A Structural Model of the Mobility Table*

ROBERT M. HAUSER, *University of Wisconsin–Madison*

ABSTRACT

We propose a multiplicative (log-linear) model for mobility tables (or other cross-classifications) which is helpful in locating cells where counts are especially dense or sparse. This specification eliminates the confounding of main effects and interaction effects, which has plagued many other methods of measuring and interpreting association in mobility tables, especially those methods based on the model of simple statistical independence. The model yields a parsimonious set of parameters which describe the table, and goodness of fit can be assessed with standard inferential procedures. For each cell of the table the model yields a useful measure of association, which we call the new mobility ratio. We illustrate the model by reanalyzing the classic British mobility table of 1949, and we use that example to compare our measure of association with other mobility measures.

Occupational mobility is a fundamental indicator of the temporal aspect of social stratification (Duncan, b). The centrality of occupational roles in the organization of contemporary and especially industrial societies is coupled with strong commonalities across time and space in the differential access of occupational incumbents to social (including economic and political) rewards (Treiman). In this way occupational incumbency may be viewed as a proxy or index of social standing and occupational mobility as an index of social mobility. The terms "proxy" and "index" are used deliberately, for we would not wish to reify the concept of occupational status nor to discourage the analysis of other aspects of social inequality.

*An earlier draft of this paper was presented at the meetings of the Research Committee on Social Stratification, International Sociological Association, 1977. This research was supported by Grants GI–31604 and GI–4436 from the National Science Foundation, MH–06275 from the National Institute of Mental Health, and the Institute for Research on Poverty at the University of Wisconsin–Madison under funds granted by the Equal Opportunity Act of 1964 and administered by the Department of Health, Education and Welfare. Peter J. Dickinson, Randy D. Hodson, and James Baron assisted in computations, using facilities of the Center for Demography and Ecology which are supported by Grant HD–5876 from the National Institute for Child Health and Human Development. I thank Halliman Winsborough, Harry P. Travis, and Otis Dudley Duncan for advice and criticism. The opinions expressed herein are those of the author.

Occupations and Stratification

Students of mobility disagree about the dimensionality of occupational classification, about the mixture of functional and evaluative elements in classification, and about the level of detail required in classification. Yet there is substantial consensus that measurements of occupational mobility across or within generations may provide insights into the openness or rigidity of a society and the interactions of that openness or rigidity with demographic metabolism and social organization. That is, a mobility regime consists of a set of rules or processes governing access to social positions which is articulated with the flow of persons through the life cycle and the social organization of production. Thus arises a basic problem in mobility analysis: How does one distinguish the rules of access from the interplay of supply and demand in the labor market or from long-term processes of societal development and transformation?

Not only does occupation have broad validity as a proxy for social standing, but occupations are a salient feature of everyday life. Thus social surveys can be used to obtain valid measurements of social mobility *qua* occupational mobility between and within generations, and these survey-based measurements can be compared across time and place. Of course, standardization of occupational classifications is only one of several factors limiting valid comparison. Crude as it is, the concept of occupational social standing may be as close as we have come to a common metric in which the social mobility regimes of differing societies or populations may be compared (Featherman et al., b; Miller). For example, the comparative measurement of income inequality is conceptually no less treacherous than that of occupational inequality, and response error as well as price and consumption differentials will complicate efforts to measure and compare intergenerational economic mobility. Yet aside from their normative or political implications, comparative measurements are necessarily an element in a theory of social mobility.

The record of sociological mobility studies is paralleled by a history of statistical analysis in which occupational mobility has often served as stimulus, object, or illustration of statistical ideas. Indeed, it is consistent with the historical pattern that sociologists were introduced to the method of path analysis primarily by way of its successful application in studies of occupational mobility (Blau and Duncan). Devices for the statistical analysis of mobility data range from simple descriptive measures, e.g., gross mobility rates or inflow and outflow rates, to complex analytic schemes, e.g., Markov processes, canonical analyses, structural equation models, or log-linear models. We make no systematic effort to review these measures and models (see Bibby; Boudon, a; Pullum). Rather, we focus almost exclusively on multiplicative (log-linear) representations of the intergenerational occupational mobility table. In so doing, we do not intend to suggest that other methods and approaches are inferior, but to exploit some features of the log-linear model which seem interesting and fruitful. The methods described here may be applied in the comparison of two or more mobility tables, and they may be applied to higher order tables. Also, they may prove useful in the analysis of cross-classified data which are not mobility data; for example, they may be applied directly to analyses of assortative mating, sibling resemblance, and other instances of relationship, preference, or interaction which have strong formal resemblance to occupational mobility.

Conceptual Issues in Mobility Analysis

One might ask what justifies an extended treatment of the bivariate mobility table when multivariate structural equation models (Duncan, d; Goldberger and Duncan) have proven useful in elucidating processes of social mobility (Blau and Duncan; Duncan et al.; Sewell and Hauser) and changes and differentials in those processes (Bielby et al.; Duncan, c,; Featherman and Hauser; Treiman and Terrell). Blau and Duncan (152–61, 194–99) are probably correct in arguing that it is more difficult to think about mobility than about status attainment. Moreover, interval measurements of status variables provide greater parsimony and greater statistical power than nominal or ordinal measurements, and both of those properties are desirable in multivariate or comparative analysis.

Without discounting these arguments a case can be made for the analysis of mobility tables. They permit a more detailed examination of rates and patterns of movement between occupations than do scalar measures of association. Likewise, the categoric measurement of occupations makes possible a more detailed representation of the occupational structure —the origin and destination distributions of the mobility classification— than does measurement on numeric scales. Finally, mobility classifications may be analyzed without first specifying a status ordering of occupational groups, and the mobility table may be used to study the interaction among several dimensions of occupational classification. In short, mobility tables are useful because they encourage a direct and detailed examination of movements in the stratification system. Within a given classification they tell us where in the social structure opportunities for movement or barriers to movement are greater or less, and in so doing provide clues about stratification processes which are no less important, if different in kind, from those uncovered by multivariate causal models.

Some analysts may have exaggerated the conceptual differences between the analysis of mobility tables and causal modeling of stratification processes. Thus, Blau and Duncan (see also Pullum, ix) draw a distinction between "concern . . . with the opportunities for success of individuals or with the occupational structure of the society. . . . The occupational structure is conceived of as consisting of the relations among its constituent subgroups; and these occupational subgroups, not the individuals composing them, are the units of analysis" (23).

We take a contrary view, that a scalar measure of association between numeric status variables is no less an indicant of "mobility structure" than is a set of coefficients pertaining to the interior cells of a mobility table. Likewise, just as the marginal distributions of a mobility table are taken to reflect the constraints of occupational origins and opportunities, the means and standard deviations of status origin and destination variables play a similar part in linear models of status attainment (McClendon). The important distinction is not between units of analysis but between levels of measurement and of detail.

We have already mentioned the dual character of the mobility table; it reflects both the relative chances of movement or stability and the constraints of occupational origins and opportunities. Students of the mobility table have long recognized this duality and have tried in numerous ways to give it a valid empirical referent. The prevailing view is well-expressed in Boudon's exhaustive review: "A good mobility index should make a distinction between the amount of mobility generated by the changes in the social structure and the amount of mobility generated by other factors. Indeed the former should be eliminated" (a, 17). Devices for separating "structural" from "circulation" components of mobility within a single table have included indexes which attempt to control the marginal (origin and destination) distributions and accounting schemes based on minimum mobility, maximum mobility, or expected mobility (under statistical independence).

One persistent conceptual error has been the tendency to regard the intergenerational mobility table as a population transformation, that is, a projection matrix which carries the labor force forward from one period, that of the fathers' employment, to a later period in which sons have replaced fathers. A heroic effort along these lines by Kahl was neatly dismembered by Duncan (a, 54–9). He demonstrated that differentials in the timing and quantity of fertility combined with the disjuncture between the succession of cohorts and that of generations to invalidate the interpretation of the mobility table as a population transformation. Rather complex demographic accounting schemes are required to reconcile occupational mobility with population replacement processes (Coleman; Matras). In spite of these complexities Boudon (b) ignored the problem by specifying arbitrary (and equal) occupation distributions of fathers and sons. Other researchers have attempted to remove "structural" mobility by equating the destination distribution of a table to that observed in the fathers' generation (Hazelrigg; Pullum, 42–6). Yet on Duncan's argument homogeneity of origin and destination distributions does not correspond to the hypothesis of no change in the occupational structure.

The origin distribution of a mobility table is better read as a distribution of occupational origins among men than as a distribution of occupations among fathers. On this reading it is still appropriate to separate the effects of occupational origins, occupational destinations, and relative mobility chances. Rather than registering changes in mobility within a single mobility table, we attempt to locate changes in the comparison of tables for different periods, cohorts, or stages of the life cycle. Thus, Hope (a, b) and Hauser et al. (a) measured changes in relative mobility chances in Great Britain and the United States, respectively, conditional on changes across cohorts or periods in both origin and destination distributions. Obversely, Hauser et al. (b) measured structural changes in mobility among American men by standardizing on a set of relative mobility chances and jointly varying origin and destination distributions to reflect the experience of different cohorts and periods.

Mobility Ratios and Mobility Models

The ratio of the observed frequency in each cell of a mobility table to that frequency expected under statistical independence has undoubtedly been the most influential of those indexes proposed to eliminate the influence of origin and destination distributions on the observed pattern of mobility. At about the same time this ratio was hit upon by Goldhamer (a) and his student Rogoff at the University of Chicago, by Glass and his colleagues in Great Britain, and by Carlsson in Sweden. In spite of substantial methodological criticism the "social distance mobility ratio" (Rogoff) or "index of association" (Glass) and aggregate measures based on it have continued in use (Goldhamer, b).

The discovery of the mobility ratio remains important because of the impetus it has given to mobility measurement and because of the multiplicative model underlying the index. Let x_{ij} be the observed frequency in the cell corresponding to the ith origin and jth destination of a mobility table, and let the row and column sums of frequencies be $\sum_j x_{ij} = x_{i.}$ and $\sum_i x_{ij} = x_{.j}$, respectively, where $\sum_i x_{i.} = \sum_j x_{.j} = N$. For the ijth cell the mobility ratio is

$$R_{ij} = x_{ij}N/x_{i.}x_{.j}. \qquad (1)$$

In keeping with our interest in separating the effects of origin distributions, destination distributions, and relative mobility (or immobility) chances, we may rewrite equation 1 as

$$x_{ij} = a\,b_i c_j d_{ij}, \qquad (2)$$

where $a = N$, $b_i = x_{i.}/N$, $c_j = x_{.j}/N$, and $d_{ij} = R_{ij}$. In this way the mobility ratio may be interpreted as if it were a parameter in a multiplicative model of the mobility table. The model includes a main effect ($a = N$), reflecting the total magnitude of the frequencies; row ($b_i = x_{i.}/N$) and column ($c_j = x_{.j}/N$) effects reflecting the prevalence of each occupation in the origin and destination distribution, respectively; and interaction effects ($d_{ij} = R_{ij}$) reflecting the tendency toward mobility or immobility in each combination of origin and destination categories. Conceptually, the model of equation 2 is appealing, but—to anticipate later developments—the interpretation of the mobility ratio as a parameter is a fundamental error. Statistically, the R_{ij} are not interaction parameters, but errors of prediction under the simple independence model, in which there are no interactions between row and column variables. It would be helpful to know the parameters of a model like equation 2 if it described the observed frequencies, but otherwise the terms in the equation are likely to mislead. The fact that mobility ratios are based on the model of statistical independence, which does not fit mobility data, accounts for the flaws of the mobility ratio as an index of association.

With reference to this ratio Rogoff wrote:

> When the actual amount of mobility coincides with the expected amount, the ratio will, of course, be unity. When twice as many sons as expected enter an occupational class, the ratio will be two. The difference between these two ratios, whether they be based on data from the same table or different tables, is in no way attributable to variations in the availability of occupational positions, since these variations have been accounted for in the definition of social distance mobility (32).

Unfortunately, R_{ij} lacks these desirable properties. By inspection of equation 1 it appears that R_{ij} varies inversely with the marginal frequencies, $x_{i.}$ and $x_{.j}$, so large ratios will be less common in rows or columns with large marginal sums. Indeed, there is a strict maximum of R_{ij} which varies among cells; the maximum is the reciprocal of the larger of the row or column proportions associated with a given cell (Tyree, 580):

$$\max(R_{ij}) = N/\max(x_{i.}, x_{.j}). \quad (3)$$

Moreover, a nonsingular matrix of R_{ij} determines the original row and column marginal frequencies of a mobility table up to a constant of proportionality representing the total sum of frequencies in the table (Blau and Duncan, 93–97; Tyree, 578–80). It is easy to show how this occurs. Suppose we multiply both sides of equation 1 by $x_{i.}$ and sum the resulting expression across rows. We have

$$\textstyle\sum_i x_{i.} R_{ij} = \sum_i (x_{ij} N/x_{.j}) = (N/x_{.j}) \sum_i x_{ij} = N, \quad (4)$$

and providing the table is square, so $\max(i) = \max(j)$, equation 4 describes a set of linear equations by which the $x_{i.}$ are determined, given the R_{ij}. A similar expression may be written for the $x_{.j}$, and once both sets of margins are known, the original joint frequency distribution can be determined from equation 1. An important consequence is that the sets of mobility ratios for two tables cannot be the same unless the marginal distributions of the two tables are the same; clearly the mobility ratios cannot be useful in the measurement of change (when marginals differ across tables) if they cannot take on values corresponding to the hypothesis of no change.

Table 1. AN HYPOTHETICAL INTERGENERATIONAL OCCUPATIONAL MOBILITY TABLE

Father's Occupation	Son's Occupation 1	2	3	Total
1	500	100	100	700
2	100	100	100	300
3	100	100	100	300
Total	700	300	300	1300

Finally, R_{ij} confounds the effects of the sizes of origin and destination categories with tendencies toward mobility or immobility. To illustrate this, following Goodman (e), we consider the hypothetical frequencies in Table 1. It is evident by inspection that all of the association in the table is due to the large entry in the (1, 1) cell. All of the other entries are the same; no index based on origin and destination categories or combinations of them which did not include the (1, 1) cell would register any association. Yet consider the mobility ratios based on these frequencies which are shown in Table 2. The mobility ratios suggest that the highest affinity between origin and destination categories occurs among origins 2 and 3 in combination with destinations 2 and 3. The ratios show a *lower* affinity between origin 1 and destination 1, and they show dissociation between origin 1 and destinations 2 and 3 and between origins 2 and 3 and destination 1. Clearly, the mobility ratios distort the pattern of association which was evident from inspection of Table 1. The distortion occurs because the large entry in the (1, 1) cell induces a relative increase in the marginal proportions of the first row and first column, and it induces a relative decrease in the marginal proportions in the other rows and columns. Consequently, the mobility ratio for a given cell is depressed or inflated inversely with the marginal proportions affecting it.

The data of Table 1 can be reproduced by a multiplicative model whose parameters do not distort the pattern of association we have observed. For example, in the model of equation 2, let $a = 1$, $b_1 = b_2 = b_3 = 10$, $c_1 = c_2 = c_3 = 10$, $d_{11} = 5$ and $d_{ij} = 1$ for all other (i, j). Clearly, this model fits the data exactly, for

$$x_{11} = (1)(10)(10)(5) = 500,$$

and for all other (i, j)

$$x_{ij} = (1)(10)(10)(1) = 100.$$

Moreover, the row and column effects of this model, as well as the interaction effects, differ substantially from those underlying the calculation of the mobility ratios in Table 2. Thus, under the independence model the effects of rows (or columns) 1, 2 and 3 are 7/13, 3/13 and 3/13, respectively, but under the revised model the effect of each row (and each column) is the same.

Table 2. MOBILITY RATIOS FOR THE DATA IN TABLE 1

Father's Occupation	*Son's Occupation* 1	2	3
1	1.33	0.62	0.62
2	0.62	1.44	1.44
3	0.62	1.44	1.44

One way to describe the difference between the independence model and our revised model for the frequencies in Table 1 is that there is an undesirable part–whole relationship between the internal frequencies and marginal (row and column) effects under the independence model. In the revised model that part–whole relationship is eliminated, and the structure of the data becomes clear. The confounding of the interaction effects and marginal effects under the independence model (when it does not fit the data) prevents us from seeing the pattern of the interaction effects when we look at the mobility ratios.

If we know the pattern of the interactions in a mobility table—as we did by inspection in the present example—we can readily obtain the marginal effects. Likewise, if we know the marginal effects, we can readily ascertain the interaction effects. For example, when the model of quasi-perfect mobility fits the non-diagonal cells of a mobility table, Goodman (e, 835–40) shows how the interactions along the main diagonal may be elucidated by a set of row and column effects which have a probability interpretation (also, see Goodman, c).

In undertaking to interpret a mobility table we may know little or nothing about either the marginal effects or the interaction effects. Assuredly we will want to be cautious in basing our interpretation of the table on received knowledge, for most received knowledge about the structure of mobility tables is based on departures from the model of statistical independence. Moreover, unlike the case of our contrived example, we cannot assume that departures from statistical independence are located in one or a few cells of the table. In general, interactions of varying size may be scattered throughout the table (White, a).

Models of Quasi-Independence

Our analytic effort may have begun to sound like a bootstrap operation. In a certain sense it is, but we do not lack appropriate statistical models and methods. In a series of papers—of which the first was published 15 years ago—Leo Goodman (a, b, c, d, e, g) developed and exposited methods for the analysis of contingency tables (including mobility tables) in which the significant interactions were localized in specified cells or sets of cells in the table (also, see Pullum; White, a, b). For example, in the case of highly aggregated (3 by 3 or 5 by 5) mobility tables Goodman showed that most of the interaction pertained to cells on or near the main diagonal (when the occupation categories were listed in order of increasing status). Goodman (b, d) proposed that the analyst ignore or "blank out" those cells where interaction was greatest (where frequencies were thought to be especially dense or especially sparse) and attempt to fit a modified model of statistical independence, termed "quasi-independence," to the remaining frequencies in the table. In the case where only diagonal cells were blanked out in a mobility table, Goodman called the model one of "quasi-perfect mobility," after the term "perfect mobility," which had earlier been applied to the model of statistical independence in a mobility table. For an early application of this model to a large (17 by 17) table see Blau and Duncan (64–67).

Goodman (b, c, d) noted that quasi-independence might hold over all cells in a table whose entries were not ignored, or it might hold within, but not between certain subsets of cells whose entries were not ignored.

By ignoring the cells where interactions are large the model of quasi-independence gives a set of expected frequencies which vary only with row and column effects (Goodman, b, e). That is, if the quasi-independence model fits the data, the expected frequencies are not subject to the confounding of marginal proportions and interactions which occurs when the hypothesis of simple statistical independence is wrong. If row and column effects are obtained from the segment of the table exhibiting quasi-independence, then expected frequencies can also be obtained for the cells which were ignored in estimating the model of quasi-independence. The ratio of the observed frequency in a diagonal cell of a mobility table to the frequency expected under an appropriate model of quasi-independence is the new index of immobility proposed by Goodman (d, e, g). In describing certain diagonal or near-diagonal entries in a mobility table as ignored or blanked out, Goodman says, in effect, that the frequencies are fitted as observed by a model for the full table. In that model the density of observations in the quasi-independent segment of the table is taken as a reference point, and in the model for the full table the indices of immobility are the multiplicative parameters pertaining to the cells which were originally ignored or blanked out.

This idea is illustrated in a simple way by our revised multiplicative model for the frequencies in Table 1. Except in cell (1, 1) the frequencies are quasi-independent; indeed, they are uniform. We chose simple row and column effects (equal to 10) to reproduce the frequencies in the other cells. Taking the interaction (or lack of it) in the remaining cells as a point of reference (by setting $d_{ij} = 1$ whenever $i \neq 1$ or $j \neq 1$), it followed that the multiplicative parameter $d_{11} = 5$, which in this case was also Goodman's index of immobility for cell (1, 1).

Models of quasi-independence have provided important insights into the structure of mobility tables. Aside from Goodman's expository papers, they have been applied in cross-national, inter-urban, and cross-temporal analyses (Featherman et al., a; Hauser et al., a; Iutaka et al.; Pullum; Ramsøy). Goodman (d) has also shown how related ideas may be applied to test any specific hypothesis about the pattern of association in a mobility table. At the same time the application of quasi-independence models in mobility analysis has been less than satisfying in some ways. Even where large numbers of cells are blocked, quasi-independence models do not fit large tables very well (Hauser et al., a; Pullum). That is, in a more detailed mobility table, it appears that association is not limited to the small number of cells on or near the main diagonal. The larger the number of entries blocked (or fitted exactly) before a good fit is obtained, the less substantively appealing is the model of quasi-independence. Moreover, by treating departures from quasi-independence in the blocked or ignored cells as parameters or indices of mobility and departures in the unblocked cells as error, the quasi-independence model attaches too much theoretical importance to occupational inheritance (Hope, b). Of course, occupational inheritance is always defined by reference to a given classification of occupations, and the problem is exacerbated by the fact that the model of quasi-independence fits best when the mobility table is based on broad occupation groups. Thus the model is of greatest validity in the measurement of immobility in classifications where the concept of occupational inheritance becomes vague.

The focus on fit on or near the main diagonal follows a traditional sociological interest in occupational inheritance, but it also draws our attention away from other aspects of association in the table. For example, one might hypothesize that certain types of mobility are as prevalent as other types of mobility or immobility. More generally, one might wish to construct a parametric model of mobility and immoblity for the full table which would recognize the somewhat arbitrary character of occupational inheritance and the possible gradations of association throughout the table.

Goodman's (g) elegant Berkeley Symposium paper was an important and influential step forward. That paper presented a general multiplicative model for mobility tables (and other cross-classifications). Within the general model Goodman proposed and applied to the classic British and Danish mobility data a number of alternative specifications, all but one of which—the simple independence model—assumed ordinality in the occupational categories. The models incorporated combinations of parameters for upward and downward mobility, for the number of boundaries crossed, and for barriers to crossing particular categoric boundaries. Many of these models—as well as problems in comparing their goodness of fit—are reviewed by Bishop et al. (Ch. 5, 8, 9) and related models are discussed by Haberman (Ch. 6). Within the same multiplicative framework we take a slightly different approach in developing models of the mobility table.

A Multiplicative Model of the Mobility Table

Let x_{ij} be the observed frequency in the ijth cell of the classification of men by their own occupations ($j = 1, \ldots, J$) and their own occupations or fathers' occupations at an earlier time ($i = 1, \ldots, I$).[1] In the context of mobility analysis the same categories will appear in rows and columns, and the table will be square with $I = J$. For $k = 1, \ldots, K$, let H_k be a mutually exclusive and exhaustive partition of the pairs (i, j) in which

$$E[x_{ij}] = m_{ij} = \alpha\beta_i\gamma_j\delta_{ij} \tag{5}$$

where $\delta_{ij} = \delta_k$ for $(i,j) \epsilon H_k$, subject to the normalization $\prod_i \beta_i = \prod_j \gamma_j = \prod_i \prod_j \delta_{ij} = 1$. The normalization of parameters is a matter of convenience, and we choose the value of α so it will hold. However, unlike the usual setup, the interaction effects are not constrained within rows or columns even though the marginal frequencies are fixed. The model says the expected frequencies are a product of an overall effect (α), a row effect (β_i), a column effect (γ_j), and an interaction effect (δ_{ij}). The row and column parameters represent conditions of occupational supply and demand; they reflect demographic replacement processes and past and present technologies and economic conditions. The cells (i,j) are assigned to K mutually exclusive and exhaustive subsets, and each of those sets shares a common interaction parameter, δ_k. Thus, aside from total, row, and column effects, each expected frequency is determined by only one parameter, which reflects the level of mobility or immobility in that cell relative to that in other cells in the table.

The interaction parameters of the model correspond directly to our notions of variation in the density of observations (compare White, a, 26). Unlike Goodman's (g) multiplicative models, this model does not assume ordinal measurement of occupations. Of course, the assumption of ordinality may help us interpret results, or our findings may be used to explore the metric properties of our occupational classification. For the model to be informative, the distribution of levels across the cells of the table must form a meaningful pattern, and one in which the parameters are identified (Haberman, 217; Mason et al.). Further, the number of levels (K) should be substantially less than the number of cells in the table. These latter properties are partly matters of substantive and statistical interpretation and judgment, rather than characteristics of the model or of the data. We have found it difficult to interpret models where the number of levels is greater than the number of categories recognized in the occupational classification.

It may be helpful to present the model of equation 5 in more than one way. There is a pronounced rightward skew in multiplicative effects because small effects are bounded between 0 and 1, while large effects are unbounded. We get a fairer picture of many empirical results by taking logs of frequencies and parameters and writing the model in additive form. Let $u = \log \alpha$, $u_{1(i)} = \log \beta_i$, $u_{2(j)} = \log \gamma_j$, $u_{12(ij)} = \log \delta_{ij}$, and $u_{3(k)} = \log \delta_k$. Then the model is

$$\log m_{ij} = u + u_{1(i)} + u_{2(j)} + u_{12(ij)}, \tag{6}$$

where $u_{12(ij)} = u_{3(k)}$ for $(i,j) \epsilon H_k$, and H_k is defined as before. Here, the normalization of parameters is $\sum_i u_{1(i)} = \sum_j u_{2(j)} = \sum_i \sum_j u_{12(ij)} = 0$.

A slight variation of equation 5, which we present in multiplicative form, is more suggestive of the way in which we have manipulated empirical data for purposes of estimation and testing. With H_k defined as before,

$$E[x_{ij}] = m_{ijk} = \alpha\beta_i\gamma_j\delta_k \qquad \text{for } (i,j) \in H_k \tag{7}$$

and

$$m_{ijk} = 0 \qquad \text{for } (i,j) \notin H_k, \tag{8}$$

subject to the normalization $\prod_i \beta_i = \prod_j \gamma_j = \prod_k \delta_k{}^{n_k} = 1$, where n_k is the number of cells assigned to the kth level. This version of the model suggests a 3-dimensional representation of the original 2-dimensional table in which $IJ(K\text{-}1)$ of the interior cells contain structural zeros, and the original IJ frequencies are fitted by row (β_i), column (γ_j) and level (δ_k) parameters, as under a model of quasi-independence (Bishop et al., 225–26; Goodman, g, 689).

To estimate and test models of the present form we have represented mobility tables as incomplete multiway arrays and used Goodman's computer program, ECTA, to estimate frequencies by iterative rescaling and to run tests of goodness of fit (and other hypotheses).[2] Under the usual sampling assumptions, e.g., that the data were obtained by independent Poisson or simple multinomial sampling, maximum likelihood estimates are obtained in this way (Bishop et al., 206–8; Goodman, g, 663–67). The likelihood ratio test statistic produced by the program has a χ^2 distribution with degrees of freedom equal to IJ, the number of cells in the array which are not structural zeros, less the number of distinct parameters which have been estimated. In general this will be $IJ - 1 - (I\text{-}1) - (J\text{-}1) - (K\text{-}1) = (I\text{-}1)(J\text{-}1) - (K\text{-}1)$, but it may be greater, depending on the arrangement of levels within the original 2-way array (for example, see Bishop et al., 227).

In presenting goodness-of-fit tests and comparing alternative models, it is convenient to use a single letter to denote each variable. We let P = father's occupation, S = son's occupation, and H = the levels to which the several cells in the mobility table are assigned in the model. Following Goodman's (f) notation, in which the highest order marginal distributions fitted under a given model are listed in a series of parentheses, we denote our model by $(P)(S)(H)$. Written in this form it becomes clear that the model is one of statistical independence, conditional on the assignment of cells in the P by S table to levels of H. Under the model the association between P and S is spurious; no association (quasi-independence) between P and S occurs within levels of H (Goodman, g, 689). We could think of the scheme as a latent factor or latent structure model in which the levels of H are latent classes. However, the assignment of cells and hence, of observations to levels of H, is strictly deterministic, so the term "manifest class" might be more fitting.

Mobility in Great Britain

Table 3 gives frequencies in a 5 by 5 classification of son's by father's occupation in Great Britain in 1949 (Glass, 183). These data are no longer of great substantive interest, but they have been so thoroughly analyzed by students of mobility that they have become a standard set of observations against which each new model or method is calibrated. (Payne et al. argue that the Glass mobility data are invalid.) While the British data were obtained by a type of random stratified sampling, we shall proceed as if they had been obtained under simple random sampling. The classification used here combines the original categories 2 and 3 and 6 and 7 used by Glass and Hall (Svalastoga). Thus, our categories are (1) professional and high administrative; (2) managerial and executive; inspectional, supervisory and other non-manual (higher grade); (3) inspectional, supervisory and other non-manual (lower grade); (4) skilled manual and routine grades of non-manual; and (5) semi-skilled manual and unskilled manual.

Table 3. OBSERVED FREQUENCIES IN THE BRITISH MOBILITY TABLE

	Son's Occupation				
Father's Occupation	*1*	*2*	*3*	*4*	*5*
1	50	45	8	18	8
2	28	174	84	154	55
3	11	78	110	223	96
4	14	150	185	714	447
5	0	42	72	320	411

Table 4 gives the design matrix of a model for the data of Table 3. Each numeric entry in the body of the table gives the level of H_k to which the corresponding entry in the frequency table was assigned. Formally, the entries are cardinal numbers, but for convenience in interpretation the numeric values are inverse to the estimated density of mobility or immobility in the cells to which they refer.

On this understanding the design says that, aside from conditions of supply and demand, immobility is highest in category 1 (at level 1) and least in category 4 (at level 6). There is a secondary peak of immobility in the second status category (at level 3), but immobility in status category 2 is less than mobility between status categories 1 and 2 (at level 2). Finally, the third and fifth status categories (at level 4) share an intermediate level of immobility. It is worth noting that 4 of the 6 density levels recognized in the design appear along the main diagonal (1, 3, 4, and 6), and these range from the most to the least dense, excluding two intermediate density levels (2 and 5). While two of the higher density levels appearing on the diagonal (1 and 3) do not pertain to any off-diagonal cells, the two lower density levels (4 and 6) also characterize the interchange of status categories 1 and 2 with status categories 3 and 5, respectively. Thus, the design says that immobility in status categories 3 and 5 is of the same magnitude as mobility between status category 3 and either category 1 or category 2. Also, it says that immobility in status category 4 is as unlikely as long distance mobility between status category 5 and either category 1 or category 2.

Table 4. DESIGN MATRIX FOR THE BRITISH MOBILITY TABLE

Father's Occupation	Son's Occupation 1	2	3	4	5
1	1	2	4	5	6
2	2	3	4	5	6
3	4	4	4	5	5
4	5	5	5	6	5
5	6	6	5	5	4

If we ignore the distinctions between density levels 2 and 3 and between levels 5 and 6, the successively lower density zones resemble a set of concentric layers, like those of an onion, surrounding the peak of immobility in the highest status category. This pattern is broken by the relatively high level of immobility at the bottom of the status hierarchy. Still there is a broad region of low density which includes immobility among skilled workers (category 4) as well as mobility between status categories 4 and 5 and all other groups in the status hierarchy. Thus, the model says that a skilled worker is almost equally likely (or unlikely) to originate in any social stratum, and the son of a skilled worker is almost equally likely (or unlikely) to rise or fall to any other social stratum. In some respects the design matrix of Table 4 parallels Levine's (chap. 4) description of the surface of the British mobility table as a saddle. However, our interpretation is more extreme, since the density reaches an absolute minimum in the fourth category, not merely a minimum among the diagonal cells. In this way our model is closer to Goodman's (d, 39; e, 846) conclusion that the British 5 by 5 table shows "status disinheritance" in the skilled category.

Finally, the design is symmetric across the main diagonal of the mobility table; it recognizes no distinction between the chances of upward and of downward mobility. While this aspect of the design is implicitly supported by the fit of the data to this model, we have directly tested the symmetry and quasi-symmetry of the British data (Bishop et al., 282–83, 286–89). These tests do not depend on our specification of the design matrix. We reject the hypothesis of complete symmetry in the British data with a likelihood-ratio test statistic of $G^2 = 46.2$ with 10 degrees of freedom ($p < .001$). This hypothesis is more inclusive, however, than that expressed in our design matrix; it says that frequencies are equal in corresponding cells above and below the main diagonal. Thus, symmetry implies homogeneity between the occupation distributions of fathers and sons as well as equality of the interactions pertaining to upward and to downward movements within each pair of occupations. Under the latter hypothesis (quasi-symmetry) we obtain $G^2 = 11.0$ with 6 degrees of freedom, for which $.10 > p > .05$. Thus, there is very little evidence of asymmetry in the interactions in the British table. The observed inequality of frequencies above and below the main diagonal is largely due to the lack of homogeneity between occupation distributions of fathers and sons ($G^2 = 35.2$ with 4 degrees of freedom, $p < .001$).

Goodness of Fit

The model of Table 4 fits the British mobility table rather well. Under the model of statistical independence we obtain a likelihood-ratio test statistic $G^2 = 810.98$, which is asymptotically distributed as χ^2 with 16 degrees of freedom. With the model of Table 4 as null hypothesis, we obtain $G^2 = 12.13$ with 11 degrees of freedom, since we use 5 degrees of freedom in creating the 6 categories of H. The model accounts for 98.5 percent of the association in the baseline (simple independence) model, and it misclassifies only 1.1 percent of the joint distribution of father's occupation and son's occupation. While the model is associated with a rather large nominal probability ($.25 < p < .50$), it cannot be taken at face value because we obtained the model by an exploratory process, i.e., by examining residuals from other models.[3]

At the same time the model of Table 4 is relatively simple, and it compares favorably in fit with more complex models of the same mobility table. To take an extreme example, after fitting conditional beta distributions to the British table McCann (182) reports a goodness-of-fit chi-square of 41.8 with only 2 degrees of freedom. That is, we fit the data much more closely with 9 fewer parameters than McCann.[4] In modeling the British table Goodman (g, 676) also obtains a less satisfactory fit. For example, the DAF model with 12 degrees of freedom yields $G^2 = 60.6$, and no model with more than 8 degrees of freedom yields G^2 as low as 12.

Table 5. LOG OF RATIO OBSERVED TO EXPECTED FREQUENCY UNDER THE MODEL OF TABLE 4: BRITISH DATA

Father's Occupation	Son's Occupation 1	2	3	4	5
1	0.00	0.04	-0.22	-0.06	0.15
2	-0.06	0.00	0.04	-0.00	-0.02
3	0.40	-0.10	-0.01	0.05	-0.05
4	0.11	0.02	-0.03	0.00	0.00
5	*	0.07	0.08	-0.03	0.01

*Undefined.

By examining residuals we can obtain a better idea of the quality of the fit and perhaps see how to improve the model. Table 5 shows a measure of the lack of fit in each cell of the mobility classification. The residuals are expressed as natural logs of the ratios of observed frequencies to those estimated under the model:

$$\log(e_{ij}) = \log(x_{ij}/\hat{m}_{ij}) = \log x_{ij} - \log \hat{m}_{ij}, \qquad (9)$$

where x_{ij} is the observed frequency and $\hat{m}_{ij}$ is the estimated frequency in the ijth cell. As long as the residuals are small, say, less than $\pm.20$, they can be interpreted approximately as proportionate errors. Thus, expressed in this way the residuals have a convenient interpretation, and positive and negative deviations are expressed symmetrically in the metric of the (log-linear) model. For example, the entry of 0.08 in the cell (5,3) says the observed mobility from unskilled to lower non-manual occupations is $e^{.08} = 1.08$ times the mobility estimated under the model. (Unsubscripted e is the base of natural logarithms and should not be confused with the sample residuals in the multiplicative model, $e_{ij} = x_{ij}/\hat{m}_{ij}$.) The entry of -0.22 in the cell (1, 3) says mobility from stratum 1 to stratum 3 is $e^{-.22} = .80$ times the mobility estimated under the model. As suggested by these two examples, the approximation is better when the residual is small in absolute value.

Under the model of Table 4 cells (1, 1) and (2, 2) each have a unique parameter, so the zero residuals in those cells convey no new information. In only four cells of Table 5—(3, 1), (4, 1), (1, 3), and (1, 5)—are the residuals greater than .10 in absolute value. These cells contain very few observations, only 41 of the total of 3,427 observations in the full table, so small numeric errors appear as large proportionate errors. However, the juxtaposition of a large positive residual in cell (3, 1) with a large negative residual in cell (1, 3) suggests an asymmetry in the flow between the highest and lowest non-manual strata in which upward mobility is more prevalent than downward mobility. In the dense cells of the table the fit appears to be satisfactory.

One disadvantage in expressing residuals as in Table 5 is that we do not take account of their sampling variability, which depends on the expected frequencies in the cells. Also, occasional cells which contain no observations (sampling zeros) will yield ratios of observed to estimated frequencies which will equal zero, whose log is undefined, as in cell (5, 1). The latter problem can easily be treated by noting the cells which contain sampling zeros and by assigning those cells to the lowest density level of the model (unless there is a compelling theoretical reason not to do so). We can take account of sampling variability in the residuals in several ways; perhaps the simplest is to form the ratio

$$z_{ij} = (x_{ij} - \hat{m}_{ij})/\sqrt{\hat{m}_{ij}} \qquad (10)$$

which is the square root of the component of the Pearson chi-square statistic for each cell of the table. The z_{ij} are (very roughly) interpretable as unit normal deviates. Under our model only two deviates are as large as 1 in absolute value, and none is larger than 2. However, there are several more cells in the table (25) than degrees of freedom under the model (11), so the expected value of z_{ij} is less than unity (see Bishop et al., 135–51).

Level Parameters and the New Mobility Ratio

The measures of fit we have examined have told us nothing about the several parameters of the model. That is, we have not shown that our suggested interpretation of the design matrix (Table 4) is substantively appealing, or even that the design correctly sorts the cells of the mobility table into zones of high and low density. Certainly we want to look at the way in which the model fits and interprets the data as well as at deviations from fitted values.

The upper panel of Table 6 shows the row, column and level parameters estimated under the model of Table 4 for the British mobility table. The parameters are expressed in additive form, that is, they are effects on log frequencies under the model of equation 6. The row and column parameters show much the same pattern among fathers and sons, clearly indicating the larger relative numbers of manual relative to non-manual origins and destinations. Of course, these parameters reflect a number of factors, including temporal shifts in the distribution of the labor force across occupations, differential fertility, and life cycle differences in occupational positions.

Table 6. PARAMETERS AND RESIDUALS (IN ADDITIVE FORM) FROM MAIN, ROW, AND, COLUMN EFFECTS IN THE MODEL OF TABLE 4: BRITISH MOBILITY TABLE

A. *Additive Parameters*

	Category of Row, Column or Level					
Design factor	*1*	*2*	*3*	*4*	*5*	*6*
Rows (father's occupation)	-2.250	-.158	.161	1.647	.600	--
Columns (son's occupation)	-2.613	-.154	.105	1.702	.960	--
Levels (density)	4.603	1.995	1.298	.274	-.678	-.952

Grand mean = 4.174

B. *Level Parameter Plus Residual* (*log* R^*_{ij})

	Son's Occupation				
Father's Occupation	*1*	*2*	*3*	*4*	*5*
1	4.60	2.04	.05	-.73	-.80
2	1.93	1.30	.31	-.68	-.97
3	.68	.18	.26	-.63	-.73
4	-.57	-.66	-.70	-.95	-.68
5	0.00	-.88	-.60	-.71	.28

The level parameters show very large differences in mobility and immobility across the several cells of the classification, and these differences closely follow our interpretation of the design matrix. Differences between parameters for different cells in the table may readily be interpreted as differences in the log frequency, net of row and column effects. For example, the estimates say that the immobility in the first stratum is 5.56 = 4.603 − (−.952) greater (in the metric of logged frequencies) than the estimated mobility or immobility in cells assigned to level 6 in the design matrix, e.g., in cell (4, 4). In multiplicative terms, immobility in the first stratum is $e^{5.56} = 258.5$ times greater than mobility or immobility at level 6. The parameters do show a very sharp density gradient across the levels of the design. The smallest difference, between levels 5 and 6, indicates a relative density $e^{-.678-(-.952)} = e^{.274} = 1.315$ times as great at level 5 than at level 6.[5]

It would be incorrect to attach too much importance to the signs of the level parameters as reported in Table 6, for they simply reflect our normalization rule that level parameters sum to zero (in the log-frequency metric) across the cells of the table. For example, while the parameter for level 6 reflects relatively low density, it is not clear that it indicates "status disinheritance" in the diagonal cells to which it pertains (compare Goodman, d, e).

We can write the sample counterpart of equation 5 as

$$\hat{m}_{ij} = \hat{\alpha}\hat{\beta}_i\hat{\gamma}_i\hat{\delta}_{ij}. \tag{11}$$

Recalling from equation 9 that

$$e_{ij} = x_{ij}/\hat{m}_{ij}, \tag{12}$$

we substitute 11 in 12 and rearrange terms to obtain

$$x^*_{ij} = \hat{\alpha}\hat{\beta}_i\hat{\gamma}_j\hat{\delta}_{ij}e_{ij}. \tag{13}$$

We divide both sides of equation 13 by the first 3 terms on the right hand side to obtain

$$R^*_{ij} = x_{ij}/\hat{\alpha}\hat{\beta}_i\hat{\gamma}_j. \tag{14}$$

We shall call R^*_{ij} the new mobility ratio, or, simply, the mobility ratio. In the case of diagonal cells R^*_{ij} is equivalent to the new immobility ratio proposed by Goodman (d, e, g; also, see Pullum, 7–8), but we suggest the ratio be computed for all cells of the table as an aid both to substantive interpretation and to the evaluation of model design.

The lower panel of Table 6 gives logs of the new mobility ratios for the model of Table 4 fitted to the British mobility table. While the entries in this panel depend on our specification of the model, neither need that specification rigidly govern our interpretation of the relative densities. With the possible exception of mild heterogeneity at level 4, the logs of the new mobility ratios do correspond very closely with our interpretation of the design matrix and with the estimated parameters. The fit is good enough so there is no overlap in densities across levels recognized in the design, and all of the negative entries are segregated in levels 5 and 6 of the design. These residuals complement our interpretation of the within-level deviations in Table 5 by confirming our allocation of cells to levels. In a model that fits well, the array of mobility ratios adds little to what we know from the design matrix and the parameters. However, it may still be useful in comparative analyses or in efforts to reduce the number of parameters used to fit the data. In exploratory analyses we have found it useful to proceed iteratively by fitting a table with a design matrix specified a priori and then using the new mobility ratios as a guide to a better specification.

Conceptually, R^*_{ij} is related to R_{ij}, Rogoff's social distance mobility ratio and Glass's index of association. Both R_{ij} and R^*_{ij} may be interpreted as ratios of observed counts to those estimated from a scale factor and row and column effects under a given statistical model (compare equations 1 and 14). The important difference between R_{ij} and R^*_{ij} is that the new mobility ratio is obtained from a model which fits the data, so row and column effects are not confounded with relative densities (interactions) in the interior of the table. For these reasons R^*_{ij} does not have the undesirable properties of R_{ij}. In general, (1) R^*_{ij} is not bounded; (2) in a square table the set of R^*_{ij} do not determine the marginal frequencies (nor the marginal effects); and (3) the set of R^*_{ij} can be symmetric, i.e., $R^*_{ij} = R_{ji}$, under any set of marginal frequencies (or effects), but the set of R_{ij} can only be symmetric when the observed frequencies are symmetric, i.e., $x_{ij} = x_{ji}$ for all i and j (compare Tyree, 577–80).

In this context it is instructive to show the relationship between the parameters of the multiplicative model and the marginal frequencies of the mobility table. The model fits the observed marginal frequency distributions, that is, $\sum_i \hat{m}_{ij} = x_{.j}$ and $\sum_j \hat{m}_{ij} = x_{i.}$, so from equation 11

$$\sum_i \hat{m}_{ij} = \sum_i \hat{\alpha} \hat{\beta}_i \hat{\gamma}_j \hat{\delta}_{ij} = \hat{\alpha} \hat{\gamma}_j \sum_i \hat{\beta}_i \hat{\delta}_{ij} = x_{.j} \tag{15}$$

and

$$\sum_j \hat{m}_{ij} = \sum_j \hat{\alpha} \hat{\beta}_i \hat{\gamma}_j \hat{\delta}_{ij} = \hat{\alpha} \hat{\beta}_i \sum_j \hat{\gamma}_j \hat{\delta}_{ij} = x_{i.}\,. \tag{16}$$

Thus, the marginal frequency in a given column (or row) is the product of the corresponding column (or row) effect, a scale factor, and a weighted sum of the row (or column) effects, where the weights are the interaction effects for corresponding rows (or columns) within the given column (or row).

Similarly, from equations 13 and 14 we can write

$$\sum_i x_{ij} = \hat{\alpha}\hat{\gamma}_j \sum_i \hat{\beta}_i \hat{\delta}_{ij} e_{ij} = \hat{\alpha}\hat{\gamma}_j \sum_i \hat{\beta}_i R^*_{ij} = x_{.j} \qquad (17)$$

and

$$\sum_j x_{ij} = \hat{\alpha}\hat{\beta}_i \sum_j \hat{\gamma}_j \hat{\delta}_{ij} e_{ij} = \hat{\alpha}\hat{\beta}_i \sum_j \hat{\gamma}_j R^*_{ij} = x_{i.}, \qquad (18)$$

so we may also think of the new mobility ratios as weights in the expressions relating marginal frequencies to corresponding marginal effects. We can write expressions in the old mobility ratios, R_{ij}, which are formally similar to equations 17 and 18; however, those expressions can be simplified to eliminate the R_{ij}, while equations 17 and 18 cannot be simplified to eliminate the R^*_{ij}. For example, from the definition of R_{ij}

$$x_{ij} = (1/N) x_{i.} x_{.j} R_{ij}, \qquad (19)$$

so

$$\sum_i x_{ij} = (x_{.j}/N) \sum_i x_{i.} R_{ij} = x_{.j}, \qquad (20)$$

but we know from equation 4 that $\sum_i x_{i.} R_{ij} = N$.

Suppose it were possible to solve for the marginal effects by writing linear equations in the R^*_{ij}, so (following Blau and Duncan, 93–4):

$$\sum_i \hat{\beta}_i R^*_{ij} = m \quad \text{for all } j \qquad (21)$$

and

$$\sum_j \hat{\gamma}_j R^*_{ij} = n \quad \text{for all } i. \qquad (22)$$

Under these conditions we could rewrite equations 17 and 18, respectively, as

$$x_{.j} = \hat{\alpha}\hat{\gamma}_j m \qquad (23)$$

and

$$x_{i.} = \hat{\alpha}\hat{\beta}_i n. \qquad (24)$$

That is, if the mobility ratios determine the marginal effects, then $x_{.j}$ is just a scalar multiple of $\hat{\gamma}_j$, and $x_{i.}$ is just a scalar multiple of $\hat{\beta}_i$, which is to imply that the model is indistinguishable from the simple independence model and R^*_{ij} is indistinguishable from R_{ij}. But in general our model is not that of simple independence; R^*_{ij} is not equal to R_{ij}; and our row and column parameters are not scalar multiples of the marginal frequencies. This says that the row and column effects under our model are not generally determined by the R^*_{ij}. In summary, the new mobility ratios appear to have properties which make them useful in the specification of models of the mobility table, in the interpretation of mobility tables, and in the comparison of mobility tables.

Discussion: Mobility Ratios and Other Measures of Association

We have directed our criticisms of mobility indexes primarily at the old mobility ratio, R_{ij}, but several other common measures of association also fail to elucidate the pattern of association in the mobility table for much the same reason that R_{ij} is defective. For reference purposes Table 7 gives the new mobility ratios (R^*_{ij}) for the model of Table 4; the entries are just the anti-logs of the entries in the lower panel of Table 6. This table provides no new information, but it may be helpful in evaluating other measures of association when the latter are presented in multiplicative form.

Table 7. NEW MOBILITY RATIOS (R^*_{ij}): BRITISH MOBILITY TABLE

Father's Occupation	Son's Occupation 1	2	3	4	5
1	99.78	7.67	1.05	.48	.45
2	6.89	3.66	1.36	.51	.38
3	1.97	1.19	1.30	.53	.48
4	.57	.52	.49	.39	.51
5	0.00	.41	.55	.49	1.33

OLD MOBILITY RATIOS

Table 8 gives the old mobility ratios (R_{ij}) for the British data under the model of simple statistical independence. Clearly, one need not resort to hypothetical data to show differences between interpretations based on

Table 8. SOCIAL DISTANCE MOBILITY RATIOS (R_{ij}): BRITISH MOBILITY TABLE

Father's Occupation	Son's Occupation 1	2	3	4	5
1	13.16	2.49	.47	.34	.21
2	1.92	2.51	1.29	.76	.38
3	.72	1.08	1.62	1.05	.64
4	.31	.71	.93	1.16	1.02
5	.00	.36	.65	.93	1.67

the old and new mobility ratios. The entries in Table 8 are similar to those which any experienced student of mobility has encountered on many occasions; for example, see Pullum's (3–7) description of the British table. From the R_{ij} one would conclude (correctly) that there is substantial immobility at both the top and the bottom of the occupational hierarchy, but far less at the top and somewhat more at the bottom than is indicated by the R^*_{ij}. The R_{ij} show a decline in immobility as one goes from the first to the fourth stratum along the diagonal. A similar monotonic pattern appears among the R^*_{ij}, but the decline is much sharper. Under our normalization of parameters immobility among skilled manual workers is actually much less than that expected from row, column and scale effects; it is as rare as very long-distance moves upward or downward in the status hierarchy. Further, while the R_{ij} show rates of interchange between the two highest strata which are comparable in volume to immobility in the second stratum, the R^*_{ij} show immobility in stratum 2 to be about half as great as upward or downward mobility between the first or second stratum. Last, the R_{ij} decline regularly as one moves away from the main diagonal in any row or column, suggesting a unidimensional scaling of inter-occupational distances. In contrast the R^*_{ij} show abrupt shifts between levels of the design and some irregularity within them. For example, there is a very low density in cell (4, 4), and the other R^*_{ij} in row 4 and column 4 are all slightly larger, but show no other pattern. The son of a skilled worker is least likely to enter a skilled trade, but he is little more likely to enter any other specific stratum. A similar statement holds for the recruitment of skilled workers. It may be instructive with regard to the conceptual distinction between social distance and similarity that the skilled occupations are distant from all occupations, including the skilled trades, while the selection and recruitment of skilled workers resemble those of semi-skilled and unskilled workers in important respects.

We can clarify the relationship between the R_{ij} and R^*_{ij} by expressing the former in terms of the latter. By definition

$$R_{ij} = x_{ij}N / x_{i.}x_{.j}. \tag{25}$$

Under the model of equation 5 and from equations 13 and 14 we can write

$$x_{ij} = \hat{\alpha}\hat{\beta}_i\hat{\gamma}_j R^*_{ij} , \tag{26}$$

so

$$N = \textstyle\sum_i \sum_j x_{ij} = \hat{\alpha} \sum_i \sum_j \hat{\beta}_i \hat{\gamma}_j R^*_{ij} . \tag{27}$$

By substitution from equations 26, 27, 17, and 18 we can rewrite equation 25 as

$$\begin{aligned} R_{ij} &= (\hat{\alpha}\hat{\beta}_i\hat{\gamma}_j R^*_{ij})(\hat{\alpha}\textstyle\sum_i\sum_j\hat{\beta}_i\hat{\gamma}_j R^*_{ij}) / (\hat{\alpha}\hat{\beta}_i\sum_j\hat{\gamma}_j R^*_{ij})(\hat{\alpha}\hat{\gamma}_j\sum_i\hat{\beta}_i R^*_{ij}) \\ &= R^*_{ij}(\textstyle\sum_i\sum_j\hat{\beta}_i\hat{\gamma}_j R^*_{ij}) / (\sum_j\hat{\gamma}_j R^*_{ij})(\sum_i\hat{\beta}_i R^*_{ij}). \end{aligned} \tag{28}$$

The double sum in the numerator of equation 28 is a scale factor which does not vary with the indexes i and j. Thus, the variable parts of the expression say that R^*_{ij} is related to R_{ij} inversely as the product of weighted averages of the column and of the row parameters, whose respective weights are the new mobility ratios in the ith row and the jth column. In general R_{ij} will be low, relative to R^*_{ij}, when the new mobility ratios in the ith row and the jth column are large, and R_{ij} will be high, relative to R_{ij}^*, when the new mobility ratios in the ith row and the jth column are small. For example, the relatively large value of R_{44}, the old immobility ratio for upper non-manual (skilled) occupations, is explained by the very low levels of association throughout the fourth row and the fourth column of the table (when that association is indexed by R^*_{ij}). In general a given row and column need not contain only high or only low R^*_{ij}, and the relationship of R_{ij} and R^*_{ij} will vary among cells in the mobility classification.

PARAMETERS OF THE SATURATED LOG-LINEAR MODEL

Like the specification of equation 6, the conventional parametric representation of the log-linear model also describes frequencies in terms of parameters for row effects, column effects, and interaction effects. Moreover, we can "saturate" the model by including all main effects and interactions, thus fitting observed counts perfectly. Some expert social statisticians have suggested to the author that interaction parameters under the saturated log-linear model would yield, by inspection alone, substantially the same interpretation as that obtained using our (tedious) methods. However, the usual normalization of parameters of the log-linear model gives priority to row and column effects relative to interactions, that is, relative to association in the interior cells of the table. Even though the saturated log-linear model fits a table completely, this conventional normalization of parameters has much the same effect on the pattern of interaction parameters as the assumption of statistical independence has on the pattern of old mobility ratios. Consequently, under the saturated log-linear model the multiplicative parameters for the interactions are no more informative than the residuals from fit under simple independence. The marginal effects are too large in rows or columns where the interactions are strong, and the marginal effects are too small where the interactions are weak. Obversely, the estimated interactions are deflated or inflated relative to a model in which row, column, and interaction effects are given equal priority.

An algebraic presentation of the conventional log-linear model may clarify this argument. Let

$$l_{ij} = \log m_{ij}, \tag{29}$$

where m_{ij} is the expected count in the ijth cell, and $\sum_i \sum_j m_{ij} = N$. The saturated log-linear model says that

$$l_{ij} = u + u_{1(i)} + u_{2(j)} + u_{12(ij)}, \tag{30}$$

subject to the constraints

$$\textstyle\sum_i u_{1(i)} = \sum_j u_{2(j)} = \sum_i u_{12(ij)} = \sum_j u_{12(ij)} = 0. \tag{31}$$

Under these constraints the u-terms are obtained by a row and column decomposition of the logs of expected frequencies paralleling that in a two-way analysis of variance with one observation per cell (Bishop et al., 24):

$$u = \textstyle\sum_i \sum_j l_{ij} / IJ, \tag{32}$$

$$u_{1(i)} = (\textstyle\sum_j l_{ij} / J) - u, \tag{33}$$

$$u_{2(j)} = (\textstyle\sum_i l_{ij} / I) - u, \tag{34}$$

and

$$u_{12(ij)} = l_{ij} - (u_{1(i)} + u_{2(j)}) + u. \tag{35}$$

Note that equation 30 and equation 6 are identical, and the constraints on row and column parameters are the same in the conventional model and in ours. The important difference between the models of equations 6 and 30 lies in the constraints on the interaction parameters ($u_{12(ij)}$) and in the implications of those constraints for the specification of equalities among subsets of interaction parameters. The specification that the interaction parameters sum to zero within every row and within every column of the table (see equation 31) is equivalent to the model of simple independence in its implications for interpreting the pattern of association in the table. By relaxing the normalization of the $u_{12(ij)}$ in equation 31 we obtain new insights into the pattern of association in the table.

The estimated multiplicative parameters of the British table under the saturated model are given in Table 9. These tell essentially the same story as the old mobility ratios in Table 8, but they are a slight improvement over the old mobility ratios. Note that the coefficients in cells (1, 1) and (5, 5) are larger than the old mobility ratios. The improvement occurs because the row and column effects under statistical independence are based on sums of frequencies, while they are based on sums of logs of frequencies in the case of the saturated log-linear model. The operation of taking logs reduces the effect of positive outliers on the row and column sums, and so reduces (but does not eliminate) the influence of the small number of large interactions, i.e., of the positive skew of frequencies, on the row and column effects. However, the array of parameters in Table 9 is still substantially misleading in respect to the pattern of association in the British data.

Table 9. MULTIPLICATIVE (τ) PARAMETERS UNDER SATURATED MODEL: BRITISH MOBILITY TABLE

Father's Occupation	Son's Occupation 1	2	3	4	5
1	29.33	1.48	.35	.29	.23
2	3.90	1.36	.86	.59	.37
3	1.71	.68	1.26	.94	.73
4	.95	.57	.93	1.33	1.48
5	.00	1.27	2.86	4.71	10.81

MARGINAL ADJUSTMENT

We shall consider one other method of inspecting the pattern of association in the full mobility table. Mosteller drew attention to Levine's use of iterative proportional rescaling to adjust British and Danish 5 by 5 mobility tables to uniform marginal distributions. This adjustment facilitated Levine's interpretation of the mobility tables and exposed similarities in the pattern of association in the two tables.[6] The iterative proportional rescaling procedure is generally attributed to Deming and Stephan, who suggested it be used to adjust sample cross-classifications to known marginal distributions; the same procedure is used to obtain maximum-likelihood estimates of frequencies in log-linear models of contingency tables when no closed-form estimates exist (Bishop et al., 83–7). The method has been applied frequently in recent years.

The adjustment procedure is straightforward. Each row entry is multiplied by the ratio of the desired row sum to the actual row sum. Then each column entry is multiplied by the ratio of the desired column sum to the actual column sum. By alternating row and column adjustments, convergence of the adjusted cell counts to both desired marginal totals is usually obtained within a few iterations. Multiplicative adjustment preserves the initial pattern of association in a table because odds-ratios are invariant to scalar transformations applied uniformly across rows and columns. For example, the upper panel of Table 10 gives hypothetical frequencies in a 2 by 2 mobility table. The odds-ratio in this table is

$$(x_{11}/x_{12})/(x_{21}/x_{22}) = (x_{11}/x_{21})/(x_{12}/x_{22}) = (x_{11}\,x_{22})/(x_{21}\,x_{12}). \qquad (36)$$

Suppose the frequencies in interior rows 1 and 2 of the table are multiplied by arbitrary non-zero constants, say, a and b, respectively. Likewise, the frequencies in the interior columns of the table are multiplied by non-zero constants c and d. The adjusted frequencies are shown in the lower panel of Table 10. Under this transformation the marginal proportions will not generally be preserved, but the odds-ratio will be unaffected, for

$$(ac\,x_{11})(bd\,x_{22})/(ad\,x_{12})(bc\,x_{21}) = (x_{11}x_{22})/(x_{12}x_{21}). \qquad (37)$$

As early as 1912 Yule recognized the desirability of constructing measures of association with this invariance property (Goodman and Kruskal). In fact both the old (R_{ij}) and the new (R^{*}_{ij}) mobility ratios have this invariance property, i.e., that they preserve the odds-ratios in the observed frequencies; this is obvious from inspection of equations 25 and 26.

In recommending adjustment to uniform marginals Mosteller implied that the adjusted frequencies elucidated the pattern of association in a table:

. . . we can interpret the resulting numbers as transitional or conditional probabilities expressed in per cents—either son's distribution given the father's category, or father's given the son's. . . . In the sense of having a common nucleus of association . . . it would be fair to say that the two occupational tables are nearly equivalent (8).

Similarly, in respect to the same example, Bishop et al. write

By comparing diagonal values we see that, except for category 1, the tendency for fathers and sons to fall into the same category is stronger in Denmark than Britain. By looking across rows we can see, for fathers in each category, in which country the sons are more mobile (100).

Again, Fienberg suggests that standardization to uniform marginals permits one "to look at the association or interaction, unconfounded by the two sets of marginal allocations" (c, 308).

Table 10. RAW AND MULTIPLICATIVEY ADJUSTED FREQUENCIES IN A HYPOTHETICAL TABLE

	Columns		
Rows	*1*	*2*	*Total*
A. *Raw frequencies*			
1.	x_{11}	x_{12}	$x_{11} + x_{12}$
2.	x_{21}	x_{22}	$x_{21} + x_{22}$
Total	$x_{11} + x_{21}$	$x_{12} + x_{22}$	
B. *Adjusted frequencies*			
1.	acx_{11}	adx_{12}	$a(cx_{11} + dx_{12})$
2.	bcx_{21}	bdx_{22}	$b(cx_{21} + dx_{22})$
Total	$c(ax_{11} + bx_{21})$	$d(ax_{12} + bx_{22})$	$c(ax_{11} + bx_{21})$ $+ d(ax_{12} + bx_{22})$

While it is strictly correct that the original marginal frequencies of a table cannot be deduced from the set of frequencies adjusted to uniform marginals, neither do the adjusted frequencies display the pattern of association in the sense intended here. For example, Table 11 gives the adjusted

Table 11. OBSERVED FREQUENCIES ADJUSTED TO EQUIPROPORTIONAL MARGINALS: BRITISH MOBILITY TABLE

Father's Occupation	Son's Occupation 1	2	3	4	5
1	3.46	1.03	.23	.18	.12
2	.92	1.87	1.12	.73	.37
3	.42	.97	1.68	1.21	.74
4	.21	.74	1.13	1.55	1.37
5	.00	.40	.85	1.34	2.42

frequencies of the data in Table 3. We chose uniform marginal sums of 5, so the condition of simple independence would yield an entry of unity in each cell. The pattern of marginally adjusted frequencies in Table 11 is substantially the same as that of the old mobility ratios in Table 8, and it is markedly different from the pattern of the new mobility ratios in Table 7. If the marginal adjustment eliminates the effects of variation in the marginal distributions, why doesn't it uncover the underlying pattern of association in the table? The problem lies in the distinction between marginal distributions ($x_{i.}, x_{.j}$) and marginal effects (β_i, γ_j); recall equations 15 and 16. Equalization of the marginal distributions does not equalize the marginal effects; the former differ from the latter because they are confounded with the underlying pattern of interaction in the table. For example, consider again the fourth row or the fourth column of Table 3 (skilled manual occupations). Because the interactions are weak in that row and column, the marginal proportions are relatively low. Consequently the adjustment procedure induces too large a relative increase in the marginal frequencies in that row and column, leading to an excessively large adjusted entry in the (4, 4) cell.

Similarly, in analyzing the British and Danish mobility tables Levine adjusted the frequencies to uniform marginals, took logs of the adjusted frequencies, and fitted curves to the logs of adjusted frequencies. Levine's model is flawed because the initial multiplicative adjustment did not reveal the underlying pattern of association in the British and Danish tables. However, if adjustment of the full table to uniform marginals does not yield the underlying interaction structure, neither is it valueless. The procedure can be used as a rough guide to similarity or dissimilarity in the odds-ratios of two or more tables, even though it does not provide a satisfactory picture of the pattern of association in any one classification.

In summary, we have evaluated measures of association which are based on the model of simple independence and other measures which are based on the saturated model. There are real differences among these measures of association. At the same time, each of the measures we have reviewed suggests essentially the same interpretation of the pattern of association in the mobility table. This interpretation is in each case fundamentally different from that suggested by the new mobility ratios (R^{*}_{ij}). This difference occurs primarily because the other measures of association confound the prevalence effects of rows and columns with the interaction structure in the interior cells of the table.

Conclusions

There are several important features of our model which we have overlooked in the present discussion. We have not discussed how one obtains a design matrix with which to begin an analysis, nor have we shown how the new mobility ratios may be used as a guide in revising a design matrix. It is best to begin with a good theory, but simple models of quasi-independence may be used in the absence of a strong theory to obtain residuals which have diagnostic value. It is possible to program a computer to proceed from an initial design matrix to a "best fitting" model, subject to certain rules for creating categories and drawing boundaries between them. There are also straightforward statistical methods for the measurement of heterogeneity within levels of a design. However, substantial caution in the use of mechanical fitting methods is advisable, for they will tend to fit chance fluctuations in sparse data; the use of smoothed data is sometimes helpful.

Models of the present type may be used to measure similarity among occupations and distances between them. They may also be used to measure changes in mobility. We have focused the present discussion on patterns of interaction within a single classification, but our methods can easily be extended to comparisons of mobility tables in an exploratory or confirmatory mode. Moreover, the marginal effects as well as interactions may be of substantive interest. For example, one might wish to show how exogenous economic or technological factors affect the prevalence of occupations. Finally, related methods may be used in the analysis of multidimensional mobility tables, such as classifications of men by their current occupation, first occupation, and fathers' occupation.

In sum we believe that models like those described here are a powerful tool for the analysis of occupational mobility tables. They can be used in exploratory or confirmatory analyses, and in either case we believe they may yield new and well-founded theoretical insights into processes of mobility. Of course, the application of these models need not be limited to mobility data, but may be extended to other instances where a similar specification of the model appears plausible.

Notes

1. We assume the familiarity of the reader with log-linear models for frequency data. Fienberg (a) and Goodman (h, i) give useful introductions, as does the comprehensive treatise by Bishop et al. We rely heavily on methods for the analysis of incomplete tables which have been developed by Goodman (c, d, g), Bishop and Fienberg, Fienberg (b, d), and Mantel; again, Bishop et al. (especially 206–11, 225–28, 282–309, 320–24) is valuable.
2. ECTA does not compute parameters for incomplete designs, nor does it compute their degrees of freedom correctly. We estimated parameters from expected frequencies using a program for multiple regression analysis.
3. We shall describe these exploratory methods elsewhere. The reader should bear in mind that the design in Table 4 is intended to illustrate a class of models; we are not proposing this design as a general description of mobility tables.
4. An even simpler model fits almost as well as that of Table 4. If we combine levels 2 with 3 and 5 with 6, we obtain $G^2 = 49.04$ with 13 degrees of freedom and account for 94 percent of the association under the baseline model. This test statistic has a low probability ($p < .001$), but given the large sample, we would expect small departures from frequencies estimated under the model to be statistically significant.
5. Exact tests of the difference between any two level parameters can be carried out in a straightforward way. Modify the design matrix to combine the two groups to be contrasted in a single level, and fit the revised model. Since the revised model is a special case of (nested within) the initial model, the difference between the likelihood-ratio test statistics (G^2) of the two models will asymptotically be distributed as χ^2 with 1 degree of freedom.
6. For further evidence and discussion of the similarity of the British and Danish tables see Goodman (d, e), and Bishop et al.

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