A Study on Generalising Bayesian Inference to Evidential Reasoning

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Abstract. In this paper the relationship between Bayes' rule and the *Evidential Reasoning* (*ER*) rule is explored. The *ER* rule has been uncovered recently for inference with multiple pieces of uncertain evidence profiled as a belief distribution and takes Dempster's rule in the evidence theory as a special case. After a brief introduction to the *ER* rule the conditions under which Bayes' rule becomes a special case of the *ER* rule are established. The main findings include that the normalisation of likelihoods in Bayesian paradigm results in the degrees of belief in the *ER* paradigm. This leads to *ER*-based probabilistic (likelihood) inference with evidence profiled in the same format of belief distribution. Numerical examples are examined to demonstrate the findings and their potential applications in probabilistic inference. It is also demonstrated that the findings enable the generalisation of Bayesian inference to evidential reasoning with inaccurate probability information with weight and reliability.

Keywords: Evidential reasoning, Belief distribution, Bayesian inference, Probabilistic reasoning, Likelihood inference, Decision making.

1 Introduction

The evidential reasoning (*ER*) rule has been established recently for conjunctive combination of independent evidence with weights and reliabilities [16]. It constitutes a general conjunctive probabilistic reasoning process and reveals that the combined degree of joint support for a proposition from two pieces of independent evidence constitutes two parts in general: the bounded sum of their individual support and the orthogonal sum of their collective support. The *ER* rule is based on the orthogonal sum operation and as such inherits the basic properties of being associative and commutative, which means that it can be used to combine multiple pieces of evidence in any order without changing the final results. It also satisfies common sense synthesis axioms that any rational probabilistic reasoning process should follow.

The *ER* rule takes the original *ER* algorithm [12, 13, 14, 15] as a special case when the reliability of evidence is equal to its weight and the weights of all pieces of evidence are normalised. It is proven that Dempster's rule in the theory of evidence

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[2, 3, 7, 9] is also a special case of the *ER* rule when each piece of evidence is fully reliable. The *ER* rule enhances Dempster's rule for combining pieces of fully reliable evidence that are highly or completely conflicting through a new reliability perturbation analysis, thus resolving the non-definition and counter intuitive problems associated with Dempster's rule [5, 6, 17].

In the *ER* rule, a frame of discernment is composed of a set of hypotheses that are mutually exclusive and collectively exhaustive as in the theory of evidence [7]. It is assumed that basic probabilities can be assigned to not only singleton hypotheses but also to any of their subsets, thereby allowing a piece of evidence to be profiled by a belief distribution (*BD*) defined on the power set of the frame of discernment. *BD* is regarded as the most natural and flexible generalisation of conventional probability distribution in the sense that the former allows inexact reasoning at whatever level of abstraction [4] and on the other hand reduces to the latter precisely if basic probabilities are assigned to singleton hypotheses only.

Bayesian inference is regarded as a classical and rigorous probabilistic reasoning process. Much attention has been paid to generalise Bayesian inference. Dempster's pioneer work [2, 3] is among the most prominent, in which Dempster asserted that the ordinary likelihood function based on a sample from a general multinomial population is proportional to the upper probability of the hypothesis. Shafer [7, 8] and Smets [11] proposed belief functions to show that the application of Dempster's rule on these belief functions can approximate Bayesian inference in general when sample size is very large [8] but only lead to the same result as Bayes' rule for a rather special case with a single frequency distribution, which however is rare in practice if any. Aickin [1] proposed to construct credibility functions and modify Dempster's rule to make likelihood inference very nearly a special case of the Dempster–Shafer theory, which leads to computations that are quite different from those of Smets. In Aickin's approach, a credibility function is generated by dividing all likelihoods with the maximum likelihood for each sample, which is consistent with Demspter's aforementioned assertion but is not meant to show that Dempster's rule can be reduced to Bayes' rule for equivalent likelihood inference from sample data.

Our research is rooted in Dempster's original work on multivalued mapping from sample space to hypothesis space. In this paper, we intend to show the novel results generated from our new research that the *ER* rule, which takes Dempster's rule as a special case when all observations are fully reliable, is the same as Bayes' rule in likelihood inference if likelihoods are normalised for mapping observations from sample space to hypothesis space. In this way, any evidence generated from observations can be equivalently profiled in the same format as belief distribution for consistent knowledge representation and inference, whilst in Bayesian inference evidence is represented in different formats of *prior* probability and likelihood. The generalisation of Bayesian inference to evidential reasoning is also investigated in the context of information acquisition from ambiguous observations and inaccurate diagnoses. Numerical examples are examined to show how evidential reasoning can be conducted to implement and generalise Bayesian inference in situations where data are not accurate. It is also shown how important evidence reliability can be in inference.

The rest of the paper is organised as follows. In Section 2, the concepts and properties of the *ER* rule are briefly introduced. In Section 3, the conditions under which the *ER* rule reduces to Bayes' rule are established. Section 4 presents a study on generalising Bayesian inference to evidential reasoning. Two numerical examples are examined. The paper is concluded in Section 5.

2 Brief Introduction to the *ER* Rule

In this section, the *ER* rule established recently [16] is briefly introduced. Suppose $\Theta = \{h_1, \cdots, h_N\}$ is a set of mutually exclusive and collectively exhaustive hypotheses. Θ is referred to as a frame of discernment. The power set of Θ consists of 2^N subsets of Θ, denoted by 2^Θ or $P(\Theta)$, as follows

$$P(\Theta) = 2^\Theta = \{\varnothing, \{h_1\}, \cdots, \{h_N\}, \{h_1, h_2\}, \cdots, \{h_1, h_N\}, \cdots, \{h_1, \cdots, h_{N-1}\}, \Theta\} \tag{1}$$

In the framework of the *ER* rule, a piece of evidence e_j is represented as a random set and profiled by a belief distribution (*BD*) as follows

$$e_j = \left\{ (\theta, p_{\theta,j}), \forall \theta \subseteq \Theta, \sum_{\theta \subseteq \Theta} p_{\theta,j} = 1 \right\} \tag{2}$$

where $(\theta, p_{\theta,j})$ is an element of evidence e_j, representing that the evidence points to proposition θ, which can be any subset of Θ or any element of $P(\Theta)$ except for the empty set, to the degree of $p_{\theta,j}$, referred to as probability or degree of belief in general. $(\theta, p_{\theta,j})$ is referred to as a focal element of e_j if $p_{\theta,j} > 0$.

Associated with evidence e_j is a reliability, denoted by r_j, which represents the ability of the information source, where e_j is generated, to provide correct assessment or solution for a given problem (Smarandache et al., 2010). The reliability of a piece of evidence is the inherent property of the evidence, and in the *ER* framework measures the degree of support for or opposition against a proposition given that the evidence points to the proposition. In other words, the unreliability of a piece of evidence sets a bound within which another piece of evidence can play a role in support for and opposition against different propositions.

On the other hand, evidence e_j can also be associated with a weight, denoted by w_j. The weight of a piece of evidence shares the same definition as that of its reliability. The former is not different from the latter if all pieces of evidence are measured in the same joint space. When different pieces of evidence are acquired from different sources and measured in different ways, however, the weight of evidence can be used to reflect its relative importance in comparison with other evidence and determined according to who uses the evidence. This means that weight w_j can be subjective and different from reliability r_j in situations where different pieces of evidence are generated from different sources and measured in different ways.

To combine a piece of evidence with other evidence, it is necessary to take into account the above three elements of the evidence: its belief distribution, reliability and weight. In the *ER* rule, this is achieved by defining a so-called weighted belief distribution with reliability as follows

$$m_j = \left\{ \left(\theta, \tilde{m}_{\theta,j}\right), \forall \theta \subseteq \Theta; \left(P(\Theta), \tilde{m}_{P(\Theta),j}\right) \right\} \tag{3}$$

where $\tilde{m}_{\theta,j}$ measures the degree of support for θ from e_j with both the weight and reliability of e_j taken into account, defined as follows

$$\tilde{m}_{\theta,j} = \begin{cases} 0 & \theta = \emptyset \\ c_{rw,j} m_{\theta,j} & \theta \subseteq \Theta, \theta \neq \emptyset \\ c_{rw,j}\left(1 - r_j\right) & \theta = P(\Theta) \end{cases} \tag{4}$$

where $m_{\theta,j} = w_j p_{\theta,j}$. $c_{rw,j} = 1/\left(1 + w_j - r_j\right)$ is a normalisation factor, which is uniquely determined to satisfy $\sum_{\theta \subseteq \Theta} \tilde{m}_{\theta,j} + \tilde{m}_{P(\Theta),j} = 1$ given that $\sum_{\theta \subseteq \Theta} p_{\theta,j} = 1$. Note that there would be $w_j = r_j$ or $m_{\theta,j} = r_j p_{\theta,j}$ if all pieces of evidence are measured in a joint space, or $p_{\theta,j}$ for each piece of evidence is given by the same probability function. Compared with Shafer's discounting method, the critical difference is that in the *ER* rule, the degree of residual support (after discounting) is earmarked to the power set for redistribution instead of assigning it specifically to the frame of discernment.

If two pieces of evidence e_0 and e_1 are independent in that the information that e_0 carries does not depend on whether e_1 is known or not and vice versa, the combined degree of belief to which e_0 and e_1 jointly support proposition θ, denoted by $p_{\theta,e(2)}$, is then generated by the orthogonal sum of their weighted belief distributions with reliability (i.e. m_0 and m_1), given as follows

$$p_{\theta,e(2)} = \begin{cases} 0 & \theta = \emptyset \\ \dfrac{\hat{m}_{\theta,e(2)}}{\sum_{D \subseteq \Theta} \hat{m}_{D,e(2)}} & \theta \subseteq \Theta, \theta \neq \emptyset \end{cases}$$

$$\hat{m}_{\theta,e(2)} = \left[(1 - r_1) m_{\theta,0} + (1 - r_0) m_{\theta,1}\right] + \sum_{B \cap C = \theta} m_{B,0} m_{C,1} \qquad \forall \theta \subseteq \Theta \tag{5}$$

The recursive formulae of the *ER* rule are also given to combine multiple pieces of evidence in any order.

It is proven that Dempster's rule is a special case of the above *ER* rule when each piece of evidence e_j in question is assumed to be fully reliable, or $r_j = 1$ for all *j*.

3 Equivalence between the *ER* Rule and Bayes' Rule

This section is aimed to provide the exact conditions under which a special case of the *ER* rule, the same as Dempster's rule, reduces to Bayes' rule.

Let e_0 stand for old evidence that is profiled with the *prior* probabilities of the hypotheses in the frame of discernment $\Theta = \{h_1, \cdots, h_N\}$, or

$$e_0 = \left\{ \left(h_i, p_{i0}\right), i = 1, \cdots, N, \sum_{i=1}^{N} p_{i0} = 1 \right\} \tag{6}$$

where p_{i0} is the probability to which evidence e_0 points to hypothesis h_i, or $p_{i0} = p(h_i|e_0)$.

Let c_{ij} stand for the likelihood to which the j^{th} test result (e_j) is expected to occur given that the i^{th} hypothesis (h_i) is true and evidence e_0 is known, or $c_{ij} = p(e_j|h_i, e_0)$, with $\sum_{j=1}^{L} c_{ij} = 1$ for $i = 1,\cdots,N$, as shown in Table 1. Given that a test result e_1 is observed as new evidence, Bayes' rule can be used to generate *posterior* probability that both e_0 and e_1 support hypothesis h_i as follows

$$p(h_i|e_1,e_0) = \frac{p(e_1|h_i,e_0)p(h_i|e_0)}{\sum_{n=1}^{N} p(e_1|h_n,e_0)p(h_n|e_0)} \tag{7}$$

Table 1. Likelihoods

Hypothesis	Test result				
	e_1	…	e_j	…	e_L
h_1	c_{11}	…	c_{1j}	…	c_{1L}
⋮	⋮	⋱	⋮	⋱	⋮
h_i	c_{i1}	…	c_{ij}	…	c_{iL}
⋮	⋮	⋱	⋮	⋱	⋮
h_N	c_{N1}	…	c_{Nj}	…	c_{NL}

While Bayes' rule is rigorous, the combination of old evidence e_0 with new evidence e_1 in Equation (7) is not symmetrical [7], in the sense that the old evidence is profiled as a probability distribution over the set of hypotheses h_i for $i = 1,\cdots,N$, whilst the new evidence is characterised by likelihoods over the set of test results e_j ($j = 1,\cdots,L$) for a given hypothesis. This asymmetry underpins Bayesian inference as a process of updating knowledge once new evidence becomes available. However, this can cause confusion if multiple pieces of evidence are not particularly classified as old and new and need to be combined in any order. Nevertheless, it is desirable that both old and new evidence is represented in the same format for combination.

Let p_{ij} stand for the degree of belief that test result e_j points to hypothesis h_i, with $\sum_{i=1}^{N} p_{ij} = 1$ for $j = 1,\cdots,L$. Test result e_j can then be profiled over the set of hypotheses symmetrically in the same way as for evidence e_0 as follows

$$e_j = \left\{(h_i, p_{ij}), i = 1,\cdots,N, \sum_{i=1}^{N} p_{ij} = 1\right\} \qquad j = 1,\cdots,L \tag{8}$$

p_{ij} can be generated from likelihood c_{ij}. The following results establish the equivalence conditions under which Bayes' rule is a special case of the *ER* rule with $r_j = 1$ for all *j*, which constitutes a symmetrical evidence combination process.

Theorem 1. If all tests to generate likelihoods in Table 1 are conducted independently, the relationship between likelihood c_{ij} and degree of belief p_{ij} is given by

$$p_{ij} = c_{ij} \Big/ \sum_{n=1}^{N} c_{nj} \qquad \text{for } i = 1, \cdots, N \text{ and } j = 1, \cdots, L \tag{9}$$

Let $p_{h_i, e(2)}$ stand for the combined degree of belief to which both e_0 and e_j support hypothesis h_i. We then have the following result.

Corollary 1. Under the same conditions as for Theorem 1, if probability is assigned only to singleton hypothesis, each piece of evidence is fully reliable and the degrees of belief are given by Equations (9), the *ER* rule reduces to Bayes' rule, or

$$p_{h_i, e(2)} = p(h_i | e_j, e_0) \tag{10}$$

The numerical example below is used to demonstrate how the above results could be applied to symmetrical Bayesian inference via equivalent evidential reasoning.

Example 1. Suppose independent tests and diagnoses for a sample of 10000 persons in a population are shown in Table 2. We are interested to find the probability to which a person from the population already has *AIDS* if the person has his first *HIV* test that is positive.

Table 2. Experimental Data

Sample Data		Test Result		Total Diagnosis
		HIV Positive (e_1)	*HIV* Negative (e_2)	
Hypotheses	*AIDS* (h_1)	95	5	100
	No AIDS (h_2)	990	8910	9900
Total Test		1085	8915	10000

What needs to be identified is the degree of belief, denoted by $p_{h_1, e(2)}$, to which h_1 is supported by both pieces of evidence: the *prior AIDS* distribution of the population as revealed by the experiment (e_0) and a positive *HIV* test result (e_1). From Equation (9), the *prior* probabilities $p_{10} = p(h_1|e_0)$ and $p_{20} = p(h_2|e_0)$, and likelihoods c_{11} and c_{21} for the two pieces of evidence e_0 and e_1 can be generated from the experimental data given in Table 2 as follows

$$p_{10} = p(h_1|e_0) = \frac{100}{10000} = 0.01, \quad p_{20} = p(h_2|e_0) = \frac{9900}{10000} = 0.99;$$

$$c_{11} = p(e_1|h_1, e_0) = \frac{95}{100} = 0.95, \quad c_{21} = p(e_1|h_2, e_0) = \frac{990}{9900} = 0.1$$

$$p_{11} = \frac{c_{11}}{c_{11} + c_{21}} = \frac{0.95}{0.95 + 0.1} = \frac{0.95}{1.05} = 0.9048, \quad p_{21} = \frac{c_{21}}{c_{11} + c_{21}} = \frac{0.1}{1.05} = 0.0952$$

Equation (5) with $r_0 = r_1 = 1$ can then be used to calculate $p_{h_1,e(2)}$ as follows

$$p_{h_1,e(2)} = \frac{p_{11}p_{10}}{p_{11}p_{10} + p_{21}p_{20}} = \frac{0.9048 \times 0.01}{0.9048 \times 0.01 + 0.0952 \times 0.99} = 0.0876$$

From the conventional Bayesian analysis, the same result can be generated as follows

$$p(h_1|e_1, e_0) = \frac{p(e_1|h_1, e_0)p(h_1|e_0)}{p(e_1|h_1, e_0)p(h_1|e_0) + p(e_1|h_2, e_0)p(h_2|e_0)} = \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.1 \times 0.99} = 0.0876$$

4 Generalisation of Bayesian Inference to Evidential Reasoning

Bayesian inference as shown in the previous section is rigorous but requires accurate *prior* probabilities and likelihoods in the sense that each test must lead to exactly one of the *L* test results and each test result must be diagnosed to belong to exactly one of the *N* hypotheses. Such accuracy is desirable and should always be pursued. However, ambiguous test results and inaccurate diagnoses are common in real experiments. This section is aimed to investigate how the above "accurate" and "rigorous" Bayesian inference can be generalised for rigorous reasoning with evidence generated from ambiguous tests and inaccurate diagnoses.

Let θ stand for a proposition representing a set of diagnoses, $c_{\theta,j}$ for the generalised likelihood to which the j^{th} test result (e_j) is expected to occur given proposition θ, with $\sum_{j=1}^{L} c_{\theta,j} = 1$ for any $\theta \subseteq \Theta = \{h_1, \cdots, h_N\}$, and $p_{\theta,j}$ for the belief degree to which the j^{th} test result points to proposition θ, with $\sum_{\theta \subseteq \Theta} p_{\theta,j} = 1$ for any $j = 1, \cdots, L$. Belief degree $p_{\theta,j}$ can be generated from generalised likelihood $c_{\theta,j}$ as follows.

Corollary 2. Suppose the same conditions as for Theorem 1 are held. If all tests for generating generalised likelihood $c_{\theta,j}$ are conducted independently, the relationship between $c_{\theta,j}$ and $p_{\theta,j}$ is given by:

$$p_{\theta,j} = c_{\theta,j} \Big/ \sum_{A \subseteq \Theta} c_{A,j} \qquad \text{for } \theta \subseteq \Theta \text{ and } j = 1, \cdots, L \tag{11}$$

Example 2. Suppose there are imprecise experimental data for a population, as shown in Table 3. It is also assumed that the experimental data can represent the *prior AIDS* distribution of the population with a 95% level of reliability and an *AIDS* diagnosis from a *HIV* test can be regarded to be 98% reliable. What is the probability to which a person from the population already has *AIDS* if the person's first *HIV* test turns out to be positive, given that the person's *HIV* test result and the experimental data are regarded to be of equal importance in the inference?

Table 3. Experimental Data under Uncertainty

Diagnosis		*HIV* test result			Total diagnosis
		Positive e_1	*Negative* e_2	*Unknown* e	
AIDS	h_1	95	5	0	100
No AIDS	h_2	980	8860	10	9850
Unknown	$\Theta = \{h_1, h_2\}$	5	7	38	50
Total test		1080	8872	48	10000

The belief degrees for the evidence of the *prior AIDS* distribution (e_0) for the population are given by $p_{10} = 100/10000 = 0.01$, $p_{20} = 9850/10000 = 0.985$, $p_{\Theta 0} = 50/10000 = 0.005$, as shown in Table 3.

The generalised likelihood $c_{\theta 1}$ and belief degree $p_{\theta 1}$ for the evidence of positive *HIV* test result (e_1) are calculated in Table 3 by $c_{11} = 95/100 = 0.95$, $c_{21} = 980/9850 = 0.0995$, $c_{\Theta 1} = 5/50 = 0.1$, and then

$$p_{11} = \frac{c_{11}}{c_{11} + c_{21} + c_{\Theta 1}} = \frac{0.95}{1.1495} = 0.8264,\ p_{21} = \frac{0.0995}{1.1495} = 0.0866,\ p_{\Theta 1} = \frac{0.1}{1.1495} = 0.087.$$

The reliabilities and weights of e_0 and e_1 are given by r_0 =0.95, r_1 =0.98 and $w_0 = w_1 = 0.5$. Note that the weights are normalized here with $w_0 + w_1 = 1$ for illustration purpose. In general, such normalisation is not always required. The degrees of individual support for θ from e_0 and e_1 are calculated by

$$m_{h_1,0} = w_0 p_{10} = 0.5 \times 0.01 = 0.005,\ m_{h_2,0} = w_0 p_{20} = 0.4925,\ m_{\Theta 0} = w_0 p_{\Theta 0} = 0.0025;$$

$$m_{h_1,1} = w_1 p_{11} = 0.5 \times 0.826 = 0.4132,\ m_{h_2,1} = w_1 p_{21} = 0.0433,\ m_{\Theta 1} = w_1 p_{\Theta 1} = 0.0435$$

Equation (5) can then be used to combine e_0 and e_1 to count their joint support by

$$\hat{m}_{h_1,e(2)} = (1 - r_1) m_{h_1,0} + (1 - r_0) m_{h_1,1} + m_{h_1,0} m_{h_1,1} + m_{h_1,0} m_{\Theta 1} + m_{\Theta 0} m_{h_1,1} = 0.0241$$

$$\hat{m}_{h_2,e(2)} = (1 - r_1) m_{h_2,0} + (1 - r_0) m_{h_2,1} + m_{h_2,0} m_{h_2,1} + m_{h_2,0} m_{\Theta 1} + m_{\Theta 0} m_{h_2,1} = 0.0549$$

$$\hat{m}_{\Theta,e(2)} = (1 - r_1) m_{\Theta 0} + (1 - r_0) m_{\Theta 1} + m_{\Theta 0} m_{\Theta 1} = 0.0023$$

The belief degrees to which e_0 and e_1 both support θ are finally generated by

$$p_{h_1,e(2)} = \frac{\hat{m}_{h_1,e(2)}}{\hat{m}_{h_1,e(2)} + \hat{m}_{h_2,e(2)} + \hat{m}_{\Theta,e(2)}} = \frac{0.0241}{0.0241 + 0.0549 + 0.0023} = \frac{0.0241}{0.0813} = 0.2964$$

$$p_{h_2,e(2)} = \frac{0.0549}{0.0813} = 0.6753, \text{ and } p_{\Theta,e(2)} = \frac{0.0023}{0.0813} = 0.0283$$

The ambiguity and inaccuracy in the experiment are retained by $p_{\Theta,e(2)}$ in the above final results. As such, the probability to which the person has *AIDS* is not precise but between 0.2964 and 0.3247 ($p_{h_1,e(2)} + p_{\Theta,e(2)}$). The probability to which the person does not have *AIDS* is between 0.6753 and 0.7036 ($p_{h_2,e(2)} + p_{\Theta,e(2)}$).

It should be noted that the reliability of evidence plays an important role in inference and should be estimated with care and rigor. For instance, if both pieces of evidence are assumed to be fully reliable in Example 2, or $r_0 = r_1 = 1$, it can be shown that there will be $p_{h_1,e(2)} = 0.0716$, $p_{h_2,e(2)} = 0.926$ and $p_{\Theta,e(2)} = 0.0024$, meaning a much smaller probability (0.0716 to 0.0740) of having *AIDS* with much smaller ambiguity (0.0024). Such results are quite different from the results generated above for r_0 =0.95 and r_1 =0.98, but justifiable as evidence e_0 is against the first hypothesis "*AIDS*" much more than evidence e_1 against the second hypothesis "*No AIDS*".

5 Concluding Remarks

In this paper, the recently established evidential reasoning (*ER*) rule was briefly introduced. The relationship between Bayes' rule and the *ER* rule was then investigated and their equivalence conditions were provided. This study shows that Bayesian inference can be conducted in a symmetrical process in the *ER* paradigm where each piece of evidence is profiled in the same format of belief distribution. This on one hand facilitates the combination of evidence in any order for Bayesian inference. On the other hand, experimental data can be used to acquire evidence. In this study, Bayesian inference was generalised to take into account ambiguous test results and inaccurate diagnoses in experiment. This can help conduct inference in a realistic yet rigorous manner without having to make unnecessary assumptions about inaccurate or missing data. The two examples demonstrated the implementation processes of Bayesian inference in the *ER* paradigm. Finally, it is important to note that the reliability of evidence plays an important role in inference and needs to be estimated using domain specific knowledge with care and rigor.

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