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Tolerancing decisions can profoundly impact the quality and cost of the mechanism. To evaluate the impact of tolerance on mechanism quality, designers need to simulate the influences of tolerances with respect to the functional requirements. This paper proposes a mathematical formulation of tolerance analysis which integrates the notion of quantifier: "***For all*** *acceptable deviations (deviations which are inside tolerances),* ***there exists*** *a gap configuration* ***such as*** *the assembly requirements and the behavior constraints are verified*" & "***For all*** *acceptable deviations (deviations which are inside tolerances), and* ***for all*** *admissible gap configurations, the assembly and functional requirements and the behavior constraints are verified''.* The quantifiers provide a univocal expression of the condition corresponding to a geometrical product requirement. This opens a wide area for research in tolerance analysis. To solve the mechanical problem, an approach based on optimization is proposed. Monte Carlo simulation is implemented for the statistical analysis. The proposed approach is tested on an over-constrained mechanism.

1. Introduction

1.1. Context

Mechanical product reliability is an important product quality factor and is dependent on different parameters among which tolerance design is an important activity. Proper tolerance design enables complex mechanical assemblies consisting of numerous parts to assemble and work together in a proper manner so that they fulfill their design objectives. As technology increases and performance requirements continually tighten, the cost and the required precision of assemblies increase as well. There is a strong need for increased attention to tolerance design in order to enable high-precision assemblies to be manufactured at lower costs.

To improve the tolerancing process in an industrial context, there exists a strong need for tolerance analysis to estimate the probability expressed in ppm (defected product per million) with high-precision computed at lower cost. The engineers need tolerance analysis methods:

- to improve product quality,
- to decrease the manufacturing cost,
- to reduce scrap in production (eco-aware attitude), and customer returns.

1.2. Related works

A substantial amount of research has been devoted to the development of tolerance analysis. Tolerance analysis concerns the verification of the value of functional requirements after tolerance has been specified on each component.

There are three main issues in tolerance analysis.

(1) The models for representing the geometrical deviations and gaps: the variation of the real entity from the ideal entity in 3D can be described in any one of the following manners:
  - With the help of the vectors [1] or vectorial tolerancing [2],
  - By the torsors of the small displacements [3,4],
  - By matrices [5,6],
  - By kinematic formulation [7] or a kinematic approach [8,9],
  - By stream of variations (SOVA) [10].
(2) A mathematical model for calculating the system behavior with deviations,
(3) The development of the solution techniques or analysis methods, such as worst-case searching and statistical analysis [11,1,12,13].

Worst-case analysis considers the worst possible combination of each deviation and examines the functional characteristic. Consequently, worst-case tolerancing can lead to excessively tight part tolerances and hence high production costs [12]. Statistical tolerancing on other hand works by setting the tolerances so as to assure a desired yield. By permitting a small fraction of assemblies to not assemble or function as required, an increase in tolerances for individual dimensions may be obtained, and in turn, manufacturing costs may be reduced significantly [13].

The Tolerance analysis methods are divided into two distinct categories based on the type of accumulation input: displacement accumulation or tolerance accumulation. The aim of displacement accumulation is to simulate the influences of deviations on the geometrical behavior of the mechanism. Usually, tolerance analysis uses a relationship of the form [13]:

$$Y = f(X), \tag{1}$$

where Y is the response (characteristic such as gap or functional characteristics) of the mechanism and $X = \{x_1, x_2, \ldots, x_n\}$ are the values of some characteristics (such as situation deviations or/and intrinsic deviations) of the individual parts or subassemblies making up the mechanism. The function f is the mechanism response function which represents the deviation accumulation. The relationship can exist in any form for which it is possible to compute a value for y given values of $X = \{x_1, x_2, \ldots, x_n\}$. It could be an explicit analytic expression or an implicit analytic expression. In a particular relative configuration of parts of a mechanism consisting of gaps without interference between parts, the composition relations of displacements in some topological loops of the mechanism permits to determine the response function f. Mechanism can be divided into two main categories in terms of degree of freedom, Iso-constrained mechanisms, and over-constrained mechanisms [14]. Given their impact on the response function formulation for the problem of tolerance analysis, a brief discussion of these two types is given by Ballu et al. [15]:

> "Isoconstrained mechanisms are quite easy to grasp. Geometrical deviations within such products do not lead to assembly problems; the deviations are independent and the degrees of freedom catch the deviations. When considering small deviations, functional deviations may be expressed by linear functions of the deviations".
>
> "Considering overconstrained mechanisms is much more complex. Assembly problems occur and the expression of the functional deviations is no more linear.
>
> Depending on the value of the manufacturing deviations:
>
> • the assembly is feasible or not;
>
> • the worst configuration of contacts is not unique for a given functional deviation.
>
> For each overconstrained loop, events on the deviations have to be determined:
>
> • events ensuring assembly,
>
> • events corresponding to the different worst configurations of contacts.
>
> As there are different configurations, the expression of the functional deviation cannot be linear".

For over-constrained mechanism, determination of explicit function f is very complex, whereas this determination is easy for an open kinematic chain [14].

For statistical tolerance analysis, the input variables $X = \{x_1, x_2, \ldots, x_n\}$ are continuous random variables which enable to represent part deviations. In general, they could be mutually dependent. A variety of methods and techniques (Linear Propagation (Root Sum of Squares), Non-linear propagation (Extended Taylor series), Numerical integration (Quadrature technique),

Monte Carlo simulation …) are available for estimation of the probability distribution of Y and the probability of the respect to the geometrical requirement [13].

The aim of tolerance accumulation is to simulate the composition of tolerances i.e. linear tolerance accumulation, 3D tolerance accumulation. Based on the displacement models, several vector space models map all possible manufacturing variations (geometrical displacements between manufacturing surfaces or between manufacturing surface and nominal surface) into a region of hypothetical parametric space. The geometrical tolerances or the dimensioning tolerances are represented by deviation space [16–19], T-Map® [20,21] or specification hull [22,23]. These three concepts are a hypothetical Euclidean volume which represents all possible deviations in size, orientation and position of features. For tolerance analysis, this mathematical representation of tolerances allows calculation of accumulation of the tolerances by Minkowsky sum of deviation and clearance domains [16,18,20]; to calculate the intersection of domains for parallel kinematic chain; and to verify the inclusion of a domain inside other one. The methods based on this mathematical representation of tolerances are very efficient for the tolerance analysis.

Current methods of displacement and tolerance accumulation are believed to have major drawbacks that reduce the accuracy of tolerance stack-up evaluation. These drawbacks are:

• The limited scope of the statistical approaches: explicit functions without gap and numerical simulations without gap. To use a statistical approach, it needs to simplify the mechanical model of an over-constrained system with gaps. This simplification is the current industrial practice.

• The limited scope of the tolerance accumulation approaches: linear problem (linear accumulation by Minkowsky sum).

1.3. Issue

This paper focuses on the development of an analysis method. Moreover, the proposal is independent of the adopted solution of geometrical modeling (vector, torsor, matrix, . . .), it includes a mathematical formalization based on the quantifier notion and an implementation based on optimization and Monte Carlo simulation. The quantifier notion translates the concept that a functional requirement must be respected in at least one acceptable configuration of gaps (existential quantifier "there exists"), or that a functional requirement must be respected in all acceptable configurations of gaps (universal quantifier "for all") [11,22,23]. A configuration is a particular relative position of parts of a mechanism consisting of gaps without interference between parts. In previous papers, this formalization based on quantifier allows to deduce some rule for the definition of the virtual boundary (Least Material Condition and Maximum Material Condition) [23] and to propose a first approach for tolerance analysis based on Quantified Constraint Satisfaction Problem for linear problem [11]. An extension of the mathematical formulation [23] based on the quantifier with help of formal logic in order to provide an integrated formalization is detailed in Section 2.

The quantifier impacts the result of the tolerance analysis [11]. Therefore, we propose an algorithm to compute this mathematical formulation based on coupled optimization and Monte Carlo simulation. This algorithm and its application are detailed in Section 3.

2. Quantifier and mathematical formulation of tolerance analysis

The aim of this section is to formalize the tolerance analysis problem based on the quantifiers. The quantified formulation is illustrated with geometrical requirement and assembly requirement. Moreover, an extension is proposed with the help of formal logic.

2.1. Quantifier notion for geometrical product requirement

A mechanism is a set of parts with joints. Most of joints have functional gap. These gaps induce displacements between parts. Each relative position defines a configuration of the joint. A configuration is a particular relative position of parts of a mechanism consisting of gaps without interference between parts. The product geometrical requirement limits the variation between two surfaces of the mechanism, which are in functional relation. This requirement is a condition on the functional characteristic between these surfaces. For any given mechanism with gaps, the relative orientation or position of these surfaces depends on the configuration, which is not single. Therefore, the value of the functional characteristic depends on the configuration of the mechanism. There is an ambiguity in the expression of the requirement because the considered configuration is not described. In order to address this problem, it is necessary to specify: in which configuration, the condition of the geometrical requirement must be checked. The expression of the geometrical product requirement is not univocal [24]. So, to define a univocal expression of the condition corresponding to a geometrical product requirement, this expression is completed by a quantifier (∃ or ∀). The quantifier translates the concept that the condition must be respected in at least one configuration of the mechanism (∃), or that the condition must be respected in all configurations of the mechanism (∀).

- In the case of the quantifier ∃, **if there exists** one configuration of the mechanism **such as** the value of the functional characteristic is less than or equal to the tolerance, **then** the geometrical product requirement is respected.
- In the case of the quantifier ∀, **if for all** configurations of the mechanism, the value of the functional characteristic is less than or equal to the tolerance, **then** the geometrical product requirement is respected.

2.2. Mathematical formulation of tolerance analysis for geometrical product requirement

The approach used in this paper is a parameterization of deviations from theoretic geometry, the real geometry of parts is apprehended by a variation of the nominal geometry. The substitute surfaces model these real surfaces. This parameterization of variations is detailed in the following sub section, and it enables us to define a variations parametric space, in which each coordinate system axis represents a parametric variable.

The mathematical formulation of tolerance synthesis takes into account not only the influence of geometrical deviations on the geometrical behavior of the mechanism and on the geometrical product requirements, but also the influence of the types of contacts on the geometrical behavior; all these physical phenomena are modeled by hulls (compatibility hull, interface hull and functional). With this description by hulls, a mathematical expression of the admissible deviations of parts is detailed in the section Relations between hulls.

2.2.1. Geometrical description by variations parametric space

The geometrical behavior model needs to be aware of the surface deviations of each part (situation deviations and intrinsic deviations) and relative displacements between parts according to the gap (gaps and functional characteristics). Compared with the nominal model, each substitute surface has situation variations and intrinsic variations:

- The **situation deviations** define the orientation and position variations between a substitute surface and the nominal surface,

Table 1
Subspace description.

Subspace name	Column vector	Designation
Situation	**S**	Space of all situation deviations of parts
Intrinsic	**I**	Space of all intrinsic deviations of parts
Gap	**G**	Space of all gaps between parts
Functional characteristic	**Fc**	Space of all functional characteristics between parts

- The **intrinsic deviations** of substitute surface are specific to their type. They define the surface variations. For instance, the intrinsic variation of a substitute cylinder is radius variation between the substitute cylinder and the nominal cylinder,

also two types of relative displacements between parts:

- The **gaps** define the orientation and position variations between two substitute surfaces in contact,
- The **functional characteristics** define the orientation and position variations between two substitute surfaces in functional relation.

The deviation of part surfaces, the gaps between parts and the functional characteristics between parts are described by parameters. Thereafter, the geometrical behavior of parts will be defined in space such as each coordinate axis corresponds to a parameter that is the variations parametric space. Four types of subspace corresponding to the four types of parameters are defined in Table 1 [22]:

2.2.2. Geometrical behavior description by convex hulls

The tolerance synthesis model is based on the expression of the geometrical behavior of the mechanism; various hulls modeling the geometrical behavior of the mechanism are defined for 1D and 3D applications.

- **the compatibility hull** ($D_{compatibility}$)

Composition relations of displacements in the various topological loops express the geometrical behavior of the mechanism [11,4]. The composition relations define compatibility equations between the situation deviations and the gaps. The set of compatibility equations, obtained by the application of composition relation to the various cycles, makes a system of linear equations. So that the system of linear equations admits a solution, it is necessary that compatibility equations are checked. These compatibility equations characterize some hyperplanes in the Situation × Gap × Functional characteristic space.

- **the interface hull** ($D_{interface}$)

Interface constraints limit the geometrical behavior of the mechanism and characterize non-interference or association between substitute surfaces, which are nominally in contact [11,17, 18,20,22,23,25,26]. These interface constraints limit the gaps between substitute surfaces. These constraints define the interface hull in Gap × Intrinsic space. In the case of floating contact, the relative positions of substitute surfaces are constrained technologically by the non interference, the interface constraints result in inequations defined in Gap × Intrinsic space. In the case of slipping and fixed contact, the relative positions of substitute surfaces are constrained technologically in a given configuration by a mechanical action. An association models this type of contact; the interface constraints result in equations defined in Gap × Intrinsic space.

- **the functional hull** ($D_{functional}$)

The functional requirement limits the orientation and the location between surfaces, which are in functional relation. This requirement is a condition on the relative displacements between these surfaces. This condition could be expressed by constraints, which are inequations. These constraints define the functional hull in Functional characteristic × Intrinsic space.

2.2.3. Relations between hulls

The objective of this mathematical formulation is to formalize the necessary and optimal constraints on deviations of each part, i.e. the vectors **s** and **i**. The previous geometrical behavior description and the quantifier expression enable to define the admissible deviations of parts such as the functional requirement is respected. These admissible deviations form a hull in situation and intrinsic spaces called specification hull. To define it, we formalize a textual relation and a mathematical relation between various hulls [11,22,23].

For assembly requirement, the quantifier is $\exists$. The specification hull is defined as:

"The deviations are admissible" **is equivalent to** *"there exists an admissible gap configuration of the mechanism* **such as** *the geometrical behavior and the assembly requirement (interface constraints) are respected"*. The mathematical expression of this equivalence is:

$$
\begin{aligned}
&(\mathbf{s}, \mathbf{i}) \in D_{specification} \\
&\Leftrightarrow \exists\, \mathbf{g} \in Gap \quad : (\mathbf{s}, \mathbf{g}, \mathbf{i}) \in D_{compatibility} \cap D_{interface}.
\end{aligned}
\tag{2}
$$

For functional requirement, the quantifier is $\forall$, the specification hull is defined as:

"The deviations are admissible" **is equivalent to** *"for all admissible gap configurations of the mechanism,* **there exists** *a functional characteristic* **such as** *the geometrical behavior and the functional requirement are respected"*. The mathematical expression of this equivalence is:

$$
\begin{aligned}
&(\mathbf{s}, \mathbf{i}) \in D_{specification} \\
&\Leftrightarrow \forall \mathbf{g} \in \{\mathbf{g} \in Gap : (\mathbf{s}, \mathbf{g}, \mathbf{i}) \in D_{compatibility} \cap D_{interface}\}, \\
&\qquad \exists\, \mathbf{fc} \in Functional\ characteristic \\
&: (\mathbf{s}, \mathbf{g}, \mathbf{i}, \mathbf{fc}) \in D_{compatibility} \cap D_{interface} \cap D_{functional}.
\end{aligned}
\tag{3}
$$

This quantifier notion enables to formalize the relations between hulls (compatibility hull, interface hull and functional hull) and specification hull. These relations are a theoretical formulation of tolerance analysis. Moreover, it is possible to impose the requirement is true for each point of the functional surface, or simultaneously for all point of the functional surface, it is possible to express the specification hull with many requirements (The requirements are translated into constraints which characterize the functional hull).

2.3. Mathematical formulation based on first order logic

The basic syntactic elements of First-order logic (FOL) are connectives, quantifiers, predicates, functions and the variables. The theory presented here generalizes the quantifier based tolerance expression and using the framework and syntax of formal logic, generalizes the quantifier based expression into a logical expression of tolerance analysis for mechanism. In order to formalize the problem, we proceed by adopting the semantics of the generic model: V represents the set of all variables in the tolerance analysis problem. V consists of sets of Situation and Intrinsic deviations, Gaps and functional characteristics. The universe or domain D for the tolerance analysis problem includes the possible assignments for the members of the variable V. These assignments play an important role in the analysis problem as they contribute to or control the search for the design solution. These assignments include the values that the variables in functional characteristics space can take on as established from the client requirements or as needed by the different constraints. The domain also includes the assignment values for the variables related to the nominal dimensions. The interpretation functions or constraints for the tolerance analysis problem are based on the expression of the geometric behavior of the mechanism. The

mathematical form of these constraints is in terms of linear or non-linear expressions involving members of V. The relations may be of type equality or inequality. The relations coming under the compatibility Hull $D_{compatibility}$ are in the form of linear equations where as the relations from interface hull and functional hull ($D_{interface}$ and $D_{functional}$) are in the form of inequality or equality.

The objective of the mathematical formulation for the tolerance analysis problem is to formalize the necessary and optimal constraints on deviations of each part. In order for a mechanism to assemble successfully, the different components in the presence of deviations should assemble without interference and should have a specific set of gaps that characterize the instance of the mechanism. An acceptable solution s_g can than be defined as a solution that allows the mechanism which validates the existence of gaps with values from universe such that all the constraints are satisfied. This condition stipulates the use of an existential quantifier for an initial search for the existence of a feasible configuration of gaps. Therefore using the existential quantifier, the solution s_g is defined as:

"the deviations are admissible" is equivalent to *"there exists an admissible gap configuration of the mechanism and a functional characteristic such that the geometrical behavior and the functional requirement are respected"*. It can be translated as:

$$
\exists\, s_g \in S : \quad
\begin{aligned}
&s_g = D_{assembly} \vDash (D, C) \\
&s_g \vDash \exists GC(V, \bar{a}) : \bar{a} \in D.
\end{aligned}
\tag{4}
$$

In the same way, the solution s_{Fc} is defined textually as:

"The deviations are admissible" is equivalent to *"for all admissible gap configurations of the mechanism; there exists a functional characteristic such that the geometrical behavior and the functional requirement are respected"*. This may be written as:

$$
\exists\, s_{FR} \in S : \quad
\begin{aligned}
&s_{FR} = D_{FR} \vDash (D, C) \\
&s_{FR} \vDash \forall G\, \exists\, Fc\, C(V, \bar{a}) : \bar{a} \in D.
\end{aligned}
\tag{5}
$$

Based on this formalization, we will propose in the following section an algorithm for statistical tolerance analysis of complex systems (over-constrained mechanism with non linear behavior).

3. Statistical tolerance analysis based on optimization and Monte Carlo simulation

In order to ensure the robustness of design, it is necessary to simulate and study the effect of variations on mechanism due to variation on parts. Variations may take place in any random pattern concurrently in the concerned dimension and may affect the assemblability and function of the mechanism. In the following section, the approach discussed above will be modified and integrated with the Monte Carlo simulation tool to obtain an algorithm which performs the tolerance analysis of a mechanical assembly from a sample population of components generated by Monte Carlo simulation.

The tolerance analysis method adopted in this article is based on using Monte Carlo simulation and quantifier notion to a solution of optimization in order to calculate the probability of assembly and functioning of a given assembly. The application of this approach to 3D tolerance analysis will be discussed.

3.1. Transformation of the formalization

In order to implement the proposed formulation by quantifier, we need to transform it into mathematical expression. First of all, we consider the more general framework of quantified constraint satisfaction problems formalization, which are defined as follows, and we illustrate its application for the transformation of the formalization.

The quantified constraint satisfaction problem (QCSP) is an extension of the Constraint Satisfaction Problem (CSP) in which variables are totally ordered and quantified either existentially or universally [27]. QCSP provides a better expressiveness for modeling problems. The goal in a QCSP can be either to determine satisfiability or to find a consistent instantiation of the existential variables for all instantiations of the universal ones. A Quantified Constraint Satisfaction Problem (QCSP) is a formula of the form QC where Q is a sequence of quantifiers $Q1 \times 1 \ldots Qn \times n$, where each Qi quantifies ($\exists$ or $\forall$) a variable xi and each variable occurs exactly once in the sequence. C is a conjunction of constraints ($c1 \ldots cm$) where each ci involves some variables among $x1, \ldots, xn$.

Based on the QCSP formalization, the mathematical expression of tolerance analysis for assembly requirement is: *"For all acceptable deviations (deviations which are inside tolerances), there exists a gap configuration such as the assembly requirement (interface constraints) and the compatibility equations are verified".*

$$\forall x_1, \forall x_2, \forall x_3, \ldots, \forall x_n, \qquad \exists x_{n+1}, \ldots, \exists x_m \; ;$$
$$D(x_1), D(x_2), D(x_3), \ldots, D(x_n), D(x_{n+1}), \ldots, D(x_m) \; ;$$
$$C_1, \ldots, C_p$$

- $x_1, x_2, \ldots, x_n$ are the variables which represent each part deviation (s, i),
- $x_{n+1}, \ldots, x_m$ are the variables which represent each gap between parts (g),
- the mathematical representation of geometrical specifications is a set of intervals which limit each part deviation like vectorial tolerancing: $x_i \in D(x_i)$ with x_i a part deviation and $D(x_i)$ its tolerance interval, $D(x_1) \times \cdots \times D(x_n) = D_{specification}$
- the mathematical representation of interface constraints is a set of inequations which limit each gap: $x_j \in D(x_j)$ with x_j a gap and, $D(x_{n+1}) \times \cdots \times D(x_m) = D_{interface}$
- the mathematical representation of the compatibility equations is a set of constraints: $C_1, \ldots, C_p$.

The expressive power of QCSP integrates the notion of quantifier in the expression. But this expression does not integrate the stochastic aspect of the statistical tolerance analysis. To do so, the first term of the expression *"For all acceptable deviations (deviations which are inside tolerances)"* is modified for computation of the defect probability:

For assembly requirement, the mathematical expression is: *"For each sample (Monte Carlo simulation) of acceptable deviations (deviations which are inside tolerance limit), there exists a gap configuration such as the assembly requirement (interface constraints) and the compatibility equations are verified".*

For functional requirement, the mathematical expression is: *"For each sample (Monte Carlo simulation) of acceptable deviations (deviations which are inside tolerances), the worst case of functional characteristics must be respected the functional requirements such as the interface constraints and the compatibility equations are verified".*

3.2. Algorithm for statistical tolerance analysis by Monte Carlo simulation and optimization

A new algorithm is proposed based on statistical sampling power of Monte Carlo simulation and on optimization to find the worst gap configuration. The following section details the general description of the algorithm.

A general flow chart describing the module for tolerance analysis is shown in Fig. 1. The main principle behind the algorithm is to simulate and evaluate the influence of the manufacturing deviations on the nominal dimensions of an assembly. In order to achieve this, Monte Carlo simulation is used to simulate the deviations and the optimization to identify the worst gap configuration. This process is repeated recursively for a large
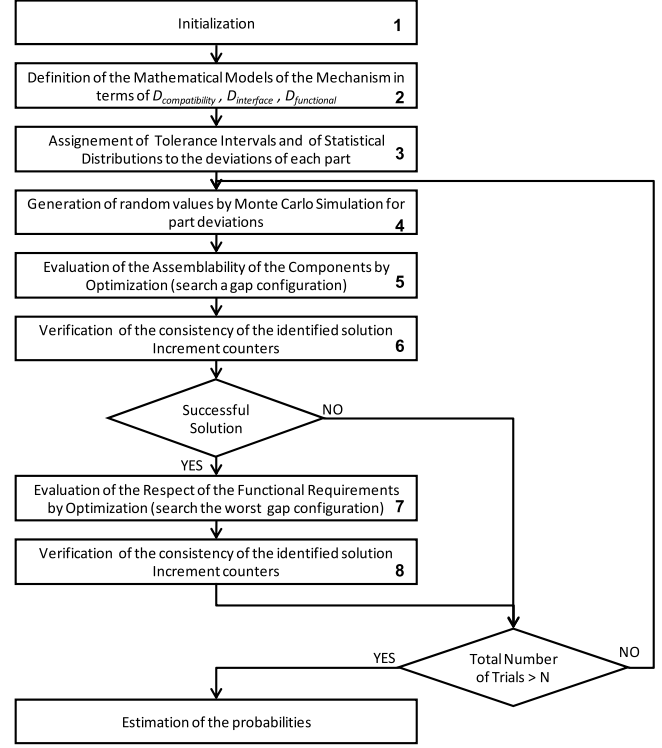

Fig. 1. General scheme of tolerance analysis with Monte Carlo simulation.

sample of deviations to estimate assembly probability in order to perform the tolerance analysis of any given mechanism consisting of sub components.

A mathematical model of the mechanism is expressed in form of the compatibility, interference and functional hulls (step 2 of the algorithm), it is defined by a set of variables and a set of constraints on subsets of the variables:

Constraints describing the compatibility hull generally formulated as:

$$C_c(s, g, fc) = 0 \Leftrightarrow (s, g, fc) \in D_{compatibility}. \tag{6}$$

Constraints describing the interface hull:

$$C_i(i, g) \leq 0 \quad \text{and} \quad C_i(i, g) = 0 \Leftrightarrow (i, g) \in D_{interface}. \tag{7}$$

Constraints describing the functional hull

$$C_f(i, fc) \leq 0 \Leftrightarrow (i, fc) \in D_{functional}. \tag{8}$$

The part deviations (s, i) are then simulated recursively within the algorithm. Monte Carlo simulation is used to generate random variables simulating the part deviations (step 4 of the algorithm) with all the generated deviations being within the $D_{specification}$. A sample of part deviations is noted:

$$s' = \{s'_1, s'_2, \ldots, s'_n\}, \qquad i' = \{i'_1, i'_2, \ldots, i'_o\}. \tag{9}$$

For any given instance of iteration, the part deviations generated by Monte Carlo simulation should satisfy the set of constraints:

$$(s', i') \in D_{specification}. \tag{10}$$

For verifying assembly requirement (steps 5 & 6 of the algorithm), the aim is to verify the existence of gap configuration. For each sample (instance of part deviations), we verify if there exists an admissible gap configuration of the mechanism such as the assembly requirement (interface constraints) and the compatibility equations are verified:

$$(s', i', g) \in D_{compatibility} \cap D_{interface}. \tag{11}$$

This check is performed with help of translation of the relation (2). In the algorithm, for an instance of iteration, its mathematical form becomes

$$\exists\, \mathbf{g} : (\mathbf{s}', \mathbf{i}', \mathbf{g}) \in D_{compatibility} \cap D_{interface}. \tag{12}$$

Depending on this decision process it may be desirable:

- to determine whether a solution exists (verify the consistency of the Constraint Satisfaction Problem),
- to find one solution, to compute the space of all solutions of the Constraint Satisfaction Problem,
- or to find an optimal solution relative to a given cost function which respects all constraints ($\mathbf{C_c}$ and $\mathbf{C_i}$).

In our case, the goal is not to find a particular solution. But the check of the solution existence is made possible by using optimization to reduce the computing time. To do that, a numerical algorithm is used to find the global minimum of sum of the gaps subject to the constraints. By this method, a minimum solution (worst case) is obtained for gap configuration such that the assembly requirement is verified. In fact, the aim is to find if there are values of gaps such that all the constraints are satisfied and gaps have values more than 0 (there is no interference). That means the system is assembled. The result of this step effectively establishes if the individual parts with the simulated deviations would be able to assemble.

The constraints are a logical combination of equalities, inequalities, and domain specifications. This optimization problem includes non-linear constraints which may be equalities or inequalities. An example of non linear constraint is detailed in the following section: Application. Therefore we use Numerical Nonlinear Global Optimization techniques. Numerical algorithms for constrained nonlinear optimization can be broadly categorized into gradient-based methods and direct search methods. Gradient-based methods use first derivatives (gradients) or second derivatives (Hessians). Direct search methods do not use derivative information. The functions used in this case implement several algorithms for finding constrained global optima (Tool box of Mathematica). The methods are flexible enough to cope with functions that are not differentiable or continuous and are not easily trapped by local optima. The implemented algorithms include a tolerance for accepting constraint violations. Therefore, we add a step to check the consistency of the identified solution.

To evaluate the respect of the functional requirements (steps 7 & 8 of the algorithm) of each sample (instance of part deviations) that assembles, we verify if for all admissible gap configuration of the mechanism there exists functional characteristics such as the functional requirements are verified:

$$(\mathbf{s}', \mathbf{i}', \mathbf{g}, \mathbf{fc}) \in D_{compatibility} \cap D_{interface} \cap D_{functional}. \tag{13}$$

This check is performed with help of translation of the relation (3). In the algorithm, for an instance of iteration, its mathematical form becomes

$$\forall g \in \{\mathbf{g} \in Gap : (\mathbf{s}, \mathbf{g}, \mathbf{i}) \in D_{compatibility} \cap D_{interface}\}$$
$$\exists fc \in D_{functional}$$
$$: (s', i', g, fc) \in D_{compatibility} \cap D_{interface} \cap D_{functional}. \tag{14}$$

Depending on this decision process, it may be desirable to determine the space of all solutions of the Constraint Satisfaction Problem, or to find an optimal solution relative to a given cost function which respects all constraints ($\mathbf{C_c}$, $\mathbf{C_i}$ and $\mathbf{C_f}$).

Two alternative approaches to translate relation (3) have been formalized which are (a) optimization of worst-case values [28], (b) interval propagation with help of interval arithmetic to reduce the domain of the variables (functional characteristics). To reduce
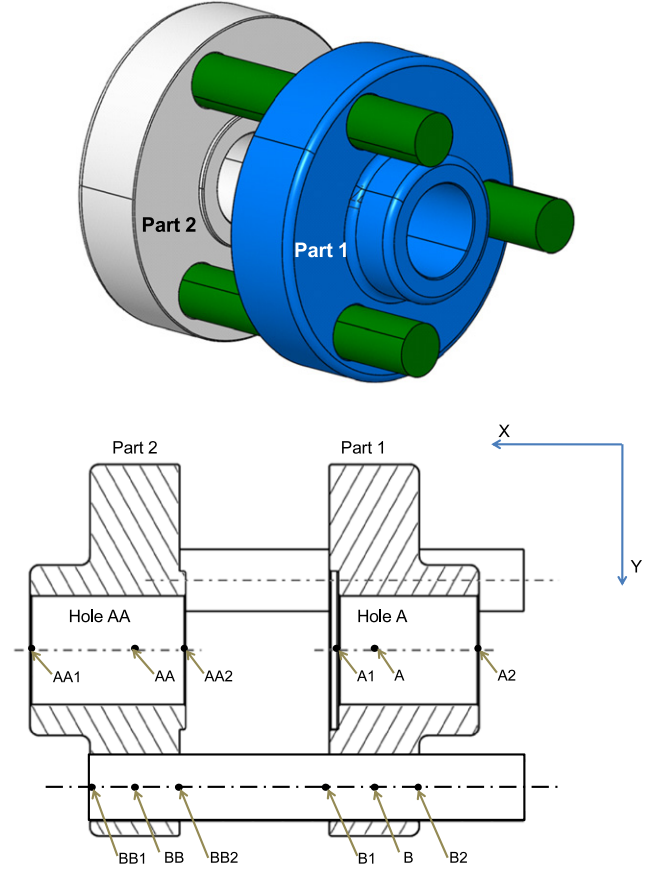

Fig. 2. Case study.

the computing time, we use the optimization approach to find the worst case values of the functional characteristics:

$$\mathbf{fc}_{max} = \mathrm{Max}(\mathbf{fc}(\mathbf{g}, \mathbf{s}', \mathbf{i}'))$$

S.T.

$$\mathbf{C_c}(\mathbf{s}', \mathbf{g}, \mathbf{fc}) = 0$$
$$\mathbf{C_i}(\mathbf{i}', \mathbf{g}) \leq 0 \quad \text{and} \quad \mathbf{C_i}(\mathbf{i}', \mathbf{g}) = 0$$
$$\mathbf{C_f}(\mathbf{i}', \mathbf{fc}) \leq 0$$
$$\mathbf{fc}_{min} = \mathrm{Min}(\mathbf{fc}(\mathbf{g}, \mathbf{s}', \mathbf{i}'))$$

S.T.

$$\mathbf{C_c}(\mathbf{s}', \mathbf{g}, \mathbf{fc}) = 0$$
$$\mathbf{C_i}(\mathbf{i}', \mathbf{g}) \leq 0 \quad \text{and} \quad \mathbf{C_i}(\mathbf{i}', \mathbf{g}) = 0$$
$$\mathbf{C_f}(\mathbf{i}', \mathbf{fc}) \leq 0$$
$$\mathbf{fc}_{max} \in D_{functional} \quad \text{and} \quad \boldsymbol{fc}_{min} \in D_{functional}. \tag{15}$$

For this step, we use the same Numerical Nonlinear Global Optimization techniques followed by the verification of the assembly requirement.

The result evaluates if the individual parts with the simulated deviations would be able to assemble as well as if the resultant mechanism would respect the functional requirements.

4. Application to an over-constrained mechanism

The approach developed for the tolerance analysis for 3D mechanical assemblies has been applied and validated over different models with and without GD & T specifications. For the sake of brevity and clarity of application, in this paper, a simple over-constrained mechanism as shown in Fig. 2 is taken as an example. This example is the simplified version of a forging tool
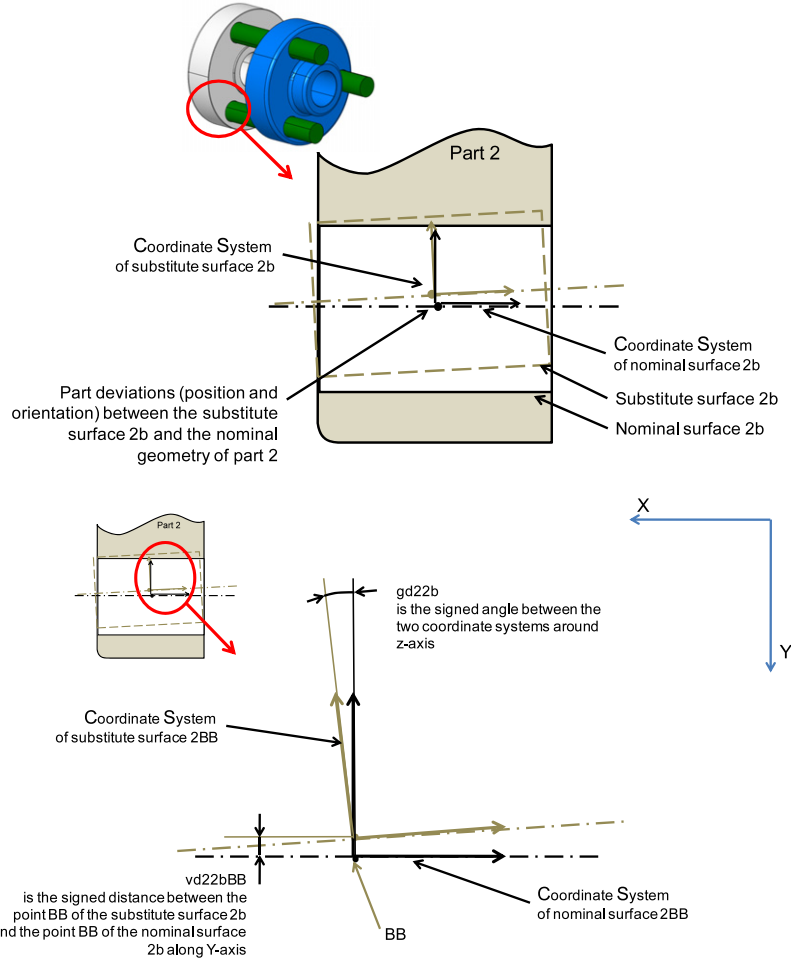
Fig. 3. Deviation modeling.

with omission of some parts. Fig. 2 illustrates the different views of the case study mechanism. The two main parts are assembled by three guide shafts. The contact between the shafts and the part 2 is fixed, and the contact between the shafts and the part 1 is floating. The functional characteristic (FC) is coaxiality between the center holes of the two parts. In terms of the tolerance analysis, this example is not simple. The position of the three guide shafts (120°) generate dependence of the displacements along y and z axes and of the rotations around the three axes regarding to the non linear constraints of non interference (gaps between the shafts and part 1).

This example includes 32 part deviation variables, 24 gap variables, 6 functional characteristic variables, 3 assembly requirement topological loops therefore 18 assembly requirement compatibility equations (linear equations), 3 functional requirement topological loops therefore 18 functional requirement compatibility equations (linear equations), 6 non interference constraints (6 non linear inequations), 6 fixed joint constraints (12 linear equations), and 2 functional requirement constraints (2 non linear inequations).

4.1. Geometrical description

As discussed in Section 2.2, the deviation of part surfaces, the gaps between parts and the functional characteristics between parts are described by four types of parameter. The proposed formalization is tool independent but for the sake of illustration, in this section the tool chosen to model the geometrical deviation is the Small Displacement torsors (SDT).

The SDT $D_{ka/k,M}$ defines the part deviation (position and orientation) between the substitute surface ka and the nominal geometry of part k, expressed at the point M. The components of a SDT can also be seen as different parameters for orientation and position: $D_{ka/k,M} = \{a_{dka/k}, b_{dka/k}, g_{dka/k}, u_{dka/k,M}, v_{dka/k,M}, w_{dka/k,M}\}$. $a_{dka/k}, b_{dka/k}, g_{dka/k}$ are the rotation deviation parameters and $u_{dka/k,M}, v_{dka/k,M}, w_{dka/k,M}$ are the position deviation parameters expressed at the point M. Fig. 3 illustrates this geometrical modeling.

In the same way, the gap can also be modeled by SDT. The SDT $G_{ka/ib,M}$ defines the gap (position and orientation) between the substitute surface ka of part k and the substitute surface ib of part i. The gap parameters characterize the displacement between the situation features of the two contact surfaces. In the case of a cylindrical joint, the gap parameters characterize the displacement between the two axes of the cylinders. Fig. 4 illustrates this gap modeling.

As mentioned in Section 2.2, the vector **s** is defined by all situation deviation parameters of all surfaces. For the case study, the vector **s** is given as:

$\mathbf{s} = \{bd11a, gd11a, vd11aA, wd11aA, bd11b, gd11b,$

$\quad vd11bB, wd11bB, bd11c, gd11c, vd11cC, wd11cC,$

$\quad bd11d, gd11d, vd11dD, wd11dD, bd22a, gd22a,$

$\quad vd22aAA, wd22aAA, bd22b, gd22b, vd22bBB,$

$\quad wd22bBB, bd22c, gd22c, vd22cCC, wd22cCC,$
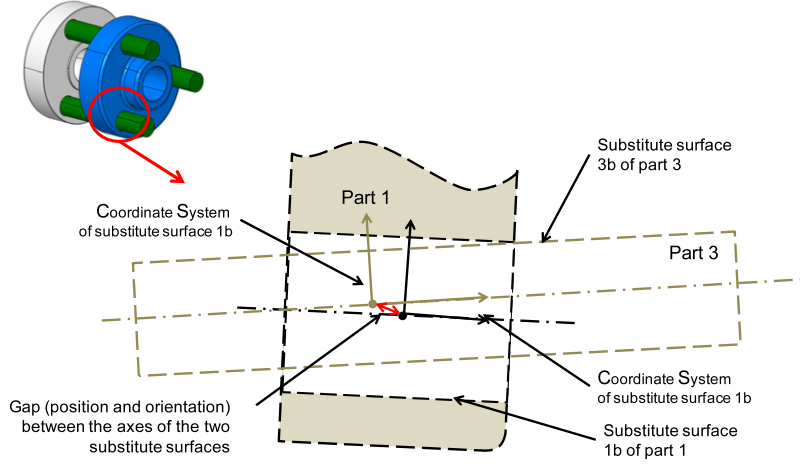
$\quad bd22d, gd22d, vd22dDD, wd22dDD\}.$

Fig. 4. Gap modeling.

The vector **i** is defined all intrinsic deviation parameters of all surfaces. For the case study, the vector **i** includes the diameter of each cylinder:

$$\mathbf{i} = \{d1b, d1c, d1d, d4, d5, d6\}.$$

In the same way, the vector **g** and **fc** is defined:

$$
\begin{aligned}
\mathbf{g} = \{ & ag2b4b, ag1b4b, bg1b4b, gg1b4b, ug2b4bBB, \\
& ug1b4bB, vg1b4bB, wg1b4bB, ag2c5c, ag1c5c, \\
& bg1c5c, gg1c5c, ug2c5cCC, ug1c5cC, vg1c5cC, \\
& wg1c5cC, ag2d6d, ag1d6d, bg1d6d, gg1d6d, \\
& ug2d6dDD, ug1d6dD, vg1d6dD, wg1d6dD \}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{fc} = \{ & afc1a2a, bfc1a2a, gfc1a2a, ufc1a2aA, \\
& vfc1a2aA, wfc1a2aA \}.
\end{aligned}
$$

This geometrical parameterization allows the modeling of the geometrical behavior which is detailed in the following section.

4.2. Geometrical behavior

The geometrical behavior of the mechanism is expressed by the composition relations of small displacements in the various loops of mechanism graph. The sum of the deviation situations and gaps along a loop of a mechanism graph must be equal to zero:

AR1: $D1/1d + G1d/6d + G6d/2d + D2d/2 + D2/2c$
$\qquad + G2c/5c + G5c/1c + D1c/1 = 0$

AR2: $D1/1d + G1d/6d + G6d/2d + D2d/2 + D2/2b$
$\qquad + G2b/4b + G4b/1b + D1b/1 = 0$

AR3: $D1/1c + G1c/5c + G5c/2c + D2c/2 + D2/2b$
$\qquad + G2b/4b + G4b/1b + D1b/1 = 0$

FR1: $D1/1b + G1b/4b + G4b/2b + D2b/2 + D2/2a$
$\qquad + G2a/fc + fc + Gfc/1a + D1a/1 = 0$

FR2: $D1/1c + G1c/5c + G5c/2c + D2c/2 + D2/2a$
$\qquad + G2a/fc + fc + Gfc/1a + D1a/1 = 0$

FR3: $D1/1d + G1d/6d + G6d/2d + D2d/2 + D2/2a$
$\qquad + G2a/fc + fc + Gfc/1a + D1a/1 = 0.$

The following equations show the set of equations generated through the compatibility hull:

$-ag1c5c + ag1d6d + ag2c5c - ag2d6d = 0$

$-bd11c + bd11d + bd22c - bd22d - bg1c5c + bg1d6d = 0$

$-gd11c + gd11d + gd22c - gd22d - gg1c5c + gg1d6d = 0$

$240 \cdot \sqrt{3} \cdot bd11c + 240 \cdot \sqrt{3} \cdot bd11d - 240 \cdot \sqrt{3} \cdot bd22c$
$\qquad - 240 \cdot \sqrt{3} \cdot bd22d + 240 \cdot \sqrt{3} \cdot bg1c5c + 240 \cdot \sqrt{3} \cdot bg1d6d$
$\qquad + 240 \cdot gd11c - 240 \cdot gd11d - 240 \cdot gd22c + 240 \cdot gd22d$
$\qquad + 240 \cdot gg1c5c - 240 \cdot gg1d6d - ug1c5cC + ug1d6dD$
$\qquad + ug2c5cCC - ug2d6dDD = 0$

$-240 \cdot \sqrt{3} \cdot ag1c5c - 240 \cdot \sqrt{3} \cdot ag1d6d$
$\qquad + 240 \cdot \sqrt{3} \cdot ag2c5c + 240 \cdot \sqrt{3} \cdot ag2d6d$
$\qquad - 300 \cdot gd22c + 300 \cdot gd22d - vd11cC$
$\qquad + vd11dD + vd22cCC - vd22dDD - vg1c5cC + vg1d6dD = 0$

. . .

(32 linear constraints)

$240, 240 \cdot \sqrt{3}, 300$ are some nominal distances. These nominal distances amplified the effect of the rotation at the considered point.

An interface constraint limits the geometrical behavior of the mechanism and characterizes non-interference constraint or press fit constraint. For joints 1/4, 1/5, 1/6, the gap is limited by the non-interference constraint. Fig. 5 illustrates a worst configuration of gap.

The following equations show the set of equations generated through the interface hull:

$(100 \cdot gg1b4b + vg1b4bB)^2 + (-100 \cdot bg1b4b + wg1b4bB)^2$
$\qquad \le ((d1b - d4)/2)^2$

$(-100 \cdot gg1b4b + vg1b4bB)^2 + (100 \cdot bg1b4b + wg1b4bB)^2$
$\qquad \le ((d1b - d4)/2)^2$

$(100 \cdot gg1c5c + vg1c5cC)^2 + (-100 \cdot bg1c5c + wg1c5cC)^2$
$\qquad \le ((d1b - d5)/2)^2$

$(-100 \cdot gg1c5c + vg1c5cC)^2 + (100 \cdot bg1c5c + wg1c5cC)^2$
$\qquad \le ((d1b - d5)/2)^2$

$(100 \cdot gg1d6d + vg1d6dD)^2 + (-100 \cdot bg1d6d + wg1d6dD)^2$
$\qquad \le ((d1b - d4)/2)^2$

$(-100 \cdot gg1d6d + vg1d6dD)^2 + (100 \cdot bg1d6d + wg1d6dD)^2$
$\qquad \le ((d1b - d4)/2)^2$

100 is a nominal distance.

In the same way, the functional requirement is modeling. The distances between the two axes at the extreme points (A1 and A2) of the FR must be smaller than the FR. Fig. 6 illustrates a product configuration and the considered distance to formalize the coaxiality between the two holes (the datum is the hole of the part 2).
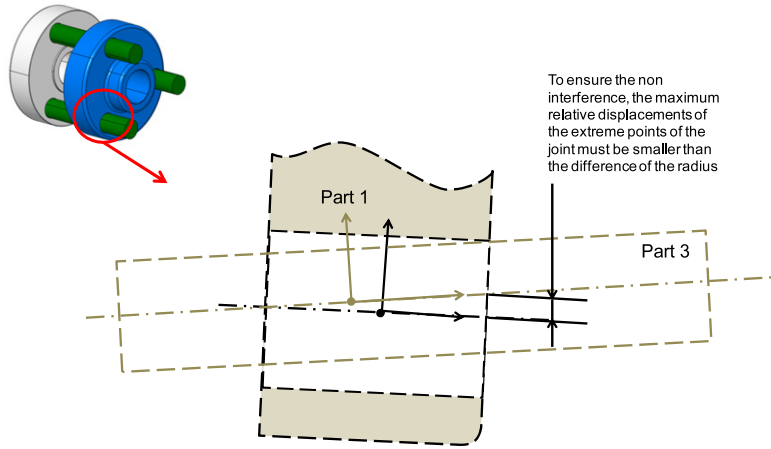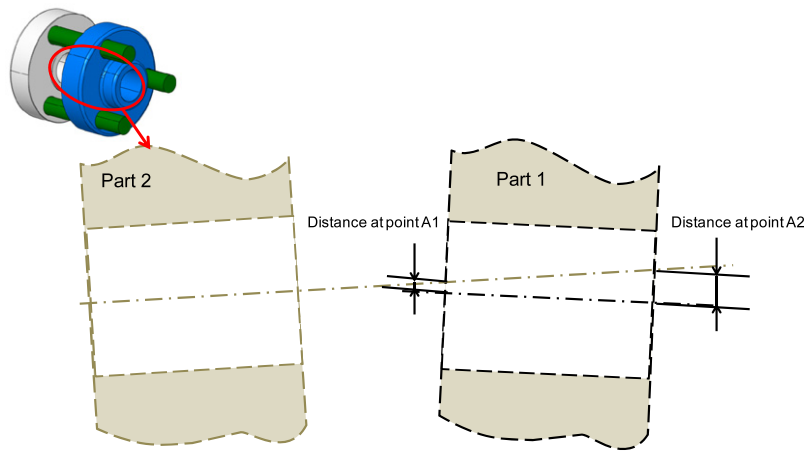
Fig. 5. Gap configuration.

Fig. 6. Functional requirement.

4.3. Optimization

For the sake of brevity, the algorithm is not detailed; to illustrate the non linear optimization problem, the related part is detailed: the maximization of one functional characteristic:

Maximization of $(-100 \cdot gfc1a2a + vg1a2aA)^2$

$+ (100 \cdot bfc1a2a + wg1a2aA)^2$

S.T.

$-ag1c5c + ag1d6d + ag2c5c - ag2d6d = 0$

$-bd11c + bd11d + bd22c - bd22d - bg1c5c + bg1d6d = 0$

$-gd11c + gd11d + gd22c - gd22d - gg1c5c + gg1d6d = 0$

$240 \cdot \sqrt{3} \cdot bd11c + 240 \cdot \sqrt{3} \cdot bd11d - 240 \cdot \sqrt{3} \cdot bd22c$

$- 240 \cdot \sqrt{3} \cdot bd22d + 240 \cdot \sqrt{3} \cdot bg1c5c$

$+ 240 \cdot \sqrt{3} \cdot bg1d6d + 240 \cdot gd11c - 240 \cdot gd11d$

$- 240 \cdot gd22c240 \cdot gd22d + 240 \cdot gg1c5c$

$- 240 \cdot gg1d6d - ug1c5cC + ug1d6dD + ug2c5cCC$

$- ug2d6dDD = 0$

$-240 \cdot \sqrt{3} \cdot ag1c5c - 240 \cdot \sqrt{3} \cdot ag1d6d$

$+ 240 \cdot \sqrt{3} \cdot ag2c5c + 240 \cdot \sqrt{3} \cdot ag2d6d$

$- 300 \cdot gd22c + 300 \cdot gd22d - vd11cC$

$+ vd11dD + vd22cCC - vd22dDD - vg1c5cC$

$+ vg1d6dD = 0$

...

$(100 \cdot gg1b4b + vg1b4bB)^2 + (-100 \cdot bg1b4b + wg1b4bB)^2$

$\leq ((d1b - d4)/2)^2$

$(-100 \cdot gg1b4b + vg1b4bB)^2 + (100 \cdot bg1b4b + wg1b4bB)^2$

$\leq ((d1b - d4)/2)^2$

$(100 \cdot gg1c5c + vg1c5cC)^2 + (-100 \cdot bg1c5c + wg1c5cC)^2$

$\leq ((d1b - d5)/2)^2$

$(-100 \cdot gg1c5c + vg1c5cC)^2 + (100 \cdot bg1c5c + wg1c5cC)^2$

$\leq ((d1b - d5)/2)^2$

$(100 \cdot gg1d6d + vg1d6dD)^2 + (-100 \cdot bg1d6d + wg1d6dD)^2$

$\leq ((d1b - d4)/2)^2$

$(-100 \cdot gg1d6d + vg1d6dD)^2 + (100 \cdot bg1d6d + wg1d6dD)^2$

$\leq ((d1b - d4)/2)^2.$

A numerical algorithm was applied to find the global maximum of the functional characteristic subject to the constraints.

4.4. Results

The tolerance analysis of the mechanism shown in Fig. 2 was performed using normal distribution, for each displacement of the extreme points of each hole, with mean at zero and dimension specific standard deviations derived from specified tolerance. The program also calculated the worst case values of the gaps "**g**" for which the assembly conditions and functional requirements by optimization were respected.

Table 2
Results for the over-constrained case study for 5 different runs.

Run	Nominal dimensions (mm)	Standard deviation (mm)	Number of accepted samples	Number of constraint violations	Probability of AR (%)	Probability of AR & FR (%)
1	$d1i = 20$; $d4 = 19.5$	0.03	9 996	4	99.81	95.98
2	$d1i = 20$; $d4 = 19.8$	0.03	8 898	1102	59.87	59.87
3	$d1i = 20$; $d4 = 19.8$	0.05	8 786	1214	22.65	22.64
4	$d1i = 20$; $d4 = 19.8$	0.08	9 440	560	3.55	3.42
5	$d1i = 20$; $d4 = 19.8$	0.01	10 000	0	99.91	99.91

The program was tested with 10 000 simulations for different nominal values and standard deviations. The results show the importance of the verification step. The worst case is 10% of constraint violation (see Table 2).

5. Discussion

We introduce a new approach about tolerance analysis, and we compare this tolerance analysis approach with the mathematical approaches developed by Davidson and Shah [20], Giordano [17], Teissandier [18]. In these approaches, the geometrical tolerances, the dimensioning tolerances or the contact constraints are represented by deviation domain/clearance domain or T-Map®. These three concepts are a hypothetical Euclidean volume which represents all possible deviations in size, orientation and position of features. For tolerance analysis, this mathematical representation of tolerances allows to calculate the composition of tolerances by Minkowsky sum of deviation and clearance domains; to calculate the intersection of domains for parallel kinematic chain; and to verify the inclusion of a domain inside other one. The methods are very efficient for the tolerance analysis, but the computational cost depends on the number of Minkowsky sums. Moreover, these methods have a major drawback: the limited scope. We can only use these methods for linear problem (linear accumulation by Minkowsky sum).

In the proposed approach, we use the same mathematical representation, and we add the compatibility domain which represents the composition relations of displacements in the various topological loops, and the mathematical formulation based on the quantifier notion. Therefore, we do not use Minkowsky sum for the resolution, we use a hybrid approach which include Monte Carlo simulation and optimization.

Moreover, we compare this tolerance analysis approach with the mathematical approach developed by Ballu [15]. In fact, Ballu proposes a mathematical approach for tolerance analysis of over-constrained mechanism. The differences between the two approaches are the format of the functional response function and the probabilistic approach (FORM/SORM for Ballu's approach and Monte Carlo simulation for this approach). To apply Ballu's approach, we need an explicit function; that reduces the scope of this approach. For explicit function, Ballu's approach is very efficient; using the approach FORM/SORM can reduce the computational cost.

This approach allows the tolerance analysis of over-constrained mechanisms without explicit response function and without linearization of the non linear constraints.

6. Conclusion

Tolerancing decisions can profoundly impact the quality and cost of the mechanism. To evaluate the impact of a tolerance on the mechanism quality, designers need to simulate the influences of this tolerance with respect to the functional requirements. Therefore, the objective of this paper is *"How to analyze the impacts of geometrical deviations of over-constrained mechanism with non linear behavior?"*

The following are the outcome of the study:

- Mathematical formulation of tolerance analysis with integration of the Quantifier notion is a new technique that uses the notion of the universal quantifiers ∀ and ∃ which provide a univocal expression of the condition corresponding to a geometrical product requirement. The application of tolerance analysis developed in this work relies on the integrated concept of quantifiers to quantify, control and verify the respect of the required functional requirement (∃ quantifier) as well as the geometrical product requirement (∀ quantifier). This addition adds a qualitative and quantitative nature to the tolerance analysis process allowing the user to specify the control elements in a model as well providing the necessary tools for validation via probability based statistical analysis.
- The algorithm developed in this research paper addresses the 3D application of the tolerance analysis for complex system (over-constrained mechanism with non linear behavior). The fundamental steps of the algorithm however remains same for all the applications i.e. the definition of the product parametric model with help of variation parametric space in either 3 dimensions using convex hulls, use of Monte Carlo simulation in case of statistical tolerance analysis, application of "there exists" quantifier to ascertain that the model with generated deviations conforms to different convex hulls i.e. respect of compatibility hull, interface hull and functional hull and display of results of calculation regarding the conformity of the assembly with respect to the individual hulls. Monte Carlo simulation has been used in conjunction with the optimization to calculate the probability of the functional operation of an assembly.

For the consistent and reliable application of the Monte Carlo simulation to the statistical tolerance analysis, the number of samples is the key of precision. By a large number of samples, the precision can be improved, but the computational cost will be increased. The improvement of this approach should be an area for some intense research on stochastic methods coupled with worse case methods.

References

[1] Chase KW, Parkinson AR. A survey of research in the application of tolerance analysis to the design of mechanical assemblies. Research in Engineering Design 1991;3:23–37.

[2] Roy U, Li B. Representation and interpretation of geometric tolerances for polyhedral objects. Computer Aided Design 1999;31(4):273–85.

[3] Bourdet P, Mathieu L, Lartigue C, Ballu A. The concept of small displacement torsor in metrology. In: Advanced mathematical tool in metrology. Advances in Mathematics for applied sciences, vol. 40. World Scientific; 1996.

[4] Ballot E, Bourdet P. A computation method for the consequences of geometric errors in mechanisms. In: CIRP seminar on computer aided tolerancing. 1997.

[5] Gupta S, Turner JU. Variational solid modeling for tolerance analysis. IEEE Computer Graphics & Application 1993;13(3):64–74.

[6] Desrochers A. Modeling three dimensional tolerance zones using screw parameters. In: 25th design automation conference, ASME. 1999.

[7] Desrochers A. Modeling three dimensional Tolerance zones using screw parameters. In: 25th design automation conference, ASME. 1999.

[8] Pasupathy KTM, Morse EP, Wilhelm RG. A survey of mathematical methods for the construction of geometric tolerance zones. Journal of Computing and Information Science in Engineering 2003;3:64–75.

[9] Kyung MH, Sacks E. Nonlinear kinematic tolerance analysis of planar mechanical systems. Computer Aided Design 2003;35:901–11.

[10] Huang W, Lin J, Bezdecny M, Kong Z, Ceglarek D. Stream-of-variation modeling—part i: a generic three-dimensional variation model for rigid-body assembly in single station assembly processes. Journal of Manufacturing Science and Engineering 2007;129:821–31.

[11] Dantan JY, Qureshi J. Worse case and statistical tolerance analysis based on quantified constraint satisfaction problems and Monte Carlo simulation. Computer Aided Design 2009;41(1):1–12.

[12] Hong YS, Chang T-C. A Comprehensive review of tolerancing research. International Journal of Production Research 2002;40(11):2425–59.

[13] Nigam SD, Turner JU. Review of statistical approaches to tolerance analysis. Computer Aided Design 1995;27(1):6–15.

[14] Mhenni F, Serré P, Mlika A, Romdhane L, Rivière A. Dependency between dimensional deviations in overconstrained mechanisms. Conception et Production intégrées CPI 2007, Rabat, Maroc, October, 2007. p. 22–4.

[15] Ballu A, Plantec J-Y, Mathieu L. Geometrical reliability of overconstrained mechanisms with gaps. CIRP Annals—Manufacturing Technology 2009;57: 159–62.

[16] Giordano M, Duret D. Clearance space and deviation space, application to three-dimensional chain of dimensions and positions. In: CIRP seminar on computer aided tolerancing. 1993.

[17] Giordano M, Samper S, Petit JP. Tolerance analysis and synthesis by means of deviation domains, axi-symmetric cases. In: 9th CIRP international seminar on computer aided tolerancing, held at Arizona State University. 2005. p. 10–2.

[18] Teissandier D, Couétard Y, Gerard A. A computer aided tolerancing model: proportioned assembly clearance volume. Computer Aided Design 1999; 31(13):805–17.

[19] Mansuy M, Giordano M, Hernandez P. A new calculation method for the worst case tolerance analysis and synthesis in stack-type assemblies. Computer Aided Design 2011;43(9):1118–25.

[20] Davidson JK, Shah JJ. Using Tolerance-Maps to represent material condition on both a feature and a datum. In: the 8th international CIRP seminar on computer aided tolerancing. 2003. p. 92–101.

[21] Ameta G, Davidson JK, Shah JJ. Using tolerance-maps to generate frequency distributions of clearance and allocate tolerances for Pin–Hole assemblies. ASME Transactions, Journal of Computing and Information Science in Engineering 2007;7(4):347–59.

[22] Dantan JY, Ballu A. Assembly specification by gauge with internal mobilities (G.I.M.): a specification semantics deduced from Tolerance synthesis. Journal of Manufacturing Systems 2002;21(3):218–35.

[23] Dantan JY, Mathieu L, Ballu A, Martin P. Tolerance synthesis: quantifier notion and virtual boundary. Computer Aided Design 2005;37(2):231–40.

[24] Dantan JY, Mathieu L, Ballu A. Geometrical product requirement: uncertainty and expression. In: 8th CIRP international seminar on computer aided tolerancing. 2003. p. 199–208.

[25] Sacks E, Joskowicz L. Parametric kinematic tolerance analysis of general planar systems. Computer Aided Design 1998;30(9):707–14.

[26] Morse EP, You X. GapSpace multi-dimensional assembly analysis. In: CIRP seminar on computer aided tolerancing. 2009.

[27] Bordeaux L, Monfroy E. Beyond NP: arc-consistency for quantified constraints. In: Proceedings of CP-2002. 2002.

[28] Kamali nejad M, Vignat F, Villeneuve F. Simulation of the geometrical defects of manufacturing. The International Journal of Advanced Manufacturing Technology 2009;45:631–48.