

1 *A study of film thickness and hydrodynamic entrance length in*
2 *liquid laminar film flow along a vertical tube*

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Abstract

The liquid film thickness and hydrodynamic entrance length in a vertical tube was studied experimentally and numerically. Measurements using distilled water, 30 wt% MEA and 40 wt% sugar solutions were carried out to investigate the effects of liquid flow rate on the formation of the liquid film. The experimental results validate the new Navier-Stokes based equation in cylindrical coordinates (Eq.16) and the volume of fluid (VOF) model giving a competitively high prediction of the liquid film thickness especially in the low Reynolds number region. In addition, a new empirical model and an improved minimal surface model have been firstly proposed for calculation of the hydrodynamic entrance length, with a relatively reasonable average absolute relative deviation (AARD) of 3.03% and 6.83%, respectively. Furthermore, the effects of the hydrodynamic entry length on the gas-liquid interfacial area calculated by the improved minimal surface model were comprehensively studied, and can be ignored if the ratio of the liquid film length (y) and the hydrodynamic entrance length (λ_E) is lower than 10. However, it should be noted that the hydrodynamic entrance length cannot be ignored in packed columns in which the liquid flow is very complex due to the packings with different structures and materials.

Keywords: Vertical tube; Film thickness; Hydrodynamic entrance length; Falling film.

1 Introduction

45 Liquid film flow, which has been widely applied for gas-liquid contactors such as
46 tray towers and packed columns, has attracted much attention since the last century.
47 Liquid film thickness is one of the key parameters which has high energy and mass
48 transfer potential due respectively to its high latent heat and mass transfer area.
49 Therefore, it is essential to understand the mechanism of liquid film flow along a
50 vertical tube.

51 Theoretically, the fluid flow along the vertical tube can be divided into three parts:
52 the entrance flow region, the fully developed flow region, and the uniform flow region
53 (downstream of the asymptotic limit). The entrance flow region, including both the
54 thermal and hydrodynamic entrance regions, is vitally important for heat and mass
55 transfer in a gas-liquid contactor, especially for lab scale apparatus^[1]. In addition, it can
56 increase the pressure drag and create skin friction drag which affects the characteristics
57 of the flow. It is reported that the length of the entrance flow region is one of the
58 important factors used to accurately predict the liquid velocity profiles, boundary layer
59 expanding, and gas-liquid contact area. This length is termed as the hydrodynamic
60 entrance length, which can be seen as the intersection point where the film thickness is
61 equal to the boundary layer thickness. Equally, researchers have focused on studies of
62 this developing region of the liquid film since last century. The dimensionless
63 hydrodynamic entrance length of a liquid film has been firstly studied based on different
64 assumed liquid velocity profiles. Andersson et al.^[2] proposed a semi-parabolic velocity
65 profile to describe a dilatant fluid, as well as a sinusoidal velocity profile and a third-
66 order polynomial approach to describe a Newtonian fluid. Ruschak et al.^[3] and Tekić^[4]

67 calculated the dimensionless hydrodynamic entrance length of a falling liquid film
68 based on a semi-parabolic velocity profile. Roy et al. [5] also calculated the
69 dimensionless hydrodynamic entrance length based on a third-degree polynomial
70 velocity profile which was solved by a Runge-Kutta method to determine the boundary
71 layer thicknesses. Then, the dimensionless hydrodynamic entrance length of a laminar
72 thin-film flow along a vertical plate was determined based on a fourth-degree
73 polynomial velocity profile by Schlichting et al.[6]. Trela et al.[7] proposed that the
74 relationship between dimensionless entrance length with the initial film thickness, h_0 ,
75 in the range of 0.1 to 2 was based on a similar parabolic velocity profile, and indicated
76 that the initial film thickness could be the only parameter that affects the dimensionless
77 entrance length^[7]. However, most of the existing empirical or semi-empirical models
78 for dimensionless hydrodynamic entrance length are based on a key parameter $\beta_0=h_0/h_m$
79 (h_0 and h_m representing the initial film thickness and average film thickness,
80 respectively)^[2, 8] without discussing the relationship between the hydrodynamic
81 entrance length and Reynolds number. Furthermore, experiments were done using only
82 a specific fluid (such as water) which might limit their application. In this investigation,
83 the relationship between hydrodynamic entrance length with Reynolds number and
84 surface tension have been studied and discussed to provide better insight to the
85 hydrodynamic entrance length exploration.

86 Furthermore, film thickness is one of the most important parameters for calculating
87 the hydrodynamic entrance length, as well as a prime and vital parameter governing
88 heat and mass transfer area^{[9] [10]}. Besides, the hydrodynamic behavior of film thickness

89 can also be used to define the transition region between laminar and turbulent flow
90 regimes. Many methods have been used to measure the film thickness of liquid film
91 flow, which could be divided into two main types: direct method and indirect method.
92 Measurements with a micrometer screw or probe are normally taken as direct
93 methods^[11]. The measurements by using radioactive tracer^[12], electrical capacitance^[13]
94 and shadow photographs^[14] are the widely used indirect methods.

95 Many researchers have proposed various models for calculating the film thickness
96 in terms of theory. Nusselt was the first to propose equations for the falling film flow
97 along a vertical tube^[15] but these were found to fail to predict the film thickness
98 correctly, even for steady laminar conditions of flow^[16]. Kapitsa has proposed a model
99 for predicting the film thickness on a plate^[17], but it fails to predict the film thickness
100 for turbulent flow^[18]. Then, Bird et al.^[19] gave a correlation to calculate the liquid film
101 thickness along a vertical plate^[19], and this has become the most commonly used
102 equation. Also, Grigoreva et al.^[20], and Grossman et al.^[21] have presented a series of
103 equations to calculate the film thickness for flow along a vertical tube. Min et al.^[22]
104 made a comparison of those different equations, and found that all of them show
105 inconsistency. Later, Hassan et al.^[23] proposed a direct relation between the film
106 thickness and the distance traveled by the liquid flow, while Murty et al.^[24] presented
107 an analysis about the flow along an inclined wall that took into consideration the
108 interfacial shear. Although both experimental methods and theories have significantly
109 improved during the past years, there is still little literature about the investigation of
110 liquid film thickness of flow along a cylindrical tube.

111 In this study, the flow along a cylindrical tube with free surface is firstly
112 investigated experimentally and numerically. The VOF model in CFD software is used
113 to simulate the film flow, because with it, one can obtain both the film thickness and
114 hydrodynamic entrance length^[25]. The comparisons between simulation and
115 experimental results are made in order to explore the characteristics of the flow along
116 a cylindrical tube. Finally, both an empirical equation and a minimal surface model are
117 proposed and developed to predict the hydrodynamic entrance length, and the effects
118 on the gas-liquid contact area.

119 **2 Experimental setup and procedures**

120 **2.1 Experimental apparatus**

121 In this work, distilled water, 30wt% MEA solution and 40wt% sugar solution were
122 used as working fluids, and their physical properties of density, viscosity and surface
123 tension are presented in Table 1. A vertical cylinder contactor made of polished stainless
124 steel with outer diameter of 10 mm and total exposed length of 75 mm was used for the
125 experimental tests, as shown in Figure 1. The contactor is placed horizontally in an open
126 environment and operated with liquid flow entering at the top, passing through a liquid
127 distributor inside the apparatus and flowing to the bottom along the stainless steel
128 vertical cylinder with its free surface in stationary air. Additionally, a high speed camera
129 with a frame rate of 10000fps, shutter of 1/142000s, resolution of 256 × 256px, and
130 calibration of 8.3±0.1µm/px was used to take the frames of liquid film. A sequence of
131 frames was obtained in five different positions from the vertical cylinder contactor
132 along the liquid flow. The schematic diagram of the experimental setup for hydraulic

133 experiment is presented in Figure 2.

134 **Table 1.**

135 **Figure 1.**

136 **Figure 2.**

137 **2.2 Experimental producer**

138 In order to have good and reproducible laminar flow along the vertical cylinder
139 contactor, a set of hydraulic experiments were performed at 298.15K and ambient
140 pressure to obtain the limits for the laminar flow region, wavy surface region, and the
141 turbulent region. Figure 2 shows the diagram of hydraulic experiments by using the
142 shadow-photographs method, which involves a high speed camera to test the hydraulic
143 phenomena in different systems. The shadow pictures obtained from the high speed
144 camera show the liquid film phenomena, which includes the variations in the liquid film
145 thickness as a function of liquid flow rate, as illustrated in Figure 3.

146 **Figure 3.**

147 As shown in Figure 3a, which is the laminar flow at the top of the cylinder near the
148 entrance at a very low liquid flow rate, the hydrodynamic entrance effect obviously
149 does exist. However, the hydrodynamic entry length only takes no more than 5% of the
150 whole exposed length and can be considered to be negligible. Figure 3b shows the
151 laminar flow in the other parts of the cylinder from which the film thickness of the
152 laminar flow can be accurately acquired. However, a rippling behavior occurs as the
153 Reynolds number increased as shown in Figure 3c. It is important to avoid the waving
154 effects when the film flow is being studied at the gas liquid absorption interface since
155 it greatly influences the contact area, and makes the film thickness to become non-

156 uniform. Consequently, the waving behavior of the liquid film was avoided in this study,
157 and the Reynolds number which causes the rippling of the liquid film is seen as a critical
158 point between the laminar flow region and the transition region.

159 The width, measured as pixel points of liquid film shadows at various liquid flow
160 rates can be obtained when the distance between the cylinder and high speed camera is
161 fixed. The diameter of the stainless steel cylinder is known as d_0 and its pixel point pp_0 ,
162 can be obtained by using an Image Software. Then, the film thickness can be easily
163 calculated using Eq. 1:

$$164 \quad \delta = \frac{d_0}{pp_0} \cdot pp - d_0 \quad (1)$$

165 When the uniform flow enters the vertical cylinder contactor, a boundary layer
166 begins to develop along the pipe due to the effect of the viscosity of solution. The
167 hydrodynamic entrance length is defined as a distance, that a flow travels after entering
168 a pipe for internal flow before the flow becomes fully developed, as shown in Figure
169 4a; or a gap for external flow before the film thickness has reached its asymptotic value
170 within a deviation of 1% [26, 27], as shown in Figure 4b.

171 **Figure 4.**

172 **3 Models and simulation**

173 **3.1 CFD model**

174 CFD (computational fluid dynamics) is usually used to solve thin fluid film
175 problems as it can overcome the limitations of the classical Reynolds equation and
176 evaluate the data quickly. CFD has become more capable of readily modeling fluid flow
177 as computational ability has rapidly increased and become able to offer the potential of

178 fine-mesh and detailed simulations. In this investigation, the model of the liquid film
179 flow along the vertical tube was developed by Fluent 6.3 and the 2D model was
180 designed by Gambit software. The fluent solvers are based on the finite volume method
181 in which the domain is discretized into a finite set of control volumes or cells, and the
182 general conservation (transport) equations for mass, momentum, energy and so on are
183 discretized into algebraic equations.

184 3.1.1 Grid generations and validation

185 In general, quad/hex meshes are normally selected to create simple geometries as
186 they can provide higher-quality solutions with fewer cells than a comparable tri/tet
187 meshes. In the present work, the quad mesh was selected to create a 2D model. In
188 addition, numerous grids with varying fineness of mesh were created for grid
189 independence testing. The film thickness and hydrodynamic entrance length of water
190 with different number of grids for which the flow rate was set to be 54.25 mL/min were
191 compared as shown in Table 2. It can be concluded that the simulation results can be
192 seen to be independent of the number of grids. However, it costs more time but results
193 in poorer accuracy for computations with a very large number of grids. Therefore,
194 635110 was chosen as the mesh number in this case.

195 **Table 2.**

196 3.1.2 Simulation set-up

197 3.1.2.1 Solver set-up

198 In general, the model of a low-speed incompressible flow selects the pressure-
199 based approach as the numerical method in which the pressure field is extracted by

200 solving pressure correction equations obtained by manipulating continuity and
 201 momentum equations. The velocities are obtained from the momentum and continuity
 202 equations.

203 3.1.2.2 Multiphase model selection

204 Regarding the simulation of a two-phase flow along the vertical tubes with free
 205 surface, VOF (volume of fluid) has been verified to be a suitable model since it can
 206 simulate two or more immiscible fluids by solving a single set of momentum equations
 207 and tracking the volume fraction of each of the fluids throughout the domain. One of
 208 its typical applications is to predict steady or transient tracking of any liquid-gas
 209 interface^[22].

210 3.1.2.3 Viscous model selection

211 The laminar flow model was selected in this study due to the low Reynolds number
 212 and the fact that the fluid flow is in the laminar region. Mass and momentum
 213 conservation equations were solved, as described in Eqs. 2 and 3, respectively.

$$214 \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = S_m \quad (2)$$

$$215 \quad \frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho g_i + F_i \quad (3)$$

216 The stress tensor can be calculated by Eq. 4

$$217 \quad \tau_{ij} = \left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] - \frac{2}{3} \mu \frac{\partial u_i}{\partial x_i} \delta_{ij} \quad (4)$$

218 3.1.2.4 Boundary conditions and solutions controls

219 Velocity at the inlet and outlet flow were set in order to be suitable for the
 220 incompressible fluid in this case. PISO algorithm (Pressure Implicit with Splitting of

221 Operator) was selected as the numerical method pressure-velocity coupling for its wide
 222 use for unsteady flow problems. Second-order upwind scheme was chosen as the
 223 discretization method due to using larger ‘stencil’ for 2nd order accuracy. The various
 224 CFD settings are summarized in Table 3 below:

225 **Table 3.**

226 3.2 Model of film thickness

227 Considering the flow of a fluid along a flat surface (as shown in Figure 5 (a)), Bird
 228 et al.^[19] reported a correlation (Eq. 5) to calculate the liquid film thickness along a
 229 vertical wall. Since there is no model given for the film flow along a cylinder, as shown
 230 in Figure 5 (b), it has been assumed that the width of the plate is equal to the
 231 circumference of the vertical cylinder. This enables the prediction of the liquid film
 232 thickness along the vertical cylinder. The present work proposes a formula (Eq. 15 and
 233 Eq. 16) based on cylindrical coordinates which gives a more reasonable calculation.

234 **Figure 5.**

$$235 \quad \delta = \sqrt[3]{\frac{3\mu Q}{\rho g W}} = \sqrt[3]{\frac{3\mu Q}{2\rho g \pi R}} \quad (5)$$

236 According to the Navier-Stokes equation, the equation of motion for laminar flow along
 237 a cylinder as shown in Figure 5 (b) can be simplified as:

$$238 \quad \mu \frac{1}{r} \frac{d}{dr} \left(r \frac{du}{dr} \right) = -\rho g \quad (6)$$

239 The general solution to Eq. 6 can easily be obtained as:

$$240 \quad \mu \frac{du}{dr} = -\frac{1}{2} \rho g r + \frac{c}{r} \quad (7)$$

241 and

242
$$u = \frac{1}{4} \frac{\rho g}{\mu} (R^2 - r^2) + \frac{c}{\mu} \ln\left(\frac{r}{R}\right) \quad (8)$$

243 Here the no-slip boundary condition $u|_{r=R} = 0$ has been used. From Eq. 7 and Newton's
 244 law, the shear stress can be expressed as:

245
$$\tau_w = -\frac{1}{2} \rho g R + \frac{c}{R} \quad (9)$$

246 On the other hand, from force balance, we have

247
$$\tau_w (2\pi R H) = \rho g \pi [(R + \delta)^2 - R^2] H \quad (10)$$

248 Then

249
$$R \tau_w = \frac{1}{2} \rho g (2R\delta + \delta^2) \quad (11)$$

250 From Eq. 8 and 10:

251
$$c = \frac{1}{2} \rho g (R + \delta)^2 \quad (12)$$

252 Hence,

253
$$\begin{aligned} u &= \frac{\rho g}{4\mu} (R^2 - r^2) + \frac{\rho g}{2\mu} (R + \delta)^2 \ln\left(\frac{r}{R}\right) \\ &= \frac{\rho g R^2}{4\mu} [2(1 + \alpha)^2 \ln(\varphi) + 1 - \varphi^2] \end{aligned} \quad (13)$$

254 Where $\varphi = r/R$ and $\alpha = \delta/R$. Thus,

255
$$\begin{aligned} Q &= 2\pi \int_R^{R+\delta} r u dr = \frac{\pi \rho g R^4}{2\mu} \int_1^{1+\alpha} \varphi [2(1 + \alpha)^2 \ln(\varphi) + 1 - \varphi^2] d\varphi \\ &= \frac{\pi \rho g R^4}{8\mu} [4(1 + \alpha)^4 \ln(1 + \alpha) - \alpha(2 + \alpha)(2 + 6\alpha + 3\alpha^2)] \end{aligned} \quad (14)$$

256 These can be rewritten as:

257
$$\frac{3\mu Q}{\rho g (2\pi R)} = \frac{3R^3}{16} [4(1 + \alpha)^4 \ln(1 + \alpha) - \alpha(2 + \alpha)(2 + 6\alpha + 3\alpha^2)] \quad (15)$$

258 and

259
$$\frac{3\mu\bar{u}}{\rho g R^2} = \frac{3}{8} \left[4 \frac{(1+\alpha)^4 \ln(1+\alpha)}{(2+\alpha)\alpha} - (2+6\alpha+3\alpha^2) \right] \quad (16)$$

260 The correlation of Bird et al. for liquid film on flat plate can be transformed from
 261 Eq. 5 as

262
$$\frac{3\mu Q}{\rho g W} = \frac{3\mu Q}{\rho g (2\pi R)} = \delta^3 = R^3 \alpha^3 \quad (17)$$

263 or
$$\frac{3\mu\bar{u}}{\rho g} = \delta^2 = R^2 \alpha^2 \quad (18)$$

264 4 Results and Discussion

265 The film thickness and hydrodynamic entrance length were obtained
 266 experimentally, as tabulated in Tables 4-6. All the experimental results of film thickness
 267 and hydrodynamic entrance length were used to determine if the entrance length effect
 268 should be neglected. As presented in the tables 4-6, it can be observed that the
 269 hydrodynamic entry length only takes less than 8.22% of the whole length (75mm) of
 270 the cylinder. Therefore, it can be concluded that the hydrodynamic entry length effect
 271 is negligible when calculating the whole contact surface area, especially for differential
 272 reactors. However, in the case of very fast reactions, the heat and mass transfer will
 273 take place at very short range close to the entrance; in which case, the hydrodynamic
 274 entry length effect should be taken into account. In addition, the experimental data
 275 obtained under different materials and liquid flow rates were used to assess the results
 276 of simulation/modeling.

277 **Table 4.**

278 **Table 5.**

279 **Table 6.**

280 4.1 Liquid film thickness

281 In order to avoid the impact of entrance effect, only 60 mm of the total cylinder
282 length (75mm) from the bottom was taken into consideration to calculate the average
283 film thickness. For the experiments, the liquid film thicknesses for different liquid flow
284 rates were measured by using Image Software. Figure 6 presents a comparison of the
285 film thickness calculated by Bird et al. equation and the model (Eq.16) proposed in this
286 work. It can be seen from Figure 6 that there is a significant difference between the two
287 calculations at the high liquid flow rate region. On the other hand, the deviation in the
288 range of low liquid flow rates is insignificant. Since experimental liquid flow rates are
289 in the range of 0-200 mL/min, it can be seen that there is no significant difference
290 between the two methods, as shown in Figure 6. Consequently, both Bird's equation
291 and Eq. 16 can be used in this calculation. However, for those cases for which the liquid
292 flow rate is large enough, the model developed from cylindrical liquid film flow based
293 on the cylindrical coordinates (Eq. 16) might be suitable than Bird's equation which is
294 based on the flat plate liquid flow in this case, due to the experiments in this work were
295 carried out in a cylindrical setup.

296 **Figure 6.**

297 The comparisons of experimental liquid film thickness for three systems and the
298 thickness calculated using Bird's equation, Eq. 16 and CFD simulations are illustrated
299 in Figure 7. It can be seen that there is no significant difference among the experimental
300 liquid film thickness and values calculated by Bird's equation and Eq.16 proposed in
301 this work, as well as the CFD simulation results. This demonstrates that both proposed

302 Eq.16 and CFD model can be used to accurately predict the liquid film thickness at low
303 Reynolds number under the experimental conditions for different liquid systems.

304 **Figure 7.**

305 **4.2 Hydrodynamic entrance length**

306 It can be seen from Figure 8 that the hydrodynamic entrance length using water,
307 MEA, and sugar solution (40 wt%) increased as the flow rate increased for both
308 simulation results and the experiment data. This phenomenon indicates that the
309 hydrodynamic entrance length is influenced by the flow rate, or Reynolds number. It
310 can also be clearly seen that there are significant differences between experimental
311 results and simulation, and the hydrodynamic entrance lengths obtained from the
312 simulation are always larger than the experimental results. The main reason is that the
313 roughness in the experimental setup is neglected in simulation whereas it has been
314 verified that the boundary layer thickness will increase as the roughness decreases^{[28,}
315 ^{29]}. Besides, the wall shear stress is also neglected in the simulation whereas it has also
316 been verified that the boundary layer thickness will increase as the wall shear stress is
317 increased^[3]. The hydrodynamic entrance length will decrease as the boundary layer
318 length increases according to the literature^[6]. Therefore, it implies that the
319 hydrodynamic entrance length acquired from the simulation will be larger than in the
320 real case. Thus, it can be concluded that the CFD simulation using VOF model is not
321 able to adequately simulate the hydrodynamic entry length properly, implying that a
322 better numerical model is required.

323 **Figure 8.**

324 4.2.1 Empirical equation

325 It is well known that many investigations regarding empirical models for predicting
326 the hydrodynamic entry length exist. However, the models, so far, are still very
327 complicated and have low prediction ability. The dimensionless hydrodynamic entry
328 length (λ) is defined as the ratio between the hydrodynamic entry length (λ_E) and the
329 radius of cylinder (R). Figure 9 shows the variation of the experimental dimensionless
330 hydrodynamic entry lengths with Reynolds number. It can be seen from the figure that
331 for each solution, with different viscosity and surface tension, the hydrodynamic entry
332 length increases with Reynolds number non-linearly. This phenomena can be correlated
333 to the hydrodynamic entry length model obtained in the liquid pipe flow^[30, 31].
334 Therefore, there seems to be a relationship between liquid pipe flow and liquid falling
335 film flow with a free surface. In addition, relationships of the parameters in a linear
336 correlation can be observed as functions of the product of viscosity and surface tension
337 for each liquid system. Therefore, the empirical model of dimensionless hydrodynamic
338 entry length (λ) of falling film flow can be written as:

$$339 \quad \lambda = a \cdot \text{Re} + b \quad (19)$$

$$340 \quad a = 0.0008 \cdot e^{8197 \cdot \mu \cdot \sigma} \quad (20)$$

$$341 \quad b = -1749.3 \cdot \mu \cdot \sigma + 0.8632 \quad (21)$$

342 here, μ is the liquid viscosity (Pa·s) and σ is the liquid surface tensor (N/m).

343 **Figure 9.**

344 Figure 10 shows comparisons between calculated and experimental hydrodynamic
345 entry length, with an AARD of 3.03%. However, the application of this empirical model

346 is still very limited since the dimensionless hydrodynamic entry length becomes a
 347 function of two physical properties instead of dimensionless numbers. Thus, a more
 348 theoretically-based model is needed in order to predict the hydrodynamic entry length
 349 with fewer and more reasonable parameters.

350 **Figure 10.**

351 4.2.2 Minimal surface model

352 **Figure 11.**

353 As shown in Figure 11b, the shape of the liquid surface near the hydrodynamic
 354 entrance is actually a rotating surface formed by generatrix $\kappa(\zeta)$ rotating once on axis
 355 ζ . Based on the differential geometric theory, the mean curvature of the rotating surface
 356 can be expressed as:

$$357 \quad 2C = \frac{\zeta''}{(1+\zeta'^2)^{3/2}} + \frac{\zeta'}{\kappa(1+\zeta'^2)^{1/2}} = \frac{\kappa\zeta'' + \zeta'(1+\zeta'^2)}{\kappa(1+\zeta'^2)^{3/2}} \quad (22)$$

358 Here C is the mean curvature, and the ζ' and ζ'' can be expressed as:

$$359 \quad \begin{aligned} \zeta' &= \frac{d\zeta}{d\kappa} \\ \zeta'' &= \frac{d^2\zeta}{d\kappa^2} \end{aligned} \quad (23)$$

360 According to Figure 11a,

$$361 \quad \zeta = \frac{z}{R} \quad \delta = \frac{r}{R} \quad \kappa = \frac{R + \delta}{R} \quad (24)$$

362 When the liquid flow rate is very small, the liquid surface area has a tendency to
 363 be minimized due to surface tension effect. When the rotating surface area becomes a
 364 minimum, the mean curvature C is now zero; therefore, the equation 25 can be obtained
 365 based on Eq.22.

366
$$\kappa\zeta'' + \zeta'(1 + \zeta'^2) = 0 \quad (25)$$

367 Thus, the 1st order derivative of generatrix $\kappa(\zeta)$ can be given as:

368
$$\zeta' = \pm \frac{\beta}{\sqrt{\kappa^2 - \beta^2}} \quad (26)$$

369 and

370
$$\zeta = \beta \ln \left(\frac{\kappa_0 + \sqrt{\kappa_0^2 - \beta^2}}{\kappa + \sqrt{\kappa^2 - \beta^2}} \right) \quad (27)$$

371 Here, β is the undetermined parameter which relates to the fluid properties and flow
 372 state, κ_0 is the dimensionless radial position at the entrance of the cylinder.

373 Actually, Eq. 27 expresses a suspended chain curve, which shows that κ decreases
 374 initially to a minimum value, and then increases after the minimum value with
 375 increasing ζ . In this case, this minimum value should be κ_∞ which also equals to the
 376 liquid film thickness of steady flow. That means, based on Eq.19, this extreme point
 377 should be $\beta = \kappa_\infty$ in order to make $\kappa' = 0$ (or $\zeta' = \infty$). Therefore Eq. 27 can be expressed
 378 as

379
$$\zeta = \kappa_\infty \ln \left(\frac{\kappa_0 + \sqrt{\kappa_0^2 - \kappa_\infty^2}}{\kappa + \sqrt{\kappa^2 - \kappa_\infty^2}} \right) \quad (28)$$

380 Here, κ_∞ is the dimensionless radial position at the infinite position of the cylinder.

381 Thus, the dimensionless hydrodynamic entrance length can be obtained by:

382
$$\lambda = \kappa_\infty \ln \left(\frac{\kappa_0 + \sqrt{\kappa_0^2 - \kappa_\infty^2}}{\kappa_\infty} \right) \quad (29)$$

383 Obviously, the minimal surface model is only suitable for the situation where the
 384 liquid flow rate is very small. For the cases of the whole range of liquid flow rates, a
 385 correction factor is needed and can be introduced as:

386
$$\zeta = \gamma \kappa_{\infty} \ln \left(\frac{\kappa_0 + \sqrt{\kappa_0^2 - \kappa_{\infty}^2}}{\kappa + \sqrt{\kappa^2 - \kappa_{\infty}^2}} \right) \quad (30)$$

387 or

388
$$\lambda = \gamma \kappa_{\infty} \ln \left(\frac{\kappa_0 + \sqrt{\kappa_0^2 - \kappa_{\infty}^2}}{\kappa_{\infty}} \right) \quad (31)$$

389 This correction factor, γ , should relate to the fluid properties and flow states. It can be
 390 observed from the hydrodynamic experiments that the correction factor is proportional
 391 to the ratio of shear stress and gravity as shown in Figure 12, and can be expressed as:

392
$$\gamma = mX + n \quad (32)$$

393 Here, m and n are undetermined parameters, and X is the ratio of shear stress and gravity.

394
$$X = \frac{\mu \bar{v}}{\rho g R^2} \quad \bar{v} = \frac{Q}{\pi R^2} \text{ or } \frac{Q}{\pi R^2 (\kappa_0^2 - 1)} \quad (33)$$

395 **Figure 12.**

396 It can be found that the parameters m and n for water, 30wt% MEA solution and
 397 40wt% sugar solution are all linear with their Bond number and density, respectively.

398
$$m = c_1 Bo + c_2 \quad (34)$$

399
$$n = c_3 \rho + c_4 \quad (35)$$

400 here

401
$$Bo = \frac{\sigma}{\rho g R^2} \quad (36)$$

402 The ratio of shear stress and gravity (X), Bond number (Bo), and the parameters m
 403 and n for water, 30wt% MEA solution and 40wt% sugar solution are presented in Tables
 404 7 and 8. In addition, the parity plot in Figure 13 shows that the experimental
 405 dimensionless hydrodynamic entry lengths were in good agreement with the predicted

406 values calculated by the developed minimal surface model in this work with AARD of
407 6.83%, which is in an acceptable range.

408 **Table 7.**

409 **Table 8.**

410 **Figure 13.**

411 **4.3 Gas-liquid interfacial area**

412 In gas absorption process, gas-liquid interfacial area reflects available effective
413 interfacial area for gas and liquid, and thus has a great impact on the mass transfer
414 performance. In general, this gas-liquid interfacial area can be regarded as the cylinder
415 surface area by rotating the liquid film length. In practice, the gas-liquid interfacial area
416 cannot be simplified as the calculated cylindroid area, due to the influence of having
417 different hydrodynamic entrance lengths for different liquid systems. Therefore, the
418 minimal surface model proposed in section 4.2.2 is suitable to accurately calculate the
419 gas-liquid interfacial area in the present work. The relationship between $(A_1-A_2)/A_2$
420 values and y/λ_E values is given in Figure 13. Here, the A_1 and A_2 are the superficial area
421 with and without consideration of the hydrodynamic entrance length, respectively; y is
422 the liquid film length and λ_E is the hydrodynamic entrance length.

423 **Figure 14.**

424 It can be concluded from Figure 14 that the hydrodynamic entrance length has a
425 significant effect on the gas-liquid interfacial area. The influence of y/λ_E value on the
426 $(A_1-A_2)/A_2$ value can be negligible when the ratio of liquid film length y and
427 hydrodynamic entrance length λ is lower than 20. In contrast, there is a sudden drop in

428 the values of $(A_1-A_2)/A_2$ with the decrease of y/λ_E values. Thus, the effects of
429 hydrodynamic entrance length must be considered to calculate the gas-liquid interfacial
430 area, which is associated with the mass transfer coefficients of packed columns. In
431 addition, the hydrodynamic entrance length should be considered in gas absorption
432 process when the value of $(A_1-A_2)/A_2$ is higher than 1% in which the y/λ_E value is below
433 10. However, it should be noted that the hydrodynamic entrance length cannot be
434 ignored in packed columns in which the liquid flow is very complex due to the packings
435 with different structures and materials.

436 **5 Conclusion**

437 The film thickness of laminar flow has been thoroughly investigated as it is a vital
438 factor of heat and mass transfer processes. The behavior of liquid film falling around a
439 vertical tube was investigated for the determination of liquid film thickness and
440 hydrodynamic entrance length using the shadow-photographs method. Here, three
441 liquids, namely, distilled water, 30wt% aqueous MEA solution and 40wt% sugar
442 solution were used to estimate the effect of liquid flow rate on the formation of liquid
443 film. In addition, a developed Navier-Stokes equation (Eq.16) and the volume of fluid
444 (VOF) model using computational fluid dynamics (CFD) simulation were developed
445 and used to predict the values of experimental film thickness with high predictability,
446 especially in the low Reynolds number region.

447 However, the comparison between the predicted values and experimental data of
448 the hydrodynamic entrance length indicated that the VOF model is not applicable for
449 prediction of hydrodynamic entrance length. Based on the experimental data, a new

450 empirical model and an improved minimal surface model were developed and used to
451 predict the hydrodynamic entrance length, with AARD of 3.03% and 6.83%,
452 respectively. However, the application of empirical correlations are still quite limited,
453 and the minimal surface model has a little bit less prediction ability but a much larger
454 applicable range.

455 Furthermore, the gas-liquid interfacial area is an important factor in separation
456 process area. Thus, the effects of the hydrodynamic entry length on the gas-liquid
457 interfacial area were also studied. The calculated values based on the improved minimal
458 surface model showed that the hydrodynamic entry length effect can be neglected in
459 calculating the whole contact surface area if the hydrodynamic entry length only takes
460 less than 8.22% of the whole length. However, in the case of very fast reaction, the heat
461 and mass transfer will take place at a very short range close to entrance; in which case,
462 the hydrodynamic entry length effect should be taken into account.

463

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476 **NOTATION**

477	Bo	Bond number
478	C	mean curvature
479	d_0	diameter of the stainless, m
480	D	diameter of pipe, m
481	F_i	external volume force in the direction of i , N
482	H	Henry's law constant, -
483	P	static pressure, N
484	pp	pixel points of the shadow of the stainless steel when the liquid fall flow along
485		the cylinder, -
486	pp_0	pixel points of stainless steel without liquid flow, -
487	Q	volumetric flowrate, m ³ /h
488	r	radial position of cylinder, m
489	R	radius of cylinder, m
490	Re	Reynolds number, -
491	S_m	mass which come from dispersed second phase to first phase, kg
492	u	linear velocity of liquid, m/s
493	\bar{u}	mean velocity of liquid, m/s
494	V	volume of the cell, m ³
495	W	circumference of cylinder in wetted wall column, m
496	X	ratio of shear stress and gravity, -
497	z	axial position of cylinder, m

498 **Greek letters**

499 γ correction factor in Eq. 32, -

500 δ film thickness, m

501 δ_0 boundary film thickness, m

502 $\bar{\delta}$ average film thickness, m

503 μ viscosity, kg/m/sec

504 ρ density, kg/m³

505 τ_w shear stress, N

506 λ dimensionless hydrodynamic entrance length, -

507 λ_E hydrodynamic entrance length, mm

508 κ dimensionless radial position, -

509 σ surface tension, N/m

510 ζ dimensionless axial position, -

511

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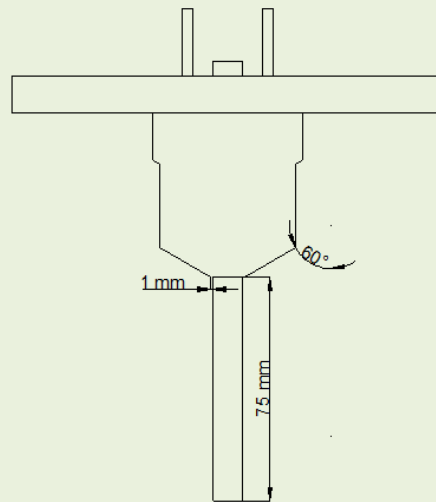
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576 **Figures**

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578



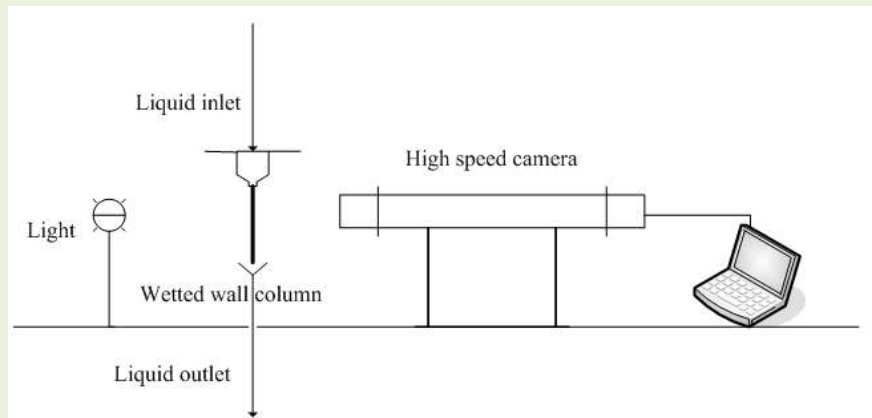
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Figure 1. Vertical cylinder contactor.

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Figure 2. Schematic diagram of the hydraulic experiment setup.

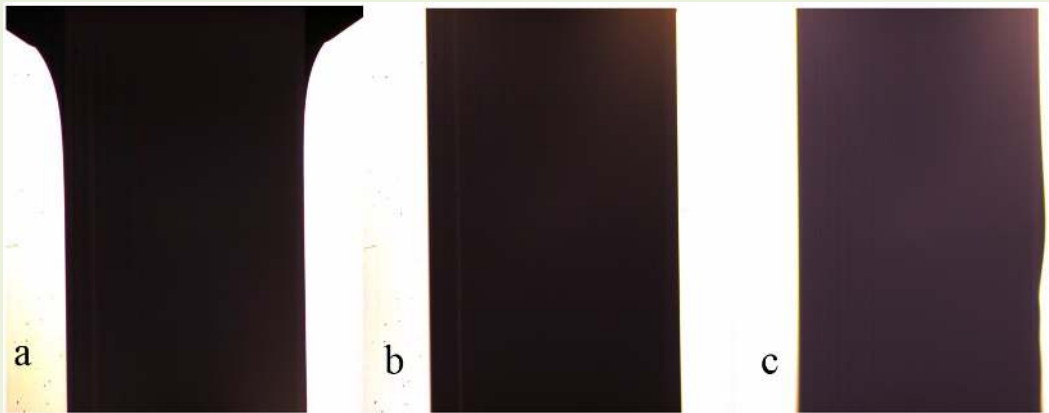
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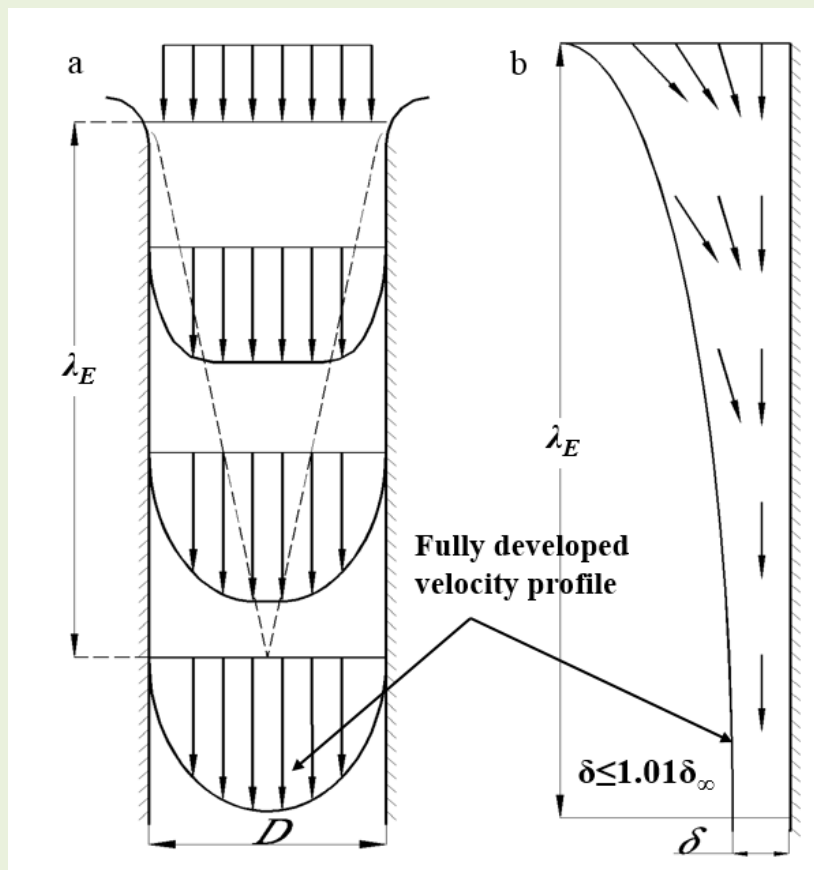
Figure 3. Picture from high speed camera showing film plus steel rod:

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a) The position at the entrance; b) the position in the middle of the cylinder; c) the position in the middle of the cylinder with rippling behavior.

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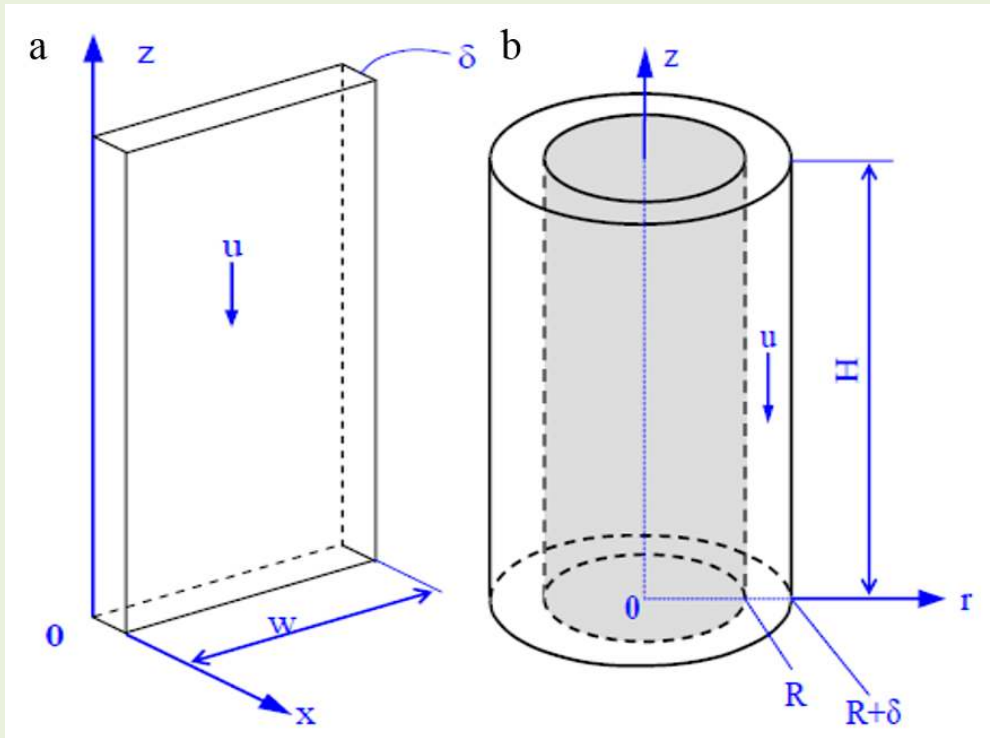
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Figure 4. Schematic diagram for the hydrodynamic entrance length in (a) a pipe and (b)

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a gap.



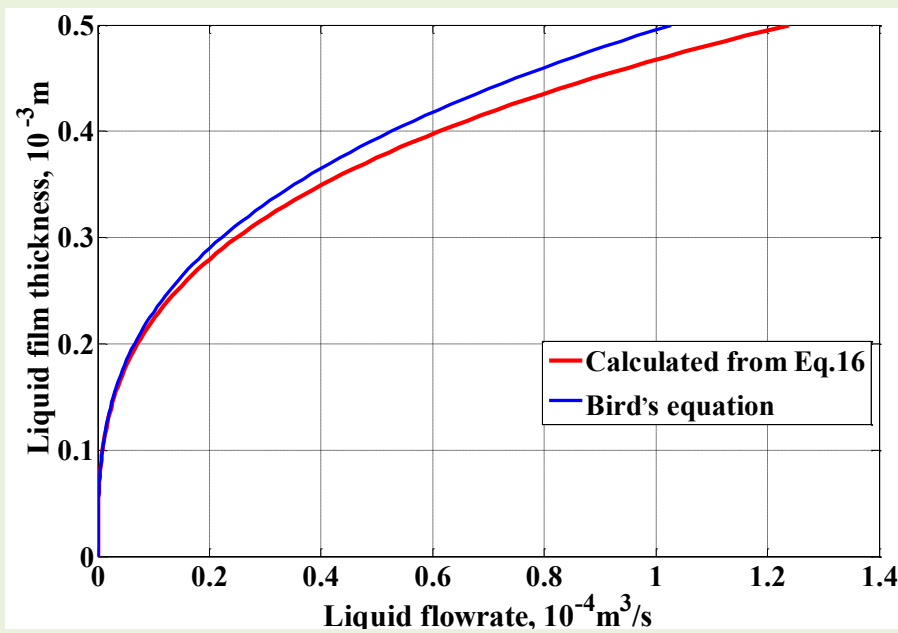
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600 Figure 5. Sketch of a laminar flow along the external surface of (a) a vertical flat plate and

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(b) a vertical cylinder.

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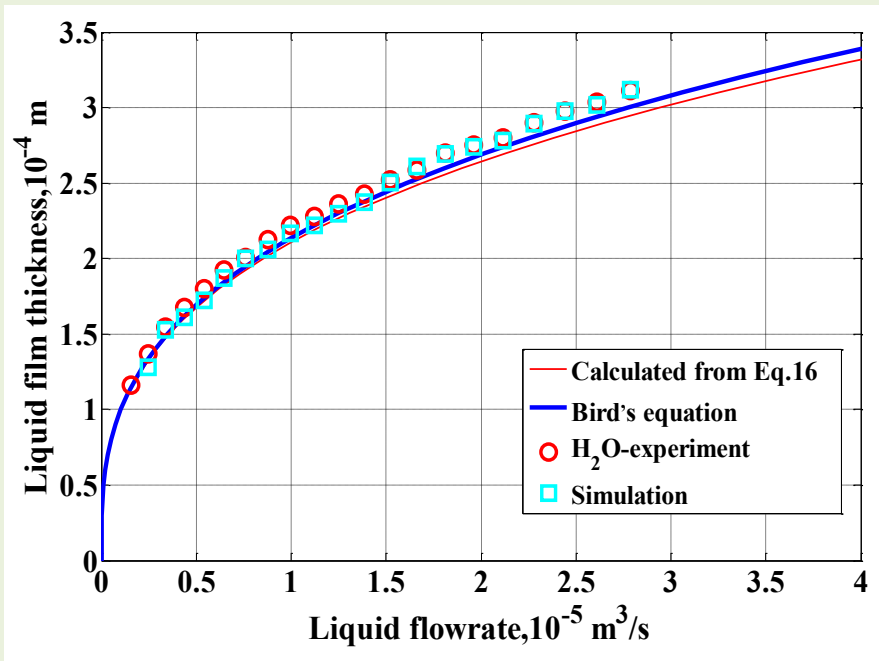
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604 Figure 6. Comparison of calculated film thickness of water between Bird's equation and

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Eq.16 proposed in this work.

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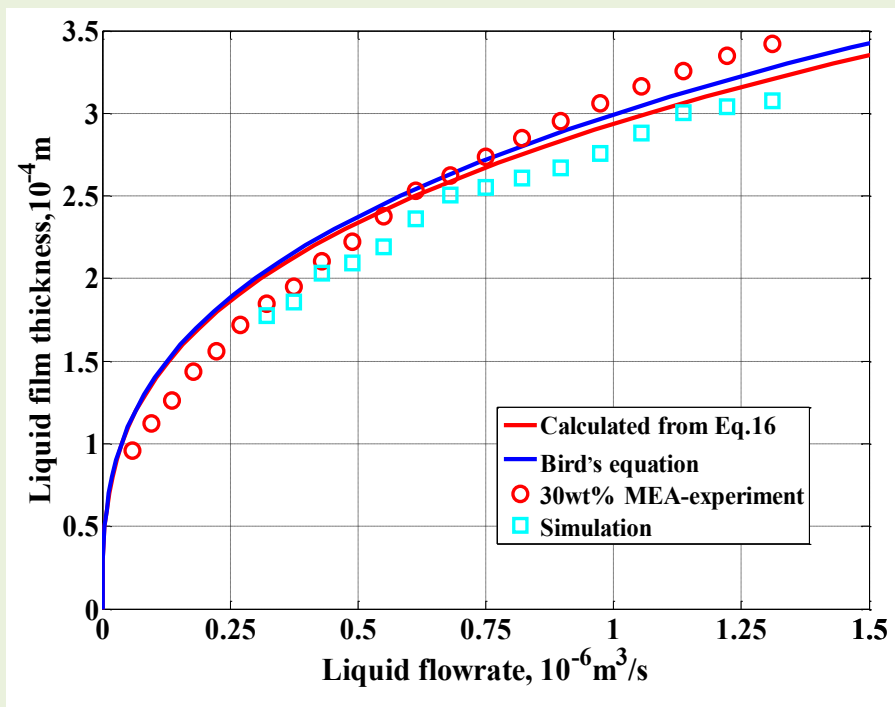


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Figure 7a. The liquid film thickness comparison results of H₂O.

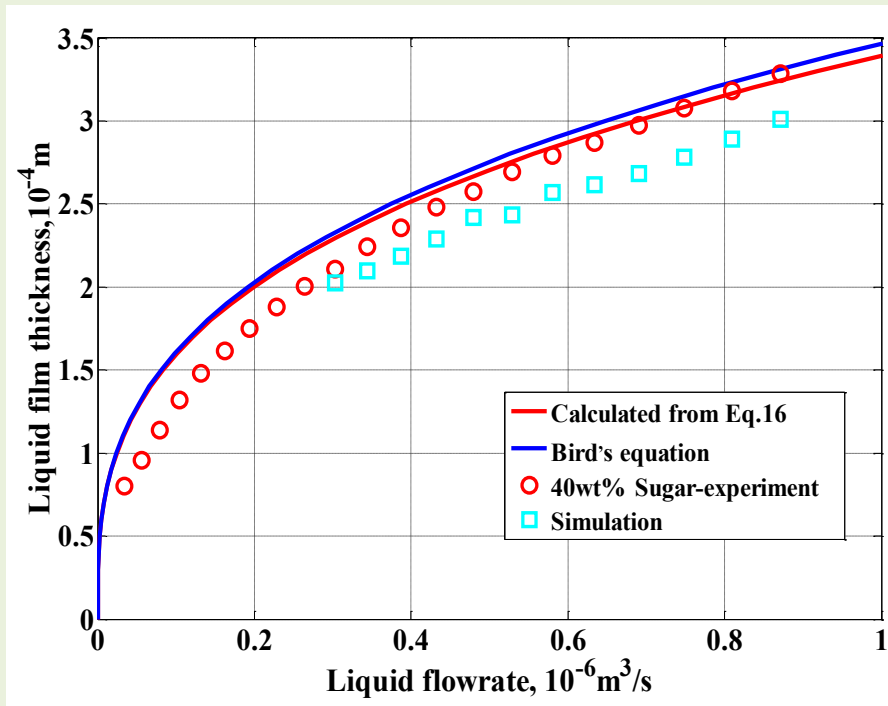
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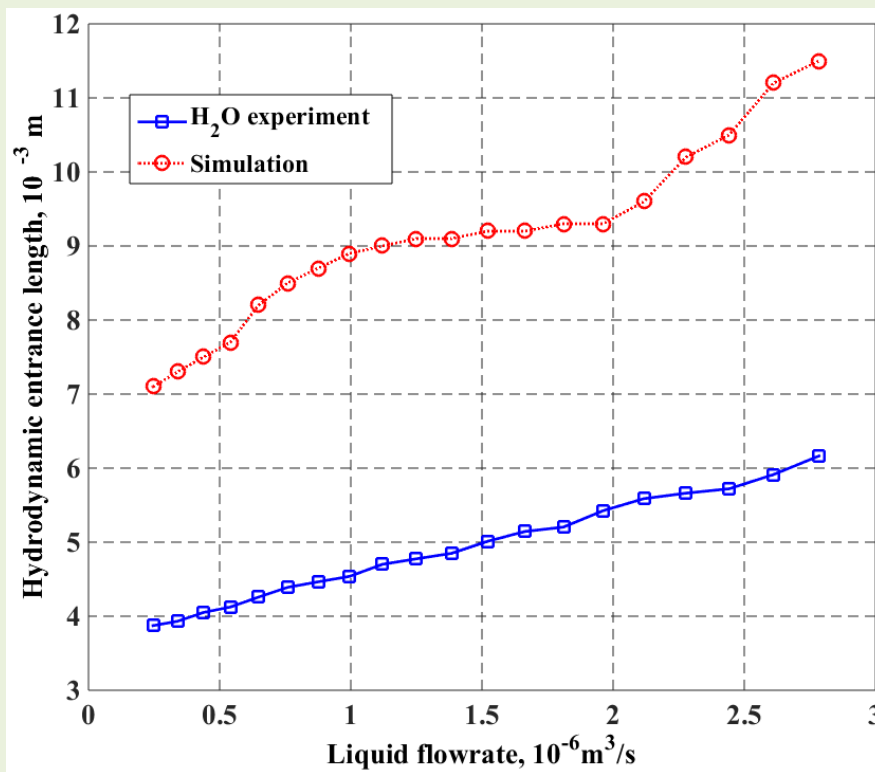
Figure 7b. The liquid film thickness comparison results of 30 wt% MEA.



612 Figure 7c. The liquid film thickness comparison results of 40 wt% Sugar.

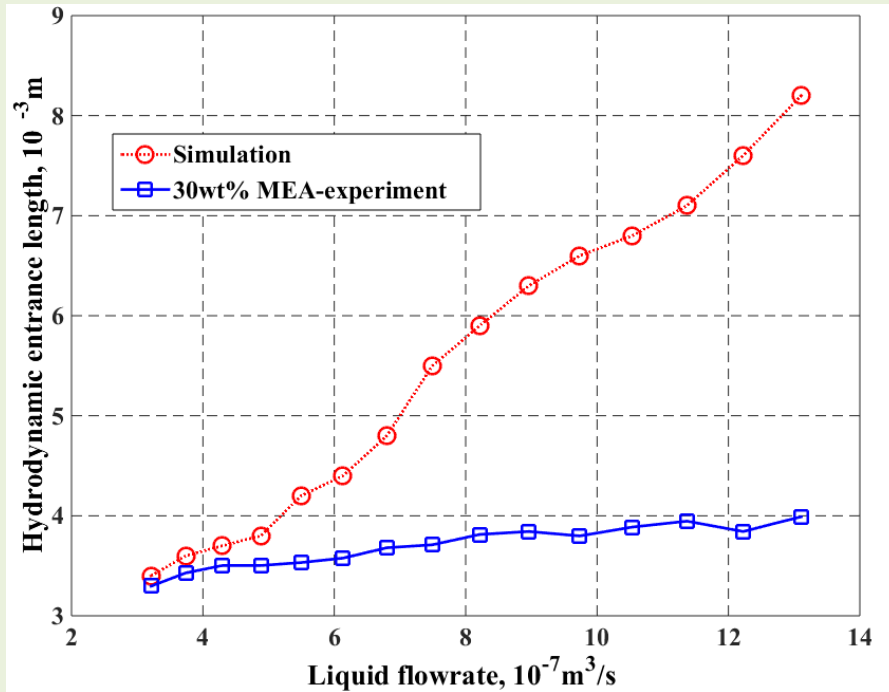
613 Figure 7. Calculated liquid film thickness by Eq.16 and Bird's equation compared with
614 experiment measurements and simulation result.
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617 Figure 8a. The hydrodynamic entrance length comparison results of H₂O.
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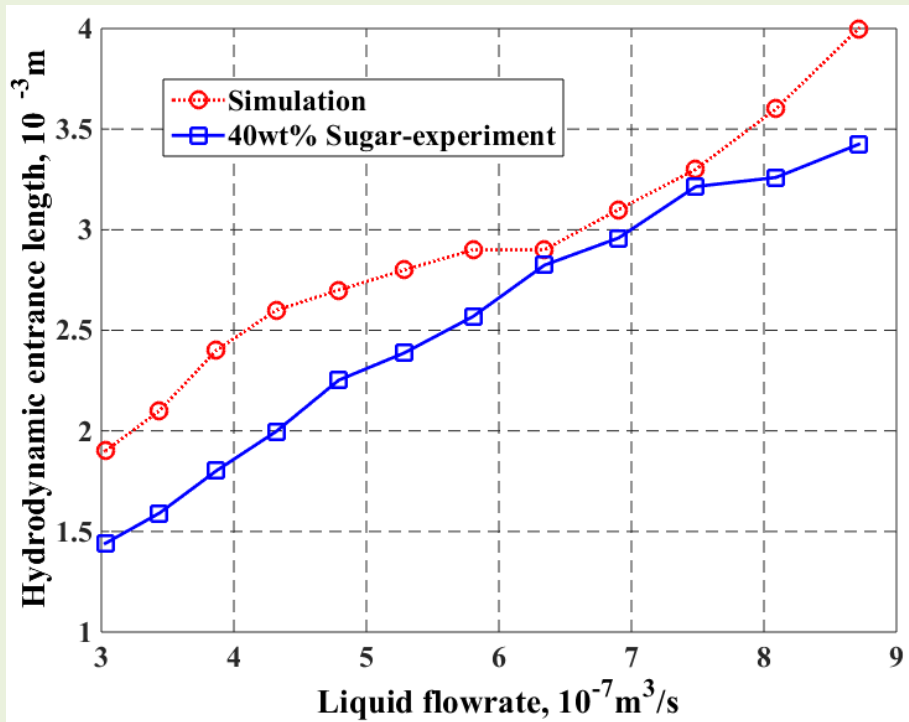
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Figure 8b. The hydrodynamic entrance length comparison results of 30wt% MEA.



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Figure 8c. The hydrodynamic entrance length comparison results of 40 wt% Sugar

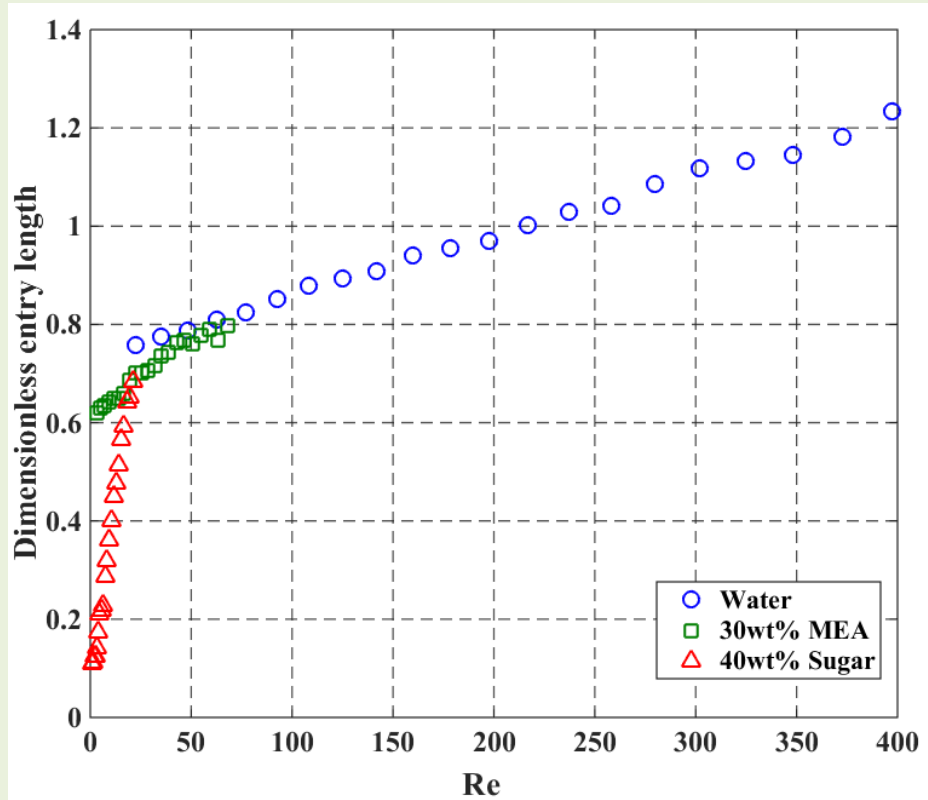
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Figure 8. Comparison of Hydrodynamic entrance length between experiments and

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simulation results.

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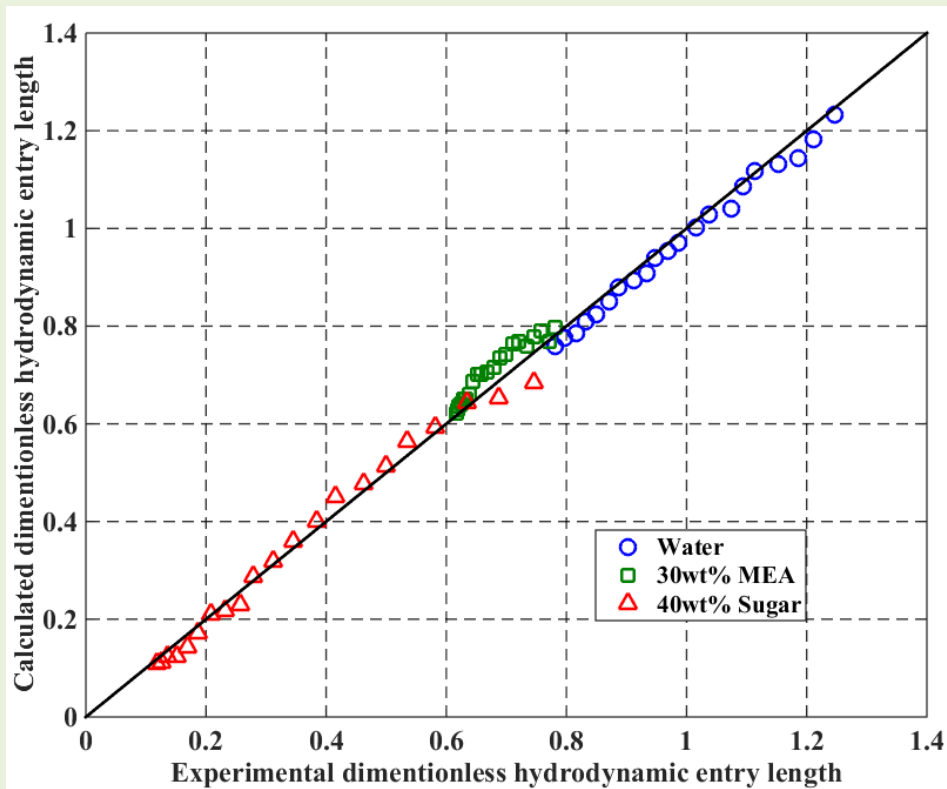


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Figure 9. The relationship between the experimental hydrodynamic entrance length and the Reynolds number.

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Figure 10. Crossplot between calculated and experimental hydrodynamic entrance length.

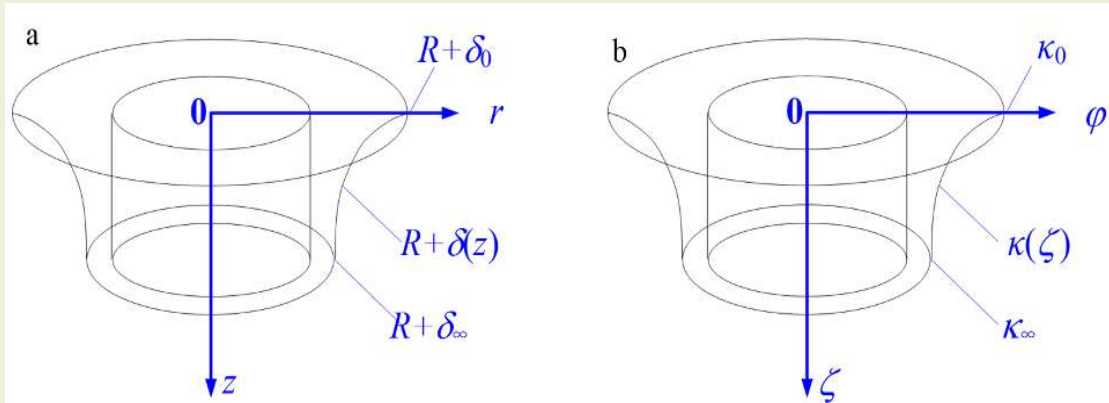
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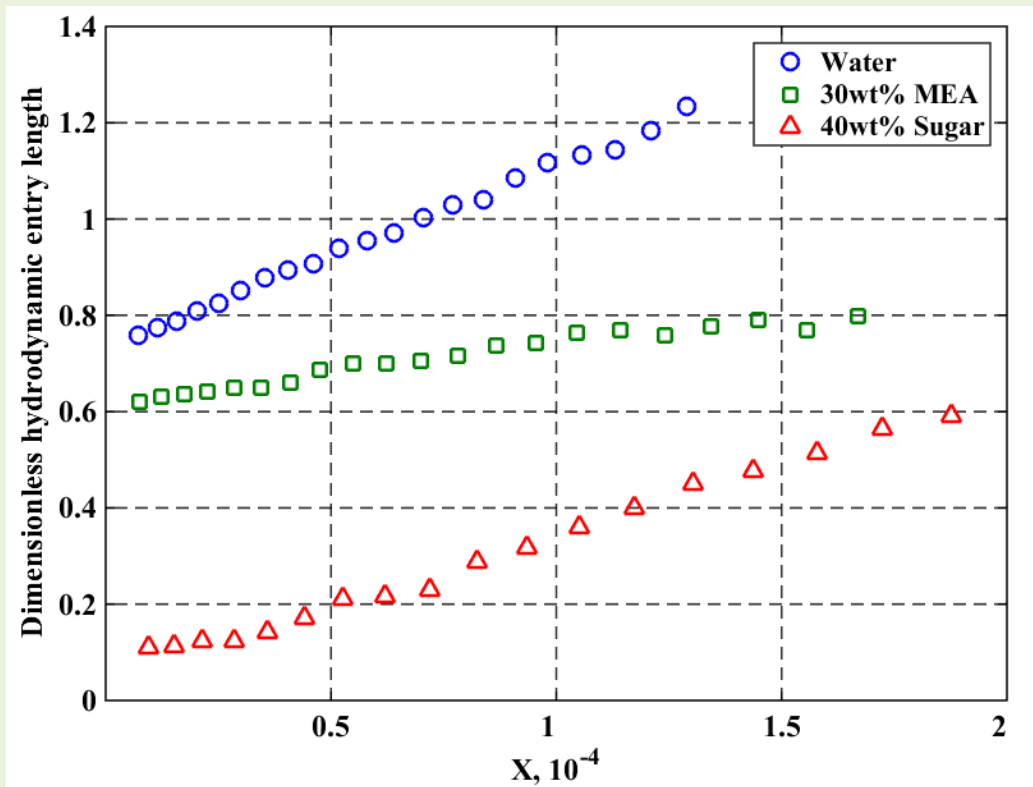


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638 **Figure 11. The schematic diagram of liquid film surface in the entrance region with**
 639 **normal and dimensionless coordinates.**

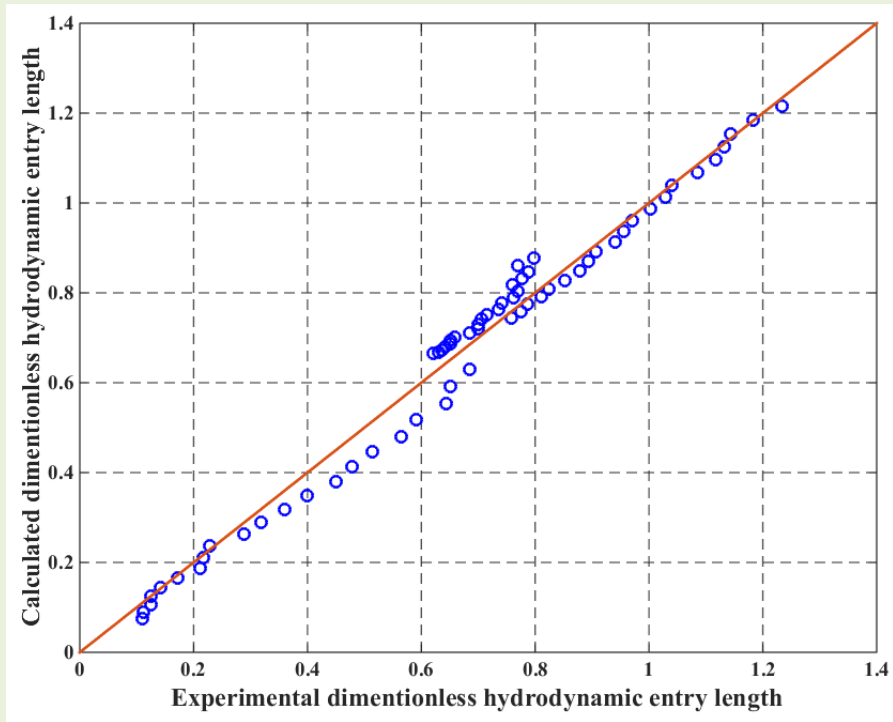
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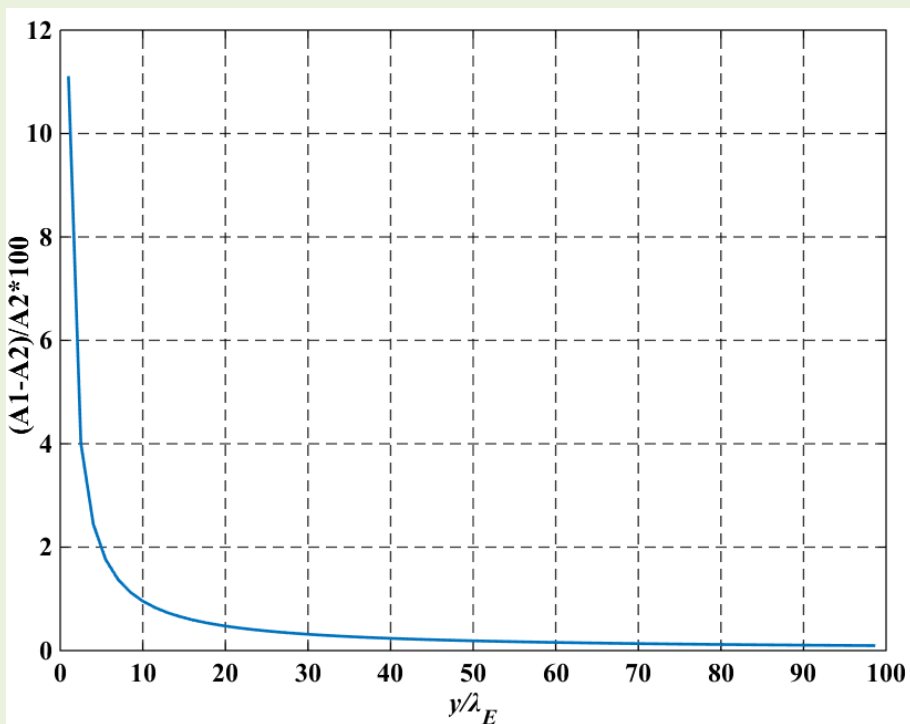
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643 **Figure 12. The experimental dimensionless hydrodynamic entry length varied with Q.**



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645 **Figure 13. The parity plot of experimental and calculated dimensionless hydrodynamic**
 646 **entrance length.**



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Figure 14. Relationship between $(A_1-A_2)/A_2$ and y/λ_E .

650 **Tables**

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654 **Table 1. Properties of working fluids at 298.15 K and a pressure of 101.325 kPa**

Working fluid	ρ (kg·m ⁻³)	μ (mPa·s)	σ (mN·m ⁻¹)
distilled water	997.057 ³	0.8899 ³	72.01 ⁴
40wt% sugar solution ⁶	1177.0	6.162	74.90
30wt% MEA solution	1010.6 ⁵	2.48 ⁵	60.41 ⁴

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658 **Table 2. The results of meshes number independent test**

Meshes No.	δ (mm)	λ_E (mm)
44890	0.2057	7.6
635110	0.2062	7.7
1796700	0.2066	7.7

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Table 3. Important information used for the simulations.

Settings	Choice
Simulation type	2D, unsteady
Solver	Pressure based and implicit
Multiphase model	VOF and implicit
Viscous model	Laminar
Materials	Water & air, MEA (30wt%) & air, Sugar (40wt%) & air
Operating conditions	Standard pressure, gravity
Boundary conditions	Velocity inlet, outflow
Solution controls	PISO, second order upwind

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Table 4. Film thickness and hydrodynamic entrance length of distilled water

No.	Re	δ (mm)	λ_E (mm)
1	22.3779	0.1161	3.7935
2	35.1278	0.1366	3.8730
3	48.5086	0.1547	3.9322
4	62.5205	0.1675	4.0504
5	77.1634	0.1802	4.1243
6	92.4373	0.1926	4.2574
7	108.3422	0.2006	4.3904
8	124.8781	0.2127	4.4643
9	142.0451	0.2219	4.5383
10	159.8430	0.2278	4.7009
11	178.2720	0.2360	4.7748
12	197.3320	0.2426	4.8487
13	217.0230	0.2524	5.0113
14	237.3450	0.2587	5.1443
15	258.2980	0.2698	5.2035
16	279.8820	0.2750	5.4252
17	302.0971	0.2801	5.5878
18	324.9431	0.2901	5.6617
19	348.4202	0.2979	5.7209
20	372.5283	0.3034	5.9130
21	397.2674	0.3111	6.1643

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Table 5. Film thickness and hydrodynamic entrance length of 40 wt% Sugar

No.	Re	δ (mm)	λ_E (mm)
1	0.8417	0.0799	0.5471
2	1.3545	0.0959	0.5621
3	1.9205	0.1139	0.6222
4	2.5396	0.1317	0.6222
5	3.2119	0.1482	0.7124
6	3.9373	0.1618	0.8627
7	4.7159	0.1749	1.0581
8	5.5477	0.1881	1.0882
9	6.4325	0.2006	1.1432
10	7.3706	0.2109	1.4409
11	8.3618	0.2241	1.5912
12	9.4061	0.2355	1.8016
13	10.5036	0.2479	1.9970
14	11.6542	0.2573	2.2525
15	12.8580	0.2695	2.3878
16	14.1149	0.2790	2.5681
17	15.4250	0.2872	2.8236
18	16.7882	0.2972	2.9589
19	18.2046	0.3079	3.2144
20	19.6741	0.3179	3.2595
21	21.1968	0.3285	3.4248

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Table 6. Film thickness and hydrodynamic entrance length of 30 wt% MEA

No.	Re	δ (mm)	λ_E (mm)
1	3.0909	0.0960	3.1038
2	4.9991	0.1125	3.1481
3	7.0482	0.1263	3.1777
4	9.2380	0.1435	3.2073
5	11.5686	0.1562	3.2516
6	14.0400	0.1718	3.2516
7	16.6522	0.1846	3.2959
8	19.4052	0.1952	3.4290
9	22.2989	0.2105	3.5029
10	25.3334	0.2223	3.5029
11	28.5088	0.2376	3.5324
12	31.8249	0.2529	3.5768
13	35.2818	0.2625	3.6802
14	38.8794	0.2739	3.7098
15	42.6179	0.2852	3.8132
16	46.4971	0.2954	3.8428
17	50.5172	0.3060	3.7985
18	54.6780	0.3161	3.8871
19	58.9796	0.3255	3.9463
20	63.4220	0.3346	3.8428
21	68.0051	0.3418	3.9906

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Table 7. The ratio of shear stress and gravity X and the correction factor γ in**Eq.32**

No.	X			γ		
	Distilled	30 wt%	40 wt%	Distilled	30 wt%	40 wt%
	water	MEA	Sugar	water	MEA	Sugar
1	7.2687E-06	7.5895E-06	7.9080E-06	1.2757	1.0445	0.1848
2	1.1408E-05	1.2275E-05	1.2727E-05	1.3129	1.0687	0.1917
3	1.5754E-05	1.7307E-05	1.8044E-05	1.3418	1.0867	0.2139
4	2.0309E-05	2.2684E-05	2.3862E-05	1.3902	1.1039	0.2155
5	2.5063E-05	2.8406E-05	3.01781E-05	1.4232	1.1259	0.2484
6	3.0026E-05	3.4475E-05	3.6994E-05	1.4764	1.1323	0.3028
7	3.5193E-05	4.0883E-05	4.4310E-05	1.5296	1.1539	0.3737
8	4.0563E-05	4.7648E-05	5.2124E-05	1.5624	1.2066	0.3866
9	4.6137E-05	5.4757E-05	6.0438E-05	1.5951	1.2388	0.4086
10	5.1897E-05	6.2210E-05	6.9260E-05	1.6592	1.24483	0.5181
11	5.7921E-05	7.0007E-05	7.8560E-05	1.6922	1.2611	0.5753
12	6.4083E-05	7.8148E-05	8.8386E-05	1.7252	1.2831	0.6553
13	7.0478E-05	8.6633E-05	9.8691E-05	1.7896	1.3260	0.7303
14	7.7104E-05	9.5462E-05	1.0950E-04	1.8441	1.3429	0.8281
15	8.3915E-05	1.0465E-04	1.2081E-04	1.8722	1.3863	0.8830
16	9.0912E-05	1.1418E-04	1.3262E-04	1.9592	1.4034	0.9548
17	9.8141E-05	1.2404E-04	1.4492E-04	2.0253	1.3930	1.05569
18	1.0555E-04	1.3428E-04	1.5774E-04	2.0594	1.4318	1.1122
19	1.1315E-04	1.4486E-04	1.7104E-04	2.0882	1.4598	1.2146
20	1.2098E-04	1.5568E-04	1.8486E-04	2.1658	1.4277	1.2383
21	1.2905E-04	1.6702E-04	1.9917E-04	2.2656	1.4890	1.3084

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Table 8. Bond number (Bo), and the parameters m and n of working fluids

Working fluids	Bo	m	n
distilled water	0.2610	5364.4685	0.0848
30wt% MEA solution	0.2944	7864.1437	1.2319
40wt% sugar solution	0.2437	2798.6857	1.0535

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