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A Study of the Electric Field in an Open Magnetospheric Model

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The qualitative properties of an open magnetosphere and its electric field are examined and compared to a simple model of a dipole in a constant field and to actual observations. Many such properties are found to depend on the separatrix, a curve connecting neutral points and separating different field-line regimes. In the simple model it turns out that the electric field in the central polar cap tends to point from dawn to dusk for a wide choice of external fields. Near the boundary of the polar cap electric equipotentials curve and become crescent-shaped, which may explain the correlation of polar magnetic variations with the azimuthal component of the interplanetary magnetic field, reported by Svalgaard. Modifications expected to occur in the actual magnetosphere are also investigated: in particular, it appears that bending of equipotentials may be reduced by cross-field flow during the merging of field lines and that open field lines connected to the polar caps emerge from a long and narrow slot extending along the tail.

INTRODUCTION

In the "open" model of the magnetosphere, field lines from the earth's polar caps are directly connected to the interplanetary magnetic field. This model, suggested in a simple form by Dungey [1961], requires that some of the interplanetary field lines convected by the solar wind flow establish connection with the geomagnetic field by means of a merging process at a magnetic neutral point and sever this connection again slightly later by a similar process at another neutral point. For simple cases (e.g. southward interplanetary magnetic field) the model predicts a dawn-to-dusk electric field across the polar cap and this is in agreement with current patterns deduced from magnetic variations and also from barium vapor releases and electric field probes. Direct magnetic observations, it may be added, have not yet resolved the question whether polar magnetic field lines do or do not connect to the interplanetary field [Fairfield and Ness, 1972].

This is a study of the qualitative properties of the electric field produced in the polar cap by a simple model of the open field, the polar cap being *defined* (in this work) as the region on earth from which field lines connect to the interplanetary magnetic field. The model consists of the sum of a dipole and a constant field ; a somewhat similar model (but including an image dipole and other modifications) has been previously studied by Forbes and Speiser [1971] who investigated its configuration and the location and displacement of its neutral points, under various conditions.

The main virtue of this model is that it probably represents the simplest magnetic field with the appropriate topology and limits (at $r \rightarrow 0$ and $r \rightarrow \infty$). At the same time — as will be discussed — its use constitutes a gross simplification of actual conditions, and because of this, only general conclusions can be drawn. In fact (as will be shown) the model contains an internal inconsistency which hinders the derivation of a complete solution for the velocity field; this could be overcome by a more elaborate model, but it was felt that such an elaboration would not produce any different qualitative behavior.

In what follows it will be shown that many properties of the open model are related to a special curve, the separatrix. Because field lines passing near that curve become contorted, it turns out that near the boundary of the polar cap the electric field tends to align itself orthogonally to the boundary; this may explain the correlation between the dawn-dusk component B_{iy} of the interplanetary magnetic field and polar magnetic variations, deduced by Svalgaard [1968, 1972], Mansurov [1969] and their co-workers. Finally, a tentative model of the real magnetosphere will be developed, in which particular attention will be given to the discrepancy in magnitude between polar and interplanetary electric fields and to the geometry of merging field lines.

THE SIMPLE MODEL

The prototype of the open magnetospheric model [Dungey, 1961, 1963] is given by a dipole in a southward-pointing constant field (Figure 1). Because such a field is axially symmetric, every meridional cross section in it presents the same appearance and the neutral points A and B will lie on a circular neutral line along which $\underline{B} = 0$.

In general, however, one expects not neutral lines but neutral points. Consider a model field lacking any symmetry (e.g. a dipole in an arbitrarily oriented constant field): the condition for a neutral point is the vanishing of the three components of $\underline{B}$. This gives three equations for the three coordinates of the neutral point, and these generally only yield isolated solutions. Dungey [1961] realized that even in the situation portrayed by figure (1) a solar wind arriving from the left would destroy the symmetry by shifting both A and B to the right, and accordingly he assumed only A and B to be neutral. Later work [Levy et al., 1964 ; Brice, 1967] investigated details and consequances of this model.

The main evidence for the open model comes from the polar electric field. In the interplanetary medium the solar wind, flowing with a velocity $\underline{v}$, sets up an electric field

$$\underline{E} = -\underline{v} \times \underline{B} \qquad (1)$$

If conductivity is high along magnetic field lines and negligible across them, such lines will be equipotentials. Dungey then showed that in situations resembling figure (1) the electric field in the earth's polar cap will point from dawn to dusk.

In principle it might be argued that the conductivity of the ionosphere could short-circuit this electric field. Actually, if even a small ($\sim 10^{-11}$ M.K.S.) cross-field conductivity exists in interplanetary space due to wave noise and other causes, a steady current may be generated by unipolar induction between the moving solar wind and the stationary ionosphere. Assuming that the ionospheric conductor forms the major resistive part of this circuit, one then obtains the same dawn-to-dusk electric field as before, which in fact resembles the one observed.

In what follows a system of cartesian coordinates will be used with the origin at the dipole, the z axis along the dipole axis and the solar wind flow (far from the dipole) antiparallel to the x axis. If one imposes then an external (or "interplanetary") magnetic field parallel or antiparallel to the y axis, the field line configurations will resemble those given in figures (2) and in each case there will only exist two neutral points, marked A and B .

If equation (1) is to hold with equipotential field lines, then $\underline{v}$ must undergo modification in the region near the origin. We defer here the derivation of this modification and the question whether it is at all possible to meet these conditions, but *assume* that the equation does indeed hold and that field lines are equipotentials. It is then possible to derive the potential distribution at the polar caps simply by following open field lines connected to the earth until they are well beyond the dipole's influence. At such far-away points one may assume $\underline{v}$ to have its undisturbed value $-v\hat{\underline{x}}$ (v = constant) , which allows the potential on such lines to be derived and thus also provides the potential of their end points on the polar cap.

In particular, this method may be applied to field lines passing the neutral points in figures (2). It is then easily seen from the figures that in both cases the dawn edge A' of the polar cap is at a more positive potential than the dusk edge B' , causing in both cases (and both polaes) a dawn-to-dusk electric field across the polar cap.

The same procedure has been applied numerically to other orientations of $\underline{B}_i$ and the results will be described later. We only note here that while it turns out that the x component of $\underline{B}_i$ can rotate the electric field in the middle of the polar cap out of the plane $x = 0$, the y component of the electric field there, in all of the cases which were computed, always pointed from dawn to dusk.

THE SEPARATRIX

In the present model there exist four types of field lines — closed ones, open lines connected to the northern polar cap, open lines connected to the southern polar cap and unlinked field lines with no connection to the dipole. Similarly, space may be divided into four regions, each containing only field lines of one type. Since in the model discussed here, except perhaps for special cases, closed field lines do not extend to arbitrarily large distances (the numerical study, at least, disclosed no such cases) the region occupied by closed field lines will be finite, although the other three regions will extend to infinity.

Now Dungey has shown [Dungey, 1963] that in models of the type discussed here the four regions generally meet along a line, which will be termed here the separatrix (a similar analysis has been applied to sunspot fields by Sweet [1958]) . The behavior of field lines approaching the separatrix is shown schematically in figure (3-a), where P is a point on the separatrix and Q' and Q'' are two points straddling the boundary of the northern polar cap. As is evident, the open field line L_1 ending at Q' and the closed field line L_2 ending at Q'' will both pass close to the separatrix before heading towards earth. Actually, the field configuration in this case is not at all planar, as will become clear in a moment, but it is hard to draw a more accurate picture in two dimensions.

Now even though the field near P resembles that existing near an X-type neutral point, the field intensity at P does not vanish, since there will in general exist a non-vanishing field component along the separatrix. In the actual magnetosphere, due to the presence of sheet currents such as one observes in the magnetopause, the point P may actually be stretched into a line, as shown schematically in figure (3-b), but this modification will not be treated.

Near earth the open field line L_1 will tend to follow the closed one L_2 and since the dipole component dominates there, both will tend to stay in a fixed meridional plane. Similarly, at points far from earth, the line L_1 will tend to follow the same direction as some unlinked interplanetary field line L_3 (figure 3-a) . Near P , however, the dominant component of the field is tangential to the separatrix, so that the line L_1 , as it approaches from infinity and turns towards the northern polar cap, will suffer there a large displacement along the separatrix.

By considering the contributions of the two component magnetic fields it may be shown, for the cases shown in figures (2), that the magnetic field along the separatrix always flows outwards from the more southern neutral point and into the more northern one, i.e. from A to B in (2-a) and from B to A in (2-b). There will always exist two branches of this line, one on the sunward side of the dipole and the other on the night side (figure 4). Thus if the field is as shown in figures (2-a) and (4), the shift suffered by the field line L_1 will cause it to arrive at the polar cap closer to B' and further away from A', compared to what might be considered the "most direct connection."

Suppose for simplicity that this "most direct connection" corresponds to a uniform polar electric field with equipotentials as in figure (5-a); then the shifts of Q' described above will have (qualitatively) the effect of distorting the equipotentials to pattern (5-b) for (2-a) and to (5-c) for (2-b), for the northern polar cap. In the southern polar cap the distortions will be oppositely oriented. In other words, the equipotentials will become crescent-shaped near the polar cap boundary, the orientation of the crescents reversing in opposite polar caps and also with opposite signs of the y component of the external magnetic field.

CURRENT FLOWS

In recent years Svalgaard [1968] and his co-workers [Friis-Christensen et al., 1971, 1972] and independently Mansurov [1969] discovered a component of the daily magnetic variation which correlates very strongly with the y component of the interplanetary magnetic field. Initially the effect was mainly evident in the vertical component at near-polar stations -- Thule in the north and Vostok in the south. At Thule the vertical component decreased during daytime hours for $B_{iy} > 0$ and increased for $B_{iy} < 0$, while at Vostok the opposite occured. The effect was approximately proportional to B_{iy} and displayed a strong seasonal variation: in wintertime it became almost undetectable, suggesting its control by ionospheric conductivity. Later and more detailed work [Svalgaard, 1972a, 1972b] suggested that the cause was a relatively narrow current circulating in the polar ionosphere during daytime hours, along a constant magnetic latitude of about 82°. The sense of rotation of this current was opposite for the two polar caps and also for opposing signs of B_{iy}, being counterclockwise around the northern magnetic pole for $B_{iy} > 0$.

Such currents are expected in the models developed here, assuming the polar cap boundary is located around 80° magnetic latitude. The crescent pattern of equipotentials in figures (5-b) and (5-c) will produce and electric field which near the boundary is approximately radial and very strong (equipotentials are crowded there and, in fact, on the boundary itself the field becomes singular, a point which will be discussed later on), directed inwards for (5-b) and outwards for (5-c). It may also be shown that in general if (5-b) holds for the northern polar cap, (5-c) is appropriate for the southern one, and vice versa. The ionospheric Pedersen current then flows radially near the cap boundary, but the associated Hall current along $\underline{B} \times \underline{E}$ will generally be several times larger and will produce the main magnetic effects. This latter current will have the location and flow direction deduced by Svalgaard.

One interesting feature of the model might be worth mentioning here. If (5-b) and (5-c) represent the potential distribution on two opposite polar caps, it will be realized that except for singular points the polar cap boundaries will have opposite electrical polarities and thus might give rise to a pole-to-pole electrical current.

In the idealized model this paradox is resolved as follows. Evidently, any pole-to-pole flow occurs mainly outside the ionosphere: its electric current would tend to flow from the boundary of (5-b) to the "first" closed field line and then follow that line to a point adjacent to the boundary of (5-c). Now from figure (3-a) it is evident that such "first" closed field lines will pass very close to the separatrix and will therefore become shifted there, in a manner very similar to the one deduced earlier for "first" open field lines. It then turns out that the closed field lines receiving current from the boundary of (5-b) all originate near point A' of (5-c), which has the same potential. Ideally such field lines experience no potential drop and carry no current; in practice, of course, the field undergoes considerable modification near the boundary (as will be discussed later) and this limits the applicability of the preceding argument.

NUMERICAL RESULTS

Polar electric potential plots resembling (5-b) were obtained numerically for the simple model of a dipole in a constant field, using the field line tracing method described earlier. In the cartesian frame of figures (2) an array of points was selected on the northern polar cap, at one degree intervals in x and y (the curvature of the surface was neglected — all tracings started at $z = 1$). A field line was then traced from each of the points: lines which returned to earth were marked as being connected to points outside the polar cap, while others were followed until the projection of their radius vector on the direction of the external field reached 200 earth radii.

Now at large distances from earth the contribution of the dipole may be neglected and the field tends to a constant vector

$$\underline{B}_i = B_{ix}\hat{\underline{x}} + B_{iy}\hat{\underline{y}} + B_{iz}\hat{\underline{z}} \qquad (2)$$

The solar wind is also assumed to be constant

$$\underline{v} = -v\hat{\underline{x}} \qquad (3)$$

Equation (1) then shows that the electric field can be derived from a scalar potential

$$\phi = v(B_{iz}y - B_{iy}z) \qquad (4)$$

The quantity in parentheses was derived for the end point of each open field line, of those traced, and was then printed on a map of the polar cap corresponding to the point at which the field line originated.

Some typical results are shown in figure (6), where the components of $\underline{B}_i$ were taken as $5 \cdot 10^{-5}$ of the dipole field at the pole, which comes to about 3γ. As can be seen the crescent pattern is evident in all cases. It appears that the x component of $\underline{B}_i$ tends to rotate the pattern around the pole and also that the z component controls the size of the polar cap, which is largest for fields with southward (negative) B_{iz}. This latter result resembles one obtained by Forbes and Speiser [1971].

Another interesting result concerns the variation of $\underline{E}$ along a dawn-dusk cross section in the central part of the polar cap. In all cases investigated this field was weakest near the singular point at which all crescents met and became several cases stronger on the opposite side (this variation does not take into account the boundary effect discussed in the preceding section, which produces very strong fields near the boundaries). This behavior was in complete agreement with the variation of the central polar electric field observed aboard OGO-6 [Heppner, 1972], which exhibited correlations with the y component of the interplanetary magnetic field. The significance of this agreement is not clear because (as will be pointed out in a later discussion of this point) the true magnetosphere differs considerably from this model.

In order to test the effects of the asymmetry introduced by an extended magnetospheric tail the dipole was replaced by an asymmetric field with nightside field lines stretched out and dayside ones slightly compressed. This field was expressed in Euler potentials

$$\underline{B} = \nabla\alpha \times \nabla\beta \qquad (5)$$

with

$$\alpha = \sin^2\theta / r - \lambda \sin^6\theta \, (1 - \cos \varphi)^3$$
$$\beta = \varphi \qquad (6)$$

and it reduces to the dipole field (with unity dipole moment) in the limit $\lambda = 0$. Taking $\lambda = 0.005$ then creates an asymmetry roughly comparable to that of the observed magnetosphere, but the polar potential aptterns for such a field model remain close to those obtained with a pure dipole field. The main difference seemed to be that the polar cap now extended about 25% further on the night side. As a result, the crescents formed between adjacent equipotentials became asymmetric, widening on their night sides and narrowing on their day sides.

As an added test the integration was extended in one calculation to distances of both 200 R_e and 300 R_e. The difference in derived electric potentials was of the order of 1% .

FIELD LINE MERGING

So far the model has taken into account the solar wind velocity $\underline{v}$ only at large distances from the dipole, using it for deriving the electric field there and then propagating $\underline{E}$ to the polar caps by regarding field lines as equipotentials.

Actually, of course, $\underline{v}$ has to agree with $\underline{E}$ and $\underline{B}$ throughout all space; the relation between these observables is expected to tend to the form of equation (1), except in regions in which special conditions prevail. The form of $\underline{B}$ itself depends on the physical processes which occur : since in the present model $\underline{B}$ is prescribed a-priori, only the topology of the field is expected to be valid. Similarly, the values of $\underline{v}$ derived to fit such a model will resemble the actual flow pattern only in a most general way. Nevertheless, such solutions for $\underline{v}$ represent the first step towards understanding the flow pattern created by the interaction of the solar wind with the magnetosphere.

This raises the question whether such solutions exist. Solutions which satisfy (1) for the closed magnetosphere can be found: they have a velocity field which flows tangentially to the boundary on its outside (like a fluid flow encountering an obstacle) and which vanishes inside the boundary. In this case $\underline{E}$ vanishes on closed field lines and is discontinuous on the boundary; because of (1), however, the external electric field is everywhere orthogonal to the boundary and may be maintained by a suitable charge distribution. Of course, such a model has no polar electric field and therefore does not fit the interpretation of observations assumed here.

In an open model of the magnetosphere the open ends of field lines connected to the polar caps move with the solar wind, and therefore such lines must have been initially unlinked (open at both ends) and will ultimately become unlinked again. The process by which the connectivity changes is <u>field line merging</u> [<u>Dungey</u>, <u>1953</u> ; <u>Stern</u>, <u>1966</u>, and references cited there; <u>Yeh and Axford</u>, <u>1970</u>] . The merging of field lines in the model field of figure (2-a) is schematically shown in figure (7) ; similar stages may occur in a reversed sequence when open polar field lines reconnect and become unlinked again.

Allowing the process of merging and reconnection to take place, one can now derive an appropriate velocity field satisfying equation (1), for open field lines connected to the polar caps, by noting that this equation implies that $\underline{v}$ is a field line velocity. One consequence of this — the only one that counts here — is that particles moving with $\underline{v}$ and sharing the same field line will continue doing so at all times. The remaining property of field line motion, namely flux preservation, is automatically insured if at infinity $\underline{v}$ reduces to the undisturbed solar wind flow.

On open polar field lines an appropriate $\underline{v}$ is thus obtained [e.g. Levy et al. 1964] if particles close to earth move so as to continue to share the same field lines with distant particles embedded in the undisturbed solar wind. On unlinked field lines the motion should be continuous with that on open field lines and should be obtainable in the same fashion. Unfortunately, the fact that unlinked lines have both ends in the solar wind leads to an internal inconsistency of this particular model. In the situation shown in figure (2-a), for instance, if particles are assumed to stay on the same field lines as solar wind particles at either $y \to \infty$ or $y \to -\infty$, different values of $\underline{v}$ are obtained in each case.

Another way of viewing this problem is by considering the electrostatic potential ϕ, which at points far from the origin, in figure (2-a), is proportional to z. If both ends of any unlinked field line are to be at the same potential, they must asymptotically approach the same value of z. In fact, the dipole field contribution near the plane $z = 0$ is always northward, so that field lines approaching from $y \to -\infty$ will always be raised near the origin to higher levels of z and their asymptotic values of z (and of ϕ) will <u>not</u> match. Consequently their asymptotic values of $\underline{E}$ do not match, either, and this by equation (1) leads to the discrepancy in $\underline{v}$ noted before.

It might be possible to eliminate this discrepancy by a more refined model in which every unlinked field line has matching asymptotic directions, although the practical derivation of such a model in analytical form appears to be rather difficult. We do not believe that such a refinement would lead to different qualitative results, and since this work is mainly concerned with qualitative properties, we shall not attempt to include it. In the real magnetosphere, of course, such matching is expected to occur, and there will exist additional field sources to produce it ; one possibility for such sources are the currents in the magnetopause. In figure (7) where the merging sequence of an unlinked field line is schematically given, matching asymptotic directions are indeed assumed.

CONVECTION

In the idealized model, the entire boundary of the polar cap (except for one singular point) is an equipotential and therefore $\underline{E}$ is normal to the surface covered by field lines emanating from it. In analogy with the closed model, therefore, it should be possible to find a solution in which both $\underline{E}$ and $\underline{v}$ vanish on closed field lines. Unfortunately, $\underline{E}$ also becomes singular as the boundary is approached, so this simple result is not possible because of physical reasons.

Two ways exist for resolving this difficulty in the actual magnetosphere. It is possible that the separatrix is in fact a neutral line, due to fields produced by additional currents not included in the simple model : then merging can occur at all points along it and merging field lines suffer no contortion. In this case equipotentials are no longer tangential to the polar cap boundary , causing $\underline{E}$ to penetrate to closed field lines and producing there a flow field $\underline{v}$, which would be the well-known convection flow deduced from observations [Axford, 1969, and references cited there] .

Alternatively, the field at the separatrix may not vanish but have a preferred direction, so that merging remains confined to the neutral points, as drawn in figure (7). However, as the equipotentials are pressed together during the merging process (in what may be a relatively weak magnetic field) current flow between them takes place. The points on the polar cap boundary then no longer have the same potential but each of them assumes the potential appropriate to a point located somewhat poleward from it, producing the pattern shown schematically in figure (8-a) . This again extends $\underline{E}$ onto closed field lines and creates a convective flow.

Existing theories are usually based on the field of figure (1) or on modifications of it and tend to assume the first possibility. However, the existence of the "Svalgaard effect" suggests that the second one is appropriate for the sunward side of the magnetosphere. The absence of the effect at night and the existence of a long magnetospheric tail indicate that the situation might be somewhat different on the night side, and this will be discussed in the next section.

With this modification the equipotentials in the polar cap might appear as the solid lines in figure (8-b), where straight lines were drawn on the night side of the cap in the absence of a more detailed model. The ionosphere will extend the equipotentials in a way shown schematically by the broken lines in the drawing, leading by eq. (1) to a convective flow on closed field lines. In any actual derivation of such equipotentials one must take into account, among other things, the potentials of the opposite polar cap and also the shear of closed field lines which pass near the separatrix (which, as was noted before, is expected to counteract the tendency for currents to flow from one polar cap to the other). The final result, of course, is that field lines and particles attached to them move in such a way that their "roots" follow the closed equipotentials, as suggested in convection theories.

A REALISTIC MODEL FOR THE TAIL

In any attempt to deduce the role of the magnetospheric tail in an open magnetosphere one must take into account the discrepancy in the magnitude of $\underline{E}$ between predictions based on simple models and what is actually observed.

In the computer calculations described earlier the magnitude of the voltage drop across the entire polar cap was in all cases of the order of $4 - 7 \cdot 10^5$ volts, which is $10 - 20$ times larger than the observed value. This discrepancy is rather fundamental and can be inferred from simple considerations of magnetic flux. If the polar cap extends 1300 km from the pole (in agreement with observations and with the model used) the total magnetic flux through it is about $3.3 \cdot 10^{12}$ gauss m^2. In interplanetary space, with an ambient magnetic field of 5γ, the same bundle of field lines has a cross section of $6.6 \cdot 10^{16}$ m^2, which is approximately the area of a square with a side of 40 R_e. Now with $v = 300$ km/sec the electric field is about 10^4 volt/R_e, which would indicate a voltage of at least $4 \cdot 10^5$ volts across the bundle and consequently also across the polar cap.

The most plausible explanation here [Morfill and Scholer, 1972] is that the same forces which stretch out the geomagnetic tail also elongate the bundle to the same length as the tail -- for instance, into a slab 400 R_e long in the direction of $\underline{v}$ and 4 R_e wide in the direction of $\underline{v} \times \underline{B}$. The polar bundles of open field lines are thus mostly stretched out along the sides of the tail, shedding their flux gradually with increasing distance from earth. This agrees with tail models derived from the arrival of solar protons at the polar caps [Van Allen et al., 1971 ; Scholer, 1972 ; Morfill and Scholer, 1972].

The physical cause of this deformation is not clear. Levy et al. [1964] who were the first ones to introduce it (implicitely) believed that the cause was a lack of efficiency in the merging process, allowing only a fraction of the magnetic flux intercepted by the front of the magnetosphere (in their work, 20%) to be merged with geomagnetic field lines, while the remainder would be shoved aside.

A point worth noting in this connection is that the deformation of the "exit slot" from a 40 R_e square to a rectangle 4 R_e wide decreases the impedance of the external circuit of polar cap currents by a factor of 100 . A simple estimate of sheath conductivity gives the order of $2 \cdot 10^{-9}$ M.K.S., which indicates that at least an appreciable fraction of these currents will close in the magnetosheath, thus shielding the bulk of the interplanetary electric field from the "short circuiting" effect of the polar ionosphere. It is possible that the elongation of the bundle reflects some physical process which promotes such shielding.

A tentative cross section of the tail for the same external field direction as in (6-b) is given in figures(9). One interesting feature of this model is that there might not exist any detectable boundary between open polar field lines and closed field lines in the high latitude tail. There will, of course, be a current across the tail, with its circuit completed by flows on the top and bottom of open field lines; the flow of this current across the exit slot may then be partially carried by the magnetization current at the boundary of the dense sheath plasma and partially by guiding center transport in the region of sharply kinked field lines (upper left in figure 9-b).

In the case drawn in figures (9) the north-south component of $\underline{B}_i$ vanishes. If that component were to point southward, the connection of the interplanetary field to the top and the bottom of the tail would be more direct, while if it pointed northward, the connection would be more contorted. It could be that this factor is related to the observed correlation between geomagnetic activity and southward $\underline{B}_i$. Of course, for reasons given earlier, the field lines bounding each slot must ultimately tend to the same asymptotic direction as corresponding lines on the other side of the tail.

Two final points may be noted. First of all, the effects discussed so far do not explain the correlation between B_{iy} and the polar electric field observed by Heppner [1972] . Using electric field probes aboard OGO-6 , Heppner noted that for $B_{iy} > 0$, E_y at the center of the northern polar cap was stronger on the dawn side, while for $B_{iy} < 0$, E_y was stronger on the dusk side. At the southern polar cap the correlation was reversed.

In the computer experiments shown in figures (6) such a behavior is indeed evident, but in view of the great distortion of the actual magnetosphere compared to the model , discussed earlier in this section, not too much reliance should be placed on this fit. What actually happens may be the following.

Consider two strips on the northern polar cap, aligned with the noon-midnight direction and cutting equal amounts of magnetic flux (figure 10) . If $B_{iy} > 0$, then Heppner's observations indicate that the electric field in strip A is stronger than that in strip B . By the preceding arguments, this implies that the field lines of A enter the interplanetary field along a wider "slot" than those of B . However, since the fluxes cut by the strips are equal, this also means that the exit cross section of A is the shorter of the two — e.g., the field lines of A leave the tail along a distance of 200 R_e while those of B leave along a distance of 400 R_e . Referring to figure (9) one sees that this is a plausible situation, since the field lines of A are located on that side of the tail which is closer to the exit. Thus the "exit slot" of the bundle of open polar field lines may well be not rectangular but rather taper down with distance along the tail.

The second point concerns the forces acting on the tail current as it flows across the plasma sheet and both exit slots (figure 9) : in all three cases $\underline{j} \times \underline{B}$ points earthward. Presumably this balances whatever force extends the tail backwards, which apparently is not electromagnetic — e.g.,entry of sheath particles into the tail region would produce such a force.

SUMMARY

In this work an attempt was made to derive the qualitative properties of the open magnetosphere and of its electric field. This was done in two stages. First, a simple model was investigated, consisting of a dipole in a constant field; later on this model was modified in ways suggested by observations of the actual magnetosphere.

The simple model pointed out the role of the separatrix, the curve (possibly broadened into a sheet) marking the boundary between four basic types of field lines — closed lines, lines unlinked to the earth and open lines ending in either the southern or the northern polar cap. The simple model suggested that regardless of the external magnetic field the polar electric field would always tend to point from dawn to dusk, but it also showed that equipotentials tended to curve in crescent-shaped patterns. These crescents had opposite orientations in opposite polar caps and also with opposite signs of B_{iy} , suggesting that they might explain Svalgaard's correlation between polar magnetic variations and B_{iy} .

In the simple model field lines that approach the boundary of the polar cap behave in a singular manner. In evaluating modifications of this model it was realized that such field lines could not remain electric equipotentials and a simpler pattern was suggested: one consequence of this simplification was that it required plasma motion on closed field lines, in agreement with existing theories of magnetospheric convection. Another discrepancy existed in the magnitude of the polar electric field: the solution, already suggested by research on access of solar protons to the magnetosphere, seems to be that the polar field lines emerge into the interplanetary magnetic field in an elongated but narrow bundle, which probably tapers down with increased distance from earth.

In order to derive a complete picture it was necessary to use a model which was not completely self-consistent , and also to bridge some theoretical gaps by observation or guesswork. The most significant gaps were (1) the lack of a 3-dimensional theory on field line merging along a separatrix; (2) the lack of a realistic theory of the magnetopause, providing an indication of the mechanism by which the separatrix might actually be broadened into a sheet; and (3) the lack of a good understanding of the forces extending the earth's magnetic tail, which might perhaps explain the formation of the long and narrow "windows" through which the polar caps appear to be connected to the interplanetary field. It is hoped that future work will clarify these points and also provide a better model than the simple one used here.

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CAPTIONS TO FIGURES

Fig. 1 Schematic view of the field of a dipole immersed in a constant southward field.

Fig. 2 Schematic view of the field of a dipole immersed in a constant field orthogonal to its axis, with the solar wind moving into the plane of the drawing: (a) external field parallel to y axis ; (b) external field antiparallel to y axis.

Fig. 3 Schematic cross section of separatrix, with the projections of some field lines that pass close to it: (a) the case of a linear separatrix; (b) a separatrix broadened into a sheet.

Fig. 4 Schematic view of the separatrix in figure (2-a).

Fig. 5 Schematic view of polar cap equipotentials: (a) if field lines are connected to the polar cap by the "most direct route" ; (b) with field lines shifted near separatrix towards dusk side ; (c) with a similar shift towards the dawn side.

Fig. 6 Actual plot of equipotentials at intervals of 10^5 volts across the northern polar cap, assuming the simple model and an interplanetary electric field of 10^4 volt/R_e . Axes meet at the pole and are marked at 2 degree intervals, while the polar cap boundary is marked by broken lines. With the dipole field at the pole normalized to unity, the figures represent the following cases: (a) external field of $5 \cdot 10^{-5}$ in the y direction, as in figure (2-a) ;

(b) with component of $5 \cdot 10^{-5}$ added to (a) in the direction antiparallel to x axis, giving typical field of "away" sector ; (c) with southward component of $5 \cdot 10^{-5}$ added to (b) ; (d) with northward component of $5 \cdot 10^{-5}$ added to (b) .

Fig. 7 Schematic view of four stages in the merging of an unlinked field line with a dipole field.

Fig. 8 Expected form of polar equipotentials if equation (1) is violated on field lines close to merging: (a) schematic form from general considerations ; (b) form expected in actual magnetosphere, with "Svalgaard effect" occuring only on day side. Broken lines trace continuation of equipotentials onto closed field lines.

Fig. 9 Schematic view of configuration of magnetospheric tail, with an external field corresponding to an "away" sector: (a) configuration of external field lines near magnetosphere ; (b) cross section of tail, with circled dots representing field lines rising from the plane of the drawing and circled crosses representing field lines descending into the plane of the drawing. Lines ending in circled dots or crosses represent field lines in the plane of the figure which bend away from that plane.

Fig. 10 (no caption)

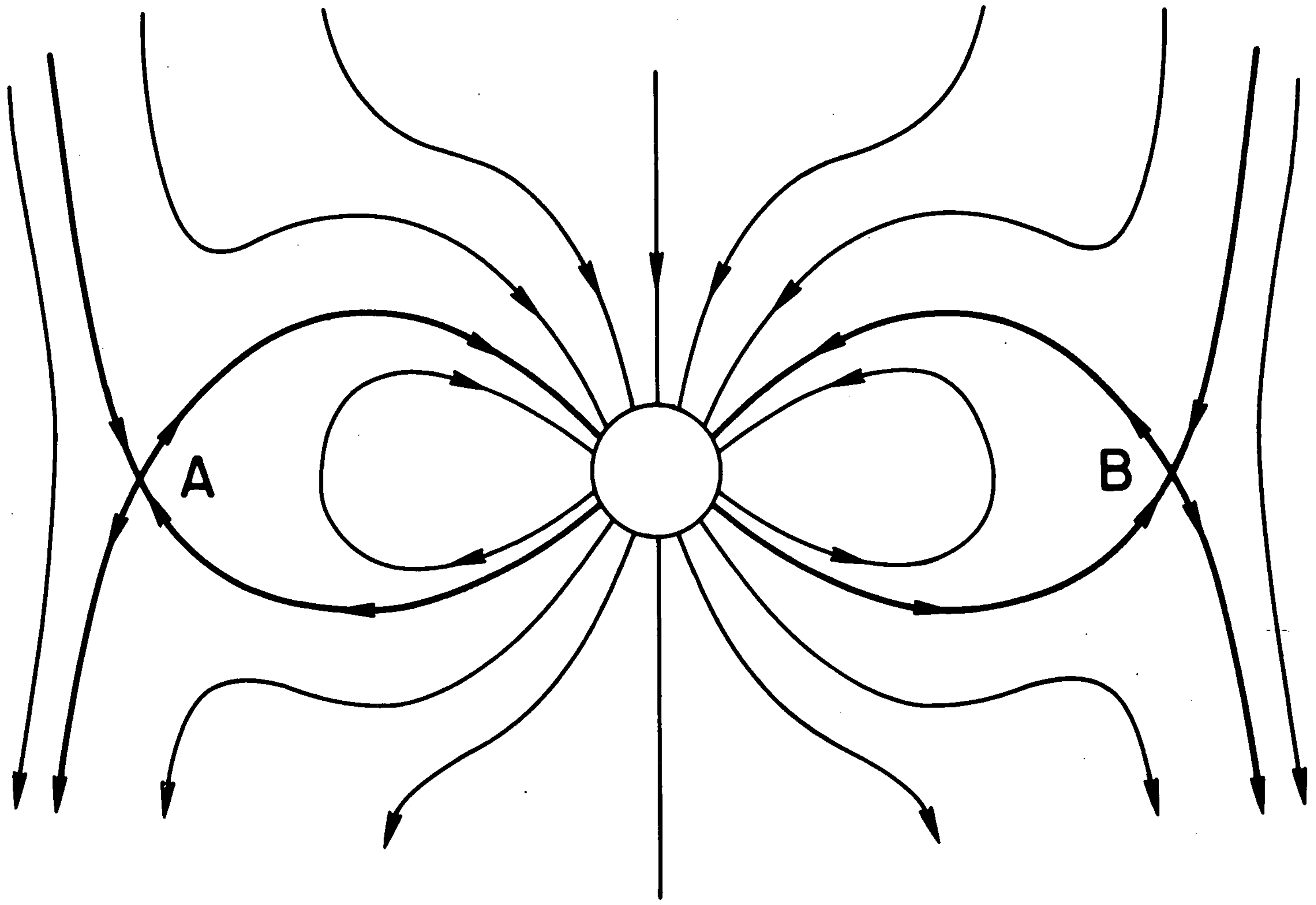
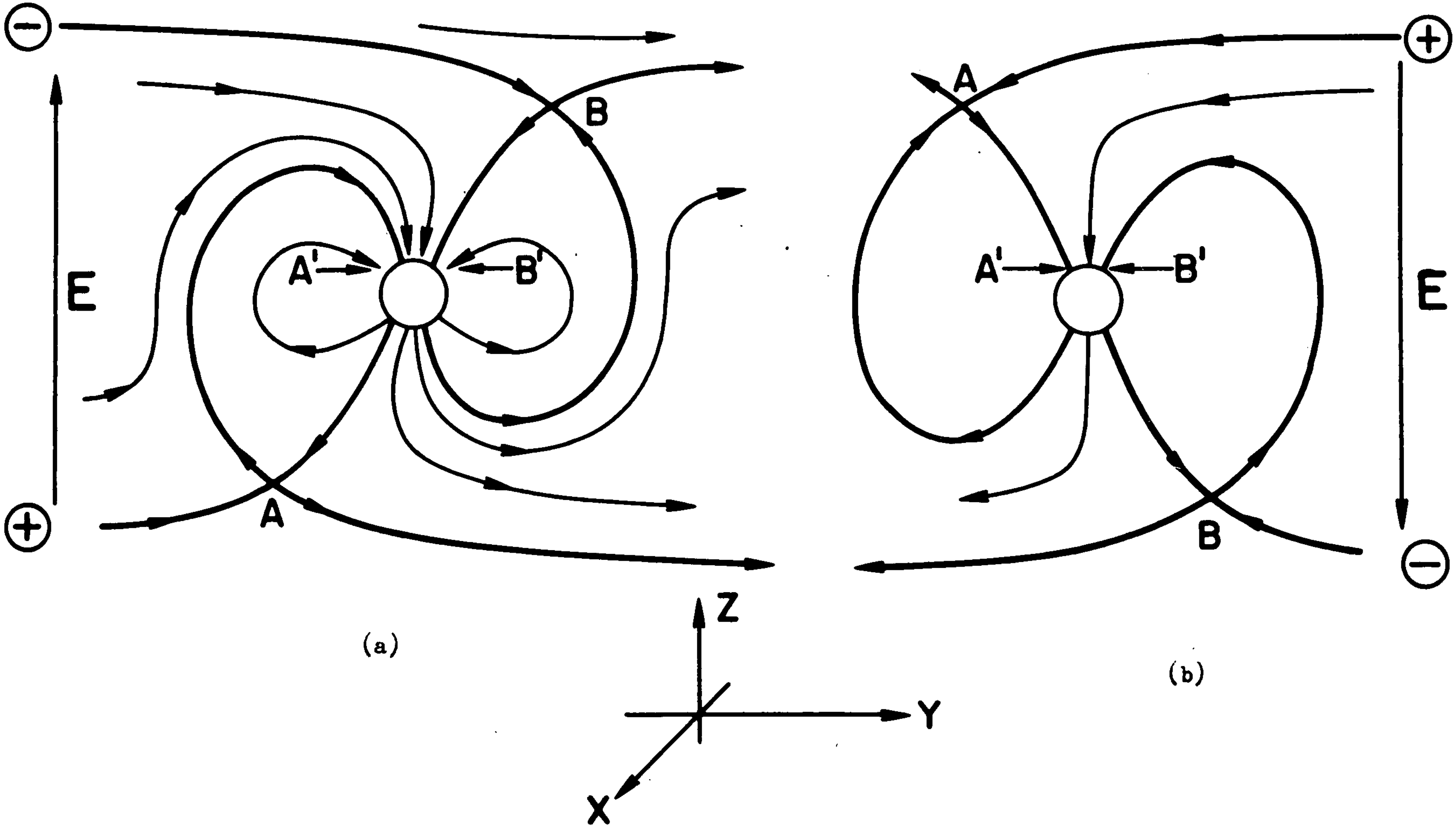


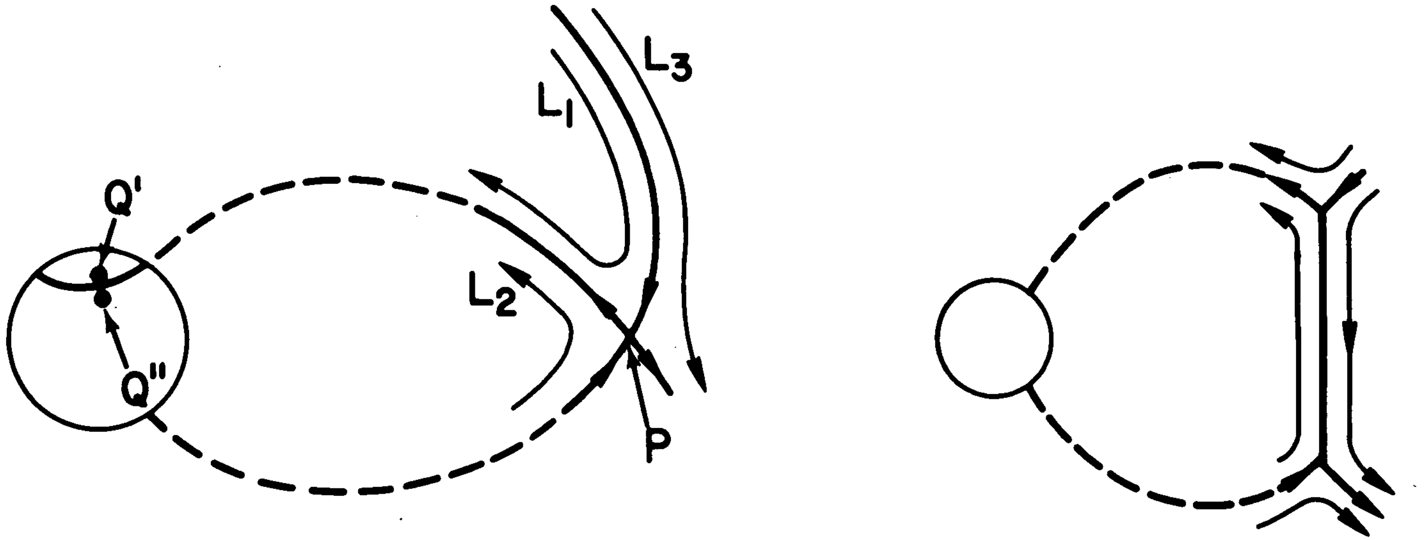
Figure 1



(a)

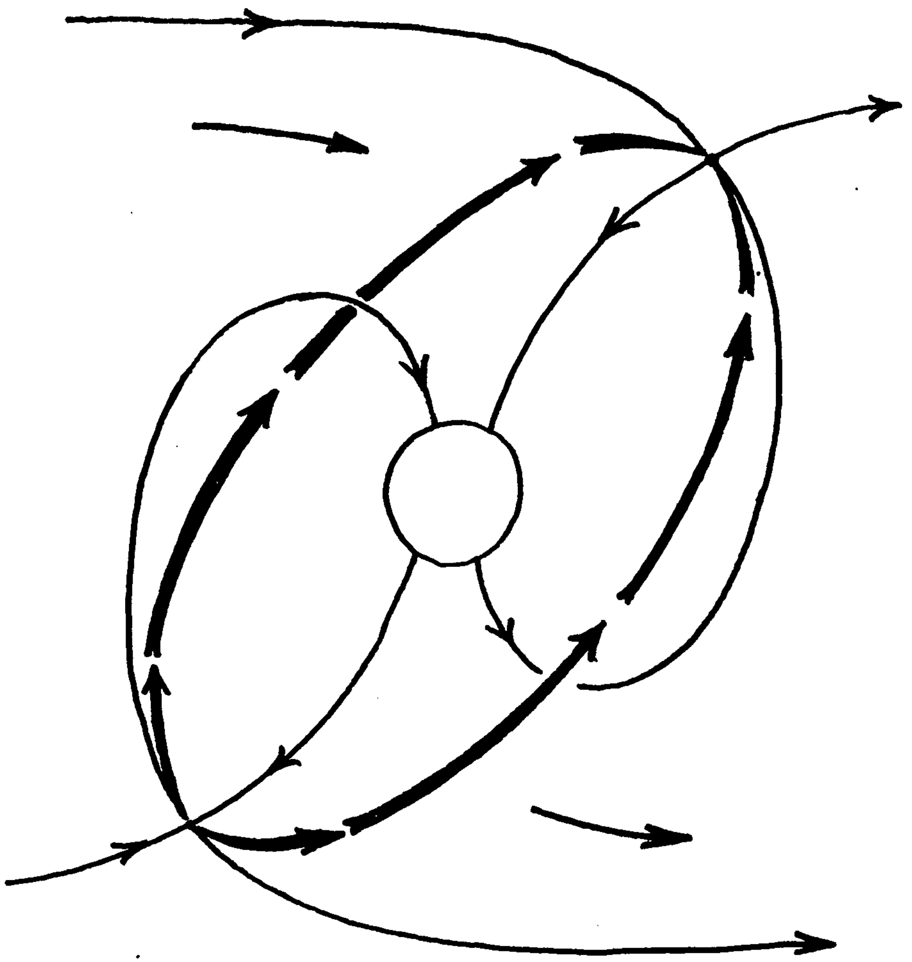
(b)

Figure 2



(a) (b)

Figure 3



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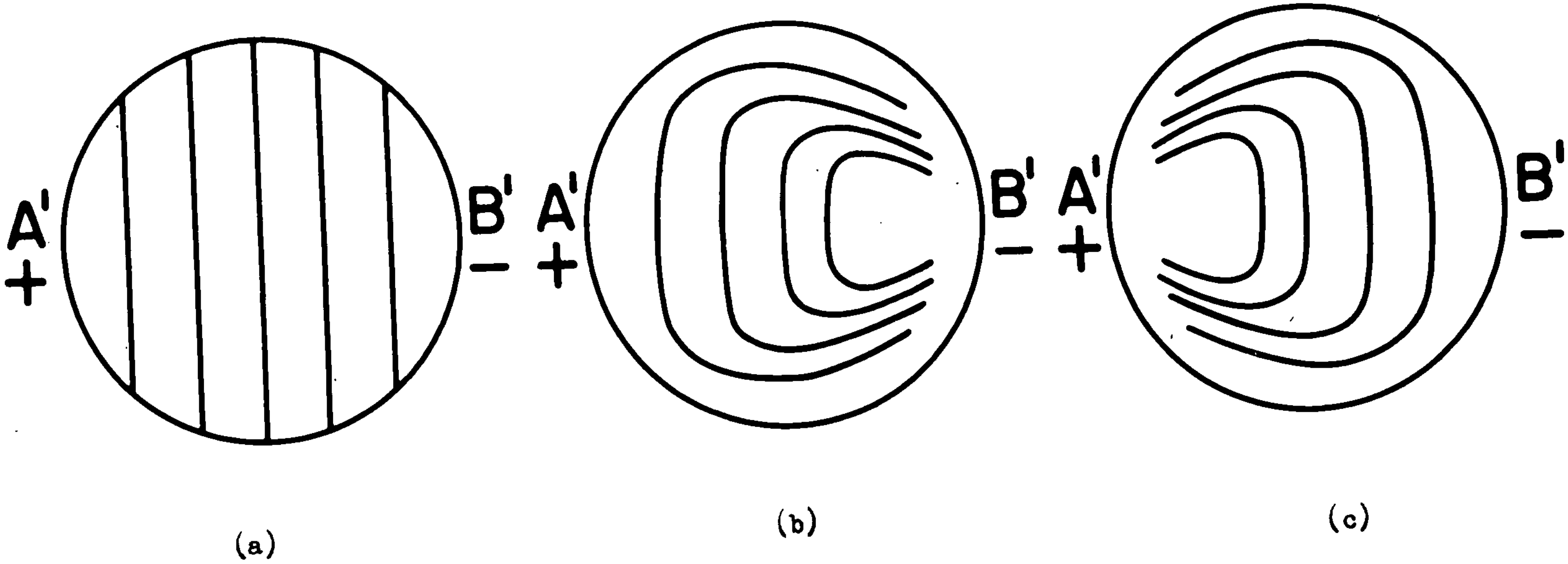


(a) (b) (c)

Figure 5

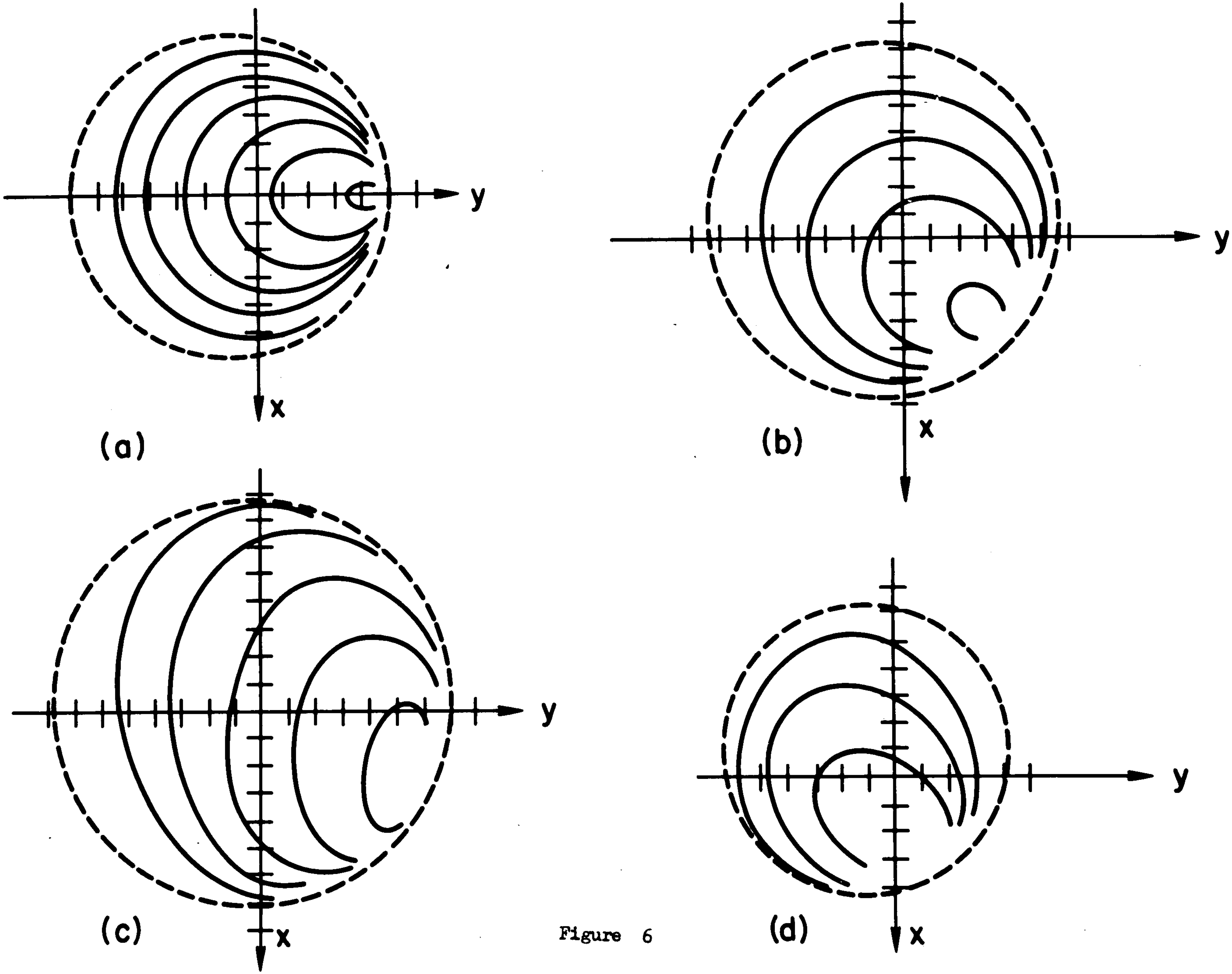


Figure 6

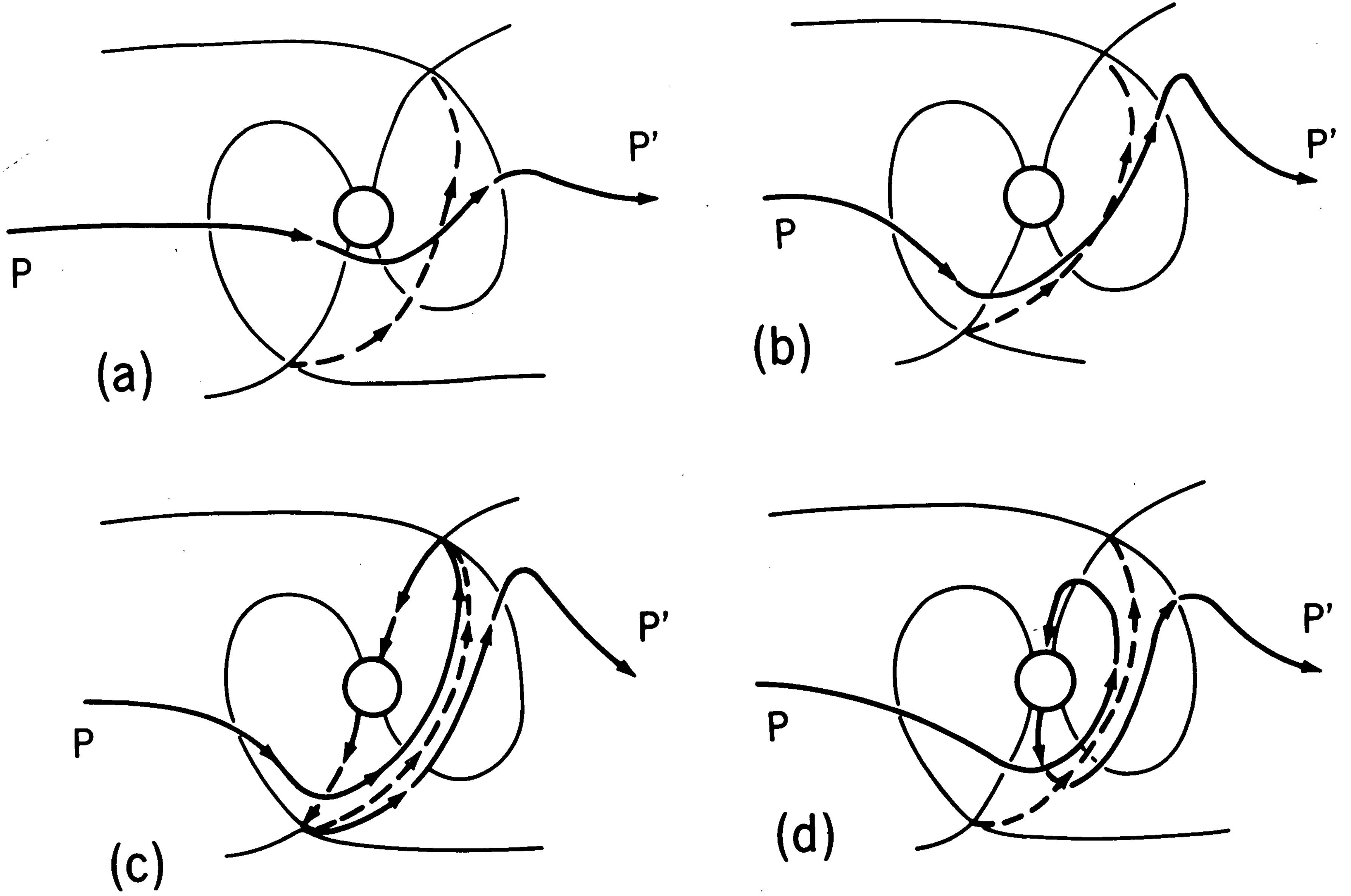
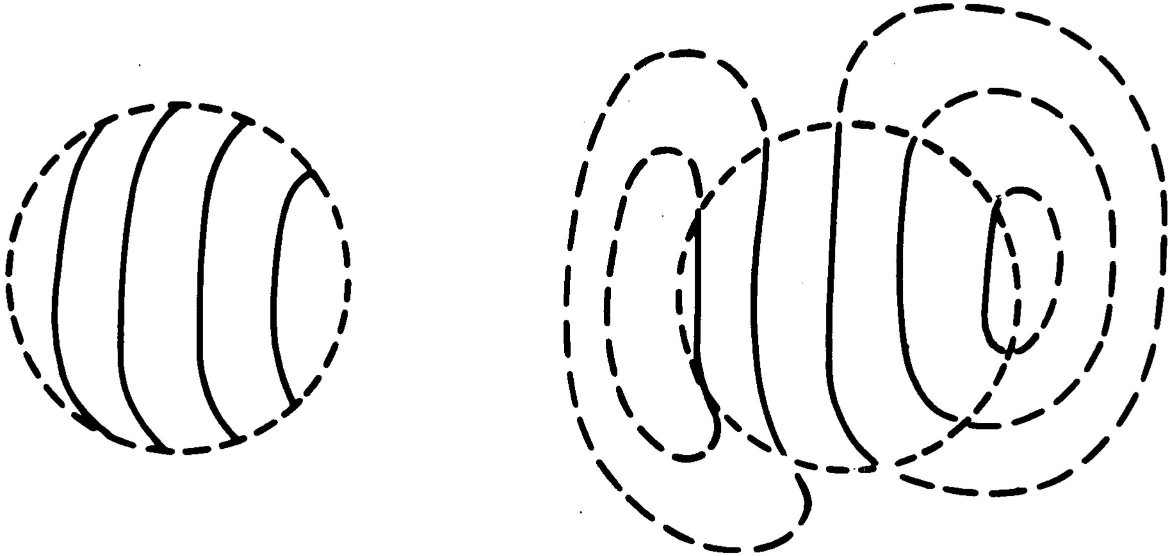


Figure 7



(a) (b)

Figure 8

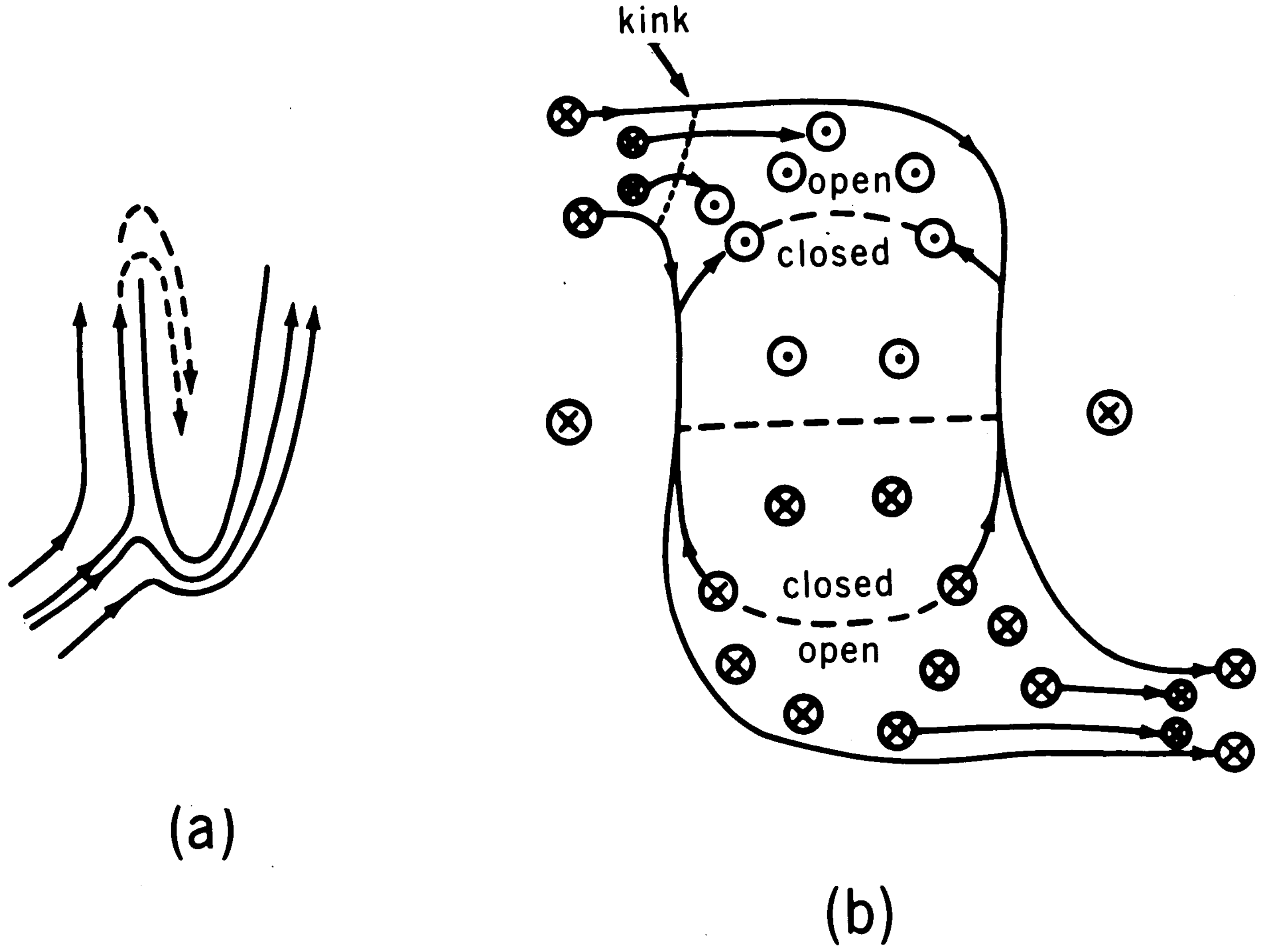


Figure 9

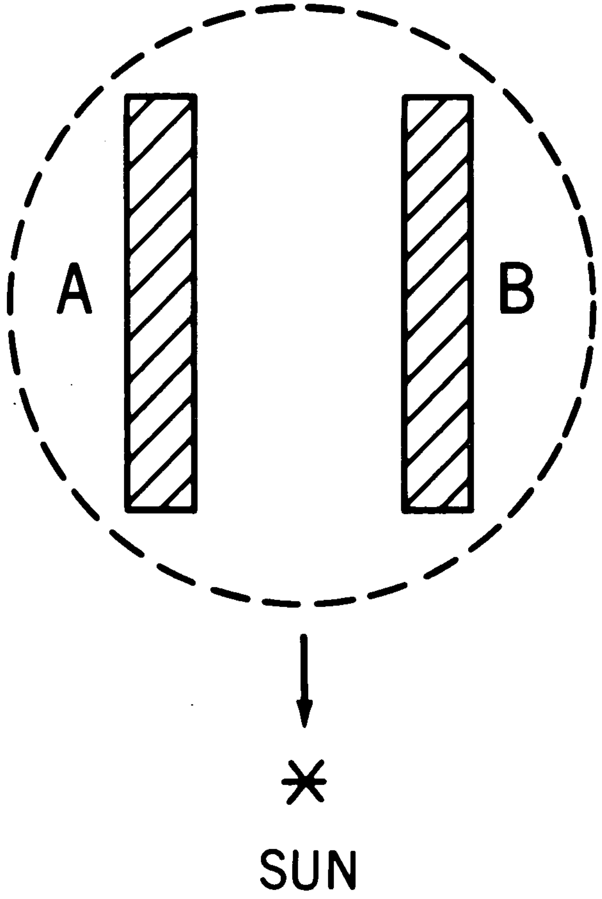


Figure 10

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