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ABSTRACT

Assessed was the performance of subjects who had been exposed to transformational geometry treatment. The performance test consisted of geometry achievement and spacial ability measures. Second and third grade subjects were randomly assigned to experimental and control groups. The experimentals received a twelve lesson treatment and the controls a one lesson treatment concerning rigid motion and congruence concepts. Pretest and posttest results indicated that the experimentals performed significantly better than the controls on the achievement test, but not on a space test. [Not available in hardcopy due to marginal legibility of original document.] (Author/CT)

A STUDY OF TRANSFORMATIONAL GEOMETRY INSTRUCTION IN THE PRIMARY GRADES

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ABSTRACT

This study assessed the performance of subjects who had been exposed to transformational geometry treatment on geometry achievement and spatial ability measures. Second and third grade subjects were randomly assigned to experimental and control groups. The experimentals received a twelve lesson treatment and the controls a one lesson treatment concerning rigid motion and congruence concepts. Pretest and posttest results indicated that the experimentals performed significantly better than the controls on the achievement test, but not on the space test. The results imply that children can learn transformational geometry skills, but they cannot apply these skills to more general tasks.

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The Cambridge Conference Report of 1969 and the geometry report of the Ontario Institute for Studies in Education (1967) are among sources giving a variety of arguments favoring the study of geometry, in particular transformational geometry, in the elementary grades. Transformational geometry includes the study of rigid motion -- translation, rotation, and reflection -- and congruence, or rigid motion invariance. The concepts of transformation and invariance are considered to be rudimentary notions which pervade all of mathematics and science (Dienes and Golding, 1967; Kapur, 1970). Mathematical properties from the various branches of geometry (topology, projective geometry, affine geometry, Euclidean geometry) can be described in terms of transformations which may be represented through several types of manipulative activities. The manipulative activities of Euclidean geometry can be described in terms of slides (translations), turns (rotations), or flips (reflections) which, alone or in combination, leave a figure or object unchanged except for its position.

Transformational geometry topics may be approached quite naturally through the manipulation of concrete objects or figure drawings. Such an approach is consistent with a general consensus among psychologists and educators that knowledge at a very basic level can be gained through personal involvement interaction. Piaget and Inhelder (1956) describe an action component of cog functioning which is built upon sensorimotor actions, but goes beyond them. Initially, the child performs actions upon objects. But eventually, after th objects become distinct images, the child is able to perform mental transf (actions) upon his images. Piaget (1964) maintains that imagery evolves f initial level of reproductive images based completely upon past perception level of true anticipatory images which are imagined to be the result of an transformation.

> The imagery itself becomes more mobile, becomes anticipatory. Now it becomes an instrument of representation capable of serving the operations. It is a symbolic instrument and an auxiliary instrument which is not an element of thought itself, but is simply a tool, an aid to the progress of thought -- an aid that takes the form of figurative representation (Piaget, 1964, p. 31).

The fact that imagery manipulations may be described in terms of g transformations leads to a question concerning effects of the study of tr mational geometry upon one's imagery or spatial abilities. Will the st slides, flips, and turns upon objects or drawings increase the ability to mental manipulations upon objects or drawings? The research of Brow and Meyers (1953) give inconsistent results concerning the effects of tr the spatial ability of high school and college students. Despite the fact committee reports and other recommendations have made specific stat favoring transformational or motion geometry in the elementary school, research evidence exists to support the recommendations. A study by S and some work by Walter (1966) do relate the successful teaching of som geometry topics in the elementary grades.

The purpose of the present study was to ascertain information concerning two major questions: (1) To what degree do second and third grade children learn transformational geometry concepts under specific instructional conditions?, and (2) What are the effects of transformational geometry instruction upon children's spatial abilities?

Method

Subjects

A sample of 63 subjects were selected from a population of 106 second and third grade pupils from six classrooms of two schools in Jackson County, Georgia. The 106 pupils were identified by their teachers as being of average or above average ability in terms of general classroom performance. Random procedures were used to assign 15 second graders and 16 third graders to an experimental group and 16 second graders and 16 third graders to a control group. Two levels of I. Q. were specified; above average children were those whose I. Q. scores ranged from 104 - 144 and average children were those whose scores ranged from 81 - 103. The mean I. Q. scores were 105.6 for the experimental group and 106.7 for the control group. No significant I. Q. differences were detected between the experimental and control groups at either level of grade or I. Q.

Curriculum program

An experimental instructional unit was designed to teach the mathematical concepts of congruence and rigid motion. The activities of the unit were divided into three sections: congruent figures, rigid motion, and congruence and motion.

Activities within the congruent figures section required the children to identify congruent objects or drawings or to construct congruent matches of given drawings. The section on motion was designed to teach children the mechanics of performing slides, turns, and flips upon given drawings to obtain appropriate image figures. The final section emphasized the matching of corresponding points of given pairs of congruent figures, and the study of particular ways (slide-congruence, turn-congruence, and flip-congruence) in which pairs of congruent drawings could be made to match. In general, the lesson activities progressed from those requiring manipulation of real objects, to the manipulation of tracing illustrations, and finally to the mental manipulation of figure drawings. Many activities involved the use of worksheets which were similar to the Motion Geometry materials of the University of Illinois Committee on School Mathematics (Phillips and Zwoyer, 1969).

A single lesson control treatment dealing with terminology and with a brief overview of the experimental unit was constructed. The control lesson included at least one exercise with each rigid motion type.

Instrumentation

An achievement test was constructed to measure the objectives of the experimental unit. The test included 44 items related to objectives (2) through (6) given in Table 1 which were scored as being either right (one point for a correct response) or wrong (no points for an incorrect response), and six multiple choice items related to objective (1) for which partial credit could be obtained. A single composite score was devised for this six item cluster. The 6 multiple choice items and 22 of the 44 dichotomous (either right or wrong) items were classified as "comprehension" items, because they measured behavior in situations analagous to those seen in the instructional treatment. The remaining 22 dichotomous items were classified as "application" items because they measured behaviors which required pupils to perform in situations more remote from activities encountered in the experimental treatment.

Table 1

BEHAVIORAL OBJECTIVE STATEMENTS

A student is able to...

(1) Identify those figures in a given collection of drawings which are congruent to a stimulus figure;

(2) Produce or complete a figure which is congruent to a given figure;

(3) Produce the image of a given figure under an indicated motion;

(4) Complete the image of a given figure under an indicated motion;

(5) Identify corresponding points of a given pair of congruent figures;

(6) Identify all congruences of two congruent figures; and

(7) Specify point matches or identify motions relating a given pair of congruent figures.

The space test consisted of four ten-item subtests and was designed to measure the ability to perform mental spatial manipulations. One subtest which included items from the Revised Minnesota Paper Form

Board measured the ability to mentally construct a puzzle from a pictured loose array of puzzle pieces; the second subtest consisted of items taken from the Space subtest of the Differential Aptitude Tests and measured the ability to visually construct a three-dimensional figure from a pictured two dimensional pattern; the ten items of the third subtest were from the Abstract Reasoning subtest of the Differential Aptitude Tests which required the ability to recognize sequential patterns that could be described in terms of rigid motions; the fourth subtest which measured the ability to visualize the unfolded appearance of a pictured piece of paper which had been folded and then punched was taken from the Paper Folding subtest of the Kit of Reference Tests for Cognitive Factors. A single point was scored for each space test item.

Procedure

Prior to the main study, the instructional and testing materials were revised through a pilot examination. The achievement and space tests were administered to all subjects both before and after instruction. The achievement test required two 30 to 40 minute sessions, and the space test was presented in a single 30 minute session. The experimental unit was administered in twelve 25 to 30 minute sessions which were held approximately 3 times a week over a period of 4 to 5 weeks. For each experimental session the experimenter (the author) removed the experimental subjects from their regular classes, whereas the control subjects remained in class with their regular teachers. During the experimental sessions the control subjects were usually involved in reading lessons or seat work related to language arts. During the instructional sessions the teacher demonstrated activities or directed pupil demonstrations and aided those who appeared to have difficulty. Some pupils who finished worksheet activities early were permitted to help others or to demonstrate the activities at the overhead projector. Approximately one week prior to the completion of the experimental treatment, the control treatment was administered.

Analysis

An item analysis was performed on all of the dichotomous achievement items. A point biserial correlation coefficient, a difficulty index, and a Phi coefficient were computed for each item; also, an internal-consistency reliability coefficient was computed for all items in the item analysis. Program MUDAID (Multivariate, Univariate, and Discriminant Analysis of Irregular Data) was used for the multivariate and univariate analysis of variance of both the achievement and space test data (Applebaum and Bargmann, 1967). MUDAID provides an analysis of each response variable for combinations of the independent variables taken two at a time. Hence, for each variable an analysis for treatment versus grade, treatment versus I.Q., and grade versus I.Q. was printed out. Gain scores of the treatment group on the achievement and space tests were compared with those of the control group by using t tests.

Results

Item analysis

The internal consistency reliability coefficient was computed to be .80 for the pretest and .89 for the posttest. These values indicate a high degree of internal consistency on both the pretest and posttest. In Table 2 and Table 3 the dichotomous achievement item statistics for the prestest and posttest data are listed, respectively. The point biserial coefficients indicate how performance on an individual item correletes with performance on all dichotomous items. The fact that no items had negative point biserial correlations indicates that no items were negative discriminators.

The Phi coefficients reflect how the difficulty indices differ. A negative Phi coefficient indicates that the item is easier for the control subjects than the experimental subjects and vice-versa for a positive Phi coefficient. The data in Table 2 reveals that, on the pretest, no significant differences existed between experimental and control subjects on any of the items. The data in Table 3 indicates that, on the posttest, no items favored the control group, and 39 items favored the experimental group; also 15 items were answered by a significantly larger percentage of the experimental subjects than control subjects. Of the 15 significant items favoring the experimental group, 3 (items 12, 26, and 27) were classified as application items and 12 as comprehension items. The 3 application items represented 14% of all application items, and the 12 comprehension items represented 55% of all dichotomous comprehension items.

Analysis of variance and t-tests

The data in Table 4 reveals that for the multiple choice composite score related to objective (1) no differences between levels of treatment (experimental and control) existed on the pretest, but differences favoring the experimental subjects did exist on the posttest. A significant grade difference and a treatment by I.Q. interaction occurred on both the pretest and posttest multiple choice composite scores. The significant treatment by I.Q. interaction indicates that the performance of experimental subjects was not like the performance of control subjects across the two levels of I.Q. For the achievement pretest, significant grade differences occurred in both analyses which involved grade. However, significant I.Q. differences occurred in only one analysis involving I.Q. For the achievement posttest, the experimental group significantly outperformed the control group, but no other differences were detected. The t-statistics given in Table 5 indicate that the experimental subjects gained significantly more then the control subjects.

The results given in both Table 4 and Table 5 indicates that before instruction no achievement differences between experimental and control subjects existed; however, after instruction the experimental group scored significantly higher than the control group. This implies that the experimental treatment induced changes in behavior covering the stated instructional objectives. In order to determine the effects of the experimental treatment across each of the stated objectives, an analysis of related item clusters was performed. Twelve item clusters were determined by grouping together [illegible]criminating and non-discriminating items (as determined by significant or non-significant Phi coefficients given in Table 3) across the seven objectives being measured by the achievement test. (Note: single item clusters measured objectives 1 and 7.) No treatment differences existed on any of the item clusters on the pretest; however, on the posttest nine of the 12 item clusters did significantly favor the experimental subjects over the control subjects. The experimental subjects surpassed the control subjects on all item clusters related to objectives (1), (3), (6), and (7) of Table 1, and the experimental subjects scored significantly higher than the controls on the discriminating item clusters related to objectives (2), (4), and (5) of Table 1. A multivariate analysis using the item clusters as separate response variables confirmed that the experimental subjects differed significantly from the controls on the posttest but not on the pretest. Although the third grade experimental subjects scored significantly higher than the second grade experimental subjects on the posttest, no significant grade or I.Q. gain score differences within the experimental group were detected.

Table 6 reveals that before instruction significant grade (favoring grade 3) and treatment (favoring the experimental group) differences existed on the space test; however, after instruction grade and I.Q. differences occurred but not treatment differences. Similarly, a multivariate analysis using the four space subtests as the multiple variables indicated that the experimental group scored significantly higher than the control group on the pretest but not on the posttest. A t-test of the space gain scores revealed no significant differences between the experimental and control groups. All statistical results do not imply that the experimental treatment induced better performance on the space test than the control treatment. Also, within the experimental group, no significant grade or I.Q. space gain score differences were detected.

Discussion

The results of this study imply that the experimental subjects learned aspects of transformational geometry. No treatment group differences were detected on the achievement pretest. However, the experimental subjects surpassed the control subjects on the total achievement posttest and on nine of the 12 posttest item clusters, thus indicating that the experimental treatment was somewhat successful in attaining the instructional objectives. The experimental subjects scored significantly better on at least one item cluster related to each of the seven instructional objectives given in Table 1. Despite the rather large differences on the total achievement posttest and pretest item clusters, the item analysis revealed that only 15 of the 44 dichotomous items were answered correctly by a significantly larger percentage of experimental subjects. The fact that three of these items were classified as application items and 12 as comprehension items suggests that the experimental subjects were taught to perform particular transformational geometry skills to a greater degree than they were taught to apply such skills towards the solution of more general exercises. Several possible hypotheses can be given. Perhaps too little emphasis was devoted to the application of rigid motion skills to more general situations, or perhaps the subjects were required to learn too many different skills during the treatment -- if only a single type of motion, such as the flip motion, had been taught, maybe the children would have had more time to internalize the mathematical relation expressed between figures and their images; and finally, many of the application items may have been too difficult -- four application items (items 29, 30, 31, and 33 of Table 3) were answered correctly by fewer than 5% of the subjects.

The results of the study do not indicate that the experimental treatment increased the subjects' spatial abilities. Although the experimental subjects surpassed the control subjects on both the space pretest and posttest, a significant difference was detected on the pretest but not on the posttest. No suitable explanation can be given to account for the lack of treatment effect upon children's spatial abilities. Perhaps the space test items were not sensitive enough to detect instructional transfer effects; or perhaps the treatment served to impair the experimental subjects' ability to work with tasks which were not completely analogous to the exercises used in the treatment. The author did note that on the space posttest the control subjects appeared to be more familiar with the space test and the space testing procedure, whereas, very few experimental subjects remembered having taken this test previously.

In conclusion, the experimental subjects did learn to execute manual procedures to produce transformation images, but they did not learn to mentally perform transformations from one state to another. In terms of Piaget's theory, the subjects did not exhibit operations upon imagery. Instead, they were able to fashion reproductive images based upon the perception of original figures and upon manipulative techniques, rather than produce anticipatory images resulting from operative thought.

Recommendations

This study was performed with a small group of elementary children, which limits external generalizability. Consequently, more research is needed to provide further information regarding questions of the present study. Moreover, better instructional and testing materials need to be developed and used with different types of children of various ages or backgrounds. Different instructional objectives and teaching sequences related to transformational geometry need to be tried. Further work is definitely needed to investigate instructional effects upon children's spatial abilities. The results of further study should prove useful to curriculum specialists who wish to make decisions on the basis of empirical evidence, to psychologists and educators who are concerned with treatment effects upon mental abilities, and to mathematics educators who wish to understand the role and significance of mathematics instruction.

Table 2

ACHIEVEMENT ITEM STATISTICS: PRETEST

Item	r_{pb}	Difficulty E	Difficulty C	Phi	Item	r_{pb}	Difficulty E	Difficulty C	Phi
7	.41**	.29	.16	.161	29	.00	.00	.00	.000
8	.48**	.26	.13	.169	30	.00	.00	.00	.000
9	.22	.10	.06	.063	31	.00	.00	.00	.000
10	.49**	.19	.06	.197	32	.42**	.10	.16	-.089
11	.43**	.74	.75	-.009	33	.00	.00	.00	.000
12	.45**	.07	.03	.078	34	.46**	.39	.19	.220
13	.46**	.32	.13	.238	35	.39**	.55	.53	.017
14	.13	.71	.88	-.204	36	.41**	.68	.72	-.045
15	.18	.29	.28	.010	37	.59**	.42	.38	.045
16	.34*	.58	.78	-.216	38	.28*	.45	.31	.143
17	.15	.10	.09	.005	39a	.24	.65	.75	-.114
18	.45**	.07	.03	.078	39b	.27*	.13	.22	-.118
19	.00	.00	.00	.000	39c	.33*	.07	.03	.078
20	.10	.03	.00	.129	40a	.31*	.16	.09	.101
21	.00	.00	.00	.000	40b	.18	.00	.03	-.125
22	.45**	.03	.06	-.071	40c	.00	.00	.00	.000
23	.43**	.45	.41	.046	41	.29*	.55	.44	.111
24	.36*	.71	.75	-.045	42	.34*	.26	.34	-.093
25	.47**	.71	.50	.214	43	.15	.10	.13	-.045
26	.39**	.29	.44	-.153	44	.44**	.65	.66	-.012
27	.40**	.07	.06	.004	45	.35*	.26	.25	.009
28	.25	.07	.03	.078	46	.01	.23	.16	.089

* $p < .05$, ** $p < .01$

Table 3

ACHIEVEMENT ITEM STATISTICS: POSTTEST

Item	r_{pb}	Difficulty E	Difficulty C	Phi	Item	r_{pb}	Difficulty E	Difficulty C	Phi
7	.52**	.58	.31	.270*	29	.00	.00	.00	.000
8	.45**	.42	.34	.078	30	.29	.03	.00	.129
9	.38**	.194	.188	.008	31	.00	.00	.00	.000
10	.44**	.29	.16	.161	32	.48*	.32	.25	.083
11	.39**	.87	.81	.080	33	.00	.00	.00	.000
12	.41**	.13	.00	.265*	34	.63**	.87	.47	.427**
13	.49**	.48	.34	.142	35	.55**	.87	.47	.427**
14	.16	.97	.91	.126	36	.36**	.97	.81	.247*
15	.33*	.48	.34	.142	37	.46**	.71	.50	.214
16	.42**	.77	.66	.131	38	.40**	.65	.50	.147
17	.27*	.36	.19	.189	39a	.27*	.94	.81	.185
18	.50**	.74	.38	.369**	39b	.48**	.65	.19	.465**
19	.28*	.16	.03	.222	39c	.60**	.32	.06	.331**
20	.[illegible]**	.68	.00	.718**	40a	.53**	.68	.31	.365*
21	.69**	.61	.06	.584**	40b	.35*	.26	.09	.216
22	.74**	.77	.13	.653**	40c	.36*	.13	.00	.265
23	.47**	.55	.34	.206	41	.32*	.65	.59	.053
24	.27*	.90	.72	.235	42	.36*	.55	.38	.174
25	.41	.68	.66	.023	42	.42**	.32	.16	.195
26	.62**	.65	.38	.201	44	.36**	.81	.53	.201
27	.68**	.55	.06	.529**	45	.27*	.39	.19	.221
28	.33*	.16	.01	.222	46	.31*	.36	.31	.045

* $p < .05$, ** $p < .01$

Table 4

SIGNIFICANT F-RATIOS FROM ANOVA OF ACHIEVEMENT TEST

Variable	Analysis	Factor	F-ratio
Multiple Choice Composite Score: Pretest	T vs. G	Grade	27.17**
	T vs. I.Q.	T x I.Q.	10.02**
	G vs. I.Q.	Grade	27.20**
Multiple Choice Composite Score: Posttest	T vs. G	Treatment	4.40*
		Grade	17.50**
	T vs. I.Q.	Treatment	4.28*
		T x I.Q.	11.93**
	G vs. I.Q.	Grade	19.31**
Achievement Pretest Dichotomous Items	T vs. G	Grade	4.16*
	G vs. I.Q.	Grade	6.98*
		I.Q.	5.28*
Achievement Posttest	T vs. G	Treatment	30.79**
	T vs. I.Q.	Treatment	27.83**

*p < .05 **p < .01

Table 5

SIGNIFICANT OBSERVED t-STATISTICS
FOR ACHIEVEMENT GAIN SCORES

Group	Mean Gain	t-statistic
Second Grade E	9.20	4.79**
Second Grade C	2.63	
Third Grade E	11.25	6.04**
Third Grade C	2.88	
Combined E	10.25	7.69**
Combined C	2.75	

**$p < .01$

Table 6

SIGNIFICANT F-RATIOS FROM ANOVA OF SPACE TEST

Variable	Analysis	Factor	F-ratio
Space Pretest	T vs. G	Treatment	6.24*
		Grade	4.10*
	T vs. I.Q.	Treatment	5.96*
	G vs. I.Q.	Grade	5.38*
Space Posttest	T vs. G.	Grade	4.67*
	T vs. I.Q.	I.Q.	4.79*
	G vs. I.Q.	Grade	7.24*
		I.Q.	7.73*

$^{*}p < .05$

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