

A stochastic Monte Carlo approach to modelling real star cluster evolution – III. Direct integration of three- and four-body interactions

M. Giersz¹* and R. Spurzem²

¹*Nicolaus Copernicus Astronomical Centre, Polish Academy of Sciences, ul. Bartycka 18, 00-716 Warsaw, Poland*

²*Astronomisches Rechen-Institut, Mönchhofstraße 12–14, D-69120 Heidelberg, Germany*

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ABSTRACT

Spherically symmetric equal-mass star clusters containing a large number of primordial binaries are studied using a hybrid method, consisting of a gas dynamical model for single stars and a Monte Carlo treatment for relaxation of binaries and the setup of close resonant and fly-by encounters of single stars with binaries and binaries with each other (three- and four-body encounters). What differs from our previous work is that each encounter is being integrated using a highly accurate direct few-body integrator which uses regularized variables. Hence we can study the systematic evolution of individual binary orbital parameters (eccentricity, semi-major axis) and differential and total cross-sections for hardening, dissolution or merging of binaries (minimum distance) from a sampling of several tens of thousands of scattering events as they occur in real cluster evolution, including mass segregation of binaries, gravothermal collapse and re-expansion, a binary burning phase and ultimately gravothermal oscillations. For the first time we are able to present empirical cross-sections for eccentricity variation of binaries in close three- and four-body encounters. It is found that a large fraction of three- and four-body encounters result in merging. Eccentricities are generally increased in strong three- and four-body encounters and there is a characteristic scaling law $\propto \exp(4e_{\text{fin}})$ of the differential cross-section for eccentricity changes, where e_{fin} is the final eccentricity of the binary, or harder binary for four-body encounters. Despite these findings the overall eccentricity distribution remains thermal for all binding energies of binaries, which is understood from the dominant influence of resonant encounters. Previous cross-sections obtained by Spitzer and Gao for strong encounters can be reproduced, while for weak encounters non-standard processes such as the formation of hierarchical triples occur.

Key words: stellar dynamics – methods: numerical – binaries: general – galaxies: star clusters.

1 INTRODUCTION

Dynamical modelling of globular clusters and other collisional stellar systems (such as galactic nuclei, rich open clusters and rich galaxy clusters) still poses a considerable challenge for both theory and computational requirements (in hardware and software). On the theoretical side the validity of certain assumptions used in statistical modelling based on the Fokker–Planck (henceforth FP) and other approximations is still poorly known. Stochastic noise in a discrete N -body system and the impossibility to directly model realistic particle numbers with the presently available hardware, are a considerable challenge for the computational side.

A large number of individual pairwise forces between particles needs to be explicitly calculated to properly follow relaxation ef-

fects based on the cumulative effects of small-angle gravitational encounters. Therefore, modelling of globular star clusters over their entire lifetime requires a star-by-star simulation approach, which is highly accurate in following stellar two- and many-body encounters. Distant two-body encounters, the backbone of quasi-stationary relaxation processes, have to be treated properly by the integration method directly, while close encounters, in order to avoid truncation errors and downgrading of the overall performance, are treated in relative and specially regularized coordinates (Kustaanheimo & Stiefel 1965; Mikkola & Aarseth 1996, 1998), their centres of masses being used in the main integrator. The world standard for such codes has been set by Aarseth and his codes NBODY n -codes ($0 \leq n \leq 7$, see Aarseth 1985, 1994, 1999a,b, 2003). Recently, another integrator KIRA intended to give high-precision has been used (McMillan & Hut 1996; Portegies Zwart, Hut & Makino 1998; Takahashi & Portegies Zwart 1998, 2000). It uses a hierarchical binary tree to handle compact subsystems instead of regularization.

*E-mail: mig@camk.edu.pl

Unfortunately, the direct simulation of most dense stellar systems with star-by-star modelling is not yet possible, although recent years have seen significant progress in both hardware and software. First, parallel (even massively parallel) computing has opened a route to increased performance at a relatively low cost and little technological advancement, such as in the case of the CRAY T3E supercomputers and for so-called Beowulf parallel PC clusters, both being fairly similar in performance. On the other hand, the already successful GRAPE special-purpose computers (Sugimoto et al. 1990; Makino et al. 1997; Makino & Taiji 1998; Makino 2002; Makino & Hut 2003) have been developed in their present generation (GRAPE-6) and aim at 100 Tfloper speeds. The cost of the latter is small in money (per Tfloper), but may be high otherwise because they are only suitable for special problems and algorithms. As a general rule one could say that every complexity different from standard force calculations in an N -body system (such as coupling to gas dynamics, many hard binaries) is a potential threat for the performance of GRAPE. Also, clever algorithms such as Ahmad–Cohen neighbour schemes (Ahmad & Cohen 1973), which would normally provide much higher efficiency, cannot yet be used in a competitive way on GRAPE. Therefore, such codes have, in the past, been most efficiently ported on to massively parallel general purpose computers (CRAY T3E and PC clusters); the prototype code NBODY6++ based on a portable MPI implementation has been described for the first time by Spurzem (1999) and was used for astrophysical problems related to the dissolution of globular clusters (Baumgardt 2001; Baumgardt et al. 2002) and the decay of massive black hole binaries in galactic nuclei after a merger (Milosavljevic & Merritt 2001; Hemsendorf, Sigurdsson & Spurzem 2002). It seems that the use of reconfigurable hardware (Kuberka et al. 1999; Spurzem & Kugel 1999; Hamada et al. 2000; Spurzem et al. 2002) helps to find an efficient way to use both GRAPE and sophisticated parallel codes. Another approach to improving the performance of direct N -body codes is the use of systolic and hyper-systolic algorithms (Lippert et al. 1996, 1998; Makino 2002; Dorband, Hemsendorf & Merritt 2003).

Despite such progress in hardware and software even the largest useful direct N -body models for both globular cluster and galactic nuclei evolution have still not yet reached realistic particle numbers ($\sim 5 \times 10^5$ for globular clusters, $N \sim 10^6$ to 10^9 for galactic nuclei). However, recent work by Baumgardt et al. (2002) and Baumgardt & Makino (2002) has pushed the limits of present direct modelling for the first time to some 10^5 using either NBODY6++ on parallel computers or NBODY4 on GRAPE-6 special-purpose hardware. There is a notable exception of a direct 1×10^6 body problem with a central binary black hole tackled by Dorband et al. (2003). Their code, although very innovative for the large- N force calculation, still lacks the fine ingredients of treating close encounters between stars and black hole particles of the standard N -body codes.

Bridging the gap between direct models and the most interesting particle numbers in real systems is far from straightforward, either by scaling (Aarseth & Heggie 1998; Baumgardt 2001) or by theory. There are two main classes of theory: (i) FP models, which are based on the direct numerical solution of the orbit-averaged FP equation (Cohn 1980; Cohn, Hut & Wise 1989; Murphy, Cohn & Hut 1990; Murphy, Cohn & Durisen 1991) and (ii) isotropic (Lynden-Bell & Eggleton 1980; Bettwieser & Sugimoto 1984; Heggie 1984) and anisotropic gaseous models (Louis & Spurzem 1991; Spurzem 1994, 1996), which can be thought of as a set of moment equations of the FP equation.

Detailed comparisons for equal-mass isolated star clusters have been performed (Giersz & Heggie 1994a,b; Giersz & Spurzem

1994; Spurzem 1996; Spurzem & Aarseth 1996) with direct N -body simulations using standard N -body codes (NBODY5, Aarseth 1985, Spurzem & Aarseth 1996; NBODY2, Makino & Aarseth 1992; NBODY4, Makino 1996; NBODY6++, Spurzem 1999). There have as yet been very few attempts to extend the quantitative comparisons to more realistic star clusters containing different mass bins or even a continuous mass spectrum (Spurzem & Takahashi 1995; Giersz & Heggie 1996).

On the side of the FP models there have been two major recent developments. Takahashi (1995, 1996, 1997) has published new FP models for spherically symmetric star clusters, based on the numerical solution of the orbit-averaged two-dimensional (2D) FP equation [solving the FP equation for the distribution $f = f(E, J^2)$ as a function of energy and angular momentum, on an (E, J^2) -mesh]. Drukier et al. (1999) have published results from another 2D FP code based on the original Cohn (1979) code. In such 2D FP models anisotropy, i.e. the possible difference between radial and tangential velocity dispersions in spherical clusters, is taken into account.

Secondly, another 2D FP model has been worked out recently for the case of axisymmetric rotating star clusters (Einsel & Spurzem 1999; Kim et al. 2002). Here, the distribution function is assumed to be a function of energy E and the z -component of angular momentum J_z only; a possible dependence of the distribution function on a third integral is neglected. As in the spherically symmetric case the neglect of an integral of motion is equivalent to the assumption of isotropy, here between the velocity dispersions in the meridional plane (r - and z -directions); anisotropy between velocity dispersion in the meridional plane and that in the equatorial plane (ϕ -direction), however, is included.

With the advent of a wealth of detailed data on globular clusters, such as luminosity functions and derived mass functions, colour-magnitude diagrams, and population and kinematical analysis, obtained by, for example, the *Hubble Space Telescope*, for extragalactic and Milky Way clusters (cf. e.g. Rubenstein & Bailyn 1997; Shara et al. 1998; Grillmair et al. 1999; Ibata et al. 1999; Piotto & Zoccali 1999; Piotto et al. 1999, to mention only the few most recent papers), easily reproducible reliable modelling becomes more important than before. For that purpose a few more ingredients are urgently required in the models in addition to anisotropy and rotation: a mass spectrum, a tidal field and the influence of stellar evolution of single stars and binaries on the dynamical evolution of the cluster.

In principle, it appears easy to include all of these in a direct N -body simulation, but it turned out that considerable effort is needed to improve the required physical knowledge and numerical codes for that purpose (Makino et al. 1997; Aarseth 1999a; Hut & Makino 1999). A notable effort to focus the experience of different groups here is the recent MODEST initiative (Hut et al. 2002; Sills et al. 2003, and the webpage at <http://www.manybody.org/modest.html>). Unfortunately, despite the enormous advances in hardware and software the modelling of say 100 000 particles is still presently a very challenging task and it is impossible to do large parameter surveys in this regime, even with NBODY6++ or NBODY4. One has to rely on lower particle numbers and prescriptions to scale the results to larger N (Aarseth & Heggie 1998; Baumgardt 2001). So we still need the fast but approximate theoretical models.

However, there is an elegant alternative way to generate models of star clusters, which can correctly reproduce the stochastic features of real star clusters, but without really integrating all orbits directly as in an N -body simulation. These so-called Monte Carlo models were first presented by Hénon (1971, 1975), Spitzer (1975) and

later improved by Stodolkiewicz (1982, 1985, 1986) and in further work by Giersz (1996, 1998, 2001). The basic idea is to have pseudo-particles, the orbital parameters of which are given in a smooth, self-consistent potential. However, their orbital motion is not explicitly followed; to model interactions with other particles such as two-body relaxation by distant encounters or strong interactions between binaries and field stars, a position of the particle in its orbit and further free parameters of the individual encounter are picked from an appropriate distribution using random numbers.

There are also Monte Carlo models by Joshi, Rasio & Portegies Zwart (2000), Watters, Joshi & Rasio (2000), Joshi, Nave & Rasio (2001), Fregeau et al. (2003) for globular clusters and Freitag (2000) and Freitag & Benz (2001) for globular clusters and galactic nuclei. They rely on the FP approximation and (hitherto) spherical symmetry, but their data structure is very similar to an N -body model.

A hybrid method using a gas dynamical model for the single stellar component, but a Monte Carlo technique for the binaries has been proposed in Paper I (Spurzem & Giersz 1996) and it was shown in Paper II (Giersz & Spurzem 2000) that such models are in good agreement with N -body and FP results where it should be expected, and that they provide a huge amount of interesting, detailed information concerning binary evolution in a star cluster (globular cluster) with a realistically large number of primordial binaries and single stars (30k binaries in cluster with 270k single stars). Another attempt to combine a direct FP model with a Monte Carlo simulation to a hybrid scheme has been reported but not continued further as yet (Huang & McMillan 2001). In this paper we present results obtained from a further improvement of our code that allows us to study in a much more realistic way the evolution of binaries in star clusters using direct three- and four-body integration in regularized variables to follow the close encounters.

It cannot be excluded that star formation in clusters predominantly produces tight binaries rather than single stars, and this is consistent with present-day binary fractions in the galactic field (Kroupa 1995). Globular cluster observations reveal present-day binary fractions of approximately 15 per cent or more, depending on the position in the cluster (Rubenstein & Bailyn 1997). Regarding the large dynamical age of (at least in the Milky Way) globular clusters we must assume that stars in them also form with a high fraction of binaries. As was reported in more detail in Paper II the dynamical modelling of large star clusters with many primordial (=initial) binaries, many of them hard (i.e. with binding energies higher than the average thermal energy of stars in the cluster, often denoted with kT), is an even harder physical and computational task than the standard N -body models. From the viewpoint of realism and the aim of finally obtaining models comparable to real clusters the main improvement here, in Paper III, is that for the first time we have detailed information concerning all orbital elements of the binaries, in particular their eccentricities.

While semimajor axes or binding energies of binaries have been subject to intensive study for the previous several decades, much less is known concerning the systematic evolution of eccentricities of binaries in cluster environments. In a pioneering study Hills (1975a,b) simulated approximately 10 000 encounters between binaries and single stars (equal masses) and discussed how binaries would interact with a star cluster environment. He found that the eccentricity of initially circular binaries after strong encounters would approach the thermal mean of $\langle e \rangle = \frac{2}{3}$ for slow encounters, and that moderately fast encounters provide even larger eccentricities. With a few thousand direct integrations of three-body scatterings Hut & Paczyński (1984) determined first cross-sections for eccentricity change of binaries with circular orbits for wide encounters.

Sigurdsson & Phinney (1993) performed stochastic binary encounters in a static cluster background (again, only encounters with circular binaries are considered) with much better statistics and unequal masses. Their results are consistent with the earlier work of Hills (1975a,b) regarding the eccentricities, that distant fly-by encounters generate little change in eccentricities, while close and exchange encounters tend to generate on average a high thermal eccentricity value of $\langle e \rangle = \frac{2}{3}$. McMillan, Hut & Makino (1991) performed small N -body simulations (100 binaries in a 1100-star model) and for the first time approximately determined differential cross-sections for three- and four-body interactions in an evolving star cluster. They also followed the evolution of eccentricity of interacting binaries. Their results are consistent with Heggie's (1975) analytical predictions for differential cross-sections and with thermal eccentricity distributions.

However, to assess the steady-state distribution of eccentricities in a globular cluster we need to know the statistical variation of eccentricities in encounters with binaries having initially all possible eccentricities (in fly-by and exchange), and we need to know how the evolving cluster background interacts with the eccentricity distribution of the binaries. Such information is crucial to judge the frequency and dynamical importance of some very important physical processes in clusters, such as the rate of mergers (producing blue stragglers), the rate of X-ray binaries and other exotic objects (binary pulsars, their decay rate due to gravitational radiation, see e.g. Rasio & Heggie 1995), the formation and evolution of products of close tidal interactions. All of these critically depend on the eccentricities of the binaries involved in them. Unfortunately, to the knowledge of the authors, there is nearly no systematic study of the evolution of initially non-circular binaries with the remarkable exception of Heggie (1975), McMillan et al. (1991) and Heggie & Rasio (1996) and some unpublished work quoted in the latter paper. Heggie & Rasio (1996) give cross-sections of eccentricity changes using an analytic approximation for very distant encounters (tidal and slow) – what is most interesting here for us is that they find for initially non-zero eccentricity an equal cross-section for positive and negative changes of eccentricities. So, the earlier picture that due to close encounters binaries tend to obtain a certain average eccentricity is complemented by wide encounters, which seem to generate a picture of a diffusion process where negative and positive changes of eccentricity are alike.

It is one of the aims of this paper to provide the missing link between the two pictures and discuss the complete binary evolution, including their eccentricities, covering all initial values, fly-bys and exchanges, and incorporate this in an evolving cluster background. The main challenge here, besides the development of our stochastic Monte Carlo model in general, is to disentangle the wealth of data produced by a model back into meaningful cross-sections and expectations for eccentricity evolution (for example).

In the following section we describe in some detail how we initialize and perform the direct integrations for binary–single and binary–binary encounters and how the outcome of the encounters is used as an input for our stochastic Monte Carlo simulation of the evolving cluster. Section 3 describes some results and Section 4 gives a discussion and conclusions.

2 BINARIES AND DIRECT THREE- AND FOUR-BODY INTEGRATIONS

In Paper II we used statistical cross-sections derived from theory or from simplified numerical studies for the changes in binding energy of a hard binary in a three- or four-body encounter (Spitzer

1987; Gao et al. 1991). Here our aim is to relax this rather severe limitation of our models, for two main reasons. One is that such cross-sections are poorly known (if at all) for the most interesting case of systems with unequal masses (spectrum of stellar masses). To prepare the simulation of large stellar systems, including a large number of primordial binaries and a mass spectrum, in our stochastic Monte Carlo model in the next Paper IV (in preparation) we have to implement and test the relevant procedures first in the equal-mass case, which is the subject of this paper. Even in the equal-mass case it is unknown how the eccentricity of the binaries changes during close three- and four-body encounters, and how the differential and total cross-sections depend on the initial eccentricity of the binaries which react with each other. Eccentricities of binaries play an important role in the minimum distance of two stars during an encounter, which will be an important tool for studying the merging of stars in clusters due to close encounters.

Our present approach is as follows. In contrast to Paper II we use a completely unbiased prescription to determine whether a binary–binary encounter is due in the sense that the probability does not assume any prescribed form of the total cross-section. Rather, we obtain a maximum impact parameter and use encounter time-scales based on it (see details below). For three-body encounters we still use a total cross-section as in Paper II to determine whether an encounter takes place, since we check that our results correctly reproduce the known cross-sections of Spitzer (1987), so it is less critical here to adopt a more general encounter probability. However, this only works for the equal-mass case. For the test runs (discussed in more details at the end of Section 2.1.1) we used the unbiased determinations of three- and four-body encounter probabilities. In future simulations for unequal-mass systems we will only use the unbiased prescription. If an individual close encounter of the three or four bodies is due, we follow the trajectory using regularized coordinates and precise N -body integration. After some time the reaction products separate again and we measure the outcome (single stars, binaries) and the internal and external energies, plus eccentricities of the binaries. Also it is possible to make some statements concerning the relative importance of the formation of hierarchical stable triples.

2.1 Direct integration of three- and four-body interactions

2.1.1 Decision as to whether an encounter takes place

At each time-step of the gas model a random number check is used to see whether a close encounter is due between a single star and a binary, or between two binaries. If a single star is needed, its parameters in the cluster frame are drawn from a Schwarzschild–Boltzmann velocity distribution, the parameters of which (density, radial and tangential velocity dispersion) at the location of the binary are given by the gaseous model variables. For binaries that are subject to a possible encounter with other binaries the nearest binary neighbour is chosen as an interaction partner (Stodolkiewicz 1982, 1986).

Whether a binary actually suffers from a close three- or four-body encounter is determined randomly for each time-step and binary. For three-body encounters a probability based on the total cross-section of Spitzer (1987) and the local density of single stars are used as in Paper II, see above. For four-body encounters a new unbiased procedure is used, based on the determination of a reasonably large maximum impact parameter p_{\max} to cover all relevant encounters. For the determination of p_{\max} we follow the prescriptions outlined in papers by Hut & Bahcall (1983), Mikkola (1983, 1984a,b) and

Bacon, Sigurdsson & Davies (1996), with a small modification described below. It means we use the following prescription:

$$p_{\max} = a(D + C/V), \quad (1)$$

where $V = v/v_c$ is a scaled relative velocity at infinity (using a critical velocity v_c , see below), a and e are the semimajor axis and eccentricity of the binary involved in a three-body encounter, or in the case of a four-body encounter the values taken from the weaker binary. For three-body interactions we use $D = 0.6(1 + e)$, $C = 4$, a choice that represents an empirical optimum, to include all relevant impact parameters for changes of the binary parameters, but to cut off too many very weak interactions which would not significantly change the internal binary parameters (note that two-body relaxation due to the cumulative effect of very many distant encounters between single stars and centres of masses of binaries is taken properly into account, see Paper II). It is clear, that interactions with very high relative velocity only matter for the binary parameters if there is a relatively small impact parameter, but, on the other hand, slow encounters involving wide and/or eccentric binaries (large apocentre separation of the binary!) need much more caution to catch all relevant encounters, i.e. larger p_{\max} is necessary. These competing effects are accommodated in the terms containing C and D , respectively, above.

The critical velocity in the case of the three-body problem is given by

$$v_c^2 = \frac{m_1 + m_2 + m_3}{m_3(m_1 + m_2)} \left(\frac{Gm_1m_2}{a} \right) = \frac{2G}{\mu_3} E_b$$

$$\mu_3 = \frac{m_3(m_1 + m_2)}{(m_1 + m_2 + m_3)}, \quad (2)$$

where m_1 and m_2 are the masses of the two members of the binary, m_3 is the mass of the incoming perturber, single star in the case of three-body encounters, E_b is the binding energy of the binary and μ_3 is a reduced mass.

In the case of four-body encounters we use $D = 0.6(1 + e_1)$, $C = 5$, following Bacon et al. (1996), with the four-body critical velocity

$$v_c^2 = \frac{m_1 + m_2 + m_3 + m_4}{(m_1 + m_2)(m_3 + m_4)} \left(\frac{Gm_1m_2}{a_1} + \frac{Gm_3m_4}{a_2} \right)$$

$$= \frac{2G}{\mu_4} (E_{b1} + E_{b2})$$

$$\mu_4 = \frac{(m_1 + m_2)(m_3 + m_4)}{(m_1 + m_2 + m_3 + m_4)}. \quad (3)$$

Now m_1, m_2, a_1, e_1 and E_{b1} denote parameters of the weaker binary in a four-body binary–binary encounter, m_3, m_4, a_2, e_2 and E_{b2} are those of the harder binary, μ_4 is a reduced mass.

From p_{\max} we determine an encounter rate \dot{N} using the simple prescription $\dot{N} = nAv_{\text{rel}}$, where n , $A = \pi p_{\max}^2$ and v_{rel} are the local binary particle density, the geometrical cross-section of p_{\max} and the actual relative velocity of the two individual binaries, respectively, chosen for a close encounter. It is crucial that the binary density be well defined, smooth and steady, otherwise spurious fluctuations will cause convergence problems in the gaseous model equations and other problems. We finally determine the binary density by smoothing out each binary over its accessible coordinate space (weighted with the orbital factor dr/v_r), and summing the contributions of all binaries in each shell to obtain the local n . To determine, in a given volume ΔV (radial shell of our gaseous model) and time-step of the

simulation Δt , whether an encounter takes place we compare $\dot{N} \Delta t$ with a random number.

When setting up our simulation and testing the code it turned out that one has to make some small modifications of the above standard procedure for the purpose of computing speed. With p_{\max} as defined above we find that our program integrates in a typical run, ranging over dozens of half-mass relaxation times and thousands of binaries (initially tens of thousands) several hundred thousands of weak four-body encounters, which lead to very small changes in the binary parameters. The computing time is dominated by the quadruple integration in an impractical and undesired way. Therefore, we limited, in the case of binary–binary encounters, the maximum impact parameter to $p_{\max} = 10 a_1$ (semimajor axis of the weaker binary). In several control runs, we checked the influence such a limitation had on the global dynamical evolution. Unfortunately, there is no simple conclusion from these experiments. While many features of global evolution (the order of the first time of collapse, the minimum and the maximum density during later gravothermal oscillations) are similar, we obtain some different results in the detailed binary evolution. Generally, fewer hard binaries develop, there is less activity caused by them, e.g. the number of escapers (single and binary) drops by some 10 per cent if the full p_{\max} is used, as compared with our standard run, using the limitation of p_{\max} . However, using large p_{\max} is not necessarily the correct way to solve the problem, because at some point weak encounters are identical to the standard two-body relaxation between two binaries and this is treated otherwise – by our standard Monte Carlo procedure. In view of these difficulties, we have decided to present our results from a standard run using the p_{\max} limitation as described above, while making the reader aware of the underlying possible problems. The solution to the ‘ p_{\max} ’ problem will be a better termination criterion for three- and four-body encounters, which will limit the integration time for rare but time-consuming cases. Unfortunately, we have not yet found any reliable termination criterion. It is worth noting that simulations up to the time equal to approximately a few hundred initial relaxation times take approximately 2 weeks and approximately 2 months with limited and unlimited p_{\max} , respectively.

2.1.2 Setting up an encounter

After we have identified three or four bodies to interact, given their masses and velocities at infinity in their centre-of-mass frame, there is some sequence of steps until the actual three- or four-body integration should start.

First, using a random sampling of impact parameter p , assuming a constant distribution in $dp^2 = 2pdp$, with $0 < p < p_{\max}$ we obtain the actual impact parameter to be used for this encounter. Secondly, the detailed integration of each encounter starts at some finite separation r_x of the reaction partners, where the perturbation of the weaker binary becomes relevant. We determine r_x by the condition that the weaker binary experiences a force perturbation $\gamma > \gamma_{\text{crit}}$ with $\gamma_{\text{crit}} = 10^{-3}$, and allowing for some generous fudge factors to make sure that under any orbital phase we still have $\gamma < \gamma_{\text{crit}}$ initially. In case of three-body encounters we use such a criterion analogously for the perturbation of the binary by a single star. To be specific, we take

$$r_x = Aa(1 + e) \left\{ 1 + \left[\frac{8m_x}{\gamma_{\text{crit}}(m_1 + m_2)} \right]^{1/3} \right\}, \quad (4)$$

where a and e are again the semimajor axis and the eccentricity of the binary (three-body) or of the weaker binary (four-body), and

$A = 10$ (three-body) or $A = 3.5$ (four-body) are the previously mentioned fudge factors. In the case of four-body encounters $m_x = m_3 + m_4$, whereas for a three-body encounter $m_x = m_3$. Note, that we present such procedures here already using individual masses m_i for all stars taking part in the encounters, in order to prepare the ground for subsequent work for multimass systems. All the results presented in this paper are obtained for systems where $m_i = \text{constant} = 1/N$ in normalized variables.

After using energy and momentum conservation to transform the encounter from infinity to r_x we start the actual three- or four-body integration at separation r_x ; first all variables are transformed into a frame in which the centre of mass of the three-body (four-body) system is at rest. Then regularized coordinates are obtained using Aarseth & Zare (1974) for three-body and Mikkola (1983) for four-body regularization. We use the integration packages TRIPLE and QUAD for unperturbed three- and four-body integration, which were in their original version kindly supplied by S.J. Aarseth with his NBODY6 program package. In both packages the minimum distance between all interacting particles is correctly tracked, even if the steps in regularized time do not exactly match the pericentre of the motion of two bodies. Secondly, termination is checked primarily against a maximum separation of the outermost body; for this parameter we choose here $1.2 r_x$, where r_x is the initial separation at the set up of the encounter as defined above. In some rare cases where the state of the remaining three bodies is not well defined we continue the integration until the next largest distance also reaches $1.2 r_x$ or a well-defined end state is reached (bound triple). If a bound quadruple is formed (three-body encounters: bound triple) we also terminate the integration.

If TRIPLE and QUAD terminate with the above distance criterion we check the resulting new configuration after the encounter. We adopt the following channels for the outcome of the reaction in the case of three-body encounters:

- (i) *Hardening*: the remaining two stars are gravitationally bound with a well-defined semimajor axis and eccentricity. The third star carries away surplus kinetic energy; this is the typical case of superelastic scattering which heats the stellar system.
- (ii) *Softening*: the binary loses binding energy at the expense of the single star. This and the following two cases are rare and occur predominantly for weak binaries.
- (iii) *Three bound*: a bound three-body subsystem is formed. For practical reasons we define a threshold of $0.1 kT$; only if the three-body system in total is bound with more than this value is it considered bound.
- (iv) *Dissolution*: the kinetic energy of the incoming star was large enough to disrupt the binary, at the end there are three single stars absorbed into the single star component.

While cases (i), (ii) and (iv) can be treated appropriately for all changes of their physical quantities (mass, internal and external energies of all reaction partners and products are properly balanced from and to single and binary components of our system), case (iii) cannot yet be correctly treated. In future work we will also keep track of bound triples, which then would be subject to further interactions, requiring eventually direct N -body integration with more than four bodies, e.g. by using the CHAIN method (Mikkola & Aarseth 1996, 1998). At present we just terminate the triple and convert it back into a binary (obtained from its two most bound stars) and a single star. The binary binding energy is reduced by the corresponding amount to lift the escaping star to zero energy. While this procedure is artificial for this particular triple it ensures that there

is no energetic error generated. The fraction of such events is very small.

For four-body encounters we employ these criteria to judge their outcome and reaction products.

(i) *Stable*: two binaries are well defined and bound after the encounter. They part from each other after the encounter with changed orbital parameters. The most probable case is that the harder binary becomes harder, and in approximately one-third of the encounters an exchange takes place.

(ii) *Dissociation*: one binary is dissolved, mostly it is the weaker binary. The possibility is not excluded, however, especially if both binaries have similar binding energies, that only the harder binary is dissolved.

(iii) *Hierarchical*: one star escapes and a hierarchical triple remains; here we check whether it fulfils the stability criterion of Mardling & Aarseth (2001). If not, the integration is continued for some time, until the third star has also escaped from a remaining binary. In very rare cases we have to terminate an encounter, because after long integration time neither a hierarchical and stable triple nor a binary will reach the escape criterion. It is interesting to note here that stable three-body orbits exist, which are not hierarchical (Heggie 2000); in this work, however, we did not check in detail the outcome of such situations, which should be performed in the future in a framework of treatment of triples in our hybrid scheme.

(iv) *Four bound*: all four stars form a bound subsystem.

Within the framework of our model we treat cases (i) and (ii) properly, as the most frequent ones. Cases (iii) and (iv) are usually decomposed into a binary and single stars forcibly, with an analogous procedure as described above for three-body encounters in order to guarantee correct energetics. In the future we will be able to deal with hierarchical triples in our stochastic Monte Carlo model.

Each full star cluster run stores a data bank of all relevant information for each encounter that occurred during the run. A typical amount of data comprises some 20k four-body interactions and 40k three-body interactions. To test our limitation of $p_{\max} = 10 a_1$ (see the final paragraph of Section 2.1.1) we created two models where we allowed for *all* three- and four-body encounters to be directly integrated within p_{\max} as defined in equation (1), no matter how weak they are. Each run comprised of approximately 70k and 500k integrated three- and four-body encounters, without significant differences from the standard one (except a better coverage of weak fly-bys, which could be seen by an increase of the cross-sections for very small changes).

Since we are interested in the statistics of binary evolution within the framework of real cluster evolution, we do not use the strategy to sample individual three- or four-body encounters in artificial settings. Rather, we simulate how real binaries suffer from encounters in an evolving cluster, but process the data of all binary scatterings afterwards in a very similar way to that of Hut & Bahcall (1983) and Bacon et al. (1996). By post-processing of the data bank we can also look at certain cross-sections, e.g. for having a minimum distance smaller than a solar radius in order to check for possible mergers. Mergers are not processed in our model, and we continue to treat all stars as point-mass objects. Looking at the fraction of events that would lead to mergers nevertheless gives us a good indication of the expected merger rates.

For the determination of total and differential cross-sections we count all events, and sort them into the different categories, using the analogous normalizations as in the cited papers.

3 RESULTS

3.1 Heggie's runs, testing

First, we repeat a number of models simulated by Heggie & Aarseth (1992), which we call Heggie's runs. They published results of full direct N -body simulations of 2500 particles with initial binary numbers of 75 (models S, SS) or 150 (models D, E), and an initial binding energy ranging from 2 to $20kT$ (models S, D), 2 to $200kT$ (model E) and 2 to $2000kT$ (model SS). These models have been carefully examined again in our Paper II using analytical cross-sections of binary encounters, and again we have, with our more detailed binary modelling, repeated the series now. Generally all results agree fairly well, with Paper II and with the original paper of Heggie & Aarseth (1992). While we do not want to show all the results here in detail, a few interesting improvements could be noticed.

Fig. 1 shows the central escape speed as a function of time in model S. Its maximum value is higher than in Paper II, and the time of the maximum is later. Both features make it more similar to the N -body results. Another difference can be seen in Fig. 2 for

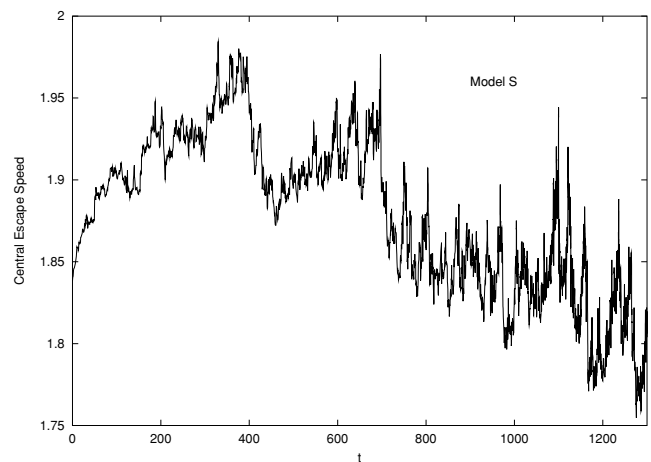


Figure 1. Model S, central escape speed as a function of time (in N -body units).

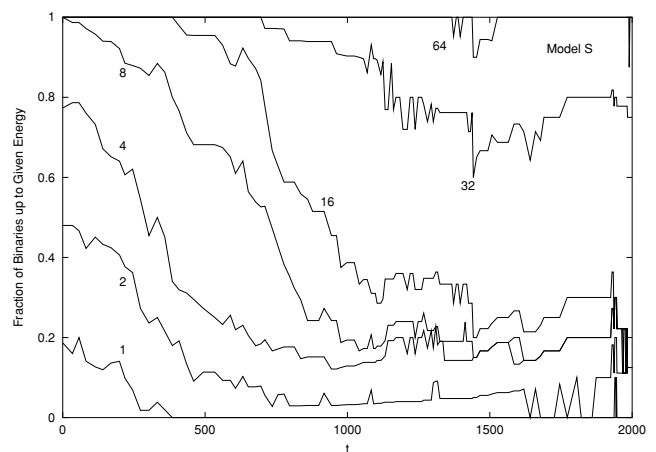


Figure 2. Model S, distribution of binding energies as a function of time (in N -body units) for all bound binaries. The unit of energy is $2kT$, i.e. one and a half times the initial mean kinetic energy of the single stars. Each line gives the fraction of these binaries with binding energy smaller than the stated value.

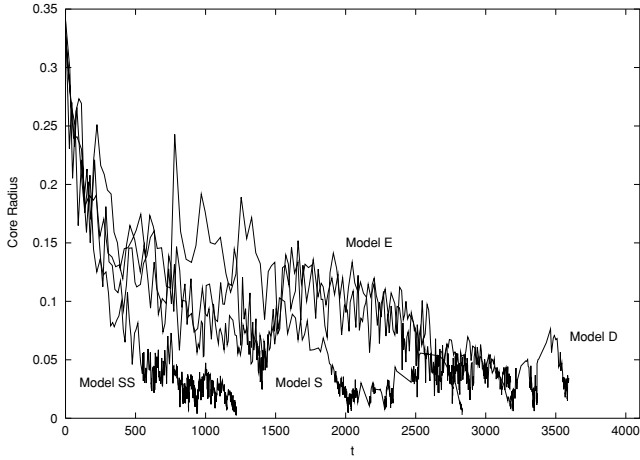


Figure 3. Models S, D, E and SS; core radius as a function of time (in N -body units).

model S, where there is a much smaller number of hard binaries (line representing $64kT$) at late times compared with Paper II. This is due to the much more efficient disruption of binaries in the initial phase by four-body encounters. Fig. 3, showing the core radii of the whole system, also shows a much smaller core radius now for model D (150 binaries) than before in Paper II, again because of less heating provided by the binaries. All features discussed in this paragraph make the results of our simulations more similar to those of Heggie & Aarseth (1992). It should be stressed here that this agreement is only qualitative. The large noise of the simulation data makes it difficult to draw firmer conclusions. The smaller values of the core radii in our models than in Heggie & Aarseth's (1992) paper mainly happen at the late phases of the evolution, when there are only a few binaries left in the system. So, the system evolution is itself more and more stochastic and difficult to compare. In the earlier stages of the evolution (core collapse and steady binary burning phase), when there is still a large fraction of binaries in the system, our results agree fairly well with those of Heggie & Aarseth (1992).

3.2 Gao's runs

Secondly, we follow our main task, to model a large star cluster with many primordial binaries. In order to compare with the previous results of Paper II and Gao et al. (1991) we use exactly the same initial conditions, i.e. a cluster of 270 000 single stars and 30 000 binaries, both distributed in a Plummer model density distribution with constant density ratio between binaries and single stars (for more details see the cited papers).

All binaries are set up initially with a so-called thermal eccentricity distribution, where orbital eccentricities are homogeneously distributed as a function of e^2 . Binding energies are distributed logarithmically homogeneous between 3 and $400kT$, and binaries and single star densities are initially obtained from a Plummer model. This is analogous to Paper II, except for the eccentricities, which did not matter there.

One of the main differences with respect to Paper II is that there is no longer a quasi-steady binary burning phase before core collapse (cf. Figs 4 and 5 with figs 19 and 21 of Paper II, respectively). Such a quasi-steady state, as was also seen by Gao et al. (1991), is no longer seen here, because binary–binary interactions have many more degrees of freedom here. While in Paper II and in Gao et al. (1991) the only assumed outcome of a binary–binary encounter is the hard-

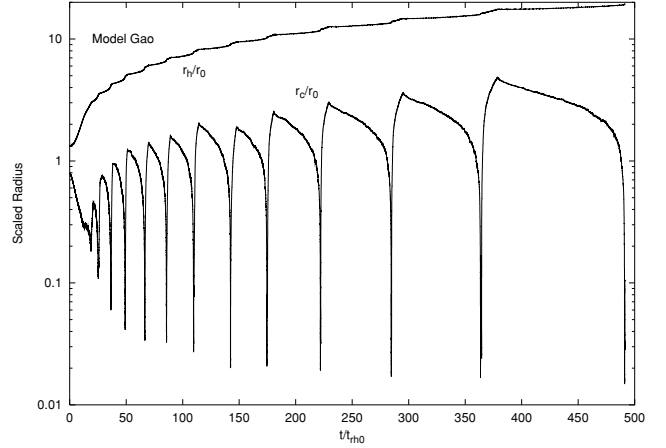


Figure 4. Evolution of the core and half-mass radii (scaled by the scale-length of the Plummer model) as a function of time (scaled by the initial half-mass relaxation time).

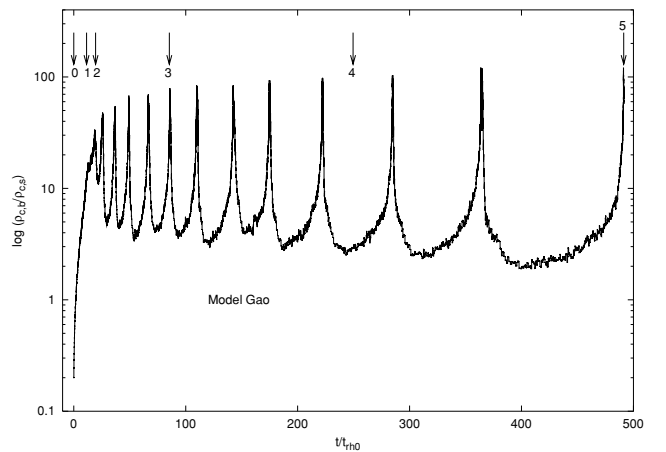


Figure 5. Ratio of central densities in binaries and singles as a function of time (scaled by the initial half-mass relaxation time). Arrows indicate the times for snapshot plots of 2D distributions of binaries bound to the system (see Fig. 9).

ening of the hard binary and destruction of the weaker binary, here encounters between moderately hard binaries can yield different and non-standard results. Both binaries can survive an encounter if their interaction is not too strong, for example. We observe that the total amount of internal energy in binaries and the total number of binary escapers is significantly smaller compared with Paper II (see Figs 6 and 7), while at the same time we have more binaries remaining in the system, partly in the outer zones. It is worth noting that the total number of escapers is larger than in Paper II, just because there are more single escapers now. This is connected with the fact that the binary destruction rate is much smaller than in Paper II, where each binary–binary encounter dissolves one of the binaries. So, in the present simulations, there are more binaries, which in binary–single interactions can remove single stars from the system. In this work binaries are not as centrally concentrated (see Fig. 8) compared with Paper II (see fig. 26 of that paper). What is not plotted here is that (again compared with Paper II) there is a much larger number of binaries surviving in the outermost parts of the cluster. We expect that most of these binaries, however, will be lost from the cluster if there the tidal field from the mother galaxy is taken into account, which

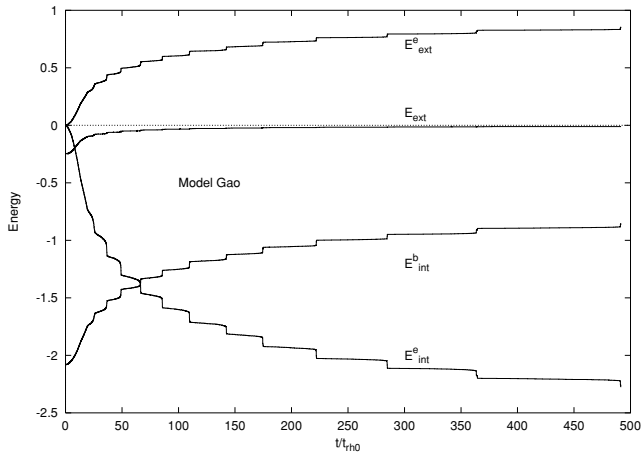


Figure 6. Energy balance as a function of time. Total internal energy of the binaries remaining bound in the system (E_{int}^b), total internal energy of those binaries which escaped (E_{int}^e), total external (i.e. potential plus translational) energy of all objects (singles and binaries) remaining bound in the system (E_{ext}^b) and escaping (E_{ext}^e).

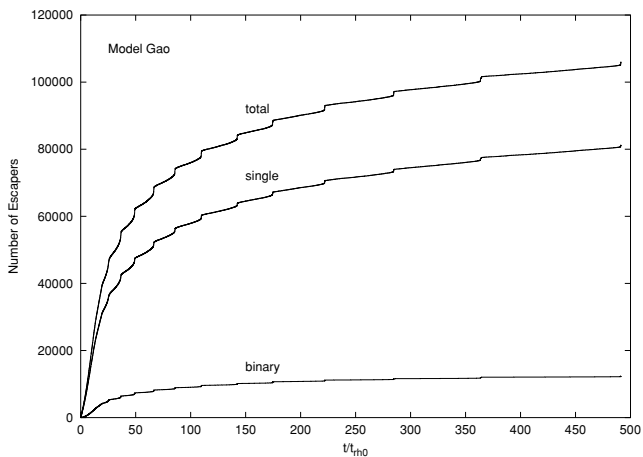


Figure 7. Number of escaping single stars, binaries and the total number of escapers as a function of time (scaled by the initial half-mass relaxation time).

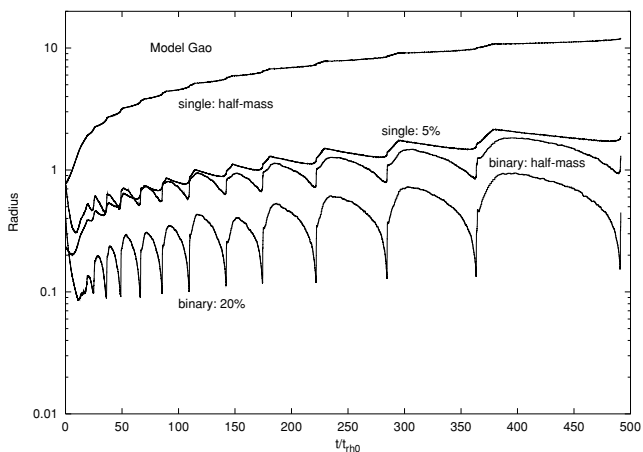


Figure 8. Evolution of Lagrangian radii containing 50 and 5 per cent of the mass of single stars and 50 and 20 per cent of the mass of binaries as a function of time (scaled by the initial half-mass relaxation time).

is a subject of our ongoing work. In conclusion, we think that any difference in the results of Paper II and this work can be ascribed to a smaller rate of binary destruction by four-body encounters here.

Now we turn to the discussion of the detailed eccentricity evolution of our binaries, for which there is no counterpart in Paper II. Fig. 9 shows, that during the entire evolution the binary density is constant as a function of e^2 , the squared eccentricity. This is consistent with the older predictions by Hills (1975a,b), McMillan et al. (1991) and Sigurdsson & Phinney (1993) that the eccentricity distribution in a large number of encounters remains thermal, with an average of $\langle e \rangle = \frac{2}{3}$. However, as a note of caution in interpreting our Fig. 9, which was obtained from a pure point mass model, in a real star cluster including stellar and binary evolution the nice thermal eccentricity distribution will not prevail, even on short time-scales. Looking, for example, in Kroupa (1995, fig. 2, top panel), one can see that in pre-main-sequence evolution high eccentricities of binaries are wiped out by what the author calls eigenevolution. For orientation, a circular binary with a semimajor axis of 1 au ($\log P \approx 2.56$ in Kroupa's figure) would correspond roughly to a $10kT$ binary in our models. In other words, in a cluster involving stellar evolution binaries with $10kT$ and eccentricity $e > 0.6$ would be subject to strong eigenevolution and may thus disturb the thermal eccentricity distribution. It is worth noting that binaries with binding energy equal to approximately $400kT$ have semimajor axis roughly equal to the solar radius (in units used in our simulations), so they will be subject to merger before interacting with other stars.

Fig. 9 shows in six panels the evolution of our initially 30 000 binaries in binding energy–eccentricity space. They all start equally distributed in a logarithmic scale of E_b and e^2 . While as expected more and more binaries are disrupted, an interesting result is that there is no tendency to accumulate anywhere in eccentricity–energy space. We always have binaries of any eccentricity, with no obvious eccentricity binding energy correlation, even in the late phases of the evolution. This is very different from the radius–energy relationship, which was plotted in Paper II in a similar way, where there was a correlation in the sense that the core does not have weak binaries, because they are destroyed. The stationary thermal eccentricity distribution of binaries is consistent with the previous scattering studies of Hills (1975a,b), McMillan et al. (1991) and Sigurdsson & Phinney (1993), but here for the first time confirmed in combination with an evolving binary population in a strongly evolving cluster.

Following every encounter in detail using our numerical integration scheme gives us the possibility of checking for closest encounter distances and possible mergers. In contrast to any other result discussed in this paper we have to fix the physical scales if we want to give meaningful results for minimum encounter distances in units of stellar or solar radii. For the scaling here we have assumed that the scaling radius of our initial Plummer model is 1.0 pc and that fixes everything in combination with the assumption that one particle (one star) has one solar mass. With our total mass of 330 000 solar masses this results in rather large central densities for our initial model, comparable only to the most concentrated globular or massive clusters near the Galactic Centre. Therefore, the discussion of ‘merging’ in any of the following figures should be seen as exemplary, since the rate of merging depends on the assumed scaling in the physical units. Any other quantities though, such as the relative distribution of minimum encounter distances in scaled N -body units or scaling behaviour of cross-sections, are independent of the selected physical units. One should also bear in mind that we do not merge any objects in our dynamical model, so it is possible that a very tight binary causes a ‘merger’ event in our simulations and

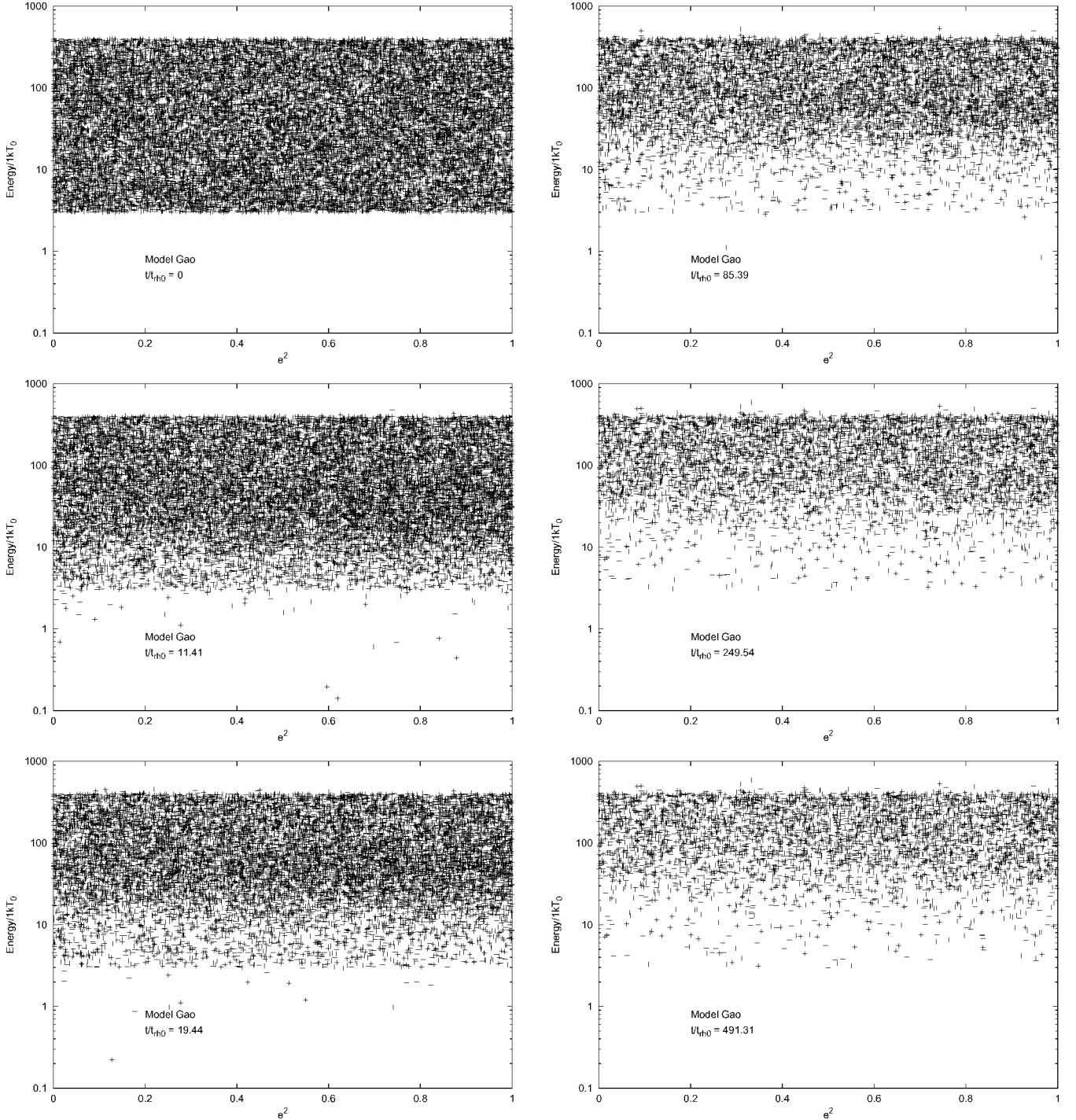


Figure 9. Snapshot plots of 2D projection of the individual data of each binary (represented by a cross) on to the energy–eccentricity squared plane. Binary binding energy is expressed in initial kT . Each plot represents a different time in units of the initial half-mass relaxation time; top left 0, top right 85.39, middle left 11.41, middle right 249.54, bottom left 19.44, bottom right 491.31. The selection of times is the same as indicated by the arrow in Fig. 5.

it is counted as a merger several times (see Figs 10 and 11). In a subsequent stage of this work merging will be included.

In Figs 10 and 11 we show each individual three- or four-body interaction, whether the minimum distance occurring during the interaction is larger than one solar radius (greater than unity) or vice versa. In the late stages the signature of density maxima in gravothermal oscillations can be seen, and in the beginning there are a lot of events of all kinds due to the large number of binaries present.

We prefer to show this rather primitive plots nevertheless, because even on that level one can easily by eye estimate that one-half of all three-body encounters, and some three-quarters of all four-body encounters would probably lead to a merger if the stars were to have solar mass and radius (but bear in mind the discussion in the previous paragraphs concerning scaling simulation units to physical units). In the late evolutionary phase with gravothermal oscillations nearly all four-body encounters lead to a merger of solar-type stars.

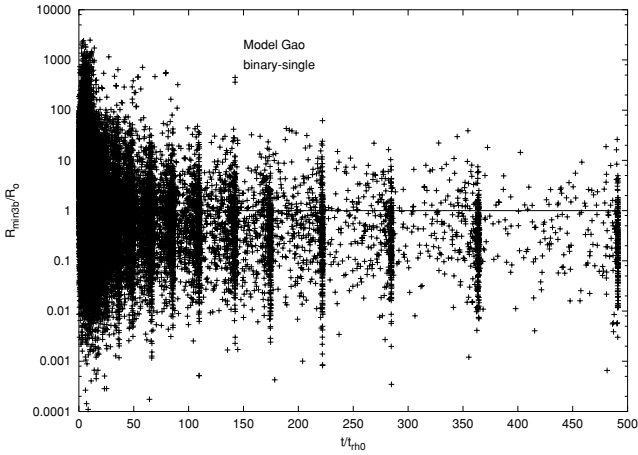


Figure 10. Minimum distance during binary–single interaction (in solar units) as a function of time (scaled by the initial half-mass relaxation time).

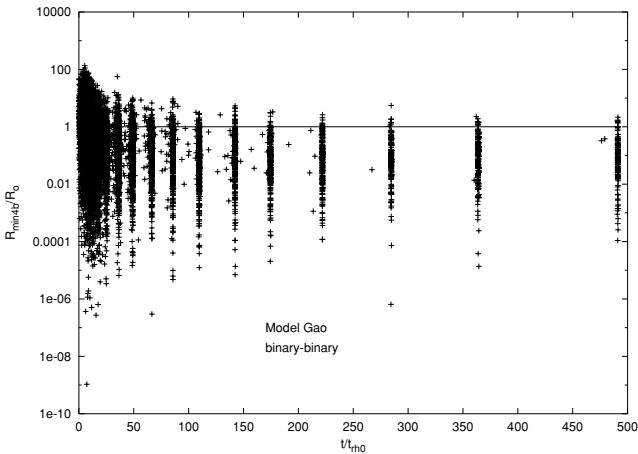


Figure 11. Minimum distance during binary–binary interaction (in solar unit) as a function of time (scaled by the initial half-mass relaxation time).

In Figs 12 and 13 we show how the initial eccentricities of the binary (the harder binary in the case of four-body encounters) influence the probability of a close encounter distance, again using the minimum distance scaled to solar values. We show four curves; the one labelled $0.0 < e < 1.0$ gives the cumulative cross-section for all encounters. The three other curves show the same cross-section for initially more circular orbits ($e < 0.3$), highly eccentric ($e > 0.7$) and moderately eccentric ($0.3 \leq e \leq 0.7$) orbits. The cross-section for merging is nearly one order of magnitude larger for encounters that involve highly eccentric binaries compared with those where there are only less eccentric or circular binaries. While this is relevant for stars with solar radii, it can be seen that if we look for much smaller minimum encounter distances, corresponding, for example, to white dwarf or neutron star mergers, that for those the merger cross-sections are totally dominated by mergers obtained from highly eccentric binaries. In other words, any merger of white dwarfs, neutron stars or other compact objects is highly likely to have originated from a very eccentric binary (not that surprising indeed).

One can also see in Figs 12 and 13 that the cumulative cross-section σ scales approximately with d for three-body encounters and with \sqrt{d} for four-body encounters, where d is the scaled min-

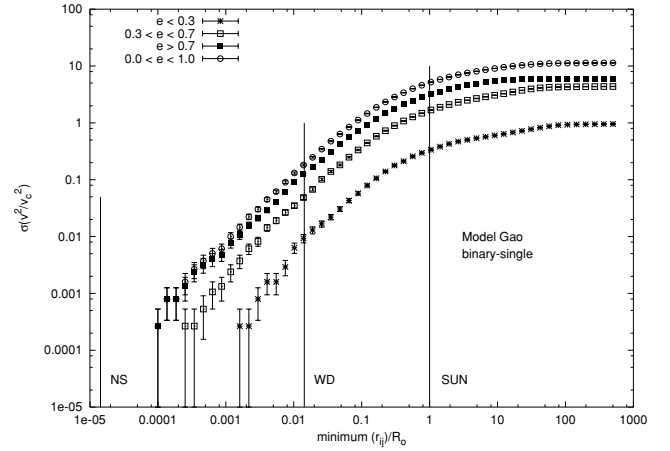


Figure 12. The cumulative cross-section for pairwise close approach during binary–single interactions. The separation is scaled to the solar radius. The cross-section is normalized to the geometrical cross-section of the binary and corrected for gravitational focusing. The normalization factor is properly averaged over all interactions (see the text for details).

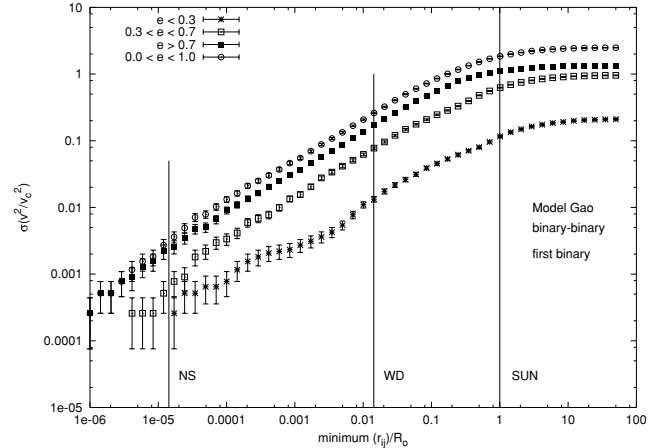


Figure 13. The cumulative cross-section for pairwise close approach during binary–binary interactions. The separation is scaled to the solar radius. The cross-section is normalized to the geometrical cross-section of the binaries and corrected for gravitational focusing. The normalization factor is properly averaged over all interactions (see the text for details).

imum encounter distance as in the figures. Hut & Inagaki (1985) discuss the fact that a linear scaling is obtained for very small d by a purely geometrical approach plus gravitational focusing, while the flattening of σ for larger d occurs due to the importance of resonant encounters, and they derive an empirical law of $\sigma \propto d^{0.4}$, which is in fair agreement with our findings. Our results tell us in connection with the discussion in Hut & Inagaki (1985) that most of our four-body encounters are indeed resonant, while the three-body encounters include many non-resonant encounters. This is a consequence that we limit p_{\max} to $10a_1$ in the case of four-body encounters as discussed above.

In Figs 14 and 15 we show, in the standard way, empirical normalized differential cross-sections as a function of $\Delta = (E'_b - E_b)/E_b$, the normalized energy change of the harder binary (E'_b , E_b binding energies after and before the encounter of the binary; in the case of dissolution or fully bound configurations we use $E'_b = 0$). The normalization is performed here over πa^2 , where a is the average

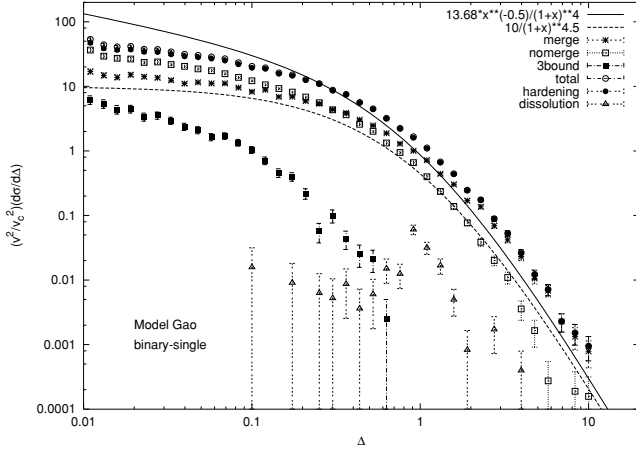


Figure 14. Differential cross-sections for relative binding energy changes (Δ) as a function of absolute value of Δ , for binary single-star interactions. The differential cross-section is normalized to the geometrical cross-section of the binary and corrected for gravitational focusing. The normalization factor is properly averaged over all interactions (see the text for details). The continuous line indicates Spitzer’s analytical estimate and the dashed line denotes Heggie’s estimate. The following symbols are for: mergers, star; no mergers, open square; three-body bound, filled square; total, open circle; hardening, filled circle and dissolution, open triangle.

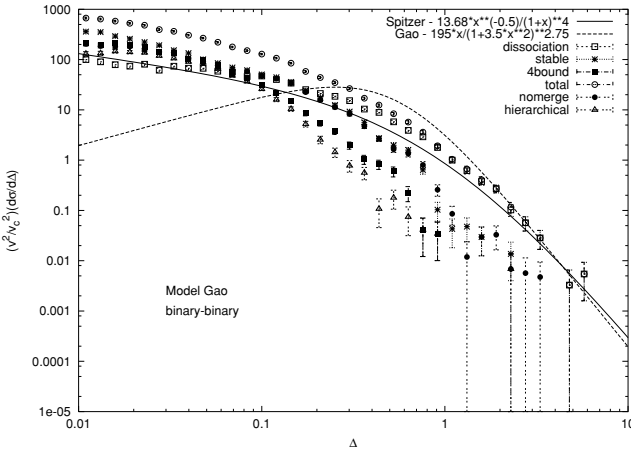


Figure 15. Differential cross-sections for relative binding energy changes (Δ) as a function of absolute value of Δ , for binary–binary interactions. The differential cross-section is normalized to the geometrical cross-section of the binary and corrected for gravitational focusing. The normalization factor is properly averaged over all interactions (see the text for details). The continuous line indicates Spitzer’s analytical estimate and the dashed line denotes Gao’s estimate. The following symbols are for: stable, star; dissolution, open square; four-body bound, filled square; total, open circle; no merger, filled circle and hierarchical, open triangle.

semimajor axis of all binaries involved in the encounters. For three-body encounters (Fig. 14) we find a fairly good agreement of the nearly entire differential cross-section with Spitzer’s expression

$$\frac{v^2}{v_c^2} \frac{d\sigma}{d\Delta} = \frac{13.86}{\Delta^{1/2}(1+\Delta)^4}, \quad (5)$$

which is plotted in the figure for comparison. For hard encounters there is also little difference from Heggie’s cross-section $\propto (1+\Delta)^{-4.5}$. Notably, however, for very small energy changes, our empirical cross-sections are different from Spitzer’s expectation. It is

not surprising that such differences occur here, because in our real cluster model there is only a limited coverage of phase space for all encounters with small energy changes (large impact parameters), different from artificial series of three-body experiments. In our real cluster time and p_{\max} are limited, so we have a smaller number of events here than in a theoretical simulation experiment of three- and four-body encounters. While this could be seen as an error from the standpoint of a strict theoretical determination of cross-sections we stress that it is of interest to derive statistical expectations of what happens to binaries in real clusters, which is truly reflected in our models. In order to avoid too many plots we have depicted in the plots multiple representations of the same data. The entire cross-section includes all events (note that we have used ‘total’ in the plot as a label for that, which is still a differential cross-section, but including all possible reaction outcomes). Our cross-sections are then subdivided in two different ways: first, differentiating between merging and non-merging events (here for one solar radius as a minimum distance) and, secondly, differentiating between physical end states such as three-bound, hardening and dissolution (in the case of four-body encounters: stable, hierarchical, triple).

First, discussing three-body cross-sections in Fig. 14 we see that three-bound and dissolution have very small probabilities, the latter occurs preferentially in strong encounters (high energy change for the binary), while the former has only a chance for small (negative) energy changes. Only for small Δ is there some gap between the entire and hardening cross-sections, at larger Δ the difference between the two is statistically insignificant. Hence, most strong three-body encounters result in a hardening, except at small energy changes – a well-known result which demonstrates the reliability of our methods. Since dissolution plays no significant role statistically (by a margin of more than two orders of magnitude in the measured cross-section at small Δ), most non-hardening events are softening (not plotted separately). The three-bound case can only occur at negative Δ , so it cannot compete with the three other cases at the same Δ . Nevertheless, we have plotted it in the same figure (using the absolute value of Δ).

Merging occurs at high Δ (strong hardening events) with higher probability than at low Δ , but for all Δ the difference in cross-section is less than an order of magnitude. This already demonstrates that there must be another factor determining close approach, which is the initial eccentricity of binaries in encounters, as was seen above for the total cross-sections as a function of minimum distance, for different initial eccentricities (Figs 12 and 13).

In the case of differential cross-sections for four-body encounters there is little analytical work to compare with. Our three- and four-body cross-sections can be compared with the pioneering work of McMillan et al. (1991) for small N -body simulations (see figs 13–16 therein). We also use Gao’s analytical cross-section (Gao et al. 1991) in Fig. 15 for comparison. It assumes that each four-body encounter results in a dissolution of the weaker binary, and a hardening of the hard binary by some fraction of the sum of the initial binding energies. For strong encounters, where dissociation dominates the cross-section, as seen in Fig. 15, Gao’s cross-section is not bad. For $\Delta < 1$ we have a competition between dissociation and stable end configurations (resulting in two surviving binaries). At small energy changes formation of bound quadruples and stable hierarchical triples is the most probable reaction channel. Since we do not yet follow the evolution of the bound four-body systems we cannot say anything definite concerning their future. However, many of them would possibly just be stripped off their fourth star and result in triples, where again there would be a considerable number of stable triples.

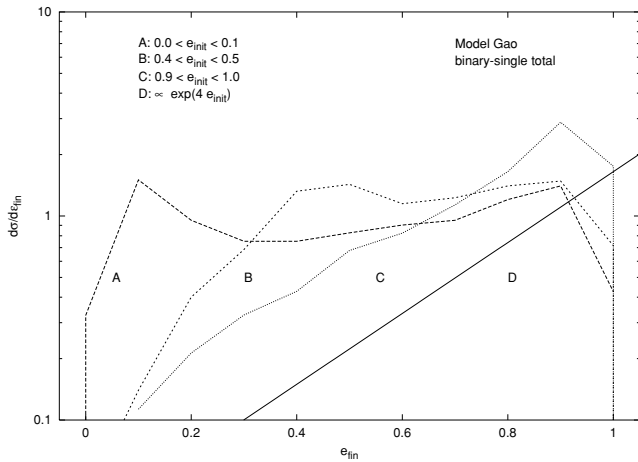


Figure 16. Differential cross-sections for eccentricity changes as a function of the final eccentricity, for all binary–single interactions. Curves labelled by A, B, C are for small initial eccentricities, medium initial eccentricities and large initial eccentricities, respectively. The line labelled by D is the exponential fitting done by eye.

Finally, we discuss a new kind of differential cross-section, which could be obtained from our data as far as we know for the first time. These are differential cross-sections for eccentricity changes as a function of the final eccentricity. These results, shown in Fig. 16 for three-body encounters, give the differential cross-section for a certain final eccentricity to occur. For initially nearly circular orbits (curves labelled A, initial eccentricity $e_{\text{init}} < 0.1$) we find that all final eccentricities after a three-body encounter occur with nearly equal probability. If there is already some initial eccentricity (see curves labelled B, for $0.1 < e_{\text{init}} < 0.5$) the probability of reaching any higher eccentricity is approximately constant, while the chance of going back to a less eccentric orbit decays exponentially. This is even more pronounced for initially highly eccentric binaries ($e_{\text{init}} > 0.9$): here the eccentricity remains high, and there is an exponential law $\propto \exp(4e_{\text{fin}})$, which could be fitted to describe the decreasing differential cross-section to reach smaller eccentricities.

Turning to four-body encounters, the entire cross-section (labelled ‘total’ in the figure) depicted in Fig. 17 shows at first glance

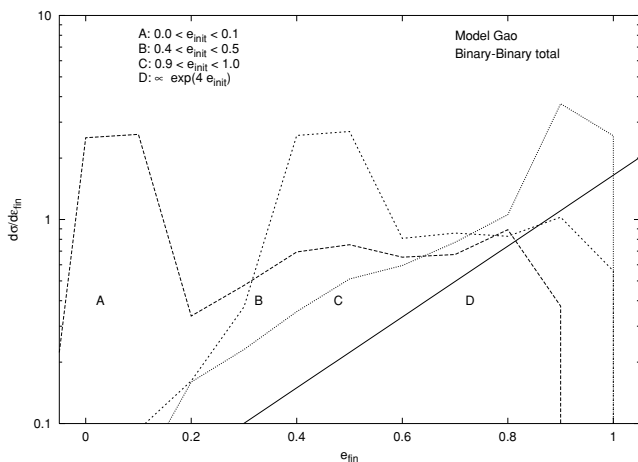


Figure 17. Differential cross-sections for eccentricity changes as a function of the final eccentricity, for all binary–binary interactions. Curves labelled by A, B, C are for small initial eccentricities, medium initial eccentricities and large initial eccentricities, respectively. The line labelled by D is the exponential fitting done by eye.

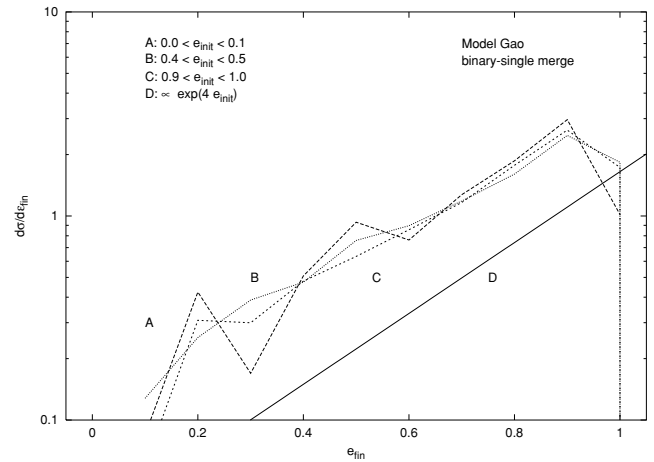


Figure 18. Differential cross-sections for eccentricity changes as a function of the final eccentricity, for binary–single interactions with mergers. Curves labelled by A, B, C are for small initial eccentricities, medium initial eccentricities and large initial eccentricities, respectively. The line labelled by D is the exponential fitting done by eye.

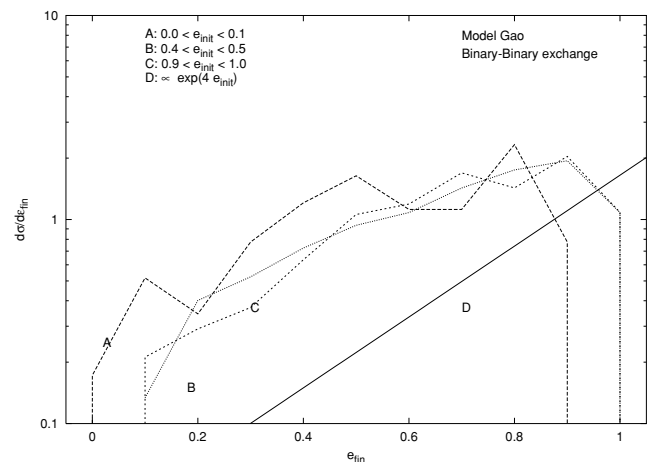


Figure 19. Differential cross-sections for eccentricity changes as a function of final eccentricity, for binary–binary interactions with exchange of stars. Curves labelled by A, B, C are for small initial eccentricities, medium initial eccentricities and large initial eccentricities, respectively. The line labelled by D is the exponential fitting done by eye.

a different behaviour. It seems there is a tendency for most binaries to keep their initial eccentricity – the differential cross-section has a maximum at $e_{\text{fin}} \approx e_{\text{init}}$! However, this is not really the case, rather we have a bimodal distribution depending on whether we look at strong encounters or weak encounters. If we discriminate strong encounters by our merger or exchange criteria (the minimum distance is smaller than the solar radius or one of the binary components is exchanged during the interaction), we see in Figs 18 and 19 that for a strong encounter the initial eccentricity is ‘forgotten’ in the sense that all differential cross-sections have a maximum at high final eccentricities and decay again with the characteristic $\propto \exp(4e_{\text{fin}})$ law already seen in three-body encounters. The maxima at $e_{\text{fin}} \approx e_{\text{init}}$ seen in Figs 16 and 17 could be ascribed totally to weak encounters (fly-bys) in which there is no strong interaction and hence no strong eccentricity change (cf. Heggie & Rasio 1996). Such a fly-by feature in the eccentricity cross-section is less pronounced in the case

of three-body encounters because we do not extend p_{\max} to large enough values (see the discussion above). For four-body encounters p_{\max} depends on the semimajor axis of the second binary, which is usually much larger than for the first one. So we can expect many more fly-by interactions than in the three-body encounters.

4 CONCLUSIONS AND DISCUSSION

We have performed a fully self-consistent numerical simulation of a point-mass star cluster consisting of 270 000 single stars and 30 000 hard binaries initially. All binaries have a thermal eccentricity distribution and a logarithmically constant binding energy distribution initially. All stars, whether members of a binary or not, have the same mass. The cluster evolution of single stars and binaries is followed using the stochastic Monte Carlo model described in Spurzem & Giersz (1996, Paper I) and Giersz & Spurzem (2000, Paper II). Instead of using statistical cross-sections to determine the outcome of close three- and four-body encounters (binary–single star and binary–binary scatterings) we use here a direct regularized three- and four-body integration, using the methods of Aarseth & Zare (1974) for three and Mikkola (1983) for four bodies (unperturbed motion). The method is tested in comparison with N -body modelling of star clusters with primordial binaries (Heggie & Aarseth 1992) and in comparison with Paper II. With the new detailed integration of binary interactions we can follow many different channels of these encounters, such as dissociation of binaries, weakening (instead of only hardening or increase of binding energy), formation of hierarchical triples and bound three- and four-body systems. Since we know the orbital elements of all binaries we can discuss differential cross-sections not only regarding binary binding energy changes but also eccentricity changes. Furthermore, we are able to monitor during encounters distances of closest approach and discuss total cross-sections for mergers to occur as a function of minimum approach distances. No mergers are considered in our models, and we just continue point-mass integration. Thus, our data only allow one to predict some fraction of encounters that would lead to mergers given a critical distance of closest approach. Subsequent evolution of the merger remnants and of the bound triple and quadruple configurations is a subject for future work. For the case of stable hierarchical triples according to the stability criterion of Mardling & Aarseth (2001) we again discuss the differential cross-sections leading to such configurations, but do not yet follow the further evolution of triples in our model.

Fregeau et al. (2003) have recently performed full Monte Carlo simulations including a large number of primordial binaries, too. They used direct integrations of binary–single encounters and cross-sections for binary–binary encounters (as in Gao et al. (1991) and as we did in Paper II). Their results show very clearly a quasi-steady binary burning phase similar to that obtained in Gao et al. 1991 and contrary to our findings. However, as they note they are unable to treat weak binaries, all of their binaries are hard (otherwise the Gao cross-sections could not be used). We think that this is the key to understanding our different results, as we have argued in Section 3.2 above. The direct integration of binary–binary encounters introduces more degrees of freedom, which are responsible for changes in the rate of binary destruction and energy generation. In particular, not every binary–binary encounter leads to one disruption, rather we could also have the formation of a hierarchy, a bound subsystem, or just one weakened and one hardened binary. Once the binaries have been burned or ejected from the core, both models exhibit gravothermal oscillations. Clearly, both our model and that of Fregeau et al. (2003) are idealized in the sense that further physics is needed to

model a real star cluster. However, we think that our treatment of integrating both three- and four-body encounters directly is more consistent, and Fregeau et al. (2003) admit that, in saying that it is their aim to do it in the same way in the future. Until they have included a detailed four-body integration procedure, and can cope with weak binaries too, it will be difficult to determine clearly the reasons for our different results. It should also be noted that Fregeau et al. (2003) used the KIRA software to integrate their three-body encounters, while we use the classical TRIPLE and QUAD procedures based on the CHAIN (Mikkola & Aarseth 1998).

The main new feature of our improved model compared with Paper II, which only used statistical cross-sections, is that binaries dissolve much quicker in a sequence of encounters, increasing their eccentricity and subsequent dissolution by four-body encounters. The time of quasi-stationary binary burning is now very much shorter than before ($\sim 50t_{\text{th}}$) and gravothermal oscillations follow (Bettwieser & Sugimoto 1984). We perform a detailed statistical analysis of all encounters between binaries and binaries and single stars, to determine total and differential cross-sections. While this is technically performed in the standard way (see Hut & Bahcall 1983; Mikkola 1983; Bacon et al. 1996) an important physical difference is that we model such encounters in the environment of a real star cluster evolution with its limitations of phase space available for initial conditions of encounters (limited time, limited maximum impact parameter). Consequently, we find that most of our empirical cross-sections agree very well with the expected analytical ones of Spitzer (1987) for three-body and Gao et al. (1991) for four-body encounters for strong encounters (large binding energy changes). For weaker interactions we find a much more differentiated result, which is important for the overall dynamics (energetics) of our cluster.

Finally, we monitor the eccentricity of all binaries. In all evolutionary stages a so-called thermal eccentricity distribution is maintained at all binary binding energies. Nevertheless, we find that strong encounters (e.g. those leading to merging and dissolution of binaries) are predominantly occurring for initially very eccentric binaries. This seems to be a contradiction (why are high eccentricities not depleted?), but is explained by our newly defined and measured cross-sections for eccentricity changes. Close three- and four-body encounters all have a high probability (maximum of the differential cross-section) of yielding high eccentricities. Having reached high eccentricity such binaries are particularly vulnerable to strong encounters, which lead to very small minimum distances, large energy changes and consequently also lead to disruptions. We find a new scaling law for the differential cross-section of eccentricity changes as a function of the final eccentricity, which scales with an exponential $\propto \exp(4e_{\text{fin}})$. This has no theoretical background yet and should be considered as purely empirical for the time being.

Having achieved one more significant step towards a full, self-consistent dynamical modelling of rich star clusters with all their binaries (and their internal degrees of freedom) we are now planning the next step. Using our detailed N -body integration for close encounters we will now be able to vary the masses of the stars (since the unknown cross-sections are not needed to determine the results), including a mass spectrum. Furthermore, we can then apply some recipes to merge stars if they overlap and increase their masses correspondingly. Also hierarchical triples should be followed in the future for their further internal evolution and perturbations by subsequent encounters with stars and binaries. This will open up paths to predicting certain types of merger remnants in star clusters, their distribution and frequency in time and distance from the cluster centre, but will also involve some rather sophisticated stellar physics

(see, e.g., Hurley et al. 2001). Finally, including an external tidal field is important for proper cluster modelling.

We think that our method of modelling very large realistic star clusters is promising as a counterpart (presumably the only one available) for the upcoming very large N -body simulations using recent generations of supercomputers (special and general purpose). Even with a few hundred Teraflops there will be no room for a large parameter survey in such models, which could be performed with our stochastic Monte Carlo model.

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