A Stochastic Power Network Calculus for Integrating Renewable Energy Sources into the Power Grid

Kai Wang, *Student Member, IEEE,* Florin Ciucu, *Member, IEEE*
Chuang Lin, *Senior Member, IEEE,* Steven H. Low, *Fellow, IEEE*

***Abstract*—Renewable energy such as solar and wind generation will constitute an important part of the future grid. As the availability of renewable sources may not match the load, energy storage is essential for grid stability. In this paper we investigate the feasibility of integrating solar photovoltaic (PV) panels and wind turbines into the grid by also accounting for energy storage. To deal with the fluctuation in both the power supply and demand, we extend and apply stochastic network calculus to analyze the power supply reliability with various renewable energy configurations. To illustrate the validity of the model, we conduct a case study for the integration of renewable energy sources into the power system of an island off the coast of Southern California. In particular, we asses the power supply reliability in terms of the average Fraction of Time that energy is Not-Served (FTNS).**

***Index Terms*—Power Grid, Renewable Energy Sources, Communication Networks, Stochastic Network Calculus**

I. Introduction

The need to reduce greenhouse gas emissions is driving the deployment of more environmentally friendly and sustainable energy sources, such as solar and wind. The next-generation grid will feature renewable energy sources to reduce the carbon footprint. A challenge, however, of solar and wind generation is their intermittency and randomness, rendering it hard to match supply and demand, which is itself variable. One way to help match uncertain supply and demand is to effectively utilize energy storage, such as batteries. It has been recently reported that the Los Angeles Department of Water and Power (LADWP) has formed a partnership with BYD Ltd. Corp. on a grid-scale battery project for renewable energy storage, which will lead to the development of a power storage unit up to 10 MWh [2]. In this paper we consider the deployment of such a large energy storage unit into a grid powered by an arbitrary number of PV panels and wind turbines, and we address the storage dimensioning problem subject to the constraint of continuously satisfying the demand for energy.

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Kai Wang is with the Dept. of Computer Science and Technology, Tsinghua University, Beijing, 100084 China. This work was done when he was a visiting student in the Dept. of Computing and Mathematical Sciences, California Institute of Technology, Pasadena, CA, 91125 USA, e-mail: kaiphd@caltech.edu.

Florin Ciucu is with Telekom Innovation Laboratories / TU Berlin, Ernst-Reuter-Platz 7, 10587 Berlin, Germany, email: florin@net.t-labs.tu-berlin.de.

Chuang Lin is with the Dept. of Computer Science and Technology, Tsinghua University, Beijing, 100084 China, e-mail: chlin@tsinghua.edu.cn.

Steven H. Low is with the Dept. of Computing and Mathematical Sciences, California Institute of Technology, Pasadena, CA, 91125 USA, e-mail: slow@caltech.edu.

A closely related system which faces a similar problem is the Internet. As a concrete example, the apparently excessive overprovisioning of buffer memory in Internet routers has led to an intense debate in the research community over the past years [3], [4], [5]. The Internet resource dimensioning problem—concerning especially bandwidth dimensioning subject to certain Quality of Service guarantees—has been traditionally formulated in the framework of queueing theory [6], and more recently in the framework of network calculus [7], [8]. We extend and apply network calculus to the problem of dimensioning of energy storage subject to certain constraints on the power supply reliability.

Network calculus uses bounds to characterize arrivals and service in a queueing system, and also to derive queueing performance measures. This bounding approach has the advantage that a very broad class of arrival processes (including deterministically regulated, Markov modulated, and even heavy-tailed and self-similar) can be analyzed. Network calculus has been mostly applied in the context of computer and communication networks, but also in other systems such as automotive [9] or avionic networks [10]. The key idea in network calculus is to transform a complex non-linear queueing system into an analytically tractable system, in a suitable $(\min, +)$ algebra. The development of network calculus has followed two interrelated directions: deterministic and stochastic. The deterministic network calculus allows a broad scope of queueing scenarios and enables the derivation of tight bounds (see [8], p. 27). A concern, however, is that these bounds can be conservative in highly multiplexed regimes or when a small probability of violation is tolerable. This motivates the stochastic extensions of the calculus (e.g., [11], [12], [13], [14], [15]), which can reap the benefit of statistical multiplexing gain, and consequently can yield efficient solutions for resource dimensioning problems.

The ability of the stochastic network calculus to model broad classes of queueing scenarios and capture statistical multiplexing gain, motivates our extension of a *stochastic power network calculus* for the power grid. In this context, arrivals can be regarded as the energy generated by the power sources, whereas statistical multiplexing manifests itself through the aggregation of many energy sources (e.g., turbines or PV panels). Such an extension of the calculus, however, is not straightforward. The reason lies in a slight conceptual difference between a conventional queueing system (e.g., an Internet router), which is described in terms of arrivals and service processes, and the power system, which is described in terms of energy supply and demand processes. More concretely, the difference lies in the concept of an energy *demand process* which is uncharacteristic to conventional queueing systems. Moreover, the power system has a very specific performance metric of critical interest, i.e., the Fraction of Time that energy is Not-Served (FTNS) accounting for the time periods during which energy demand exceeds energy supply plus storage.

To model the power system, our idea is to regard it as a queueing system where 1) arrivals are described by the energy supply process, 2) service is described by the energy demand process, and 3) the buffer is the storage capacity. To model this queueing system, we define energy supply and demand stochastic curves to model the generated energy and desired demand, respectively, in terms of probabilistic bounds. The supply curve corresponds directly to the notion of a statistical envelope, which is used in stochastic network calculus to model queueing arrivals. In turn, the demand curve has no direct correspondent in the conventional network calculus, due to the conceptual issue mentioned above. Moreover, from a technical point of view, the demand and supply curves are defined in an entirely decoupled manner. This is unlike the conventional coupling of arrival and service processes in network calculus, and creates the main technical challenge to be addressed in our power network calculus extension.

To summarize, the contribution of this paper is threefold:

- We build a stochastic power network calculus theoretical framework for the performance evaluation of power networks with renewable energy generations and storage. Our model extends the concepts of the conventional stochastic network calculus by introducing new models and properly adjusting the analytical techniques for queueing analysis.
- We derive explicit formulas of the performance metrics for the power system reliability analysis and dimensioning. In particular, the two main metrics which we consider are 1) the average Fraction of Time that energy is Not-Served (FTNS) and 2) the Waste of Power Supply (WPS) due to improper energy storage dimensioning. The obtained formulas provide fundamental guidance for the configuration of the power system with renewable energy sources and energy storage, in order to meet certain constraints like negligible FTNS and WPS.
- To illustrate the validity of our stochastic power network calculus, especially when aggregating multiple renewable energy sources, we conduct a case study for the integration of renewable-energy sources into the power system of an island off the coast of Southern California.

The rest of the paper is organized as follows. In Section II we present a description of the integrated power system, introduce notation, and provide a brief introduction into the (stochastic) network calculus. In Section III and IV we first introduce the models for the stochastic power network calculus, and then we derive formulas on the performance metrics of interest. In Section V we provide an aggregation result needed for analyzing power systems with multiple power supply sources and capturing the underlying multiplexing gain. In Section VI we conduct a real case study for the integration of renewable energy sources into the power system. Finally, some related work is discussed in Section VII, and brief conclusions are presented in Section VIII.

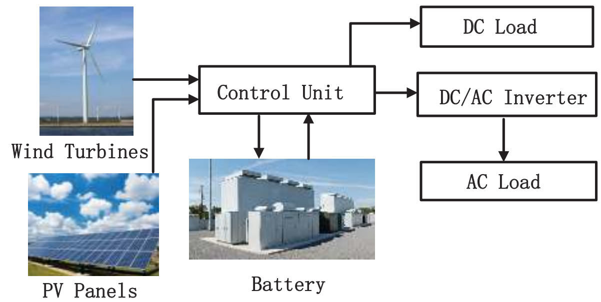


Fig. 1. Schematic of the hybrid power system

II. Formulation and Notations

A. Problem Description

Figure 1 illustrates a hybrid power system consisting of solar PV panels, wind turbines, battery storage, controller units, etc. The PV panels and wind turbines work together to satisfy the load demand. When the energy sources are abundant, the excess power generation will feed the battery until it is fully charged. Whenever there is a deficiency in power, the battery will be discharged to cover the load requirements until the energy storage is depleted.

Due to fluctuations in both power generation and demand, our goal is to investigate the effects of energy storage on the power supply reliability in configurations with different levels of renewable generation. The *reliability* of the power supply is assessed in terms of three performance metrics:

1) the Loss of Power Supply Probability (LPSP), at a given time, which quantifies the probability that an instantaneous demand cannot be met due to either a very high energy demand and/or a low level of energy supply plus storage.
2) the average Fraction of Time that energy is Not-Served (FTNS), which follows directly by averaging out the LPSP over some time scale.
3) the Waste of Power Supply (WPS), at a given time, which quantifies the amount of instantaneous wasted energy when the stored energy plus the supply exceed the energy demand.

These reliability metrics are derived as functions of the number N_p of PV panels, the number N_w of wind turbines, and the specified capacity C of battery storage; other factors, e.g., the AC/DC inverter, are ignored. The merit of these metrics, obtained explicitly, is that they can assist the decision making process for investing in the deployment of renewable energy sources and energy storage.

Our approach to the dimensioning of the hybrid solar-wind system from Figure 1, in terms of the battery storage needed to guarantee negligible FTNS and WPS, is to formulate a stochastic power network calculus, based on similar concepts from the stochastic network calculus. We do so by first modelling the individual components of the power system and then analyzing its reliability in terms of the three metrics listed above. A key feature of the calculus is that it accounts for the stochastic nature of the hybrid solar-wind system, and yet it lends itself to explicit formulas on the performance metrics of interest, e.g., FTNS and WPS.

B. Notations

We denote by $\mathcal{F}$ the set of non-negative, non-decreasing functions, i.e.,

$$\mathcal{F} = \{f(\cdot) : \forall\ 0 \leq x_1 \leq x_2,\ 0 \leq f(x_1) \leq f(x_2)\}\ ,$$

and by $\mathcal{F}^c$ the set of non-negative, non-increasing functions, i.e.,

$$\mathcal{F}^c = \{f(\cdot) : \forall\ 0 \leq x_1 \leq x_2,\ 0 \leq f(x_2) \leq f(x_1)\}\ .$$

For a random variable X, its cumulative distribution function (CDF) and cumulative complementary distribution functions (CCDF) are denoted by

$$F_X(x) = \mathbb{P}\{X \leq x\} \text{ and } F^c_X(x) = \mathbb{P}\{X > x\}\ ,$$

respectively; the former belongs to $\mathcal{F}$ and the latter belongs to $\mathcal{F}^c$.

For two numbers x and y we use the notations

$$[x]^+ = \max\{x, 0\} \text{ and } [x, y]^+ = \max\{x, y, 0\}\ .$$

For two function $f(x)$ and $g(x)$, the Stieltjes convolution is

$$f * g(x) = \int_0^x f(y) dg(x - y)\ .$$

For the same functions, their $(\min, +)$ convolution, denoted by '$\otimes$', is defined as follows:

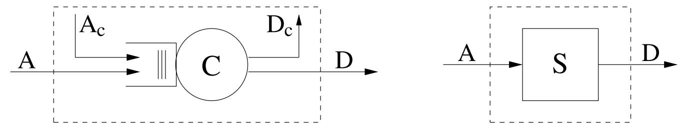
$$(f \otimes g)(x) = \inf_{0 \leq y \leq x} [f(y) + g(x - y)].$$

This convolution is characteristic to network calculus theory. We also remark that, although we adopt a discrete time model, we prefer $\inf$ and $\sup$ instead of $\min$ and $\max$ operators.

C. Network Calculus

We now provide a brief introduction into some relevant concepts and ideas from the conventional (stochastic) network calculus, by making an analogy with linear systems theory (see also [16], [17], [8], [18]).

Network calculus is an alternative to the traditional queueing theory. It was conceived by Cruz [7] in the early 1990s in a deterministic framework, and soon after, independently by Chang [19], Kurose [20], and Yaron and Sidi [21] in a probabilistic framework. Subsequently, many others have contributed to both interrelated directions of the network calculus (see [17], [8], [14] and references therein). While the development of the deterministic calculus was motivated by the need for a theory for worst-case performance guarantees (e.g., packet_delay $<$ 200 ms), the raison d'être for the stochastic network calculus was to additionally capture



(a) non−linear system (b) transformed tractable system

Fig. 2. A queueing system from the perspective of a flow A. In (a), the system is not linear; in (b), the transformed system is analytically tractable.

statistical multiplexing gains when some small violation probabilities of the performance guarantees are tolerable (e.g., $\mathbb{P}$(packet_delay > 200 ms) $\leq 10^{-3}$).

An example of a queueing scenario addressed with the network calculus is depicted in Figure 2.(a). A server with constant rate C serves two arrival flows, or aggregate of flows, A and A_c. Whenever there are more arrivals than serving capacity, the excess is temporarily stored in a shared queue. From the perspective of flow A, the queueing system transforms A's arrival process (i.e., the *input signal*, in systems theoretic terms) into a departure process (i.e., the *output signal*). Besides the characteristics of A's arrival process, this transformation depends on several external factors (i.e., the *noise*), e.g., the characteristics of the other (aggregate) flow A_c, the type of scheduling algorithm, and the queue size. Because of the complexity jointly induced by these factors, A's queueing system is generally *not linear* in the sense that the existence of an algebra such that

$$T\left(c_1 A_1 + c_2 A_2\right) = c_1 T\left(A_1\right) + c_2 T\left(A_2\right)$$

is questionable. Here, c_1 and c_2 are scalars, A_1 and A_2 are input signals, $T : \mathcal{F} \rightarrow \mathcal{F}$ transforms an input signal into an output signal, whereas the addition and multiplication operators are relative to some (unknown) algebra, yet to be discovered. Due to the lack of linearity, the analytical tractability of A's queueing system is conceivably hard.

To circumvent the lack of linearity problem, the key idea of network calculus is to transform A's original non-linear queueing system into an analytically tractable system, as depicted in Figure 2.(b). Here, S can be regarded as the *impulse-response* (in systems theoretic terms), which characterizes A's queueing system in that

$$D(t) \geq A \otimes S(t)\ , \tag{1}$$

for *all* arrival processes $A(t) \in \mathcal{F}$, where $D(t)$ is the corresponding departure process. Note the underlying $(\min, +)$ algebra over which the convolution is operated. Note also the similarity between 1) describing the system from Figure 2.(b) with a $(\min, +)$ convolution, and 2) describing conventional linear systems in terms of a (conventional) convolution (see, e.g., [22]). This similarity drives the analytical tractability of broad queueing systems with network calculus.

In network calculus terms, S is a *service* (bi-variate) *process* $S(s, t)$, and the convolution from Eq. (1) expands as

$$D(t) \geq \inf_{0 \leq s \leq t} \left[A(s) + S(s, t)\right]\ \forall t \geq 0\ .$$

In other words, S characterizes A's queueing system through a lower bound on A's received service. Because the lower bound holds for all arrival processes $A(t)$, the service process S *almost* entirely characterizes the queueing system (the characterization is not complete due to the formulation of Eq. (1) with an *inequality*, and not with an equality). We also note that, for the transformed system from Figure 2.(b), S may depend on the service capacity C, the arrivals A_c, the scheduling at the server, and possibly the queue size as well. For a survey of service processes see [23].

In addition to the concept of a service process, network calculus uses the concept of an *envelope* to characterize an arrival process $A(t)$. A version of a *stochastic sample-path envelope* (e.g., see Definition 3.11 in [14]) can be defined by a function or curve $\alpha(t) \in \mathcal{F}$, and a bounding function $\varepsilon(x) \in \mathcal{F}^c$, such that for all $t, x \geq 0$

$$\mathbb{P}\left\{\sup_{0\leq s\leq t}\left[A(s,t)-\alpha(t-s)\right]>x\right\}\leq\varepsilon(x)\ . \tag{2}$$

Once a queueing system, from the perspective of a flow A, is described with a service process $S(s,t)$ and an envelope $\alpha(t)$ with some bounding function $\varepsilon(x)$, then A's queueing performance measures of interest can be derived. Consider for instance the virtual delay process $W(t) = \inf\left[d : A(t-d) \leq D(t)\right]$ describing the delay of the last departing data unit (if any) at time t. If $S(s,t) = C(t-s)$, i.e., modelling a queueing scenario with constant-rate service given to A, then a probabilistic bound on A's delay process is for all $x \geq 0$

$$\mathbb{P}\left\{W(t) > h(\alpha+x,\beta)\right\} \leq \varepsilon(x)\ , \tag{3}$$

where $\beta(t) = Ct$ and $h(\alpha + x, \beta)$ is the maximum horizontal distance between the functions $\alpha(t) + x$ and $\beta(t)$ for $t \geq 0$ (see Theorem 5.4 in [14]).

III. Power System Modelling

In this section we introduce the stochastic power network calculus, in particular the energy demand, energy supply, and storage models for the power system from Figure 1.

The time model is discrete with 1 hour increments. Consider a time interval $[0, t]$ with $t \leq T$, where T is the maximum considered time. With abuse of notation, the process $D(t) \in \mathcal{F}$ denotes the cumulative amount of energy demand in the system (in MWh). Also, the process $S(t) \in \mathcal{F}$ denotes the cumulative amount of energy supply in the system. $D(t)$ is called the energy demand process, and $S(t)$ is called the energy supply process of the system, with initial conditions $D(0) = S(0) = 0$. The bivariate processes' extensions are $D(s,t) = D(t)-D(s)$ and $S(s,t) = S(t)-S(s)\ \forall\ 0 \leq s \leq t$. Before introducing stochastic models for these two processes, we describe the evolution of the power system in terms of the energy storage process.

A. Energy Storage

The energy storage, or battery load, is modelled by a discrete time process $b(t)$, with maximum capacity C, and which is defined recursively as follows: If the energy generated from the PV/wind system is greater than the load for a particular hour, then the surplus energy is stored in the battery and the battery is charged:

$$b(t) = \min[C, b(t-1) + [S(t-1,t) - D(t-1,t)]\eta_c]\ , \tag{4}$$

where η_c denotes the charge efficiency of the battery. When the battery reaches its maximum value C, any excess energy generated cannot be charged and is wasted.

In turn, if the energy demand is greater than the supply for a particular hour, then the battery is discharged in order to supplement the supply. In this case the recursion becomes:

$$b(t) = [b(t-1) - [D(t-1,t) - S(t-1,t)]\eta_d]^+\ , \tag{5}$$

where η_d denotes the discharge efficiency of the battery. Due to physical constraints, the minimal quantity level of battery is determined by the maximum depth of discharge. If the battery decreases to its minimum value C_{min}, then the deficient energy demand cannot be meted out from the battery system, and we refer to this event as Loss of Power Supply (LPS).

We make the initial condition $b(0) = 0$ (zero initial buffer storage), and assume for brevity $\eta_c = \eta_d = 1$ and $C_{min} = 0$. To further simplify notation, we introduce the process $C(t)$, representing the actual storage capacity for any time $t \geq 0$, by

$$C(t) = \begin{cases} C & ,\quad t > 0 \\ 0 & ,\quad t = 0 \end{cases}\ .$$

Note that, by convention, there is no storage capacity at time zero. When clear from the context, we write C for $C(t)$.

B. Energy Demand and Supply

The power queueing system described in Eqs. (4) and (5) is conceptually different from conventional queueing systems. Concretely, the energy demand process, which can be regarded using standard queueing terms as a (desired) departure process, is given as input to the power queueing system. Moreover, it is decoupled from the energy supply (i.e., arrival) process, which means that the departure process is not a function of the arrival process. In turn, in conventional queueing systems (e.g., the one from Figure 2) the two are coupled through an additional service process (e.g., see the $(\min, +)$ convolution from Eq. (1)). This conceptual difference of uncoupled vs. coupled departure and arrival processes, in power and conventional queueing systems, motivates the extension of conventional stochastic network calculus models and techniques.

To this end, we model the energy demand (i.e., the desired departure) process $D(s,t)$ using a standard network calculus model for *arrival* processes with probabilistic sample-path upper and lower bounds. In other words, we treat the energy demand process as arrivals to the queueing system.

Definition 1: (Energy Demand) An energy demand process $D(s,t)$ is said to have a stochastic upper demand curve $\alpha^u(t) \in \mathcal{F}$ with bounding function $\varepsilon_d^u(x) \in \mathcal{F}^c$, denoted by $D \sim \langle \varepsilon_d^u, \alpha^u \rangle$, if for all $t, x \geq 0$

$$\mathbb{P}\{\sup_{0\leq s\leq t}\left[D(s,t) - \alpha^u(t-s)\right] > x\} \leq \varepsilon_d^u(x)\ , \tag{6}$$

and it is said to have a stochastic lower demand curve $\alpha^l(t) \in \mathcal{F}$ with bounding function $\varepsilon_d^l(x) \in \mathcal{F}^c$, denoted by $D \sim \langle \varepsilon_d^l, \alpha^l \rangle$, if for all $t, x \geq 0$

$$\mathbb{P}\{\sup_{0\leq s\leq t}\left[\alpha^l(t-s) - D(s,t)\right] > x\} \leq \varepsilon_d^l(x)\ . \tag{7}$$

Additionally, we use the same arrival model for the energy supply process $S(s,t)$, i.e., the other input/arrival process to the power queueing system.

Definition 2: (ENERGY SUPPLY) An energy supply process $S(s,t)$ is said to provide a stochastic upper supply curve $\beta^u(t) \in \mathcal{F}$ with bounding function $\varepsilon_s^u(x) \in \mathcal{F}^c$, denoted by $S \sim \langle \varepsilon_s^u, \beta^l \rangle$, if for all $t, x \geq 0$

$$\mathbb{P}\{\sup_{0 \leq s \leq t} [S(s,t) - \beta^u(t-s)] > x\} \leq \varepsilon_s^u(x) \ , \qquad (8)$$

and it is said to have a stochastic lower supply curve $\beta^l(t) \in \mathcal{F}$ with bounding function $\varepsilon_s^l(x) \in \mathcal{F}^c$, denoted by $S \sim \langle \varepsilon_s^l, \beta^l \rangle$, if for all $t, x \geq 0$

$$\mathbb{P}\{\sup_{0 \leq s \leq t} [\beta^l(t-s) - S(s,t)] > x\} \leq \varepsilon_s^l(x) \ . \qquad (9)$$

We remark that the energy demand and supply processes, especially the upper curves, are modelled similarly to how arrival processes are modelled in conventional stochastic network calculus (see Eq. (2)). The technical consideration of entire *sample-path bounds* in all four bounds from Eqs. (6)-(9) is motivated by the simplicity of derived queueing performance metrics (see for instance the delay bound from Eq. (3)). Furthermore, the need for both upper and lower bounds for both energy demand and supply processes is motivated by the type of performance metrics of interest for the power system. In particular, the upper bound from Eq. (6) together with the lower bound from Eq. (9) are sufficient to analyze the loss of power supply probability (LPSP). In turn, the lower bound from Eq. (7) together with the upper bound model from Eq. (8) are sufficient to analyze the waste of power supply (WPS) process.

C. On Model Tightness

In practice, the shapes of the demand/supply curves and the corresponding bounding functions from Eqs. (6)-(9) should adequately capture a broad range of fluctuations in the power system. For instance, as two extreme cases, $\beta^u(t)$ and $\varepsilon_s^u(x)$ should capture maximum power generation situations (e.g., sunny all the time in the case of solar), whereas $\beta^l(t)$ and $\varepsilon_s^l(x)$ should capture a minimal level of generated energy.

The tightness of the four modelling bounds from Eqs. (6)-(8) depends on the trade-off between the shapes of the demand/supply curves and the corresponding bounding functions. For instance, when fitting or tuning $\alpha^u(t)$ and $\varepsilon_d^u(x)$, an increase in one implies a decrease in the other. This trade-off is further complicated by the need to *jointly* account for high fluctuations in both energy demand and supply, in order to produce tight bounds in the queueing analysis. To illustrate this constraint, note that the selection of very tight modelling bounds (e.g., very small $\varepsilon_d^u(x)$ in Eq. (6), which implies very large demand curve $\alpha^u(t)$, as mentioned earlier) can lead to meaningless performance measures since the power system would be incorrectly viewed as mostly in underflow. At the other extreme, i.e., smaller demand curves at the expense of bigger bounding functions, can lead to very loose performance bounds, e.g., on the loss of power supply probability. In practice, the demand/supply curves can be constructed to slightly deviate from the average rates of their demand/supply processes, and the corresponding bounding functions can be properly tuned.

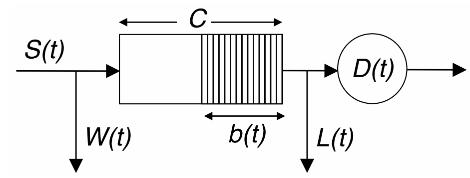


Fig. 3. A visualization of the power queueing system. $S(t)$ and $D(t)$ denote the cumulative energy supply and demand processes, respectively; $b(t)$ denotes the instantaneous buffer storage with maximum capacity C; $W(t)$ and $L(t)$ denote the instantaneous waste of power supply and loss of power supply processes.

IV. PERFORMANCE METRICS IN THE POWER QUEUEING SYSTEM

In this section we first recall a non-recursive identity for the energy storage process $b(t)$, which was defined recursively in Section III-A. The non-recursive identity will enable the analysis of the three processes of interest for the power queueing system reliability analysis: 1) the loss of power supply (LPS), 2) the Fraction of Time that energy is Not-Served (FTNS), and 3) the Waste of Power Supply (WPS).

Based on Eqs. (4) and (5), the Energy Storage Process $b(t)$ can be concisely defined as follows:

$$b(t) = \min[C, [b(t-1) + S(t-1,t) - D(t-1,t)]^+]$$

By fitting this recurrence with Eq. (3) from [24], and accounting for the initial conditions $b(0) = 0$ and $C(0) = 0$, the following non-recursive identity for $b(t)$ holds (see Theorem 1 from [24])

$$b(t) = \inf_{0 \leq s \leq t} [\sup_{s \leq u \leq t} [S(u,t) - D(u,t), S(s,t) - D(s,t) + C(s)]] \ . \qquad (10)$$

With this explicit expression we can next conduct the reliability analysis of the power system.

A. Loss of Power Supply (LPS)

Here we first derive a non-recursive formula for the loss of power supply process denoted by $L(t)$ (for visualization see Figure 3). Using this formula can then derive performance bounds on the loss of power supply probability and also on the average Fraction of Time energy Not-Served (FTNS) performance metric.

Recall that the instantaneous LPS process characterizes the deficient energy demand which cannot be meted out from the battery at some time t. According to the evolution of the power queueing system described in Section III-A, $L(t)$ is defined for all $t \geq 1$ as

$$L(t) = [D(t-1,t) - S(t-1,t) - b(t-1)]^+ \ .$$

Note that $L(t)$ would correspond to the amount of unused service capacity in a (conventional) queueing system, by regarding the demand process as a service process.

Using the explicit expression of $b(t)$ from Eq. (10), an explicit expression for $L(t)$ follows immediately:

Corollary 1: (Loss of Power Supply) The LPS process $L(t)$ satisfies for all $t \geq 1$

$$L(t) = \sup_{0 \leq s \leq t-1} [\inf_{s \leq u \leq t-1} [D(u,t) - S(u,t), \\ D(s,t) - S(s,t) - C(s)]^+] \ . \quad (11)$$

This explicit expression enables further the derivation of the instantaneous LPS probability:

Theorem 1: (LPS Probability) Given the power queueing system, assume that the energy demand process has a stochastic upper demand curve α^u with bounding function ε_d^u, i.e., $D \sim \langle \varepsilon_d^u, \alpha^u \rangle$, and the energy supply process has a stochastic lower supply curve β^l with bounding function ε_s^l, i.e., $S \sim \langle \varepsilon_s^l, \beta^l \rangle$. Then the loss of power supply probability satisfies for all $t \geq 1$

$$\mathbb{P}\{L(t) > 0\} \leq \varepsilon_d^u \otimes \varepsilon_s^l \left(C - \sup_{0 \leq s \leq t} [\alpha^u(s) - \beta^l(s)] \right) \ .$$

The theorem is quite general in the sense that it does not require a statistical independence assumption between the energy demand and supply processes. Therefore, the theorem accounts for the situation when the demand and supply processes are correlated, e.g., high energy demand implies high energy supply. The proof is based on Lemma 1 from the Appendix, which bounds the distribution of a sum of non-necessarily independent random variables.

Proof: Fix $t \geq 1$. From Eq. (11), we have

$$\mathbb{P}\{L(t) > 0\} = \mathbb{P}\{ \sup_{0 \leq s \leq t-1} [\inf_{s \leq u \leq t-1} [D(u,t) - S(u,t), \\ D(s,t) - S(s,t) + C(s)]] > 0\} \ .$$

The event from the right-hand side can be bounded as follows:

$$\begin{aligned} &\sup_{0 \leq s \leq t-1} [\inf_{s \leq u \leq t-1} [D(u,t) - S(u,t), D(s,t) - S(s,t) - C(s)]] \\ &\leq \sup_{0 \leq s \leq t} [D(s,t) - S(s,t) - C(s)] \\ &\leq \sup_{0 \leq s \leq t} [D(s,t) - S(s,t) - C] \\ &= \sup_{0 \leq s \leq t} [D(s,t) - \alpha^u(t-s) + \alpha^u(t-s) + \beta^l(t-s) \\ &\quad - \beta^l(t-s) - S(s,t)] - C \\ &\leq \sup_{0 \leq s \leq t} [D(s,t) - \alpha^u(t-s)] + \sup_{0 \leq s \leq t} [\beta^l(t-s) - S(s,t)] \\ &\quad + \sup_{0 \leq s \leq t} [\alpha^u(s) - \beta^l(s)] - C \ . \end{aligned}$$

By accounting for the assumptions in the theorem that $\mathbb{P}\{ \sup_{0 \leq s \leq t} [D(s,t) - \alpha^u(t-s)] > x\} \leq \varepsilon_d^u(x)$ and $\mathbb{P}\{ \sup_{0 \leq s \leq t} [\beta^l(t-s) - S(s,t)] > x\} \leq \varepsilon_s^l(x)$, the rest of the proof follows from Lemma 1 by regarding the two supremums as non-necessarily independent random variables. ∎

As we have previously mentioned, the result of Theorem 1 is quite general in that it holds without a statistical independence assumption between the energy supply and demand processes. Under such an additional assumption, the bound from Theorem 1 can be tightened as follows.

Theorem 2: (LPS Probability - Statistical Independence Case) With the same conditions as in Theorem 1, along with a statistical independence assumption between the energy demand and supply processes, the loss of power supply probability satisfies for all $t \geq 1$

$$\mathbb{P}\{L(t) > 0\} \leq \varepsilon_d^u(z) + \varepsilon_d^u * \varepsilon_s^l(z) \ ,$$

where $z = C - \sup_{0 \leq s \leq t} [\alpha^u(s) - \beta^l(s)]$.

The proof is similar to the proof of Theorem 1, except that right at the end one needs to apply Lemma 2 (see the Appendix), instead of Lemma 1, to bound the distribution of a sum of independent random variables (note that, according to the assumptions from the Theorem 2, the supremums $\sup_{0 \leq s \leq t} [D(s,t) - \alpha^u(t-s)]$ and $\sup_{0 \leq s \leq t} [\beta^l(t-s) - S(s,t)]$ are independent).

The second considered reliability metric, closely related to the loss of power supply probability, is the average Fraction of Time energy Not-Served. This metric, denoted by $FTNS(T)$, is defined over the entire system time period $[0,T]$:

$$FTNS(T) := \frac{1}{T} \sum_{t=1}^{T} \mathbb{P}\{L(t) > 0\} \ . \quad (12)$$

The FTNS metric will be used in our real case study from Section VI. Depending on the statistical independence between energy supply and demand, the loss of power supply probabilities from Eq. (12) can be bounded by either Theorem 1 or Theorem 2.

B. Waste of Power Supply (WPS)

The instantaneous WPS process, denoted here by $W(t)$, characterizes the amount of wasted energy at time t due to insufficient energy storage and/or demand (for visualization see Figure 3). Following the structure of the previous subsection, we first derive an explicit expression for $W(t)$ and then compute a bound on its CCDF.

To define $W(t)$ at some time t, assume that there is $b(t-1)$ remaining energy in the storage at the end of time slot $t-1$. Adding $S(t-1,t)$ supplied energy and subtracting $D(t-1,t)$ consumed energy in time slot t, it follows that there is no more than $[b(t-1) + S(t-1,t) - D(t-1,t)]^+$ remaining energy in the system at the end of time slot t. If $[b(t-1) + S(t-1,t) - D(t-1,t)]^+ > C$, then some arrivals to the power queueing system have to be dropped (i.e., energy is wasted). Formally, $W(t)$ is defined as

$$W(t) = [b(t-1) + S(t-1,t) - D(t-1,t) - C]^+ \ .$$

Note that $W(t)$ would correspond to the amount of *buffer overflow* in a (conventional) queueing system.

Recalling the explicit expression of $b(t)$ from Eq. (10), an explicit expression on $W(t)$ follows immediately:

Corollary 2: (WPS Process) For all $t \geq 1$ it holds

$$W(t) = \inf_{0 \leq s \leq t-1} [\sup_{s \leq u \leq t-1} [S(u,t) - D(u,t) - C, \\ S(s,t) - D(s,t) + C(s) - C]^+] \ . \quad (13)$$

This result corresponds directly to the (conventional) queueing result from Theorem 2 in [24].

This expression lends itself to the following upper bound on the CCDF of $W(t)$.

Theorem 3: (WPS CCDF) Given the power queueing system, assume that the energy supply process has a stochastic upper supply curve β^u with bounding function ε_s^u, i.e., $S \sim \langle \varepsilon_s^u, \beta^u \rangle$, and the energy demand process has a stochastic lower demand curve α^l with bounding function ε_d^l, i.e., $D \sim \langle \varepsilon_d^l, \alpha^l \rangle$. Then the waste of power supply probability satisfies for all $t \geq 1$ and $x \geq 0$

$$\mathbb{P}\{W(t) > x\} \leq \varepsilon_s^u \otimes \varepsilon_d^l \left(C - \sup_{0 \leq s \leq t} [\beta^u(s) - \alpha^l(s)] + x \right).$$

The proof follows the same line of argument as the proof of Theorem 1; for completeness we give it next.

Proof: For the right-hand side of Eq. (13), we have:

$$\begin{aligned}
&\inf_{0 \leq s \leq t-1} [\sup_{s \leq u \leq t-1} [S(u,t) - D(u,t) - C, \\
&\quad S(s,t) - D(s,t) + C(s) - C]] \\
&\leq \sup_{0 \leq u \leq t} [S(u,t) - D(u,t) - C, S(0,t) - D(0,t) - C] \\
&= \sup_{0 \leq s \leq t} [S(s,t) - D(s,t) - C] \\
&= \sup_{0 \leq s \leq t} [S(s,t) - \beta^u(t-s) + \beta^u(t-s) + \alpha^l(t-s) \\
&\quad - \alpha^l(t-s) - D(s,t)] - C \\
&\leq \sup_{0 \leq s \leq t} [S(s,t) - \beta^u(t-s)] + \sup_{0 \leq s \leq t} [\alpha^l(t-s) - D(s,t)] \\
&\quad + \sup_{0 \leq s \leq t} [\beta^u(s) - \alpha^l(s)] - C \ .
\end{aligned}$$

The right-hand side in the last line indicates a sufficient condition to derive $\mathbb{P}\{W(t) > x\}$. By accounting for the assumptions that $\mathbb{P}\{\sup_{0 \leq s \leq t} [S(s,t) - \beta^u(t-s)] > x\} \leq \varepsilon_s^u(x)$ and $\mathbb{P}\{\sup_{0 \leq s \leq t} [\alpha^l(t-s) - D(s,t)] > x\} \leq \varepsilon_d^l(x)$, the rest of the proof follows from Lemma 1. ■

This theorem, alike Theorem 1, is quite general in that it does not require the statistical independence between the demand and supply processes. Under such an additional assumption, the upper bound can be further strengthened as follows:

Theorem 4: (WPS CCDF - Statistical Independence Case) With the same conditions as in Theorem 3, along with an additional statistical independence assumption between the energy demand and supply processes, the waste of power supply probability satisfies for all $t \geq 1$

$$\mathbb{P}\{W(t) > x\} \leq \varepsilon_s^u(z) + \varepsilon_s^u * \varepsilon_d^l(z) \ ,$$

where $z = C - \sup_{0 \leq s \leq t} [\beta^u(s) - \alpha^l(s)] + x$.

The proof follows the same steps as the proof of Theorem 3, except for the last step application of Lemma 2 instead of Lemma 1.

V. Aggregating Different Energy Sources

In this section we provide two results concerning the aggregation of heterogeneous power supply sources. These aggregation results transform multiple supply curves into a single one, which can be then immediately used in Theorems 1-4. First we give a general result holding irrespectively of the statistical independence amongst the sources.

Proposition 1: (Energy Aggregation Property) Consider a power system that consists N power generators in parallel. If each power generator $(n = 1, 2, ..., N)$ provides a stochastic lower energy supply curve $S_n \sim \langle \varepsilon_n^l, \beta_n^l \rangle$, then the power system provides a stochastic lower supply curve $S \sim \langle \varepsilon^l, \beta^l \rangle$ with

$$\beta^l(t) = \beta_1^l(t) + \beta_2^l(t) + ... + \beta_N^l(t),$$
$$\varepsilon^l(x) = \varepsilon_1^l \otimes \varepsilon_2^l \otimes ... \otimes \varepsilon_N^l(x).$$

A similar result for a stochastic upper arrival curve appeared in [14] (p. 108), using conventional network calculus terms. The proof of our result follows the same line of argument as in [14].

Proof: Here we only consider the case with 2 energy generators, from which the proof can be easily extended to the general case with N generators. As $S(t)$ is the aggregation of 2 power supplies, we have that $S(s,t) = S_1(s,t) + S_2(s,t)$ for all $0 \leq s \leq t$. We can now write

$$\begin{aligned}
&\sup_{0 \leq s \leq t} \left[\beta^l(t-s) - S(s,t)\right] \\
&= \sup_{0 \leq s \leq t} \left[(\beta_1^l(t-s) + \beta_2^l(t-s)) - (S_1(s,t) + S_2(s,t))\right] \\
&= \sup_{0 \leq s \leq t} \left[\left(\beta_1^l(t-s) - S_1(s,t)\right) + \left(\beta_2^l(t-s) - S_2(s,t)\right)\right] \\
&\leq \sup_{0 \leq s \leq t} \left[\beta_1^l(t-s) - S_1(s,t)\right] + \sup_{0 \leq s \leq t} \left[\beta_2^l(t-s) - S_2(s,t)\right] \ .
\end{aligned}$$

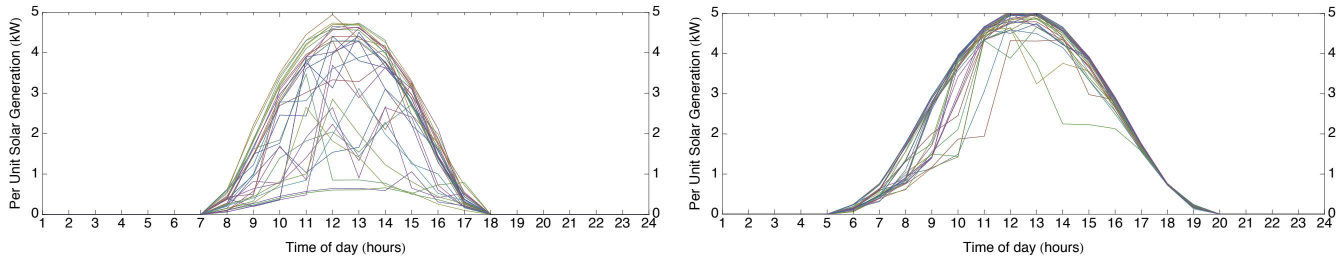
With the above assumptions, we have $\mathbb{P}\{\sup_{0 \leq s \leq t} [\beta_1^l(t-s) - S_1(s,t)]\} \leq \varepsilon_1^l(x)$, and $\mathbb{P}\{\sup_{0 \leq s \leq t} [\beta_2^l(t-s) - S_2(s,t)]\} \leq \varepsilon_2^l(x)$. Applying Lemma 1 from the Appendix concludes the proof. ■

If the individual energy supply processes are statistically independent, then a tighter bounding function can be obtained as shown in the next result.

Proposition 2: (Energy Aggregation Property - Statistical Independence Case) Under the same conditions as in Proposition 1, and assuming additionally that the energy supply process S_n $(n = 1, 2, ..., N)$ are statistically independent, then the power system provides a stochastic lower supply curve $S \sim \langle \varepsilon^l, \beta^l \rangle$ with the same $\beta^l(t) = \beta_1^l(t) + \beta_2^l(t) + ... + \beta_N^l(t)$ as in Proposition 1, but a tighter bounding function

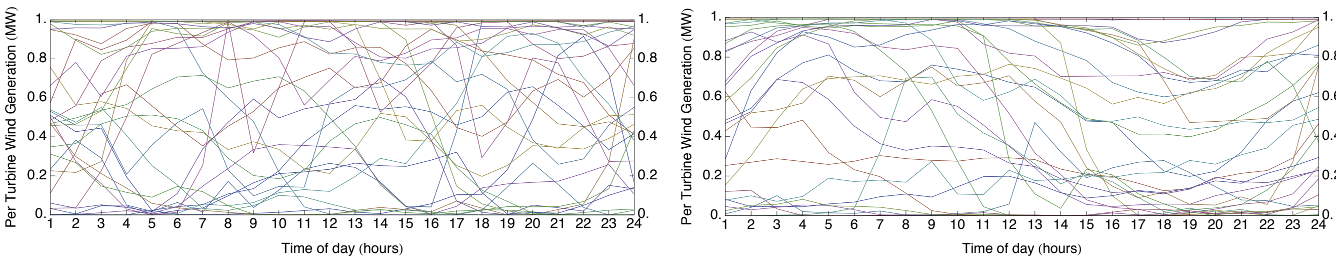
$$\varepsilon^l(x) = \sum_{n=1}^{N} \sum_{i=1}^{n} \varepsilon_1 * \cdots * \varepsilon_i(x) \ .$$

With conventional network calculus terms, a similar improvement for a stochastic upper arrival curve can be found in [14] (p. 133). The proof is similar to the proof of Proposition 1, with the difference that at the end one should invoke Lemma 2 instead of Lemma 1.



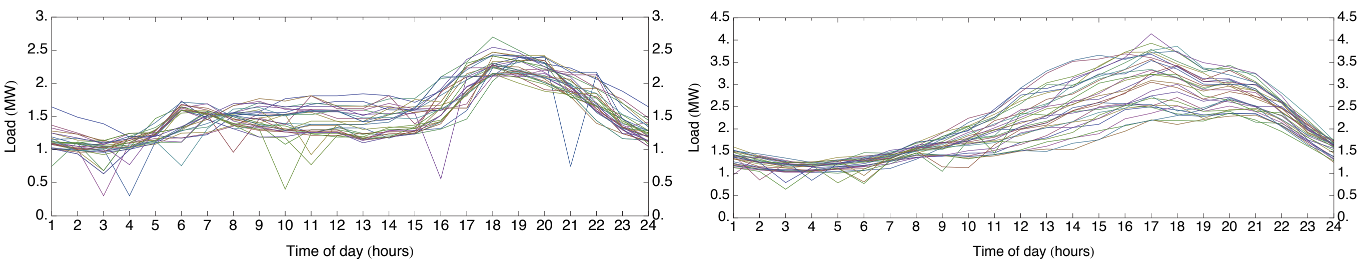
(a) January (b) July

Fig. 4. Per unit solar generation daily profiles in Long Beach, CA



(a) January (b) July

Fig. 5. Per turbine wind generation daily profiles on an island near Santa Barbara



(a) January (b) July

Fig. 6. Daily load profiles on Santa Catalina Island

VI. Case study

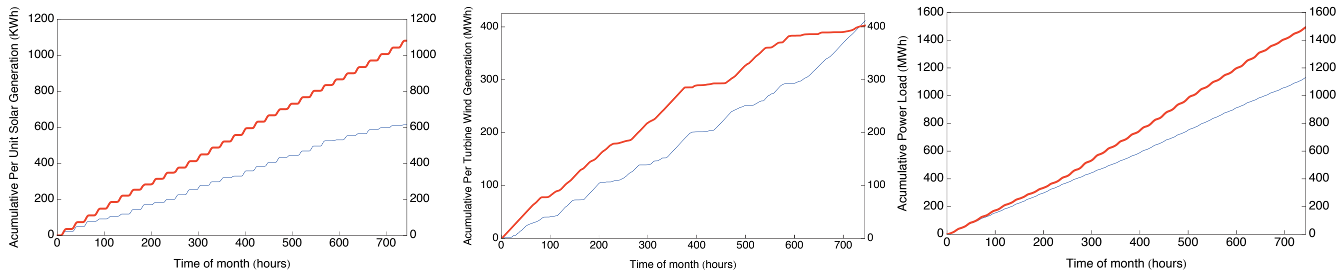
A. Description of the Data Set

As a case study, we consider Santa Catalina Island, which is located 26 miles off the coast of Southern California, USA. It has an area of 76 square miles and it has 54 miles of coastline. Currently, the electricity on Catalina is generated by a central diesel plant, and the island is served by three 12kV distribution circuits which are separated from the grid on the California mainland. It is desirable to reduce diesel-based generation for both environmental and economic reasons. This paper aims to investigate the feasibility of replacing diesel generation with generation from renewable sources.

We use data profiles including power load, solar PV generation, and wind generation for our analytical study. We consider two typical data profiles in winter and summer seasons. The hourly variations of all data profiles for the month of January 2010 are shown in Figures 4a, 5a, and 6a. In addition, the hourly variations of the data profiles for the month of July 2010 are shown in Figures 4b, 5b, and 6b. They are obtained at various locations near Santa Catalina Island with similar meteorological characteristics:

- Solar generation profile: Based on the typical meteorological year (TMY) data sets derived from the National Solar Radiation Data Base (NSRDB) archives [25], the hourly per unit ($35m^2$) solar PV energy generation data for Long Beach, California, is calculated using the System Advisor Model [26].
- Wind generation profile: The hourly energy generation data for a wind turbine located off an island near Santa Barbara, California is obtained from the Western Wind Sources data set available at the National Renewable Energy Laboratory (NREL) [27].
- Load profile: The peak values for Santa Catalina Island are obtained by personal communication with researchers from Southern California Edison [28]. The load profiles are generated from a proxy distribution circuit statistically similar to the island, whose peak is scaled to match the peak data for each of the three distribution circuits on the island.

The cumulative per unit solar generation, per turbine wind generation and load profiles are depicted in Figures 7a, 7b, and 7c. As shown in the figures, the solar PV generation in July



(a) Per unit solar generation profiles (b) Per turbine wind generation profiles (c) Cumulative load profiles on Santa Catalina Island

Fig. 7. Cumulative energy supply/demand profiles in January (blue line) and July (red line)

is significantly greater than in January due to meteorological factors. We also notice that the load in July is greater than in January, a fact which could be attributed to the abundant usage of air conditioning in the summer. For the wind generation profile, there is no significant difference in the cumulative amount; a slight difference lies in fluctuations characteristic to daily behaviors.

With the typical power load and generation profiles for a certain period, we are next going to address the following design question: *Given different configurations of renewable sources, what are the appropriate amounts of battery storage needed to ensure a certain level of power supply reliability?* To answer, we will illustrate in particular the impact of battery storage on the average Fraction of Time that energy is Not-Served (FTNS) performance metric from Eq. (12).

B. Model Fitting

From the data set given above, we first fit the stochastic demand and supply curves and the corresponding bounding functions from Definitions 1 and 2. The demand/supply curve functions are linear functions with the rate equal to the long-term mean rate of the fitted data. Once these curves are set, we next fit exponential functions for the bounding functions.

In particular, for the energy demand process, we can get a stochastic upper demand curve $\alpha^u(t)$ with bounding function ε_d^u, denoted by $D \sim \langle \varepsilon_d^u, \alpha^u \rangle$. In turn, to fit the solar power supply data, we first assume that all the PV panels are homogeneous. Then, based on the per unit data profile and given the total number N_p of PV panels, we fit a stochastic lower supply curve $\beta^p(t)$ with bounding function ε_s^p, denoted by $S^p \sim \langle \varepsilon_s^p, \beta^p \rangle$. Similarly, all the wind turbines are also assumed to be homogeneous. Based on the per turbine data profile and given the total number N_w of wind turbines, we fit a stochastic lower supply curve $\beta^w(t)$ for the wind energy supply with bounding function ε_s^w, denoted by $S^w \sim \langle \varepsilon_s^w, \beta^w \rangle$. To aggregate heterogeneous power supply sources together, we use the aggregation property from Proposition 1.

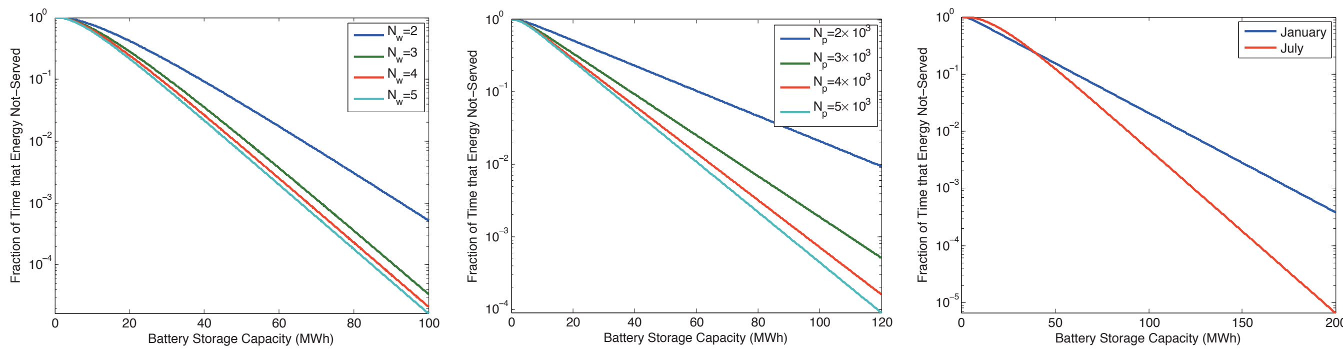
C. Numerical Results

For a given battery capacity C, the loss of power supply probability (LPSP) metric is provided by Theorem 1. Together with Eq. (12), we can get the average Fraction of Time that energy is Not-Served (FTNS) over the entire time period of the given data profile, e.g., $T = 744$ hours in January 2010. FTNS is used next to illustrate the impact of three factors, i.e., concerning wind and solar generation, and also seasonality, to the reliability of the power system.

1) Wind Generation Impact: Figure 8a depicts the FTNS metric as a function of the battery storage capacity, with a fixed level of solar generation, i.e., $N_p = 2{\times}10^3$, for different values N_w of wind turbines in January. As N_w increases, FTNS decreases with the same battery capacity and approaches a constant value. For a targeting FTNS value, say 0.01, we can readily get the amount of required battery storage for different energy configurations. For instance, when the number of wind turbines increases from 2 to 5 units, while fixing the other settings, the battery storage requirement is reduced from 66.5 MWh to 51.3 MWh, 48.4 MWh, and 46.6 MWh, respectively. This example illustrates the fact that, due to the complementary characteristics between solar and wind energy for certain locations, the hybrid solar-wind power generation system with storage banks can offer a highly reliable source of power, which is suitable for electrical loads with high reliability constraints.

2) Solar Generation Impact: Figure 8b depicts the FTNS metrics as a function of the battery storage for different levels of solar generation with a single wind turbine in January. As expected, FTNS decreases as the battery capacity increases. Similar to the decreasing rate of FTNS shown in Figure 8a by increasing N_w, the transition from high FTNS to low FTNS sharpens by increasing N_p. That means that for some targeting FTNS value, increasing the number of wind turbines would have a smaller impact on reducing the battery requirement due to the fluctuating nature of the renewable power sources. As an example, for a targeting FTNS value of 0.01, the battery capacity requirement with the configuration of one wind turbine and 2×10^3 units of PV panels is 116.8 MWh; by further increasing the number of PV units to 5×10^3 units, the battery capacity requirement decreases to 61.2 MWh.

3) Meteorological Impact: For a fixed configuration of renewable generations, we next investigate the impact of different seasons. Figure 8c shows the FTNS metric as a function of the battery storage with a single wind turbine and $N_p = 2 \times 10^3$ units of PV panels, in January and July. We notice that FTNS decreases much sharper in July than in January, beyond some critical point of the battery storage capacity. In other words, in order to guarantee a certain level of system reliability, less battery storage capacity is needed in July. This can be explained by the significantly higher



(a) $N_p = 2 \times 10^3$ (b) $N_w = 1$ (c) $N_w = 1$ and $N_p = 2 \times 10^3$

Fig. 8. FTNS as a function of battery storage capacity under various aggregation scenarios

solar PV generation in July than in January. As an example, for a targeting FTNS value of 0.01, the battery capacity requirements are $116.8\ MWh$ for January and $89.3\ MWh$ for July. We also notice that FTNS in July is greater than in January for smaller values of the battery storage than the critical point. This fact can be attributed to the increased energy demand in summer, which widens the gap between power generation and demand at the beginning of the day due to lack of any solar generation.

VII. Related Work

Aggregating stochastic power sources to achieve reliable electricity supply is a challenging problem. Various optimization techniques for hybrid PV/wind systems sizing have been proposed in the literature [29], such as probabilistic approaches [30], [31], graphical construction techniques [32], [33], artificial intelligence methods [34][35], and iterative techniques [36], [37]. For instance, the authors of [30] developed a probabilistic model of the hybrid solar-wind power system to incorporate the fluctuating nature of the resources and the load. In particular, the model convolves the probability density function of power generated by solar and wind generations, to assess the long-term performance of a hybrid system for both stand-alone and grid-connected applications. To estimate the load-shedding probability, [31] constructed a matrix for the Markov chain model based on the empirical distribution of the energy storage states, and the results derived were translated into design choices. Unlike this set of works, our analytical framework is based on very general stochastic network calculus models to capture fluctuations in both energy supply and demand.

Unlike much theoretical development in the field of the stochastic network calculus, its application to critical problems, such as the reliability of a power system, is lagging behind [38]. Recent examples of works, concerning applications of the stochastic network calculus, include for instance [39] which analyzes the delay of the IEEE 802.11 distributed coordination function (DCF), where the stochastic behavior of the DCF is characterized by a time-domain model; the end-to-end delay in some wireless networks is analyzed in [40]. The extension of the stochastic network calculus to analyze information-driven networks, developed in [41], can be regarded as an important step to bridging the gap between communication networks and information theory. Other extensions of the stochastic network calculus are developed to study the impact of network coding in acyclic networks [42] or the problem of estimating the available bandwidth in networks with random service [43]. The problem of scheduling subpiece transmission for P2P-VoD system is formalized in [44], which further analyzes its delay performance.

Lastly we mention two *parallel and closely related* works with ours. Wu *et al.* [45] also extend the stochastic network calculus to account for the supply and demand energy in a power system with renewable energy sources, which is used for the study of the stochastic energy constraint and the correlation between QoS and the uncertain energy supply. The common metric studied in both [45] and ours is the waste of power supply (WPS), albeit in [45] it is derived in a simplified queueing model with infinite storage. The main difference between the two formulations is that the energy demand in [45] is coupled with the energy supply using a stochastic service curve model alike Eq. (1), whereas this paper uses an entirely decoupled approach. The same decoupled approach is used by Le Boudec and Tomozei [46] in a deterministic framework of the network calculus. In that work, the authors investigate several problems related to battery sizes, such as the existence of necessary and sufficient conditions, and the construction of online battery charging schedules, to guarantee zero loss of power supply (LPS). For future work, it would be interesting to compare the coupled vs. decoupled formulations from [45] and ours in a stochastic framework, and to further relate them with deterministic counterparts as in [46].

VIII. Conclusion

In this paper, we have extended the stochastic network calculus framework to analyze system design in the context of the power grid. This extension was motivated by the ability of the calculus to account for high fluctuations in queueing systems, which are especially characteristic to the power grid when integrating renewable energy sources such as solar and wind. We have provided explicit formulas for various performance metrics characteristic of the power grid, such as the power system reliability depending on the number of PV cells, wind turbines, and energy storage capacity. To validate our model, we have investigated the feasibility of replacing diesel generation entirely with PV panels and wind turbines, supplemented with energy storage, in a case study on Santa Catalina Island.

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Appendix

Here we give two lemmas which are useful for the main results in the paper. The lemmas provide bounds on the distribution of a sum of two random variables, which represent instances of the energy demand or supply processes.

Lemma 1 ([14]): Let two random variables X and Y, with CDFs $F_X(x)$ and $F_Y(x)$, respectively. If $F_X^c(x) \le \varepsilon_X(x)$ and $F_Y^c(x) \le \varepsilon_Y(x) \ \forall\ x \ge 0$, for some real functions $\varepsilon_X(x)$ and $\varepsilon_Y(x)$, then for all $x \ge 0$

$$\mathbb{P}\{X + Y > x\} \le F_X^c(x) \otimes F_X^c(x) \le \varepsilon_X \otimes \varepsilon_Y(x) \ .$$

We remark that the tail bound holds irrespectively of the statistical independence between X and Y. If such an additional independence assumption holds, then the tail bound can be further improved as follows.

Lemma 2: Let two non-negative random variables X and Y such that $F_X^c(x) \le \varepsilon_X(x)$ and $F_Y^c(x) \le \varepsilon_Y(x)$ for all $x \ge 0$, and $\varepsilon_X(x) = \varepsilon_Y(x) = 1$ for all $x < 0$. Then for all $x \ge 0$

$$\mathbb{P}\{X + Y > x\} \le \varepsilon_X(x) + \varepsilon_X * \varepsilon_Y(x) \ .$$

This lemma provides a slight simplification of Lemma 6.1 from [14] and the proof follows similarly.

Proof: Fix $x \ge 0$. Then we can write

$$\begin{aligned}
&\mathbb{P}\left\{X + Y > x\right\} = \int_0^\infty \mathbb{P}\left\{X > x - y\right\} dF_Y(y) \\
&\le \int_0^\infty \varepsilon_X(x - y) dF_Y(y) \\
&= \varepsilon_X(-\infty)F_Y(\infty) - \varepsilon_X(x)F_Y(0) - \int_0^\infty F_Y(y) d\varepsilon_X(x - y) \ ,
\end{aligned}$$

after using the bound from the theorem and then integrating by parts formula for the Stieltjes integral. We can continue the last equation as follows

$$\begin{aligned}
\mathbb{P}\left\{X + Y > x\right\} &\le 1 - \int_0^\infty d\varepsilon_X(x - y) \\
&\qquad + \int_0^\infty F_Y^c(y) d\varepsilon_X(x - y) \\
&\le 1 - \varepsilon_X(-\infty) + \varepsilon_X(x) + \int_0^x \varepsilon_Y(y) d\varepsilon_X(x - y) \\
&= \varepsilon_X(x) + \int_0^x \varepsilon_Y(y) d\varepsilon_X(x - y) \ ,
\end{aligned}$$

which concludes the proof (in the third line we could restrict the domain of the integral since $\varepsilon_X(y) = 1 \ \forall y < 0$). ■

Kai Wang (S'08) is currently pursuing the Ph.D. degree in the Department of Computer Science and Technology at Tsinghua University, Beijing, China. He has been selected into the joint PhD Program by the China Scholarship Council, and since Fall 2010, he has been a visiting student in the Department of Computing and Mathematical Sciences, California Institute of Technology, Pasadena, USA. His research interests include computer networks and systems, smart grid and performance evaluation. He has published over 10 papers in refereed journals and conferences.

Florin Ciucu (S'04-M'09) Florin Ciucu was educated at the Faculty of Mathematics, University of Bucharest (B.Sc. in Informatics, 1998), George Mason University (M.Sc. in Computer Science, 2001), and University of Virginia (Ph.D. in Computer Science, 2007). Between 2007 and 2008 he was a Postdoctoral Fellow in the Electrical and Computer Engineering Department at the University of Toronto. Currently he is a Senior Research Scientist at Telekom Innovation Laboratories (T-Labs) and TU Berlin. His research interests are in the stochastic analysis of communication networks, resource allocation, and randomized algorithms. He has served on the Technical Program Committee of several conferences including IEEE Infocom, ACM Sigmetrics/Performance, or International Teletraffic Congress. Florin is a recipient of the ACM Sigmetrics 2005 Best Student Paper Award.

Chuang Lin (M'03-SM'04) received the Ph.D. degree in computer science from Tsinghua University, Beijing, China, in 1994.

He is a Professor of the Department of Computer Science and Technology, Tsinghua University, Beijing, China. He is an honorary visiting professor, University of Bradford, United Kingdom. His current research interests include computer networks, performance evaluation, network security analysis, and Petri net theory and its applications. He has published more than 300 papers in research journals and IEEE conference proceedings in these areas and has published four books.

Prof. Lin is a senior member of the IEEE and the Chinese Delegate in TC6 of IFIP. He serves as the Technical Program Vice Chair, the 10th IEEE Workshop on Future Trends of Distributed Computing Systems (FTDCS 2004); the General Chair, ACM SIGCOMM Asia workshop 2005; an Associate Editor, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY; an Area Editor, Journal of Computer Networks; and an Area Editor, Journal of Parallel and Distributed Computing.

Steven H. Low Steven H. Low (F'08) received the B.S. degree from Cornell University, Ithaca, NY, and the Ph.D. degree from the University of California, Berkeley, both in electrical engineering. He is a Professor with the Computing and Mathematical Sciences and Electrical Engineering departments at California Institute of Technology, Pasadena, and hold guest faculty position with the Swinbourne University, Australia and Shanghai Jiaotong University, China. Prior to that, he was with AT&T Bell Laboratories, Murray Hill, NJ, and the University of Melbourne, Melbourne, Australia.

Prof. Low was a co-recipient of the IEEE Bennett Prize Paper Award in 1997 and the 1996 R&D 100 Award. He was a member of the Networking and Information Technology Technical Advisory Group for the US Presidents Council of Advisors on Science and Technology (PCAST) in 2006. He was on the Editorial Board of the IEEE/ACM TRANSACTIONS ON NETWORKING, IEEE Trans on Automatic Control, ACM Computing Surveys, and the Computer Networks Journal. He is currently on the editorial boards of Foundations and Trends in Networking and is a Senior Editor of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATION.