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Research Article

Shi Yin and Baizhou Li*

A stochastic differential game of low carbon technology sharing in collaborative innovation system of superior enterprises and inferior enterprises under uncertain environment

Abstract: Considering the fact that the development of low carbon economy calls for the low carbon technology sharing between interested enterprises, this paper study a stochastic differential game of low carbon technology sharing in collaborative innovation system of superior enterprises and inferior enterprises. In the paper, we consider the random interference factors that include the uncertain external environment and the internal understanding limitations of decision maker. In the model, superior enterprises and inferior enterprises are separated entities, and they play Stacklberg master-slave game, Nash non-cooperative game, and cooperative game, respectively. We discuss the feedback equilibrium strategies of superior enterprises and inferior enterprises, and it is found that some random interference factors in sharing system can make the variance of improvement degree of low carbon technology level in the cooperation game higher than the variance in the Stackelberg game, and the result of Stackelberg game is similar to the result of Nash game. Additionally, a government subsidy incentive and a special subsidy that inferior enterprises give to superior enterprises are proposed.

1 Introduction

Global environment is an indivisible whole ecosystem. There seems to be rather compelling evidence that environmental pollution, resource depletion and global warming are issues that we seriously need to be concerned about today. Against this background, the development of low carbon technology has become an important support for global social and economic power. Responding to the development, low carbon technological innovation is playing a vital role in development of low carbon technology. How to achieve the low carbon technological innovation in enterprises is not only an important factor affecting regional

Shi Yin: School of Economics and Management, Harbin Engineering University, Harbin 150001, China,
E-mail: shyshi0314@163.com
*Corresponding Author: Baizhou Li: School of Economics and Management, Harbin Engineering University, Harbin 150001, China, E-mail: acadch@hrbeu.edu.cn

development of low carbon economy, but also the decisive factor for enterprises to acquire sustainable competitiveness and adapt to the competitive environment of future market. However, the implementation of low carbon technological innovation requires greater cost, and enterprises are faced with a great deal of pressure on capital investment. Therefore, low carbon technology sharing has become a vital role in the development of low carbon technology. Promotion of low carbon technology sharing calls for cooperation between interested enterprises. In this paper, we present a stochastic differential game of low carbon technology sharing in innovation system of superior enterprises and inferior enterprises under uncertain environment. Our objective is to find the optimal strategy of low carbon technology sharing and explore the key factors and mechanism of low carbon technology sharing.

Game theory has been used as an effective tool to study knowledge, information and technology sharing. For example, Koessler [1] provided a simple Bayesian game model for the study of knowledge sharing; the study shows that their equilibrium is always a sequential equilibrium of the associated extensive form game with communication. In 2006, Cress and Martin [2] extended the model of Koessler to study knowledge sharing and rewards based on a game-theoretical perspective. It has been found that rewarding contributions with a cost-compensating bonus can be an effective solution at the group level. Furthermore, Bandyopadhyay and Pathak [3] modelled a game of the knowledge interaction between two teams in two separate firms; it has been found that when the degree of complementarity of knowledge is higher enough, better payoffs can be achieved if the top management enforces cooperation between the employees. Wu et al. [4] established a evolutionary game model of information sharing in network organization to analyze its dynamic evolutionary procedure. Their study showed that the key factors that affect the system's evolution, cooperation profit, initial cost of the cooperation, are obtained and researched. Ou et al. [5] modelled a game theory model to analyze the impact of important factors for low carbon international technology transfer. Their study showed that reduction of the control fees and taxes and increases of domestic subsidies all effectively promote transfers. Xu and Xu [6] used prospect theory into evolutionary game theory to construct a perceived benefit matrix to explore the internal mechanism of low carbon technology innovation diffusion under environmental regulation; theoretical study and numerical simulation showed that increasing subsidy factor, carbon tax rate and regulatory effort can all induce enterprises to adopt low carbon technology innovation, and carbon tax rate has the strongest sensitivity. Gong and Xue [7] studied a game model of cooperative innovation between ICT low carbon developers and industrial enterprises. The authors considered that the sharing proportion played a key role in the cooperation, and the preferential tax policy of the government can coordinate the conflict in their cooperation, and the government incentive and regulatory penalty can promote both sides to improve input in both sharing arrangement modes.

Form the analysis of above studies, it is a mainstream trend that many scholars use game theory to study the sharing of low carbon technology. These studies have laid the method foundation for this article but we find that most of the game models established in the literature are based on the static framework. In fact, with the rapid development of science, technology and information, the frequency and speed of low carbon technology upgrading have also improved dramatically. It means that the dynamic behavior of decision maker should be considered in the study of low carbon technology sharing in the same spatio-temporal region. In addition, Gao and Zhong [8] used differential game approach to study the dynamic strategies for information sharing. Their study showed that the superior enterprises benefits most when both firms fully cooperate, but the inferior enterprises enjoys the highest integral profit when both firms only cooperate in information sharing and the lowest integral profit. Meanwhile, low carbon technology related research have been widely studied recently due to their potential applications. Zhao et al. [9] derive the optimal solutions of the Nash equilibrium without cost sharing contract and the Stackelberg equilibrium with the integrator as the leader who partially shares the cost of the efforts of the supplier. Their study showed that cost sharing contract is an effective coordination mechanism. Yu and Shi [10] used a stochastic differential game model to study knowledge sharing between enterprise and university. However, stochastic differential game model is seldom used in low carbon technology sharing and most of studies do not consider some random interference factors in sharing system. In fact, the process of decision making is often subject to various random interference factors that include the uncertain external environment and the internal understanding limitations of decision maker [11]. The random interference factors can lead to a great uncertainty in equilibrium results because

they are difficult to capture by the decision makers [12]. In this paper, we study the low carbon technology sharing in innovation system of superior enterprises and inferior enterprises under uncertain environment in the case of stochastic intervention.

The structure of this paper is organized as follows. In Section 2, stochastic differential game formulation is provided. In Section 3, we resolve models of Stacklberg master-slave game. In Section 4, we resolve models of Nash non-cooperative game. Section 5 is devoted to models of cooperative game, and comparative analysis of equilibrium results are presented in Section 6. Section 7 summarizes the paper.

2 Stochastic differential game formulation

For the sake of simplicity, enterprises of low carbon technology sharing can be divided into two interest groups: superior enterprises and inferior enterprises, which store, respectively, large quantities of low carbon technologies and heterogeneous resources of low carbon technologies. In the paper, we study a low carbon technology innovation system that consists of a single superior enterprise C and a single inferior enterprise E. In order to clarify the above problem, we further assume that decision makers are completely rational, full information, and aim to maximize their return.

Let $L_C(t)$ denote the effort level of superior enterprises at time t, and let $L_E(t)$ denote the effort level of inferior enterprises in the sharing process of low carbon technology. For further consideration, the sharing cost of low carbon technology can be denoted by $C_C(t)$ and $C_E(t)$ which are the quadratic functions of the effort level of superior enterprises and inferior enterprises at time t, respectively. Consider

$$C_C\left(L_C\left(t\right),t\right) = \frac{c_C\left(t\right)}{2}L_C\left(t\right)^2, \tag{1}$$

$$C_E\left(L_E\left(t\right),t\right) = \frac{c_E\left(t\right)}{2}L_E\left(t\right)^2, \tag{2}$$

where $c_C(t)$ and $c_E(t)$ are the cost coefficients of superior enterprises and inferior enterprises at time t, respectively.

Let $K(t)$ denote the technology level of low carbon in collaborative innovation system of superior enterprises and inferior enterprises at time t. In the sharing process of low carbon technology, the collaborative innovation between superior enterprises and inferior enterprises can improve the technology level of low carbon. Let $\sigma_C(t)$ and $\eta_E(t)$ denote the influence of the effort level of low carbon technology sharing on collaborative innovation between superior enterprises and inferior enterprises, respectively, at time t, namely, innovation capability coefficient of low carbon technology. The dynamics of technology level of low carbon are governed by the stochastic differential equation

$$\begin{cases} \dot{K}\left(t\right) = \dfrac{dK\left(t\right)}{dt} = \sigma\left(t\right)L_C\left(t\right) + \eta\left(t\right)L_E\left(t\right) - \delta K\left(t\right) \\ K\left(0\right) = K_0 \geq 0 \end{cases}. \tag{3}$$

Hence

$$\begin{cases} dK\left(t\right) = \left[\sigma\left(t\right)L_C\left(t\right) + \eta\left(t\right)L_E\left(t\right) - \delta K\left(t\right)\right]dt + \varepsilon\left(K\left(t\right)\right)dz\left(t\right) \\ K\left(0\right) = K_0 \geq 0 \end{cases}, \tag{4}$$

where δ is the attenuation coefficient of low carbon technology, $\delta \in (0, 1]$; $z(t)$ and $\varepsilon K(t)$ are the standard Wiener process and random interference factors of superior enterprises and inferior enterprises at time t, respectively.

Let $\pi(t)$ denote the total payoff of low carbon in collaborative innovation system at time t. Let $\alpha(t)$ and $\beta(t)$ denote the influence of the effort level of low carbon technology sharing on the total income of superior enterprises and inferior enterprises, respectively, at time t, namely, the marginal return coefficient of low carbon technology. Total payoff function can be expressed as

$$\pi\left(t\right) = \alpha\left(t\right)L_C\left(t\right) + \beta\left(t\right)L_E\left(t\right) + \left(\gamma + \lambda\right)K\left(t\right), \tag{5}$$

where γ is the influence of the technology innovation of low carbon on total revenue, namely, innovation influence coefficient of low carbon technology, $\gamma \in (0, 1]$; λ is the government subsidy coefficient of low carbon technology based on increments of low carbon technology level in collaborative innovation, $\lambda \in (0, 1]$.

We further assume that the total revenue is allocated between two participants, and $\theta(t)$ is the payoff distribution coefficient of superior enterprises at time t, $\theta(t) \in [0, 1]$. Although inferior enterprises have heterogeneous resources of low carbon technologies, superior enterprises store large quantities of low carbon technologies. Many practical low carbon technologies can be acquired by inferior enterprises in the sharing process of low carbon technology. Therefore, inferior enterprises need to pay much more extra sharing cost of low carbon technology. Let $\omega(t)$ denote the subsidy of low carbon technology, which inferior enterprises give to superior enterprises. The objective function of superior enterprises and inferior enterprises satisfy the following partial differential equations

$$\max_{L_C} \left\{ J_C(K_0) = E \int_0^\infty e^{-\rho t} \left[\theta(t) (\alpha(t) L_C(t) + \beta(t) L_E(t) + (\gamma + \lambda) K(t)) - (1 - \omega(t)) \frac{c_C(t)}{2} L_C(t)^2 \right] dt \right\},$$
(6)

$$\max_{L_E, \omega(t)} \left\{ J_E(K_0) = E \int_0^\infty e^{-\rho t} \left[(1 - \theta(t)) (\alpha(t) L_C(t) + \beta(t) L_E(t) + (\gamma + \lambda) K(t)) - \frac{c_E(t)}{2} L_E(t)^2 \right. \right.$$
$$\left. \left. - \frac{c_C(t)}{2} \omega(t) L_C(t)^2 \right] dt \right\}.$$
(7)

where ρ is the discount rate of low carbon technology of superior enterprises and inferior enterprises, $\rho \in (0, 1]$.

There are three control variables, $L_C(t) \geq 0$, $L_E(t) \geq 0$, $\omega(t) \in (0, 1)$, and a state variable $K(t) \geq 0$ in the sharing model of low carbon technology. Feedback control has been used more and more widely in analysis of information and economic systems [13]. Moreover, feedback control strategy has better control effect, compared with open-loop control strategy. Therefore, we use feedback control strategy to analyze sharing model of low carbon technology.

3 Resolving models of Stacklberg master-slave game

In the sharing process of low carbon technology between superior enterprises and inferior enterprises, inferior enterprises can acquire many practical low carbon technologies from superior enterprises, and then inferior enterprises need to pay much more extra sharing cost of low carbon technology. In order to promote the technology sharing of low carbon, the inferior enterprises (the leaders) determine an optimal sharing effort level and an optimal subsidy of low carbon technology sharing, and then the superior enterprises (the followers) choose their optimal sharing effort level according to the optimal sharing effort level and subsidy. This leads to a Stackelberg equilibrium.

3.1 Stacklberg master-slave solutions

Proposition 3.1. *If above conditions are satisfied, the feedback Stacklberg master-slave equilibria are*

$$L_C^S = \frac{(2 - \theta) [\alpha(\rho + \delta) + \sigma(\gamma + \lambda)]}{2c_C(\rho + \delta)},$$
(8a)

$$L_E^S = \frac{(1 - \theta) [\beta(\rho + \delta) + \eta(\gamma + \lambda)]}{c_E(\rho + \delta)},$$
(8b)

$$\omega = \begin{cases} \dfrac{2 - 3\theta}{2 - \theta}, & 0 \le \theta \le \dfrac{2}{3} \\ 0, & \text{otherwise} \end{cases}, \tag{8c}$$

where L_C^S and L_E^S are the optimal effort level of low carbon technology sharing of superior enterprises and inferior enterprises, respectively,

$$V_C^S(K) = \frac{\theta(\gamma + \lambda)}{\rho + \delta}K + \frac{\theta(1 - \theta)\phi_2}{\rho c_E(\rho + \delta)^2} + \frac{\theta(2 - \theta)\phi_1}{4\rho c_C(\rho + \delta)^2}, \tag{9a}$$

$$V_E^S(K) = \frac{(1 - \theta)(\gamma + \lambda)}{\rho + \delta}K + \frac{(1 - \theta)^2 \phi_2}{2\rho c_E(\rho + \delta)^2} + \frac{(2 - \theta)^2 \phi_1}{8\rho c_C(\rho + \delta)}, \tag{9b}$$

where $V_C^S(K)$ and $V_E^S(K)$ are the optimal sharing payoff function of low carbon technology of superior enterprises and inferior enterprises respectively, $\phi_1 = [\alpha(\rho + \delta) + \sigma(\gamma + \lambda)]^2$, $\phi_2 = [\beta(\rho + \delta) + \eta(\gamma + \lambda)]^2$.

Proof. In order to obtain the Stacklberg equilibrium, there exists a optimal sharing revenue function of low carbon technology, $V_C^S(K)$ and $V_E^S(K)$, which is a continuous differentiable function. First, we use backward induction to solve optimal control problem. The optimal sharing revenue function, $V_C(K)$, satisfies the following Hamilton-Jacobi-Bellman equation

$$\rho V_C(K) = \max_{L_C \ge 0} \left\{ \theta[\alpha L_C + \beta L_E + (\gamma + \lambda)K] - (1 - \omega)\frac{c_C}{2}(L_C)^2 + V_C'(K)(\sigma L_C + \eta L_E - \delta K) \right. \\ \left. + \frac{\varepsilon^2(K)}{2}V_C''(K) \right\}. \tag{10}$$

For solving formula (10), using extreme conditions and searching for the optimal value of L_C by setting the first partial derivative equal to zero, we can get

$$L_C = \frac{\theta\alpha + \sigma V_C'(K)}{(1 - \omega)c_C}. \tag{11}$$

Second, the optimal sharing revenue function, $V_E(K)$, satisfies the following Hamilton-Jacobi-Bellman equation

$$\rho V_E(K) = \max_{L_E \ge 0} \left\{ (1 - \theta)[\alpha L_C + \beta L_E + (\gamma + \lambda)K] - \frac{c_E}{2}(L_E)^2 - \frac{c_L}{2}\omega(L_C)^2 \right. \\ \left. + V_E'(K)(\sigma L_C + \eta L_E - \delta K) + \frac{\varepsilon^2(K)}{2}V_E''(K) \right\}. \tag{12}$$

Substituting the result of (11) into (12), we can obtain

$$\rho V_E(K) = \max_{L_E \ge 0} \left\{ (1 - \theta)\left[\frac{\alpha[\theta\alpha + \sigma V_C'(K)]}{(1 - \omega)c_C} + \beta L_E + (\gamma + \lambda)K \right] - \frac{c_E}{2}(L_E)^2 - \frac{c_C}{2}\omega\left(\frac{\theta\alpha + \sigma V_C'(K)}{(1 - \omega)c_C} \right)^2 \right. \\ \left. + V_E'(K)\left(\frac{\sigma[\theta\alpha + \sigma V_C'(K)]}{(1 - \omega)c_C} + \eta L_E - \delta K \right) + \frac{\varepsilon^2(K)}{2}V_E''(K) \right\}. \tag{13}$$

Performing the indicated maximization in (13) and searching for the optimal value of L_E and ω by setting the first partial derivative equal to zero, we can get

$$L_E = \frac{(1 - \theta)\beta + \eta V_E'(K)}{c_E}, \tag{14a}$$

$$\omega = \frac{\alpha(2 - 3\theta) + \sigma\left[2V_E'(K) - V_C'(K) \right]}{\alpha(2 - \theta) + \sigma\left[2V_E'(K) + V_C'(K) \right]}. \tag{14b}$$

Substituting the results of (11), (14a) and (14b) into (10) and (12), we can get

$$\rho V_C(K) = \left[\theta(\gamma + \lambda) - \delta V_C'(K)\right]K + \left[\theta\alpha + \sigma V_C'(K)\right]L_C - (1 - \omega)\frac{c_C}{2}(L_C)^2$$
$$+ \left[\theta\beta + \eta V_C'(K)\right]L_E + \frac{\varepsilon^2(K)}{2}V_C''(K), \tag{15}$$

$$\rho V_E(K) = \left[(1 - \theta)(\gamma + \lambda) - \delta V_E'(K)\right]K + \left[(1 - \theta)\alpha + \sigma V_E'(K)\right]L_C - \frac{c_C}{2}\omega(L_C)^2$$
$$+ \left[(1 - \theta)\beta + \eta V_E'(K)\right]L_E - \frac{c_E}{2}(L_E)^2 + \frac{\varepsilon^2(K)}{2}V_E''(K). \tag{16}$$

The solution of the HJB equation is a unary function with K as independent variable. As [11], we have

$$V_E(K) = a_1 K + b_1, \; V_C(K) = a_2 K + b_2. \tag{17}$$

where a_1, b_1, a_2 and b_2 are the constants to be solved.
Setting the first partial derivative to formula (17), we can get

$$V_E'(K) = dV_E(K)/dK = a_1, \; V_C'(K) = dV_C(K)/dK = a_2. \tag{18}$$

Substituting the results of (17) and (18) into (15) and (16), we can get

$$a_1 = \frac{\theta(\gamma + \lambda)}{\rho + \delta}, \; b_1 = \frac{\theta^2\left[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)\right]^2}{2(1 - \omega)\rho(\rho + \delta)^2 c_C} + \frac{\theta(1 - \theta)\left[\beta(\rho + \delta) + \eta(\gamma + \lambda)\right]^2}{\rho(\rho + \delta)^2 c_E}, \tag{19}$$

$$a_2 = \frac{(1 - \theta)(\gamma + \lambda)}{\rho + \delta},$$
$$b_2 = \frac{(1 - \theta)^2\left[\beta(\rho + \delta) + \eta(\gamma + \lambda)\right]^2}{2\rho(\rho + \delta)^2 c_E} + \frac{\theta(1 - \theta)\left[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)\right]^2}{(1 - \omega)\rho(\rho + \delta)^2 c_C} \tag{20}$$
$$- \frac{\omega\theta^2\left[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)\right]^2}{2(1 - \omega)^2\rho(\rho + \delta)^2 c_C}.$$

Substituting the results of a_1 and a_2 into (11), (14a) and (14b), we can further get

$$L_C^S = \frac{(2 - \theta)\left[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)\right]}{2c_C(\rho + \delta)}, \tag{21a}$$

$$L_E^S = \frac{(1 - \theta)\left[\beta(\rho + \delta) + \eta(\gamma + \lambda)\right]}{c_E(\rho + \delta)}, \tag{21b}$$

$$\omega = \begin{cases} \dfrac{2 - 3\theta}{2 - \theta}, & 0 < \theta \leq \dfrac{2}{3} \\ 0, & \text{otherwise} \end{cases}. \tag{21c}$$

Substituting the results of (17) and (18) into (7), we can get

$$V_C^S(K) = \frac{\theta(\gamma + \lambda)}{\rho + \delta}K + \frac{\theta^2\left[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)\right]^2}{2\rho(1 - \omega)(\rho + \delta)^2 c_C} + \frac{\theta(1 - \theta)\left[\beta(\rho + \delta) + \eta(\gamma + \lambda)\right]^2}{\rho(\rho + \delta)^2 c_E}$$
$$= \frac{\theta(\gamma + \lambda)}{\rho + \delta}K + \frac{\theta(1 - \theta)\phi_2}{\rho c_E(\rho + \delta)^2} + \frac{\theta(2 - \theta)\phi_1}{4\rho c_C(\rho + \delta)^2}, \tag{22a}$$

$$V_E^S(K) = \frac{(1 - \theta)(\gamma + \lambda)}{\rho + \delta}K + \frac{(1 - \theta)^2\left[\beta(\rho + \delta) + \eta(\gamma + \lambda)\right]^2}{2\rho(\rho + \delta)^2 c_E} + \frac{\theta(1 - \theta)\left[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)\right]^2}{(1 - \omega)\rho(\rho + \delta)^2 c_C}$$
$$- \frac{\omega\theta^2\left[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)\right]^2}{2(1 - \omega)^2\rho(\rho + \delta)^2 c_C} = \frac{(1 - \theta)(\gamma + \lambda)}{\rho + \delta}K + \frac{(1 - \theta)^2\phi_2}{2\rho c_E(\rho + \delta)^2} + \frac{(2 - \theta)^2\phi_1}{8\rho c_C(\rho + \delta)}, \tag{22b}$$

where $\phi_1 = [\alpha(\rho + \delta) + \sigma(\gamma + \lambda)]^2$, $\phi_2 = [\beta(\rho + \delta) + \eta(\gamma + \lambda)]^2$.

Hence, the optimal total payoff of low carbon technology sharing can be expressed as follows

$$V^S(K) = V^S_C(K) + V^S_E(K). \tag{23}$$

$\square$

Equations (21)-(22) indicate that, under model of Stacklberg game, the effort level of superior enterprises and inferior enterprises is proportional to the government subsidy of low carbon technological innovation and the innovation capability of low carbon technology; the effort level of superior enterprises and inferior enterprises is inversely proportional to the sharing cost and the discount rate of low carbon technology; the sharing payoff of low carbon technology is proportional to the marginal return of low carbon technology.

3.2 The limit of expectation and variance

From Proposition 3.1, the payoff of superior enterprises and inferior enterprises is related to the improvement degree of low carbon technical level, whose possible values are numerical outcomes of a random phenomenon by various random interference factors. Therefore, under Stacklberg game equilibrium, it is necessary to study the limit of expectation and variance.

Substituting the results of (8a) and (8b) into (4), we can get

$$\begin{cases} dK(t) = [\mu_1 + \mu_2 - \delta K(t)] dt + \varepsilon(K(t)) dz(t) \\ K(0) = K_0 \geq 0 \end{cases}, \tag{24}$$

where $\mu_1 = \frac{\sigma(2-\theta)[\alpha(\rho+\delta)+\sigma(\gamma+\lambda)]}{2c_C(\rho+\delta)}$, $\mu_2 = \frac{\eta(1-\theta)[\beta(\rho+\delta)+\eta(\gamma+\lambda)]}{c_E(\rho+\delta)}$.

For further analysis, let $\varepsilon(K(t)) dz(t) = \varepsilon\sqrt{K} dz(t)$, and then we can get Proposition 3.2 as follows.

Proposition 3.2. *The limit of expectation and variance in Stackelberg game feedback equilibrium satisfy*

$$E(K(t)) = \frac{\mu_1 + \mu_2}{\delta} + e^{-\delta t}\left(K_0 - \frac{\mu_1 + \mu_2}{\delta}\right), \lim_{t\to\infty} E(K(t)) = \frac{\mu_1 + \mu_2}{\delta}, \tag{25a}$$

$$D(K(t)) = \frac{\varepsilon^2\left[(\mu_1 + \mu_2) - 2(\mu_1 + \mu_2 - \delta K_0)e^{-\delta t} + (\mu_1 + \mu_2 - 2\delta K_0)e^{-2\delta t}\right]}{2\delta^2},$$

$$\lim_{t\to\infty} D(K(t)) = \frac{\varepsilon^2(\mu_1 + \mu_2)}{2\delta^2}. \tag{25b}$$

Proof.

Lemma 3.3 (see [14]**.** *] Itô's lemma is an identity used in Itô calculus to find the differential of a time-dependent function of a stochastic process. If* f (x) *is quadratic continuous differentiate,* t $\in$ $\forall$ *satisfy the following Itô equation*

$$f(B(t)) = f(0) + \int_0^t f'(B(s))dB(s) + \frac{\int_0^t f''(B(s))ds}{2}, \tag{26}$$

where B(t) is the Brownian motion.

According to formula (24), using Itô equation, we can get

$$\begin{cases} d(K(t))^2 = \left[\left(2(\mu_1 + \mu_2) + \varepsilon^2\right)K - 2\delta K^2\right] dt + 2K\varepsilon\sqrt{K} dz(t) \\ (K(0))^2 = (K_0)^2 \end{cases}. \tag{27}$$

We can derive the expectation value for both sides of (24) and (27), and then $E(K(t))$ and $E(K(t))^2$ satisfy the following set of non-homogeneous linear differential equations

$$\begin{cases} dE(K(t)) = [\mu_1 + \mu_2 - \delta E(K)] dt \\ E(K(0)) = K_0 \end{cases}, \tag{28a}$$

$$\begin{cases} dE\left(K\left(t\right)\right)^2 = \left[\left(2\left(\mu_1 + \mu_2\right) + \varepsilon^2\right) E\left(K\right) - 2\delta E\left(K^2\right)\right] dt \\ E\left(K\left(0\right)\right)^2 = \left(K_0\right)^2 \end{cases}. \tag{28b}$$

Solving the above non-homogeneous linear differential equation leads to

$$E\left(K\left(t\right)\right) = \frac{\mu_1 + \mu_2}{\delta} + e^{-\delta t}\left(K_0 - \frac{\mu_1 + \mu_2}{\delta}\right), \lim_{t \to \infty} E\left(K\left(t\right)\right) = \frac{\mu_1 + \mu_2}{\delta}, \tag{29a}$$

$$D\left(K\left(t\right)\right) = \frac{\varepsilon^2\left[\left(\mu_1 + \mu_2\right) - 2\left(\mu_1 + \mu_2 - \delta K_0\right) e^{-\delta t} + \left(\mu_1 + \mu_2 - 2\delta K_0\right) e^{-2\delta t}\right]}{2\delta^2},$$
$$\lim_{t \to \infty} D\left(K\left(t\right)\right) = \frac{\varepsilon^2\left(\mu_1 + \mu_2\right)}{2\delta^2}, \tag{29b}$$

where $\mu_1 = \frac{\sigma(2-\theta)[\alpha(\rho+\delta)+\sigma(\gamma+\lambda)]}{2c_C(\rho+\delta)}$, $\mu_2 = \frac{\eta(1-\theta)[\beta(\rho+\delta)+\eta(\gamma+\lambda)]}{c_E(\rho+\delta)}$. $\qquad\qquad\square$

4 Resolving models of Nash non-cooperative game

Under Nash non-cooperative game, superior enterprises and inferior enterprises will simultaneously and independently choose their optimal effort levels of low carbon technology sharing based on maximization of their profits.

4.1 Nash non-cooperative game solutions

Proposition 4.1. *If above conditions are satisfied, the feedback non-cooperative game Nash equilibria are*

$$L_C^N = \frac{\theta\left[\alpha\left(\rho+\delta\right)+\sigma\left(\gamma+\lambda\right)\right]}{c_C\left(\rho+\delta\right)}, \tag{30a}$$

$$L_E^N = \frac{\left(1-\theta\right)\left[\beta\left(\rho+\delta\right)+\eta\left(\gamma+\lambda\right)\right]}{c_E\left(\rho+\delta\right)}, \tag{30b}$$

where L_C^N and L_E^N are the optimal effort level of low carbon technology sharing of superior enterprises and inferior enterprises, respectively,

$$V_C^N\left(K\right) = \frac{\theta\left(\gamma+\lambda\right)}{\rho+\delta} K + \frac{\theta^2 \phi_1}{2\rho c_C\left(\rho+\delta\right)^2} + \frac{\theta\left(1-\theta\right)\phi_2}{\rho c_E\left(\rho+\delta\right)^2}, \tag{31a}$$

$$V_E^N\left(K\right) = \frac{\left(1-\theta\right)\left(\gamma+\lambda\right)}{\rho+\delta} K + \frac{\left(1-\theta\right)^2 \phi_2}{2\rho c_E\left(\rho+\delta\right)^2} + \frac{\theta\left(1-\theta\right)\phi_1}{\rho c_C\left(\rho+\delta\right)}, \tag{31b}$$

where $V_C^N\left(K\right)$ and $V_E^N\left(K\right)$ are the optimal sharing payoff functions of low carbon technology of superior enterprises and inferior enterprises, respectively, $\phi_1 = \left[\alpha\left(\rho+\delta\right)+\sigma\left(\gamma+\lambda\right)\right]^2$, $\phi_2 = \left[\beta\left(\rho+\delta\right)+\eta\left(\gamma+\lambda\right)\right]^2$.

Proof. According to sufficient conditions for static feedback equalization, there exists an optimal sharing revenue function of low carbon technology, which is a continuous differentiable function. The optimal sharing revenue function satisfies the following Hamilton-Jacobi-Bellman equation

$$\rho V_C\left(K\right) = \max_{L_C \geq 0}\left\{\theta\left[\alpha L_C + \beta L_E + \left(\gamma+\lambda\right) K\right] - \left(1-\omega\right)\frac{c_C}{2}\left(L_C\right)^2 + V_C'\left(K\right)\left(\sigma L_C + \eta L_E - \delta K\right)\right.$$
$$\left. + \frac{\varepsilon^2\left(K\right)}{2} V_C''\left(K\right)\right\}, \tag{32a}$$

$$\rho V_E(K) = \max_{L_E \geq 0} \left\{ (1-\theta) \left[\alpha L_C + \beta L_E + (\gamma + \lambda) K \right] - \frac{c_E}{2} (L_E)^2 - \frac{c_L}{2} \omega (L_C)^2 \right.$$
$$\left. + V'_E(K) (\sigma L_C + \eta L_E - \delta K) + \frac{\varepsilon^2(K)}{2} V''_E(K) \right\}. \tag{32b}$$

In order to maximize their profits, the inferior enterprises are so rational that they cannot accept the optimal subsidy of low carbon technology sharing, $\omega = 0$. For solving formula (32a) and (32b), using extreme conditions and searching for the optimal value of L_C by setting the first partial derivative equal to zero, we can get

$$L_C^N = \frac{\theta\alpha + \sigma V'_C(K)}{c_C}, \tag{33a}$$

$$L_E^N = \frac{(1-\theta)\beta + \eta V'_E(K)}{c_E}. \tag{33b}$$

Substituting the results of (33a) and (33b) into (32a) and (32b), we can obtain

$$\rho V_C(K) = \max_{L_C \geq 0} \left\{ \theta \left[\alpha L_C + \beta L_E + (\gamma + \lambda) K \right] - \frac{c_C L_C^2}{2} + V'_C(K)(\sigma L_C + \eta L_E - \delta K) + \frac{\varepsilon^2(K)}{2} V''_C(K) \right\}$$
$$= \max_{L_C \geq 0} \left\{ \left[\theta(\gamma + \lambda) - \delta V'_C(K) \right] K + \frac{\left[\theta\alpha + \sigma V'_C(K) \right]^2}{2c_C} + \frac{\left[\theta\beta + \eta V'_C(K) \right] \left[(1-\theta)\beta + \eta V'_E(K) \right]}{c_E} \right. \tag{34}$$
$$\left. + \frac{\varepsilon^2(K)}{2} V''_C(K) \right\},$$

$$\rho V_E(K) = \max_{L_E \geq 0} \left\{ (1-\theta) \left[\alpha L_C + \beta L_E + (\gamma + \lambda) K \right] - \frac{c_E L_E^2}{2} + V'_E(K)(\sigma L_C + \eta L_E - \delta K) + \frac{\varepsilon^2(K)}{2} V''_E(K) \right\}$$
$$= \max_{L_E \geq 0} \left\{ \left[(1-\theta)(\gamma + \lambda) - \delta V'_E(K) \right] K + \frac{\left[(1-\theta)\beta + \eta V'_E(K) \right]^2}{2c_E} \right.$$
$$\left. + \frac{\left[(1-\theta)\alpha + \sigma V'_E(K) \right] \left[\theta\alpha + \sigma V'_C(K) \right]}{c_C} + \frac{\varepsilon^2(K)}{2} V''_E(K) \right\}. \tag{35}$$

The solution of the HJB equation is a unary function with K as independent variable. As [11], we have

$$V_E(K) = a_1 K + b_1, \quad V_C(K) = a_2 K + b_2, \tag{36}$$

where a_1, b_1, a_2 and b_2 are the constants to be solved.

Substituting the result of (36) into (34) and (35), we can get

$$\rho(a_1 K + b_1) = \theta \left[\alpha L_C + \beta L_E + (\gamma + \lambda) K \right] - \frac{c_C L_C^2}{2} + V'_C(K)(\sigma L_C + \eta L_E - \delta K)$$
$$= \left[\theta(\gamma + \lambda) - \delta a_1 \right] K + \frac{\left[\theta\alpha + \sigma a_1 \right]^2}{2c_C} + \frac{\left[\theta\beta + \eta a_1 \right] \left[(1-\theta)\beta + \eta a_2 \right]}{c_E}, \tag{37}$$

$$\rho(a_2 K + b_2) = (1-\theta) \left[\alpha L_C + \beta L_E + (\gamma + \lambda) K \right] - \frac{c_E L_E^2}{2} + V'_E(K)(\sigma L_C + \eta L_E - \delta K)$$
$$= \left[(1-\theta)(\gamma + \lambda) - \delta a_2 \right] K + \frac{\left[(1-\theta)\beta + \eta a_2 \right]^2}{2c_E} + \frac{\left[(1-\theta)\alpha + \sigma a_2 \right] \left[\theta\alpha + \sigma a_1 \right]}{c_C}. \tag{38}$$

Using the $K \geq 0$ to (37) and (38), parameter values of the optimal value function can be expressed as follows

$$a_1 = \frac{\theta(\gamma + \lambda)}{\rho + \delta}, \quad b_1 = \frac{\theta^2 \left[\alpha(\rho + \delta) + \sigma(\gamma + \lambda) \right]^2}{2\rho(\rho + \delta)^2 c_C} + \frac{\theta(1-\theta) \left[\beta(\rho + \delta) + \eta(\gamma + \lambda) \right]^2}{\rho(\rho + \delta)^2 c_E}, \tag{39}$$

$$a_2 = \frac{(1-\theta)(\gamma+\lambda)}{\rho+\delta},$$

$$b_2 = \frac{(1-\theta)^2 \left[\beta(\rho+\delta)+\eta(\gamma+\lambda)\right]^2}{2\rho(\rho+\delta)^2 c_E} + \frac{\theta(1-\theta)\left[\alpha(\rho+\delta)+\sigma(\gamma+\lambda)\right]^2}{\rho(\rho+\delta)^2 c_C}$$

$$- \frac{\omega\theta^2 \left[\alpha(\rho+\delta)+\sigma(\gamma+\lambda)\right]^2}{2(1-\omega)^2 \rho(\rho+\delta)^2 c_C}. \tag{40}$$

Substituting the results of a_1, b_1, a_2 and b_2 into (33a), (33b) and (36), we can get the optimal effort level of low carbon technology sharing and the optimal sharing payoff function of low carbon technology of superior enterprises and inferior enterprises, respectively. □

4.2 The limit of expectation and variance

From Proposition 4.1, the payoff of superior enterprises and inferior enterprises is related to the improvement degree of low carbon technical level, whose possible values are numerical outcomes of a random phenomenon by various random interference factors. Therefore, under Nash equilibrium, it is necessary to study the limit of expectation and variance.

Substituting the results of (30a) and (30b) into (4), we can get

$$\begin{cases} dK(t) = \left[\vartheta_1 + \vartheta_2 - \delta K(t)\right] dt + \varepsilon(K(t)) dz(t) \\ K(0) = K_0 \geq 0 \end{cases}, \tag{41}$$

where $\vartheta_1 = \frac{\sigma\theta[\alpha(\rho+\delta)+\sigma(\gamma+\lambda)]}{c_C(\rho+\delta)}$, $\vartheta_2 = \frac{\eta(1-\theta)[\beta(\rho+\delta)+\eta(\gamma+\lambda)]}{c_E(\rho+\delta)}$.

For further analysis, let $\varepsilon(K(t)) dz(t) = \varepsilon\sqrt{K}dz(t)$, and then we can get Proposition 4.2 as follows.

Proposition 4.2. *The limit of expectation and variance in Nash non-cooperative game feedback equilibrium satisfy*

$$E\left(\bar{K}(t)\right) = \frac{\vartheta_1+\vartheta_2}{\delta} + e^{-\delta t}\left(K_0 - \frac{\vartheta_1+\vartheta_2}{\delta}\right), \lim_{t\to\infty} E\left(\bar{K}(t)\right) = \frac{\vartheta_1+\vartheta_2}{\delta}, \tag{42a}$$

$$D\left(\bar{K}(t)\right) = \frac{\varepsilon^2 \left[(\vartheta_1+\vartheta_2) - 2(\vartheta_1+\vartheta_2-\delta K_0)e^{-\delta t} + (\vartheta_1+\vartheta_2-2\delta K_0)e^{-2\delta t}\right]}{2\delta^2},$$

$$\lim_{t\to\infty} D\left(\bar{K}(t)\right) = \frac{\varepsilon^2(\vartheta_1+\vartheta_2)}{2\delta^2}, \tag{42b}$$

where $\vartheta_1 = \frac{\sigma\theta[\alpha(\rho+\delta)+\sigma(\gamma+\lambda)]}{c_C(\rho+\delta)}$, $\vartheta_2 = \frac{\eta(1-\theta)[\beta(\rho+\delta)+\eta(\gamma+\lambda)]}{c_E(\rho+\delta)}$.

Proof. The proof of Proposition 4.2 is similarly to Proposition 3.2, so we do not repeat it here. □

5 Resolving models of cooperative game

Under cooperative game, superior enterprises and inferior enterprises will choose their optimal effort levels and sharing payoff function of low carbon technology sharing based on maximization of their total payoff. Thus, low carbon technology level can be further improved through cooperation between superior enterprises and inferior enterprises.

5.1 Cooperative game solutions

Proposition 5.1. *If the above conditions are satisfied, the feedback cooperative game equilibria are*

$$L_C^C = \frac{\alpha (\rho + \delta) + \sigma (\gamma + \lambda)}{c_C (\rho + \delta)}, \tag{43a}$$

$$L_E^C = \frac{\beta (\rho + \delta) + \eta (\gamma + \lambda)}{c_E (\rho + \delta)}, \tag{43b}$$

where L_C^C and L_E^C are the optimal effort level of low carbon technology sharing of superior enterprises and inferior enterprises, respectively,

$$V^C (K) = \frac{\gamma + \lambda}{\rho + \delta} K + \frac{\phi_1}{2\rho c_C (\rho + \delta)^2} + \frac{\phi_2}{2\rho c_E (\rho + \delta)}, \tag{44}$$

where $V^C (K)$ is the optimal sharing payoff function of low carbon technology of superior enterprises and inferior enterprises, $\phi_1 = [\alpha (\rho + \delta) + \sigma (\gamma + \lambda)]^2$, $\phi_2 = [\beta (\rho + \delta) + \eta (\gamma + \lambda)]^2$.

Proof. Under cooperative game, the sharing revenue function satisfies the following equation

$$\max_{L_C, L_E} \left\{ J (K_0) = J_C + J_E = E \int_0^\infty e^{-\rho t} \left[(\alpha (t) L_C (t) + \beta (t) L_E (t) + (\gamma + \lambda) K (t)) \right. \right.$$
$$\left. \left. - \frac{c_E (t)}{2} L_E (t)^2 - \frac{c_C (t)}{2} L_C (t)^2 \right] dt \right\}. \tag{45}$$

In order to obtain the cooperative equilibrium state in this case, we assume that sharing revenue function of low carbon technology is a continuous differentiable function. The optimal sharing revenue function satisfies the following Hamilton-Jacobi-Bellman equation

$$\rho V (K) = \max_{L_C, L_E} \left\{ [\alpha L_C + \beta L_E + (\gamma + \lambda) K] - \frac{c_E}{2} (L_E)^2 - \frac{c_L}{2} (L_C)^2 \right.$$
$$\left. + V' (K) (\sigma L_C + \eta L_E - \delta K) + \frac{\varepsilon^2 (K)}{2} V'' (K) \right\}. \tag{46}$$

For solving formula (46), using extreme conditions and searching for the optimal value of L_C by setting the first partial derivative equal to zero, we can get

$$L_C^C = \frac{\alpha + \sigma V' (K)}{c_C}, \tag{47a}$$

$$L_E^C = \frac{\beta + \eta V' (K)}{c_E}. \tag{47b}$$

Substituting the results of (47a) and (47b) into (46), we can obtain

$$\rho V (K) = \max_{L_C, L_E} \left\{ \left[\frac{\alpha (\alpha + \sigma V' (K))}{c_C} + \beta L_E + (\gamma + \lambda) K \right] - \frac{c_E}{2} \left(\frac{\beta + \eta V' (K)}{c_E} \right)^2 - \frac{c_L}{2} \left(\frac{\alpha + \sigma V' (K)}{c_C} \right)^2 \right.$$
$$\left. + V' (K) \left(\frac{\sigma (\alpha + \sigma V' (K))}{c_C} + \frac{\eta (\beta + \eta V' (K))}{c_E} - \delta K \right) + \frac{\varepsilon^2 (K)}{2} V'' (K) \right\} \tag{48}$$
$$= \max_{L_C, L_E} \left\{ [(\gamma + \lambda) - \delta V' (K)] K + \frac{(\alpha + \sigma V' (K))^2}{2c_C} + \frac{(\beta + \eta V' (K))^2}{2c_E} + \frac{\varepsilon^2 (K)}{2} V'' (K) \right\}.$$

The solution of the HJB equation is a unary function with K as independent variable. As [11], we have

$$V (K) = a_1 K + b_1, \tag{49}$$

where a_1 and b_1 are the constants to be solved.

Substituting the result of (49) into (48), we can get

$$\rho \left(a_1 K + b_1 \right) = \left[\left(\gamma + \lambda \right) - \delta V'\left(K \right) \right] K + \frac{\left(\alpha + \sigma V'\left(K \right) \right)^2}{2c_C} + \frac{\left(\beta + \eta V'\left(K \right) \right)^2}{2c_E}$$
$$= \left[\left(\gamma + \lambda \right) - \delta a_1 \right] K + \frac{\left[\alpha + \sigma a_1 \right]^2}{2c_C} + \frac{\left[\beta + \eta a_1 \right]^2}{2c_E}. \tag{50}$$

Using the $K \geq 0$ to (50), parameter values of the optimal value function can be expressed as follows

$$a_1 = \frac{\gamma + \lambda}{\rho + \delta}, \, b_1 = \frac{\left[\alpha \left(\rho + \delta \right) + \sigma \left(\gamma + \lambda \right) \right]^2}{2\rho \left(\rho + \delta \right)^2 c_C} + \frac{\left[\beta \left(\rho + \delta \right) + \eta \left(\gamma + \lambda \right) \right]^2}{2\rho \left(\rho + \delta \right)^2 c_E}. \tag{51}$$

Substituting the results of a_1 and b_1 into (47a), (47b) and (49), we can get the optimal effort level of low carbon technology sharing and the optimal sharing payoff function of low carbon technology of superior enterprises and inferior enterprises, respectively.   □

5.2 The limit of expectation and variance

From Proposition 5.1, the payoff of superior enterprises and inferior enterprises is related to the improvement degree of low carbon technical level, whose possible values are numerical outcomes of a random phenomenon by various random interference factors. Therefore, it is necessary to study the limit of expectation and variance.

Substituting the results of (43a) and (43b) into (4), we can get

$$\begin{cases} dK\left(t \right) = \left[\tau_1 + \tau_2 - \delta K\left(t \right) \right] dt + \varepsilon \left(K\left(t \right) \right) dz\left(t \right) \\ K\left(0 \right) = K_0 \geq 0 \end{cases}, \tag{52}$$

where $\tau_1 = \frac{\sigma \left[\alpha \left(\rho + \delta \right) + \sigma \left(\gamma + \lambda \right) \right]}{c_C \left(\rho + \delta \right)}$, $\tau_2 = \frac{\eta \left[\beta \left(\rho + \delta \right) + \eta \left(\gamma + \lambda \right) \right]}{c_E \left(\rho + \delta \right)}$.

For further analysis, let $\varepsilon \left(K\left(t \right) \right) dz\left(t \right) = \varepsilon \sqrt{K} dz\left(t \right)$, and then we can get Proposition 5.2 as follows.

Proposition 5.2. *The limit of expectation and variance in cooperative game feedback equilibrium satisfy*

$$E\left(\bar{\bar{K}}\left(t \right) \right) = \frac{\tau_1 + \tau_2}{\delta} + e^{-\delta t} \left(K_0 - \frac{\tau_1 + \tau_2}{\delta} \right), \lim_{t \to \infty} E\left(\bar{\bar{K}}\left(t \right) \right) = \frac{\tau_1 + \tau_2}{\delta}, \tag{53a}$$

$$D\left(\bar{\bar{K}}\left(t \right) \right) = \frac{\varepsilon^2 \left[\left(\tau_1 + \tau_2 \right) - 2 \left(\tau_1 + \tau_2 - \delta K_0 \right) e^{-\delta t} + \left(\tau_1 + \tau_2 - 2\delta K_0 \right) e^{-2\delta t} \right]}{2\delta^2},$$
$$\lim_{t \to \infty} D\left(\bar{\bar{K}}\left(t \right) \right) = \frac{\varepsilon^2 \left(\tau_1 + \tau_2 \right)}{2\delta^2}, \tag{53b}$$

where $\tau_1 = \frac{\sigma \left[\alpha \left(\rho + \delta \right) + \sigma \left(\gamma + \lambda \right) \right]}{c_C \left(\rho + \delta \right)}$, $\tau_2 = \frac{\eta \left[\beta \left(\rho + \delta \right) + \eta \left(\gamma + \lambda \right) \right]}{c_E \left(\rho + \delta \right)}$.

Proof. The proof of Proposition 5.2 is similarly to Proposition 3.2, so we do not repeat it here.   □

6 Comparative analysis of equilibrium results

Proposition 6.1. *Superior enterprises can share more low carbon technologies under the condition that inferior enterprises pay much more extra cost of low carbon technology sharing. Under cooperation between superior enterprises and inferior enterprises, superior enterprises and inferior enterprises can share more low carbon technology than the other two situations. That is to say, there exist* $L_C^C \geq L_C^S \geq L_C^N$ *and* $L_E^C \geq L_E^S = L_E^N$.

Proof. From Proposition 3.1 and Proposition 4.1, inferior enterprises have the same strategy of low carbon technology sharing in both cases. However, superior enterprises have the different strategies of low carbon technology sharing. Therefore, we can get

$$
\begin{aligned}
L_C^C - L_C^S &= \frac{\alpha(\rho+\delta) + \sigma(\gamma+\lambda)}{c_C(\rho+\delta)} - \frac{(2-\theta)\left[\alpha(\rho+\delta) + \sigma(\gamma+\lambda)\right]}{2c_C(\rho+\delta)} \\
&= \frac{\theta\left[\alpha(\rho+\delta) + \sigma(\gamma+\lambda)\right]}{2c_C(\rho+\delta)},
\end{aligned}
\tag{54a}
$$

$$
\begin{aligned}
L_C^S - L_C^N &= \frac{(2-\theta)\left[\alpha(\rho+\delta) + \sigma(\gamma+\lambda)\right]}{2c_C(\rho+\delta)} - \frac{\theta\left[\alpha(\rho+\delta) + \sigma(\gamma+\lambda)\right]}{c_C(\rho+\delta)} \\
&= \frac{(2-3\theta)\left[\alpha(\rho+\delta) + \sigma(\gamma+\lambda)\right]}{2c_C(\rho+\delta)},
\end{aligned}
\tag{54b}
$$

$$
\begin{aligned}
L_E^C - L_E^S &= \frac{\beta(\rho+\delta) + \eta(\gamma+\lambda)}{c_E(\rho+\delta)} - \frac{(1-\theta)\left[\beta(\rho+\delta) + \eta(\gamma+\lambda)\right]}{c_E(\rho+\delta)} \\
&= \frac{\theta\left[\beta(\rho+\delta) + \eta(\gamma+\lambda)\right]}{c_E(\rho+\delta)}.
\end{aligned}
\tag{54c}
$$

According to the $0 \leq \theta \leq \frac{2}{3}$, we can get $L_C^C - L_C^S \geq 0$, $L_C^S - L_C^N \geq 0$, $L_E^C - L_E^S \geq 0$. □

Proposition 6.1 indicates that the government subsidy of low carbon technology is a long-term incentive mechanism which can promote low carbon technology sharing. Superior enterprises and inferior enterprises can share more low carbon technologies through this mechanism.

Proposition 6.2. *For any $K \geq 0$, under the condition that inferior enterprises pay much more extra cost of low carbon technology sharing, the optimal sharing payoff of low carbon technology of superior enterprises reaches higher than the optimal sharing payoff under the condition that inferior enterprises do not provide extra cost. Similarly, the optimal sharing payoff of low carbon technology of inferior enterprises reaches higher than the optimal sharing payoff under the condition that inferior enterprises do not provide extra cost. That is to say, there exist $V_C^S(K) \geq V_C^N(K)$ and $V_E^S(K) \geq V_E^N(K)$.*

Proof. From Proposition 3.1 and Proposition 4.1, we can get

$$
\begin{aligned}
\Delta V_C(K) = V_C^S(K) - V_C^N(K) &= \frac{\theta(\gamma+\lambda)}{\rho+\delta}K + \frac{\theta(1-\theta)\phi_2}{\rho c_E(\rho+\delta)^2} + \frac{\theta(2-\theta)\phi_1}{4\rho c_C(\rho+\delta)^2} - \frac{\theta(\gamma+\lambda)}{\rho+\delta}K \\
&\quad - \frac{\theta^2\phi_1}{2\rho c_C(\rho+\delta)^2} - \frac{\theta(1-\theta)\phi_2}{\rho c_E(\rho+\delta)^2} = \frac{\theta(2-3\theta)\left[\alpha(\rho+\delta)+\sigma(\gamma+\lambda)\right]^2}{4\rho c_C(\rho+\delta)^2},
\end{aligned}
\tag{55a}
$$

$$
\begin{aligned}
\Delta V_E(K) = V_E^S(K) - V_E^N(K) &= \frac{(1-\theta)(\gamma+\lambda)}{\rho+\delta}K + \frac{(1-\theta)^2\phi_2}{2\rho c_E(\rho+\delta)^2} + \frac{(2-\theta)^2\phi_1}{8\rho c_C(\rho+\delta)} - \frac{(1-\theta)(\gamma+\lambda)}{\rho+\delta}K \\
&\quad - \frac{(1-\theta)^2\phi_2}{2\rho c_E(\rho+\delta)^2} - \frac{\theta(1-\theta)\phi_1}{\rho c_C(\rho+\delta)} = \frac{(2-3\theta)^2\left[\alpha(\rho+\delta)+\sigma(\gamma+\lambda)\right]^2}{\rho c_C(\rho+\delta)^2}.
\end{aligned}
\tag{55b}
$$

According to the $0 \leq \theta \leq \frac{2}{3}$, we can further get $V_C^S(K) - V_C^N(K) \geq 0$ and $V_E^S(K) - V_E^N(K) \geq 0$. □

Proposition 6.2 indicates that, under the condition that inferior enterprises give a subsidy to superior enterprises, the subsidy of low carbon technology is an incentive mechanism which can promote low carbon technology sharing between superior enterprises and inferior enterprises. Superior enterprises and inferior enterprises can share more low carbon technologies through this mechanism.

Proposition 6.3. *Under cooperative game, the total payoff exceeds the total payoff of Stacklberg master-slave game, and the total payoff of Stacklberg master-slave game exceeds the total payoff of Nash non-cooperative game in collaborative innovation system. That is to say, there exist $V^C(K) \geq V^S(K) \geq V^N(K)$.*

Proof. According to Proposition 3.1, Proposition 4.1 and Proposition 5.1, we can get

$$
\begin{aligned}
V^S(K) &= V_C^S(K) + V_E^S(K) \\
&= \frac{\theta(\gamma + \lambda)}{\rho + \delta}K + \frac{\theta(1 - \theta)\phi_2}{\rho c_E(\rho + \delta)^2} + \frac{\theta(2 - \theta)\phi_1}{4\rho c_C(\rho + \delta)^2} + \frac{(1 - \theta)(\gamma + \lambda)}{\rho + \delta}K + \frac{(1 - \theta)^2\phi_2}{2\rho c_E(\rho + \delta)^2} + \frac{(2 - \theta)^2\phi_1}{8\rho c_C(\rho + \delta)} \\
&= \frac{\gamma + \lambda}{\rho + \delta}K + \frac{(4 - \theta^2)[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)]^2}{8\rho c_C(\rho + \delta)^2} + \frac{(1 - \theta^2)[\beta(\rho + \delta) + \eta(\gamma + \lambda)]^2}{2\rho c_E(\rho + \delta)^2},
\end{aligned}
\tag{56a}
$$

$$
\begin{aligned}
V^C(K) - V^S(K) &= \frac{\gamma + \lambda}{\rho + \delta}K + \frac{\phi_1}{2\rho c_C(\rho + \delta)^2} + \frac{\phi_2}{2\rho c_E(\rho + \delta)} - \frac{\gamma + \lambda}{\rho + \delta}K \\
&\quad - \frac{(4 - \theta^2)[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)]^2}{8\rho c_C(\rho + \delta)^2} - \frac{(1 - \theta^2)[\beta(\rho + \delta) + \eta(\gamma + \lambda)]^2}{2\rho c_E(\rho + \delta)^2} \\
&= \frac{\theta^2[\alpha(\rho + \delta) + \sigma(\gamma + \lambda)]^2}{8\rho c_C(\rho + \delta)^2} + \frac{\theta^2[\beta(\rho + \delta) + \eta(\gamma + \lambda)]^2}{2\rho c_E(\rho + \delta)^2}.
\end{aligned}
\tag{56b}
$$

According to the $0 \le \theta \le \dfrac{2}{3}$, we can get $V^C(K) \ge V^S(K)$ and

$$
\begin{aligned}
V^S(K) - V^N(K) &= \left(V_C^S(K) + V_E^S(K)\right) - \left(V_C^N(K) + V_E^N(K)\right) \\
&= \left(V_C^S(K) - V_C^N(K)\right) + \left(V_E^S(K) - V_E^N(K)\right).
\end{aligned}
\tag{57}
$$
$\square$

From Proposition 6.2, we can further get $V^S(K) \ge V^N(K)$.

Proposition 6.4. *Under cooperative game, the stability of the improvement degree of low carbon technical level is better than the stability of Stacklberg master-slave game, and the stability of Stacklberg master-slave game is better than the stability of Nash non-cooperative game. That is to say, there exists*

$$
\begin{cases}
E\left(\bar{\bar{K}}(t)\right) > E(K(t)) > E\left(\bar{K}(t)\right), \lim\limits_{t\to\infty} E\left(\bar{\bar{K}}(t)\right) > \lim\limits_{t\to\infty} E(K(t)) > \lim\limits_{t\to\infty} E\left(\bar{K}(t)\right) \\
D\left(\bar{\bar{K}}(t)\right) > D(K(t)) > D\left(\bar{K}(t)\right), \lim\limits_{t\to\infty} D\left(\bar{\bar{K}}(t)\right) > \lim\limits_{t\to\infty} D(K(t)) > \lim\limits_{t\to\infty} D\left(\bar{K}(t)\right)
\end{cases}.
\tag{58}
$$

Proof. According to Proposition 3.2, Proposition 4.2 and Proposition 5.2, we can get

$$
\begin{aligned}
E\left(\bar{\bar{K}}(t)\right) - E(K(t)) &= \frac{\tau_1 + \tau_2}{\delta} + e^{-\delta t}\left(K_0 - \frac{\tau_1 + \tau_2}{\delta}\right) - \frac{\mu_1 + \mu_2}{\delta} - e^{-\delta t}\left(K_0 - \frac{\mu_1 + \mu_2}{\delta}\right) \\
&= \frac{(\tau_1 + \tau_2) - (\mu_1 + \mu_2)}{\delta}\left(1 - e^{-\delta t}\right) > 0.
\end{aligned}
\tag{59a}
$$

Similarly, we can get

$$
E(K(t)) - E\left(\bar{K}(t)\right) > 0,
\tag{59b}
$$

$$
\lim_{t\to\infty} E\left(\bar{\bar{K}}(t)\right) - \lim_{t\to\infty} E(K(t)) = \frac{\tau_1 + \tau_2}{\delta} - \frac{\mu_1 + \mu_2}{\delta} = \frac{(\tau_1 + \tau_2) - (\mu_1 + \mu_2)}{\delta} > 0,
\tag{60a}
$$

$$
\lim_{t\to\infty} E(K(t)) - \lim_{t\to\infty} E\left(\bar{K}(t)\right) > 0,
\tag{60b}
$$

$$
\lim_{t\to\infty} D\left(\bar{\bar{K}}(t)\right) - \lim_{t\to\infty} D(K(t)) = \frac{\varepsilon^2(\tau_1 + \tau_2)}{2\delta^2} - \frac{\varepsilon^2(\mu_1 + \mu_2)}{2\delta^2} = \frac{\varepsilon^2[(\tau_1 + \tau_2) - (\mu_1 + \mu_2)]}{2\delta^2} > 0,
\tag{61a}
$$

$$
\lim_{t\to\infty} D(K(t)) - \lim_{t\to\infty} D\left(\bar{K}(t)\right) > 0,
\tag{61b}
$$

$$D\left(\bar{\bar{K}}(t)\right) - D\left(K(t)\right) = \frac{\varepsilon^2 \left[(\tau_1 + \tau_2) - 2\left(\tau_1 + \tau_2 - \delta K_0\right)e^{-\delta t} + \left(\tau_1 + \tau_2 - 2\delta K_0\right)e^{-2\delta t}\right]}{2\delta^2}$$
$$- \frac{\varepsilon^2 \left[(\mu_1 + \mu_2) - 2\left(\mu_1 + \mu_2 - \delta K_0\right)e^{-\delta t} + \left(\mu_1 + \mu_2 - 2\delta K_0\right)e^{-2\delta t}\right]}{2\delta^2} \tag{62a}$$
$$= \frac{\varepsilon^2 \left[(\tau_1 + \tau_2) - (\mu_1 + \mu_2)\right]}{\delta}\left(1 - 2e^{-\delta t} + e^{-2\delta t}\right).$$

The first derivative of $1 - 2e^{-\delta t} + e^{-2\delta t}$ function of t is greater than 0 for $t \in (0, \infty)$. When $t \to 0$, we have $1 - 2e^{-\delta t} + e^{-2\delta t} = 0$, and then we can get $D\left(\bar{\bar{K}}(t)\right) - D\left(K(t)\right) > 0$. Similarly, we can get

$$D\left(K(t)\right) - D\left(\bar{K}(t)\right) > 0. \tag{62b}$$

$$\square$$

Proposition 6.4 indicates that enterprises can create and bring new low carbon technologies better than in case of the Stackelberg master slave game. However, some random interference factors in sharing system can make the variance of the improvement degree of cooperation game higher than the variance of the Stackelberg master slave game. That is to say, enterprises need to bear more risk to achieve higher payoff in sharing system under the cooperative game. Similarly, the result of Stackelberg game is similar to the result of Nash game. Therefore, different game modes are chosen by enterprises with different risk preferences. Cooperative game may be chosen by some enterprises with high risk preference, while Stackelberg game may be chosen by enterprises with moderate risk preference. The risk averse entity may choose Nash non-cooperative game.

7 Conclusions

In this paper, we have shown a stochastic differential game of low carbon technology sharing in collaborative innovation system of superior enterprises and inferior enterprises under uncertain environment. In our model, we use the limit of expectation and variance of the improvement degree to identify the influence of random factors. According to Hamilton-Jacobi-Bellman equation, we get the optimal effort level of low carbon technology sharing, the subsidy of low carbon technology, the optimal sharing payoff and the total payoff of low carbon in collaborative innovation system of superior enterprises and inferior enterprises, respectively in the above game models. By comparing and analyzing of equilibrium results, we have shown that the effort level of superior enterprises and inferior enterprises is proportional to the government subsidy of low carbon technological innovation and the innovation capability of low carbon technology; the effort level of superior enterprises and inferior enterprises is inversely proportional to the sharing cost and the discount rate of low carbon technology; the sharing payoff of low carbon technology is proportional to the marginal return of low carbon technology. Moreover, we have shown that some random interference factors in sharing system can make the variance of the improvement degree of cooperation game higher than the variance of the Stackelberg master slave game. Similarly, the result of Stackelberg game is similar to the result of Nash game. By analyzing this stochastic differential game models, we have also provided a government subsidy incentive and a subsidy that inferior enterprises give to superior enterprises.

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