## "A STATISTICAL TOOL FOR BREAKWATER DESIGN"

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### Abstract

This article presents a methodology for the design of breakwater dikes which includes wave estimation, evaluation of the runup at the dike, calculation of the dynamic and pseudohidrostatic forces operating on the crown wall and finally the estimation of the stability of the armour units in the main layer.

The methodology presented is a development of existing experience in the evaluation of the effects produced by regular or monochromatic waves on structures (stability of pieces, induced forces and flow on the slope). This experience is extended to irregular or spectral waves via statistical distribution of waves, taking account of parameters such as root mean square or significant wave height, mean or peak period and water depth.

The application of the methodology described here has given satisfactory results compared with those of other methodologies and experimental data from various researchers.

# INTRODUCTION

The function of a breakwater is to provide a sheltered area, to allow certain port activities on the quay and/or to protect against sediment transport in the coastal area. Breakwaters have an outer layer that has to be stable under the wave action, it is constructed with armour units of either natural or artificial material. A core provides the support for the

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main layer. In between, there are several layers, forming a transition between the core and main layer. It is common practice, for economic or functional reasons, to construct a crown wall that reduces the extension of the main layer, the most expensive part of the dike.

Wave estimation is perhaps the most important calculation in the design of breakwaters. It is very easy to arrive at erroneous conclusions which can result in badly designed structures. The best way to determine the sea state is undoubtedly through measurements made in situ, preferably over a period of years. However this is not always possible due to the high cost of measuring equipment, its maintenance and operation as well as lack of time.

Where this information is unavailable it is possible to estimate parameters such as root mean square or significant height and its associate mean period by different means. Green (1994) compared different theoretical distributions with several of wave series. His conclusion was that Tayfun's statistical distribution for wave heights best describes the different states of the sea.

Figure 1 shows an example of best fit for different distributions of wave heights to waves corresponding to a TMA spectrum in shallow waters. As can be seen, the Tayfun's distribution offers the best fit.



Figure 1. Probability of exceedance of irregular wave against three statistical distributions.

### ESTIMATION OF RUNUP

As regards the geometric design of coastal protection structures, such as mound breakwaters, the estimation of runup is particularly important in determining the crest elevation for structures allowing overtopping and those which do not.

Based on experimental results of his own and other researchers, Losada et al (1981) developed a mathematical model to estimate runup. The original method, which only be applied to regular waves, is given by

$$Ru = H \left[Au(1 - e^{-Bu/r})\right] \tag{1}$$

where Ir is the Iribarren number defined as,

$$Ir = \frac{\tan \alpha}{\sqrt{H / L_o}}$$
(2)

H is the wave height  $L_0$  is the wave length in deep water  $\alpha$  is the slope angle

The coefficients Au and Bu depend on the type of material of which the main layer of the dike is made.

Ahrens (1988) proposes another formula for the estimation of runup

$$\frac{Ru}{H} = \frac{a \ Ir}{1 + b \ Ir} \tag{3}$$

where the coefficients a and b are obtained by a regression analysis.

Through a last square method the values of Au and Bu corresponding to various materials have been found. Table 1 gives the values found for two types of breakwater, firstly homogeneous in which there is no core, and then one with two armour units on its main layer and an impermeable core. Van der Meer (1988) defined a "porosity parameter P", that for the case of a homogeneous rubble mound breakwater corresponds to value of 0.6, and for the case of a rubble mound breakwater with two pieces in its main layer and impermeable core, P = 0.1.

			HON	MOGENEOUS	IMPERMEABLE CORE		
Material	Porosity	Au	Bu	Reference	Au	Bu	Reference
Rip-rap	0.31	1.80	0.46	Ahrens, 1975*	2.00	0.32	Ahrens, 1968
Rubble	0.40	1.37	0.60	Gumbak, 1976*	1.89	0.40	Seeling, 1980
Cubes	0.47	1.05	0.72	Jackson, 1968*			
Tetrapods	0.50	0.93	0.75	Jackson, 1968*	1.40	0.45	Dai & Kamel, 1969
Dolosse	0.56	0.70	0.82	Wallingford, 1970*	1.19	0.53	Wallingford, 1970

 Table 1 Au and Bu for different homogeneous breakwaters and breakwaters with impermeable core. \* Compiled by Losada (1991)

In figures 2 and 3 the linear relation between the Au and Bu parameters versus the porosity of the main layer of the dike can be seen. In order to find the runup produced by a dike the parameters for that type of main layer material are evaluated and Losada's exponential model is applied. If the core of the dike is neither homogeneous nor impermeable the values of Au and Bu can be estimated through an interpolation of the two extreme cases.



Figure 2. Variation in the parameter Au related to porosity of the main layer



**Figure 3.** Variation in the parameter Bu related to porosity of the main layer For homogenous dikes, the Au and Bu coefficients can be found through

$$Au = -4.706 \cdot n + 3.293 \tag{4}$$

 $Bu = -1.569 \cdot n + 0.038 \tag{5}$ 

For dikes with impermeable core, the Au and Bu coefficients can be found through

$$Au = -3.825 \cdot n + 3.344 \tag{6}$$
  

$$Bu = -1.179 \cdot n + 0.081 \tag{7}$$

where n is the porosity.

Applying the formulas of Losada et al. (1981) and Ahrens (1988), the results obtained are practically the same. This can be seen in figure 4, which shows Ru/H against lr for the case of a homogeneous dike composed of rip rap. Based on these characteristics, the parameters a and b of the Ahrens formula for different main layers was evaluated using the previous values of Au and Bu. These results are shown in figures 5 and 6. The latter gives the relation a/b against the porosity of the main layer. The relation shows a linear tendency between parameters a/b and b versus porosity.



Figure 4. Ru/H versus Ir for the Losada et al. (1981) and Ahrens (1988) formulas Ir for the case of a homogeneous dike composed of rip rap.

In order to obtain the parameters a and b the following equations are used:

For homogeneous dikes

$$a = b \ (-5.5589 \cdot n + 3.7954) \tag{8}$$

$$b = 3.9753 \cdot n - 0.6774 \tag{9}$$

For dikes with impermeable core

 $a = b \ (-3.6922 \cdot n + 3.5785) \tag{10}$ 

$$b = 1.3971 \cdot n + 0.0501 \tag{11}$$



Figure 5. Ahrens coefficient b versus porosity of the main layer.



Figure 6. Relation a/b of the Ahrens formula versus main layer porosity.

To extrapolate the results of regular wave criteria to irregular wave criteria Silva et al. (1997b) and Govaere (1997) proposed a method in which the distribution of runup is considered as the same type as that of the wave height. The wave height distribution presented by Tayfun (1981) was used, a probability distribution which takes into account wave period, root mean square wave height and local water depth at the toe of the breakwater among other parameters.

The methodology is as follows:

• Depending on the mechanical characteristics of the breakwater and the formula selected, Losada et al (1981) or Ahrens (1988), Au and Bu (figures 2 and 3) or a and b (figures 5 and 6) are chosen.

• An Ru<sub>mus</sub>, representative of the mean characteristics of the flow, is evaluated,

using the formula of Losada et al (1981)

$$Ru_{rms} = H_{rms} \left[ Au(1 - e^{\beta u \, J_{rms}}) \right] \tag{12}$$

or using Ahrens (1988) formula

$$Ru = H_{rms} \frac{a \ Ir_{rms}}{1 + b \ Ir_{rms}}$$
(13)

where,

$$J_{r_{rms}} = \frac{\tan \alpha}{\sqrt{H_{rms} / L_{o_{med}}}}$$
(14)

$$Lo_{med} = \frac{g\overline{T}^2}{2\pi}$$
(15)

$$H_{rms} = \left[\frac{1}{N}\sum_{l=1}^{N}H_{l}^{2}\right]^{1/2} = \sqrt{8}m_{o}$$
(16)

 $\overline{T}$  is the mean period

• Tayfun's statistical distribution function (1981), modified to generate a runup distribution, is applied:

$$p(\xi, N) = \xi \int_{0}^{\infty} \left[ u J_{o}^{N} \left( \frac{u}{N^{1/2}} \right) J_{o}(\xi \ u) \right] du \qquad 0 \le \xi \le N^{1/2}$$

$$p(\xi, N) = \xi \left[ I - \frac{4}{\pi} \cos^{-1} \left( \frac{N^{1/2}}{\xi} \right) \right] \int_{0}^{\infty} \left[ u J_{o}^{N} \left( \frac{u}{N^{1/2}} \right) J_{o}(\xi \ u) \right] du \qquad N^{1/2} \le \xi \le (2N)^{1/2}$$

$$(17)$$

where N is the parameter defined by Tayfun as:

$$N = \left(\frac{\pi}{7\sqrt{2}} \frac{\tanh(k_o h)}{k_o \sqrt{2} m_o}\right)$$
(18)

and  $\xi = Ru / Ru_{rms}$ 

Using this methodology it is possible to evaluate the runup for all types of main layers and for different core porosities under irregular wave attack and for whatever probability of exceedance.

Figure 7 shows a comparison of this methodology with that of van der Meer (1988), which was developed for irregular wave criteria and gives very good results but unfortunately can be applied to very few cases. As can be seen, the proposed method gives good results for whatever probability of exceedance.



Figure 7 Comparison of Ru/Hs versus Iribarren's parameter with the suggested method (•••) and van der Meer (1988) formula (------) for different probabilities of exceedance.

# **CROWN WALL DESIGN**

A conventional breakwater is generally composed of two structures which are very different in their behaviour and response to wave action, figure 8. First, there is the body of the breakwater composed of a core of loose material protected by a series of layers of larger pieces. The second is a structure embedded into the top of the former, usually a crown wall of concrete, where services are installed. Being made of loose materials the body of the breakwater is more easily deformed and the damage is ductile in nature; generally taking place over a period of time, after storms. On the other hand, the crown wall is a rigid structure; damage here is of a fragile nature; usually the result of the action of just one sufficiently large wave.

There are numerous methods to calculate the forces acting upon a crown wall, all of which are based on laboratory experiments. Martin et al. (1995) proposed a model which separates the pressures of dynamic origin from those of pseudohydrostatic origin, since these pressure are presented at different times. Figure 9 shows the forces affecting a crown wall according to this method.



Figure 8. Cross section of a crown wall

Martin's method was originally developed under regular wave criteria. The extension to the case of irregular wave criteria was made by Silva et al. (1997a) and can be summarised as follows:

### **Dynamic pressure**

The law of dynamic pressures, P<sub>d</sub>, on the breakwater can be evaluated as:



Figure 9. Laws of pressure affecting a crown wall

$P_d = \beta \rho g S$	for $A_c < z < A_c + S$		
$P_{4} = \lambda P_{4} = \lambda \beta \rho g S$	for the crown wall foundation $< z < A$ .	(21)	

With the runup,  $Ru_x$ , for the probability of exceedance chosen, equation (12) o (13), the parameters  $\alpha$  and  $\lambda$  are calculated from the following expressions:

$$\beta = 2 \operatorname{Ru}_{x} / \operatorname{H} \cos^{2} \alpha \cos \theta \tag{22}$$

where,

$$S = H (1 - A_c / Ru_x)$$
 (23)

$$\lambda = 0.8 \, \mathrm{e}^{(-10.9\mathrm{B/Lmed})} \tag{24}$$

 $\theta$  is the angle of incidence

#### Pseudohydrostatic pressure

Martin et al. (1995) proved that the law of pseudohydrostatic pressure is linear and proportional to  $\mu \rho g$ , where  $\mu$  is a factor of 1 or less, shown by them.

$$P_{\rm h}(z) = \mu \rho g \left( S + A_{\rm c} \right) \tag{25}$$

Comparison of the suggested method was made with measurements of Burcharth et al. (1995) and experiments carried out by Pedersen (1996). The proposed method gives very good results and has the advantage that it can be applied to a wide variety of crown walls of different configurations. More details can be seen in Silva et al. (1997a) and Martin et al. (1995).

#### ARMOUR STABILITY

Nowadays the methodologies most used to evaluate the stability of the main layer of a dike are those of Hudson (Shore Protection Manual, SPM (1984), the formula of Losada et al. (1979) and the van der Meer formulas (1988). The first two were developed under regular wave criteria and numerous investigations have been carried out with the idea of extrapolating these formulas to irregular wave criteria, such as that of van der Meer (1988).

After some experimental work in a laboratory, Jensen et al. (1996), suggested that when using formulas of SPM (1984) the wave height  $H_{250}$  be taken(the mean of the 250 highest waves in a sea state) to obtain the same result as with irregular wave. Vidal et al. (1995) suggest using  $H_{100}$  (the mean of the 100 highest waves in a sea state) when Losada's formula is used. Using the  $H_0$  concept is better than that of a probability of exceedance or of  $H_{1/n}$ , in that using the mean of a given number of large waves implicitly takes into account the length of the storm.



Figure 10. Example of wave height probability of exceedance.

Where the wave height distribution is unknown Tayfun's formula (1981) ean be applied, and results as presented in figure 10 are obtained. The minimum probability of exceedance given for a sea state defined by N waves is 1/N. H<sub>n</sub> would be the mean wave height found within the probability of exceedance n/N and N, as follows,

$$H_n = \frac{N}{n-1} \int_{VN}^{n/N} H \, dp \tag{26}$$

Normally, using Hudson's formula with the  $K_D$  parameter presented in the SPM (1984) gives very conservative armour sizes, as if the uses  $H_{100}$  and  $H_{250}$ . The number of stability, Ns, used in Hudson's formula and the function of stability  $\Psi$  used in the Losada's formula are related by the relation Ns<sup>3</sup> = 1/ $\Psi$ .

Figures 10 and 11 compare the methods of van der Meer (1988), Losada et al. (1979) using  $H_{100}$ , as suggested by Vidal et al. (1995), and SPM (1984) using  $H_{250}$ , again as suggested by Vidal et al. (1995) versus van der Meer's experimental results (1988). A sea state of 1000 and 3000 individual waves, respectively was considered for levels of damage, S, between 1.5 and 2.5, mass density of the rock  $\rho_s = 2650 \text{ kg/m}^3$ , mass density of the water  $\rho_w = 1025 \text{ kg/m}^3$ . The rest of the parameters are presented in table 2. In the first case, 1000 waves, the root mean square errors are: van der Meer (0.0031), Losada (0.0029) and SPM (0.0039), and for the second case, 3000 waves, the root mean square errors are: Van der Meer (0.0061), Losada (0.0075) SPM (0.0064).

Method	Parameters
Van der Meer (1988)	$K_{\rm p} = 4.0$
SPM (1984)	P = 0.4
Losada et al. (1979)	For $\cot \alpha = 3.0$ : Ir <sub>0</sub> = 0.88, A <sub>w</sub> = 0.04697, B <sub>w</sub> = -0.8084
(Values compiled in	For $\cot \alpha = 2.0$ : $Ir_0 = 1.33$ , $A_w = 0.05698$ , $B_w = -0.6627$
Losada, 1991)	For $\cot \alpha = 1.5$ : $Ir_0 = 1.77$ , $A_w = 0.09035$ , $B_w = -0.5879$

Table 2. Parameters used to evaluate figures 10 and 11, for each method.

The extension of Losada et al (1979) to irregular wave criteria and van der Meer metho take into account the wave period effect, Losada's formula in the calculation of th stability function  $\Psi$  and van der Meer's in the calculation of the stability Ns. Bot methods produce results of similar dispersion. The SPM method however does not tak into account the wave period, even so the root mean square error is similar to those c Losada and van der Meer.



Figure 10. SPM (1984), Losada et al. (1979) and van der Meer (1988) methods versus experimental data of van der Meer (1988) for dimensionless weight of the main layer.



Figure 11. SPM (1984), Losada et al. (1979) and van der Meer (1988) methods versus experimental data of van der Meer (1988) for dimensionless weight of the main layer.

# CONCLUSIONS

The statistical distribution for wave height presented by Tayfun (1981) correctly represents different sea states, having the advantage of considering wave breaking in shallow waters.

The method of Losada et al. (1981) and Ahrens (1988) can be used to estimate runup for regular waves for whatever type of armour unit used in the main layer of a dike and for different porosities of the core. The extension to the case of irregular wave criteria via the Tayfun distribution is simple and gives very good results.

Crown wall design using the methodology for irregular wave criteria given by Silva et al. (1997a) gives good results and is easy to apply.

If it is used the wave height of  $H_{250}$  is used, as suggested by Jensen et al. (1996), in Hudson's formula, or the wave height of  $H_{100}$ , as suggested by Vidal et al. (1995), in Losada's formula for the evaluation of the weight of the pieces in the main layer, almost the same results as for the van der Meer's formula.

For shallow waters, using the Tayfun distribution along with the stability function given by Losada et al. (1979) gives a significant reduction of the weight of pieces in the main layer related to those results given by the van der Meer (1988) formula.

The methodology described in this article has given very good results and presents a tremendous advantage in being applicable to many cases, which is not true of the formulas conceived under the irregular wave criteria.

### ACKNOWLEDGEMENTS

This research was funded by the Mexican Consejo de Ciencia y Tecnología, CONACYT, project *Estabilidad de estructuras permeables bajo la acción del oleaje* under contract 25114-A.

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