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A STRINGY NATURE NEEDS JUST TWO CONSTANTS

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A B S T R A C T

Dual string theories of everything, being purely geometrical, contain only two fundamental constants: c, for relativistic invariance, and a length λ, for quantization. Planck's and Newton's constants appear only through Planck's length, a "calculable" fraction of λ. Only the existence of a light sector breaks a "reciprocity" principle and unification at λ, which is also the theory's cut off.

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It is not inconceivable at present that a Dual (Super) String Theory is the theory of everything(TOE). If so all phenomena must be reducible to quantum relativistic geometry of 2-dimensional surfaces.

Witten has already pointed out [1] that ,most likely, Superstrings have no free <u>dimensionless</u> parameters. Here we shall make some simple observations on the <u>dimensionful</u> constants of nature in a dual TOE.

The (obvious ?) conclusion is that such theories only contain <u>two</u> fundamental constants ,the velocity of light , c, needed for relativity (and set equal to 1 in the following, if not explicitely written) , and a length λ, needed for quantization. It is unnecessary, in fact very unnatural, to introduce a string tension T, an energy scale, or a fundamental action $\hbar$, in the context of quantum strings.

The above conclusion is immediately reached by recalling that the Nambu-Goto (NG) string action is an area . All one needs for quantization is to convert an action into a phase factor: this is true for standard treatments of non relativistic quantum mechanics [2], or in Feynman's path integral approach [3]. Obviously, to get a phase, we only need to divide the NG action by λ^2,where λ is a fundamental length.

A question comes immediately to mind : why does one not do the same in point theories, where actions are lengths rather than areas?The problem is that even the free action of a system of point particles contains <u>several</u>

(mass) parameters :

$$S_{\text{Points}} \sim -\sum_i m_i c \int d\tau \sqrt{\dot{X}_i^2(\tau)}$$

(1) $$\dot{X}^\mu \equiv \frac{dX^\mu}{d\tau}$$

One could quantize (1) by introducing a length , which amounts to choose a unit of mass, but the functional integral, or the indetermination relations that would follow, would still contain the parameters m_i/m_j explicitely, hence will not involve purely geometrical objects. One prefers to introduce an arbitrary new unit of mass and a quantum action $\hbar$.

By contrast the classical string action does not contain parameters (there is just one superstring) .The string coordinates $X^\mu(\sigma, \tau)$ contain both the position and the momentum variables in a single object with dimensions of length. It is thus most natural not to introduce any rescaling factors in the expansion of X^μ:

$$X^\mu(\sigma,\tau) = q^\mu + \bar{p}\,\tau + \text{oscillators}$$

(2) $$\bar{p} \equiv 2\alpha' p$$

and consider $\overline{p}$ as the conjugate variable to q (we shall put a bar on the quantities we define non conventionally).

The usually adopted (and unnecessary) rescaling factor $1/2\pi T = \alpha' = dJ/dM^2$, the Regge slope parameter, has dimensions length/mass, is a classical quantity (related as we shall see to Newton's constant) , and should not be confused with λ. If we use $\overline{p}$ instead of the usual p , positions and momenta are on a more identical footing.

This is a manifestation of duality itself , which is related to σ,τ reparametrization invariance. Position variables are related to $X^\mu(\sigma)$ at a given τ ; momenta to τ evolution ,but what is σ and what is τ depends on the parametrization used. P and X' can be combined together through a reparametrization and thus they better have the same dimensions. This can also be rephrased by saying that strings obey a version of the old Born principle of reciprocity [4] ,a symmetry of nature's laws under the interchange of q's with p's, which naturally leads to harmonic oscillators and strings.

We thus proceed to quantization by Feynman's path integral:

$$(3)\quad Z_{string} = \int dX^\mu \exp\left(-\frac{i}{\pi\lambda^2}\cdot A_{rea}(X)\right) \; ; \; \lambda^2 \equiv \frac{2\alpha'\hbar}{c^2}$$

$$A_{rea}(X) \equiv \int d\sigma d\tau \sqrt{(\dot{X}X')^2 - \dot{X}^2 X'^2} \; ; \; \dot{X} \equiv \frac{\partial X}{\partial \tau} \; ; \; X' \equiv \frac{\partial X}{\partial \sigma}$$

This corresponds to a canonical quantization where:

$$
\begin{aligned}
&[X^\mu(\tau,\sigma), \overline{P}^\nu(\tau,\sigma')] = i\pi g^{\mu\nu}\lambda^2\delta(\sigma-\sigma') \\
(4)\quad &\overline{P}^\nu \equiv \dot{X}^\nu \quad ; \quad g^{ii} = -g^{00} = 1 \quad ; \quad i.e. \\
&[q^\mu, \overline{P}^\nu] = i\, g^{\mu\nu}\lambda^2 \\
&[a_n^\mu, a_m^{+\nu}] = g^{\mu\nu}\delta_{n,m}\lambda^2
\end{aligned}
$$

Thus λ^2 plays the role of Planck's constant .Quantization allows only discrete values for $\overline{p}^2$ at tree level i.e.

$$(5)\quad \overline{m}^2 \equiv \overline{P}^2 = 2N\lambda^2 = 2\, J_{Max}$$

with N an integer occupation number ($\geqslant 0$ for tachyon-free theories).Eq. (5), for N=0, breaks reciprocity since strings have non zero size ($\Delta q > \lambda$).

In order to make connection with something more familiar remember that a TOE, by definition, contains gravitation . Massive static particles attract each other inducing a gravitational acceleration given by:

$$
\begin{aligned}
(6)\quad \overline{F}_{gr.} &\equiv \overline{m}_1 a_{gr.}^{(1)} = \overline{m}_2 a_{gr.}^{(2)} = \\
&= -\overline{G}_N \frac{\overline{m}_1 \overline{m}_2}{r_{12}^2}
\end{aligned}
$$

where, evidently, in our system of units $\overline{G}_N$ is a pure number(recall that c=1). In the specific case of the Heterotic String [5] one finds for instance:

$$\overline{G}_N = \frac{e^{-2\langle D\rangle}}{16} \cdot \frac{\lambda^6}{V_6} \tag{7}$$

where $\langle D \rangle$ is the expectation value of the dilaton field and V_6 is the invariant volume of the compact manifold into which 6 of the original 10 dimensions compactify. Both should in principle come out of the theory[1].

Note that forces are dimensionless in our units. Comparing eq. (6) with Newton's formula for the gravitational acceleration (no sense in comparing forces since the units are different) we find:

$$\begin{aligned} \overline{G}_N \overline{m} &= G_N m \\ G_N &= 2\overline{G}_N \cdot \alpha' \end{aligned} \tag{8}$$

We thus see that our definition of mass is ,apart from the factor (7), the same as the so-called gravitational (Schwarzschild) radius [6] ρ_m of an object of mass m

$$\rho_m \equiv \frac{2m G_N}{c^2} \tag{9}$$

As anticipated ,this is an enterely classical concept, involving G_N but not $\hbar$. Thus our point can be rephrased by saying that string properties are best expressed in terms of sizes and gravitational radii of particles, both being quantized in units λ . On the other hand, the quantity that corresponds to λ in the standard approach is nothing but Planck's length, up to a factor. Indeed one has :

$$(10) \qquad \lambda^2 = \frac{\bar{P}}{P}\hbar = \frac{G_N \hbar}{\bar{G}_N} = \lambda_P^2 / \bar{G}_N$$

where the usual definition of the Planck length

$$(11) \qquad \lambda_P = \sqrt{\frac{\hbar G_N}{c^3}} \simeq 1.6\ 10^{-33} cm$$

has been used . Thus, using (7) :

$$(12) \qquad \lambda_P = \frac{e^{-\langle D \rangle}}{4} \left(\frac{\lambda^3}{\sqrt{V_6}} \right) \cdot \lambda$$

From general considerations [7] (see also eq. (16) below) one expects $\lambda_P = O(10^{-1} \lambda)$.

It is amusing to consider the ratio between gravitational and physical radius for a massive string:

$$(13) \quad \frac{\rho_m}{r} = \frac{2G_N m}{r} = \frac{2\bar{G}_N \bar{m}}{r} \simeq$$

$$\simeq 2\bar{G}_N \frac{O(\lambda)}{O(\lambda)} \approx \bar{G}_N \lesssim O(1)$$

They are of the same order for an elementary string state as could have been expected from reciprocity . This result indicates that strings, through the uncertainty relation (4), neatly avoid the problems due to the too singular behaviour of classical relativity. Indeed, at distances smaller than λ, heavy string exchanges modify Newton's law and make it less singular. Gravity indeed becomes fully quantum mechanical at distances smaller than λ and elementary (stringy) black holes do not probably exist.

Similar considerations also allow to determine gauge couplings obtained via a Kaluza Klein mechanism. Gauge charges are compactified momenta and again should be naturally measured in λ units. The indetermination principle gives

$$(14) \quad \bar{P}_{Comp} \gtrsim \lambda^2 / R_{Comp}.$$

but $q, \bar{p}$ reciprocity further implies

$$(15) \quad \bar{P}_{Comp.} \sim R_{Comp} \sim \lambda$$

The electromagnetic force is thus very similar to the gravitational one

one :

$$\bar{F}_{el} \equiv \bar{\alpha}\,\frac{\bar{e}_1\bar{e}_2}{r_{12}^2} \quad ; \quad \bar{e}_{1,2} = \lambda\cdot(\mathrm{Clebsh})$$

(16) $$\bar{\alpha} = \frac{1}{4}\, e^{-2\langle D\rangle}\left(\frac{\lambda^6}{V_6}\right) = 4\,\bar{G}_N$$

In our units gauge and gravitational couplings are both dimensionless (in D=4) and are "unified" for generic string states. It is only the existence of extremely light strings (in λ units) that breaks the symmetry between the two forces and reciprocity (allowing $\bar{p} \ll q$). The relation (16) is just the one deduced in the heterotic string [5] by use of modular invariance .This leads to the selection of even self dual lattices for the allowed momenta and follows directly from reciprocity at the λ scale if our units of momentum are used.

The above considerations also solve a problem in classical KK theory [8], the KK radius being kept finite by a quantum effect (a Bohr quantization rule) controlled by the new quantum length λ .

Notice that gravitational and gauge forces are classical concepts: it is just the <u>values</u> of elementary masses and charges that are quantized in (the same) units of λ . Thus a cross section such as $\sigma(e^+e^- \to \mu^+\mu^-)$ will be given in our units as

(17) $$\sigma \sim \frac{\bar{\alpha}^2\bar{e}^4}{\bar{s}} \sim \bar{\alpha}^2\,\lambda^4/\bar{s} \quad ; \quad \bar{\alpha} \simeq 1/137$$

$$(\ \bar{s} \equiv \bar{E}_{c.m.}^2 = \mathrm{length}^2\ )$$

and will be much larger than λ^2 at usual accelerator energies.

Regarding λ as the only constant controlling quantum phenomena appears to generate a paradox: Systems involving distances much larger than λ should be classical,not quantum. How about the hydrogen atom then?

The above point is true for generic string states, but breaks down for the massless string states . Somehow, these will eventually pick up a tiny mass e.g.:

$$\text{(18)} \qquad \bar{m}_{electron} \equiv \bar{m}_e = \epsilon_e \lambda \qquad \left(\epsilon_e = O(10^{-22}) \right)$$

For systems involving very light string states quantum mechanics is still very relevant (remember that the semiclassical approximation breaks down at small momenta [2]) . Take indeed the hydrogen atom. Writing Bohr's quantization condition in our units

$$\text{(19)} \qquad \bar{P}_e \cdot r_H = \lambda^2 = \bar{m}\, v\, r_H$$

as well as $\bar{F} = \bar{m}a$:

$$\text{(20)} \qquad \bar{F}_{el} = \bar{\alpha} \frac{\lambda^2}{r_H^2} = \frac{\bar{m}\, v^2}{r_H} \quad ; \quad \bar{\alpha} \simeq \frac{1}{137}$$

gives immediately the standard results (e.g. Balmer's series)

$$\langle r_H \rangle = \frac{\lambda^2}{\bar{m}_e \bar{\alpha}} \; ; \; \left(= \frac{\hbar^2}{m_e \alpha} \; , \; \alpha = \frac{\hbar c}{137} \right)$$

$$\text{(21)} \qquad \bar{E}_n = -\frac{1}{2n^2} \bar{m}_e v^2 \qquad (v = \bar{\alpha} c)$$

$$\omega_{nm} = \frac{\bar{E}_n - \bar{E}_m}{\lambda^2} = \frac{1}{2} \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \frac{\epsilon_e \bar{\alpha}^2 c}{\lambda}$$

The hydrogen atom thus satisfies $\Delta q \cdot \Delta \bar{p} = \lambda^2$ very asymmetrically:

(22) $$\Delta q \simeq 10^{24}\lambda \quad ; \quad \Delta \bar{p} \simeq 10^{-24}\lambda$$

Eq.(21) determines in principle λ in terms of Rydberg' constant .

Similarly, widths and lifetimes of ordinary strings are of the same order , while for the light states one has

(23)
$$c\tau\bar{\Gamma} = \lambda^2 \quad ; \quad c\tau \sim O(\lambda^2/\bar{m}) \gg \lambda \quad ; \quad \bar{\Gamma} \ll \lambda$$

As a final point we would like to discuss the fate of $\hbar$ in a theory that can do without it. It is not dissimilar from that of Boltzmann's constant k in thermodynamics. Replacing temperatures by energies makes the introduction of k unnecessary. Fixing a unit of temperature (e.g. the degree Kelvin) brings in k as the conversion factor.

Similarly, the (unnecessary) introduction of a unit of mass different from the fundamental length λ itself (say the gram) brings in G as the conversion factor and $\hbar$ as a new constant. $\hbar$ itself can be expressed in terms of λ and c as

(24) $$\hbar = \frac{\lambda c}{\epsilon_e} m_e$$

where ϵ_e, eq.(18), is the mass of the electron in λ units(a "calculable" number) and $m_e = 9.11\ 10^{-28}$ gr. is taken here as the definition of gram . Of course any other system can replace the electron in eq.(24), a cm ^{3} of water at the freezing point, for instance.

The fundamental quantum length λ could mean however much more than just the usefulness of a new system of units. If all probes are quantum strings it is not possible to localize something with a precision better than λ during a τ interval O(1) :

$$\Delta\left(\int_0^1 d\tau\, X(\sigma,\tau)\right) > \lambda \tag{25}$$

Since any conceivable measurement must involve some string-string scattering and this is described by a finite integral over τ s ,there is no measurement which does not involve field averages over regions of space time O(λ^4), subject themselves to quantum uncertainty. Strings thus avoid the old Bohr-Rosenfeld criticism of local fields [9]. Since momenta $\Delta\ \bar{p} > \frac{\lambda^2}{\Delta q}\ (\lesssim\lambda)$ are suppressed, the quantum constant λ is also the cut off, and the infinities of ordinary field theories are overcome. Strings also appear to achieve the goals of T.D.Lee's approach [10] based on a dynamical, discretized space time .

In conclusion string theories ,when expressed in their natural units, reduce all phenomena ,not only gravity ,to pure geometry , realizing an old dream of modern physics [11], which started with Einstein's general relativity. Although quantum mechanics plays a crucial role in dual string theories , it is also purely geometrical, as emphasized by the dimensions of the new quantum constant: maybe even Einstein would have accepted it!

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