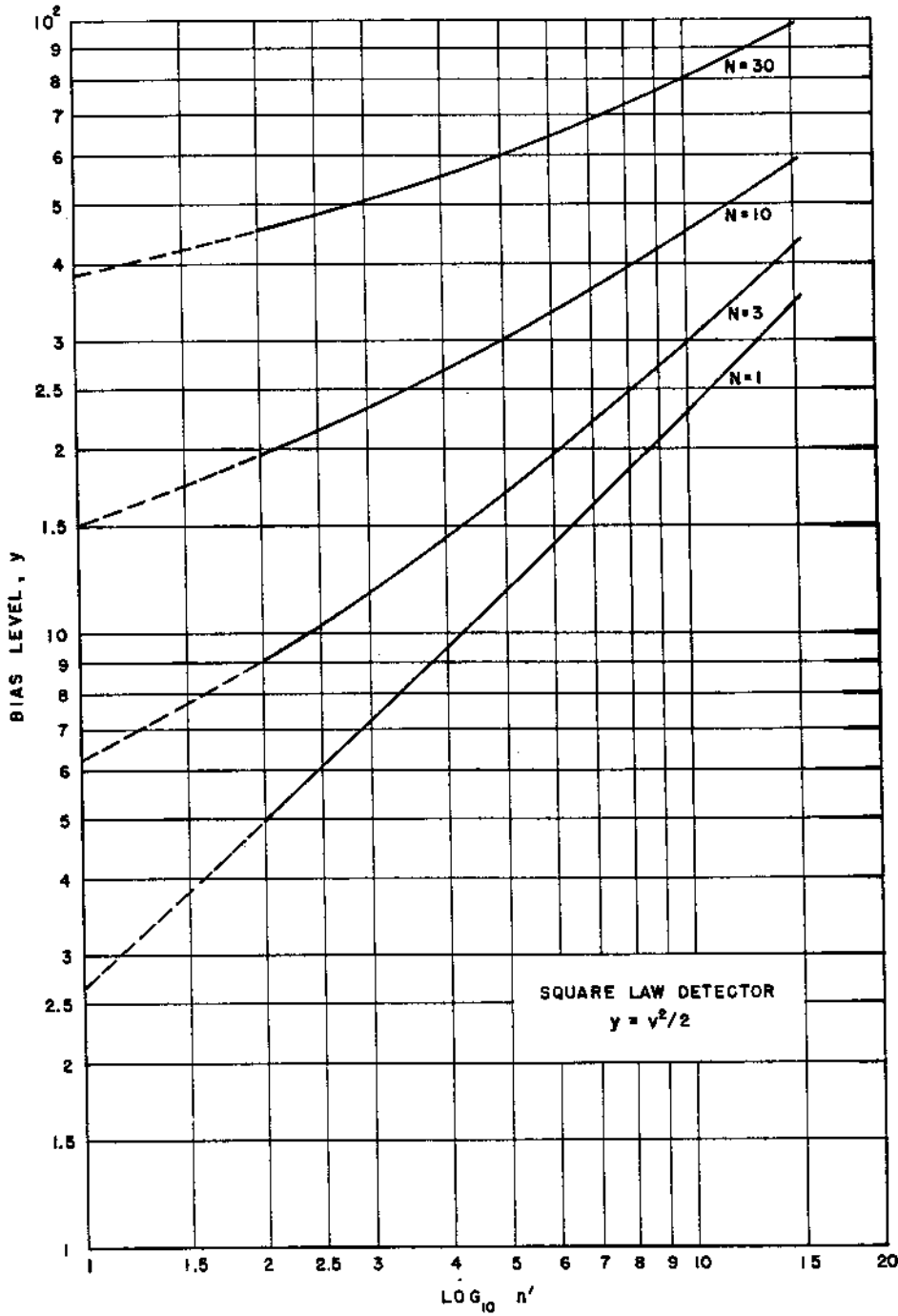


FIG. 57

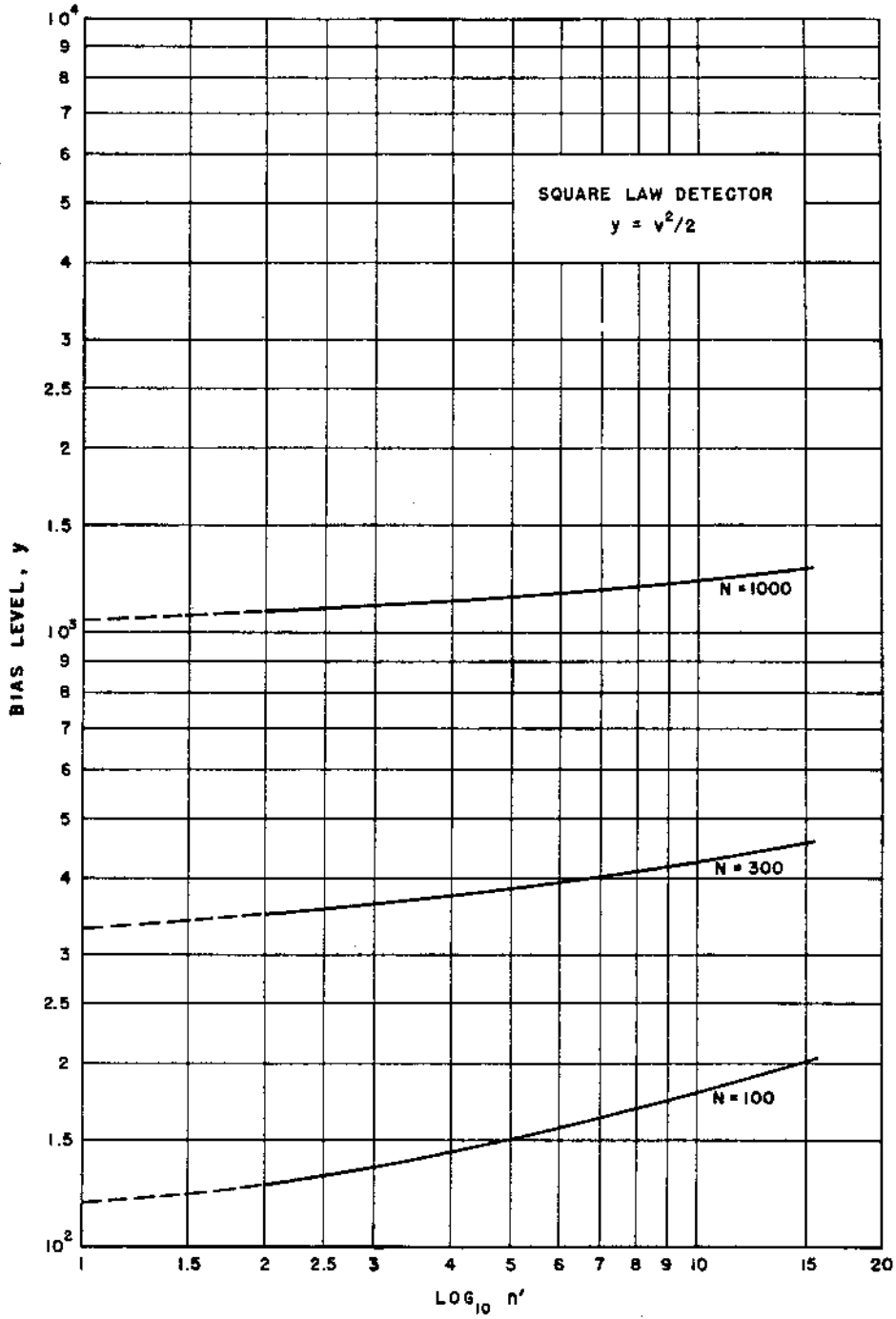


FIG. 57a

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