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A STUDY ON THE BUDGET CONSTRAINED FACILITY LOCATION MODEL CONSIDERING INVENTORY MANAGEMENT COST *

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Abstract. One of the important issues on the distribution network design is to incorporate inventory management cost into the facility location model. This paper deals with a network model making the decisions on the facility location such as the number of DCs and their locations as well as the decisions on the inventory management such as the ordering quantity and the level of safety stock at each DC. The considered model differs from the previous works by classifying the related costs into the operating cost and the investment cost. For this model, a solution procedure based on the Lagrangian relaxation method was proposed and tested for its effectiveness with various numerical examples.

Keywords. Location, inventory management, nonlinear programming, Lagrangian relaxation.

Mathematics Subject Classification. 35L05, 35L70.

1. Introduction

The design of the distribution network satisfying the demand of each retailer has been one of the important decision problems in the area of operations research and production management. Therefore, many models such as p-median model and p-center model have been developed so as to present a distribution network. The facility location model (FLM) is also one of the important models describing a distribution network.

Given the location of demand nodes (retailers) and candidate sites for the supplying facility (DC, Distribution Center), FLM decides the number of DCs and

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their locations as well as the assignment of each retailer to its supplying DC so as to minimize the considered network cost. The difference between FLM and other network models such as p-median model and p-center model is that the number of DCs is not fixed in FLM [4]. Actually, the number of DCs is considered as a decision variable in FLM whereas it is considered as a given parameter in p-median model and p-center model. And the setup cost of DCs is considered in FLM whereas p-median and p-center model do not consider the setup cost by assuming that setup costs of all DC candidates are the same. Therefore, FLM is recognized as an extended model of p-median model and p-center model. Detailed description on FLM can be found in [12].

Many researchers have dealt with FLM and various solution approaches have been suggested. Since FLM is NP-Hard [2], most researchers have focused on developing a heuristic algorithm which can find a good solution in a reasonable time. Kuehn and Hamburger [16] proposed ADD-DROP heuristic algorithm based on the greedy type methodology to solve FLM. It decides the number of DCs and their locations by adding a DC which reduces the total network cost and by dropping a DC which increases the total network cost. Each retailer is assigned to the DC providing the minimum delivery cost among the installed DCs. Erlenkotter [8] proposed a dual based approach which constructed a primal and dual formulation of FLM and induced a solution by considering the relation between the primal and dual formulation. Beasley [1] proposed an algorithm based on the Lagrangian relaxation method and various meta-heuristic algorithms including Tabu Search, simulated annealing and genetic algorithm have been proposed to solve FLM.

As more researchers are interested in FLM, it has been evolved by considering some practical issues. Capacitated facility location model assumes that each DC has the limited capacity so that the total demand assigned to the DC does not exceed its capacity [15]. And dynamic facility location model classifies the planning time horizon into several stages and selects the opening and closing DCs at each stage by considering the cost and demand at that stage [6, 9].

Another extension of FLM is to combine it with other decision problems. Shen *et al.* [27] dealt with the problem combining the facility location model and the inventory management model. Usually, the facility location model belongs to the strategic decision problem whereas the inventory management model belongs to the tactical decision problem. Traditionally, the tactical decision is made after the upper level (strategic level) decision has been made. However, by incorporating the tactical decision into the strategic decision, the decisions in different levels can be consistent so that the quality of each decision might be improved. Because of this merit, integrating multiple decision problems so as to pursue the global optimum is one of the important trends on the supply chain management. And the research of Shen *et al.* might be recognized as an example of this research trend.

In their model, the locations and demands of each retailer are given as well as the locations of the candidates of DC. The model makes the location related decisions including the number of DCs, the location of DCs and the assignment of each retailer to its supplying DC as well as the inventory related decisions including the ordering quantity, the re-order point and the safety stock level at the open DCs. It is assumed that each retailer has independent stochastic demand. The objective of the problem is to minimize the total network cost including the setup cost of DC, the delivery cost and the inventory cost. Shen *et al.* [27] formulated the problem as a non linear integer program and proposed a solution approach based on the set covering model. After that, Daskin *et al.* [5] considered the same model and proposed a Lagrangian relaxation based solution approach. Shen and Qi [26] added the routing cost to the same model. Miranda and Garido [22] and Park *et al.* [25] extended this model by considering the capacity of each DC.

Another criterion for classifying the facility location problem is the way of considering the setup cost. The objective function of the traditional FLM is to minimize the total network cost which is the sum of the delivery cost from DC to retailer and the setup cost for opening DCs. However, facility setup cost belongs to the investment cost whereas delivery cost belongs to the operating cost [19]. Usually, the operating cost is incurred continuously over a long time and can be covered by the revenue in the same time period while the investment cost is incurred intermittently over a short time and is covered by the predetermined budget. It means that summing two different types of cost might be inadequate because of their different characteristics. To overcome this difficulty, some researchers treated the setup cost as a constraint rather than an objective function [7,19,29]. Drezner [7] considered a competitive facility location model in which the budget for constructing new facilities was fixed. Wang *et al.* [29] dealt with another facility location model considering opening and closing cost of facilities. In their model, total cost of opening and closing facilities should not exceed the given budget. Melo *et al.* [19] also considered the facility location model where the facility setup cost was constrained by a given budget. Their model is different from the above by considering multi commodities and multi time phases.

Even though there are some researches on separating the investment cost and the operating cost, the author could not find any research on the combined model of the facility location and inventory management considering the budget constraint on the facility setup cost. This provided the author with the motivation for dealing with the model combining FLM and inventory management model with the budget constrained setup cost.

The rest of this paper is organized as follows. Section 2 describes the proposed problem and derives its mathematical formulation. Section 3 describes the solution approach based on the Lagrangian relaxation and Tabu Search. Section 4 discusses the results of numerical experiments to verify the performance of the proposed solution approach. Section 5 makes concluding remarks.

2. Mathematical formulation

The problem considered in this paper is now stated in detail as follows. A network which is composed of the set of retailers N having n elements and the set of candidate sites for DC location M having m elements is given. Some of

m candidate sites are selected and DCs are installed at the selected sites. Each retailer is assigned to one of the open DCs so that its demand is satisfied by the assigned DC.

Daily demands at each retailer are assumed to be independent and follow a stationary Poisson process [5, 24, 25, 27]. The Poisson distribution is known to be appropriate when demand events occur independently for a reasonable planning horizon [14, 18]. Since this paper assumes independent demand and the planning horizon is as long as a year, it might be acceptable to assume that daily demands at each retailer follow a Poisson distribution. Moreover, demands at each DC are assumed to follow a normal distribution since approximating a Poisson demand process by normally distributed demands is known to be good for sufficiently large demand values [5, 23, 24].

DC orders the products to the plant and manages its inventory so as to satisfy the demand from the assigned retailers. Inventory at DC is managed under (r, Q) policy. That is, inventory level is monitored in real time and the fixed quantity Q is ordered to the plant if the inventory level reaches the reorder point r. Ordered products are arrived after the lead time and DC holds safety stocks to meet the demand during the lead time. This paper assumes that the lead time is fixed for each DC although it may vary between DCs. The level of safety stock at each DC is determined according to the service level during the lead time.

The objective of the proposed model is to make the decisions on the facility location and on the inventory management so as to minimize the total expected network operating cost. The decisions on the facility location include the number of DCs and their locations as well as the assignment of each retailer to its supplying DC. The decisions on the inventory management include the fixed order quantity, the reorder point and the level of safety stock at DC j. The total expected network operating cost is the sum of the expected operating cost of each DC which is composed of the expected delivery costs to its retailer and the expected inventory costs at the DC. And the expected inventory management costs include the ordering cost and the inventory holding cost. The ordering cost is incurred in proportion to the number of orders and the inventory holding cost is incurred in proportion to the amount of inventory and the holding time. Since the setup cost for a DC is not included in the operating cost, it is considered as the constraint.

The considered problem is formulated as a nonlinear integer program, and the followings are the decision variables used for the formulation.

Y_j: 1 if a DC is installed on the candidate site j, and 0 otherwise.

X_{ij}: 1 if retailer i is assigned to DC j, and 0 otherwise.

Decision variables Y_j decide the location of DCs and decision variables X_{ij} decide the assignment of retailers. The number of DCs is derived by $\sum_{j \in M} Y_j$. The decisions on the inventory management can be derived from the above decision variables and the inventory related parameters. The followings are the parameters used for the mathematical formulation.

μ_i: mean of daily demand at retailer i;

χ: number of working days per year;

O_j: fixed cost per order at DC candidate site j;
h_j: annual inventory holding cost per unit at DC candidate site j;
C_{ij}: delivery cost per unit from DC candidate j to retailer i;
L_j: lead time at DC candidate j;
α: required service level at each DC.

With the above mentioned input parameters and the decision variables, the total expected delivery cost in the objective function can be derived as $\sum_{i\in N}\sum_{j\in M} C_{ij}\chi\mu_i X_{ij}$. The expected inventory management cost is composed of the on-hand inventory management cost and the safety stock management cost. Moreover, the on-hand inventory management cost is defined as a sum of the ordering cost and the inventory holding cost for the on-hand inventory. Denote by Q_j the order quantity at DC j. Since the number of ordering is $\sum_{i\in N}\chi\mu_i X_{ij}/Q_j$, the on-hand inventory management cost at DC j is derived as follow.

$$O_j \sum_{i\in N} \chi\mu_i X_{ij}/Q_j + h_j Q_j/2.$$

Since the above model is the same to EOQ model, the optimal ordering quantity at DC j is derived as follows.

$$Q_j^* = \sqrt{2O_j \sum_{i\in N} \chi\mu_i X_{ij}/h_j}.$$

The safety stock holding cost is related to the level of safety stock. Since the distribution of the demand at each retailer is assumed to be the Poisson distribution, the variance of the demand at retailer i is μ_i. Moreover, the distribution of the demand at each DC is approximated by the normal distribution. Therefore, daily demand at DC j follows the normal distribution with the mean $\sum_{i\in N}\mu_i X_{ij}$ and the variance $\sum_{i\in N}\mu_i X_{ij}$. The lead time at DC j is fixed to L_j and the service level is given at α. Therefore, if denote by Z_α the standard normal deviate, then the level of safety stock satisfying the given service level is $Z_\alpha\sqrt{L_j\sum_{i\in N}\mu_i X_{ij}}$ [5].

Since the objective function of the proposed model is derived, the proposed model can be formulated as follows.

Problem (P):

$$\min \sum_{i\in N}\sum_{j\in M} C_{ij}\chi\mu_i X_{ij} + \sum_{j\in M}\left\{\sqrt{2O_j h_j \sum_{i\in N}\chi\mu_i X_{ij}} + h_j Z_\alpha \sqrt{L_j \sum_{i\in N}\mu_i X_{ij}}\right\} \tag{2.1}$$

$$\text{s.t.} \sum_{j\in M} F_j Y_j \leqslant B \tag{2.2}$$

$$\sum_{j\in M} X_{ij} = 1 \qquad \forall i \in N \tag{2.3}$$

$$X_{ij} \leqslant Y_j \qquad \forall i \in N, \quad \forall j \in M \tag{2.4}$$

$$X_{ij}, Y_j \in \{0,1\} \qquad \forall i \in N, \quad \forall j \in M. \tag{2.5}$$

The objective function (2.1) is to minimize the sum of the expected delivery cost and the expected inventory management cost. In equation (2.2), F_j is the setup cost to open a DC at the candidate site j and B is the given budget. Therefore, constraint (2.2) requires that the total setup cost to open DCs should not exceed the given budget B. Constraints (2.3) require that each retailer is assigned to one facility. Constraints (2.4) do not allow any delivery unless the corresponding DC is open. Constraints (2.5) mean that the decision variables are binary variables.

The above model deals with the stochastic environment since daily demand follows a stochastic process. However, for the simplicity of the formulation, the model is formulated as a deterministic form by considering the expected values of each cost.

3. Solution approach

The proposed problem is NP-Hard since FLM which does not consider the inventory management cost is NP-Hard [11]. Therefore, this paper aims to develop a heuristic algorithm which can get a good solution in a reasonable time.

In this paper, the Lagrangian relaxation method which has shown good performance for various network design problems is used to solve the proposed problem [1,3]. Lagrangian relaxation method transforms a complex problem to multiple simple sub problems by relaxing some constraints. Infeasibility on the relaxed constraints is added to the objective function so that the algorithm searches towards the solution area having less infeasibility.

3.1. Lagrangain relaxation model

In this research, constraints (2.3) in problem (P) are relaxed using Lagrangian multipliers λ_i so that the following relaxed problem is derived.

Problem (LR):

$$\min \sum_{i\in N}\sum_{j\in M} C_{ij}\chi\mu_i X_{ij} + \sum_{j\in M}\left\{ INV_j \sqrt{\sum_{i\in N}\mu_i X_{ij}}\right\} + \sum_{i\in N}\lambda_i\left(1-\sum_{j\in M}X_{ij}\right) \quad (3.1)$$

s.t. (2.2), (2.4), (2.5).

INV_j inequation (3.1) corresponds to $\sqrt{2O_j h_j \chi} + h_j Z_\alpha \sqrt{L_j}$. The objective function (3.1) is derived from the objective function of problem (P) and the relaxed constraints. The objective function can be rearranged as follows.

$$\min \sum_{i\in N}\sum_{j\in M} \left(C_{ij}\chi\mu_i - \lambda_i\right) X_{ij} + \sum_{j\in M}\left\{ INV_j \sqrt{\sum_{i\in N}\mu_i X_{ij}}\right\} + \sum_{i\in N}\lambda_i. \quad (3.2)$$

Therefore, the Lagrangian relaxed problem of problem (P) can be rewritten as follows.

Step 1. Partition the set of candidate sites N as follows;
$N_1 = \{i : A_i \geqslant 0\}$, $N_2 = \{i : A_i < 0, B_i = 0\}$, $N_3 = \{i : A_i < 0, B_i > 0\}$
where $A_i = C_{ij}\chi\mu_i - \lambda_i$ and $B_i = (INV_j)^2\,\mu_i$

Step 2. Sort the elements in N_3 in increasing order of A_i/B_i ratio.

Step 3. Compute the following partial sums.

$$PS_m = \sum_{i\in N_2} A_i X_{ij} + \sqrt{\sum_{i\in N_2} B_i X_{ij}} + \sum_{i=1, i\in N_3}^{m} A_i X_{ij} + \sqrt{\sum_{i=1, i\in N_3}^{m} B_i X_{ij}}$$

where $m = 1, 2, \ldots, |N_3|$.

Step 4. Select the value of m resulting in the minimum value of PS_m, and set X_{mj} to be 1.

FIGURE 1. Solution procedure for problem (LR j).

Problem (LR′):
min (3.2)
s.t. (2.2), (2.4), (2.5)

If a decision variable $Y_{j'}$ in problem (LR′) is set to 0, then the decision variables $X_{ij'}$ also should be set to 0 for all retailers i because of constraints (2.4). If a decision variable $Y_{j'}$ in problem (LR′) is set to 1, then the decision variables $X_{ij'}$ may be set to 0 or 1. Moreover, the optimal value of $X_{ij'}$ where $Y_{j'}$ is set to 1 can be derived by solving the following sub problem.

Problem (LR j):

$$\begin{aligned} \min \quad & \sum_{i\in N} (C_{ij}\chi\mu_i - \lambda_i)\, X_{ij} + INV_j \sqrt{\sum_{i\in N} \mu_i X_{ij}} \\ \text{s.t.} \quad & X_{ij} \in \{0,1\} \quad \forall i \in N. \end{aligned}$$

The optimal solution of problem (LR j) can be derived by using the following solution procedure in Figure 1 proposed by Shen *et al.* [27].

Denote by V_j the optimal objective function value of problem (LR j). Then, the optimal value of Y_j in problem (LR′) can be obtained by solving the following problem.

Problem (LR Y):

$$\begin{aligned} \min \quad & \sum_{j\in M} V_j Y_j \\ \text{s.t.} \quad & \sum_{j\in M} F_j Y_j \leqslant B \\ & Y_j \in \{0,1\} \quad \forall j \in M. \end{aligned}$$

Problem (LR Y) is a knapsack problem which is known to be NP-Complete [11]. However, many algorithms which can solve the knapsack problem in efficient time have been developed. Moreover, the number of variables in problem (LR Y) is not a very big value since it is the number of candidate sites for DC. Therefore, this paper uses the algorithm proposed by Martello and Toth [17] which is known to solve a knapsack problem having more than 100 000 variables in a few seconds.

In summary, the optimal solution (X^*_{ij}, Y^*_j) of problem (LR′) can be obtained from the optimal solution X'_{ij} of problem (LR j) and the optimal solution Y'_j of problem (LR Y) as follows.

$$Y^*_j = Y^{'}_j \tag{3.3}$$

$$X^*_{ij} = \begin{cases} X^{'}_{ij}, \text{ if } Y^*_j = 1 \\ 0, \quad \text{ if } Y^*_j = 0. \end{cases} \tag{3.4}$$

Lagrangian multipliers reflect the infeasibility caused by the relaxed constraints and they are added to the objective function as a form of the penalty function. In this paper, the subgradient method which is adopted to various problems [10] is used to update Lagrangian multipliers.

3.2. Lagrangian heuristic

The solution obtained by equations (3.3) and (3.4) is the optimum to the relaxed problem. However, it may not satisfy the retailer assignment constraints since constraints (2.3) were relaxed. Therefore, the process constructing a feasible solution of the original problem from the obtained solution is needed and it is called the Lagrangian heuristic.

In this paper, the Lagrangian heuristic is composed of two parts; the construction phase and the improvement phase.

3.2.1. Construction phase

The construction phase aims to find a feasible solution satisfying the retailer assignment constraints.

The construction phase

Step 1. Check the assignment of retailer

1.1. Count the number of assignments of each retailer.

1.2. If retailer i' is assigned to more than 2 DCs, then compare the assignment cost for each DC and assign retailer i' to the DC providing the lowest cost.

1.3. If retailer i' is not assigned to any DCs, then it can be assigned to any of the current open DC or a DC candidate which is not open yet but its opening cost satisfies the budget constraint. After comparing the assignment cost, retailer i' is assigned to the DC providing the lowest cost. If the selected DC j' is not open yet, then set the decision variable $Y_{j'}$ to 1.

Step 2. Update the open facility

2.1. Count the number of assigned retailers for each open DCs.

2.2. If no retailer is assigned to DC j', then set the decision variable $Y_{j'}$ to 0.

3.2.2. Improvement phase

The construction phase produces a feasible solution of the original problem. In the improvement phase, the solution obtained from the construction phase is improved by searching its neighborhood solutions. In this paper, Tabu Search, which is a well-known local search algorithm, is adapted to this phase.

A local search algorithm is a meta-heuristic method for solving various combinatorial optimization problems. It moves the current solution to the most promising neighborhood solution until the termination condition is satisfied. The local search algorithm is widely applied to various combinatorial optimization problems since it is easy to understand and implement. However, it is usually stuck at a local optimum solution.

Tabu Search enhances the performance of a local search algorithm by incorporating a memory based strategy called Tabu list. Tabu list prevents the solution search from becoming trapped at a local optimal solution [13]. Since Tabu Search is a meta-heuristic which can be adapted to various problems, it is important to consider the characteristics of the proposed problem in designing the components. The design issues of Tabu Search in this paper are briefly discussed.

Move

Neighborhood in Tabu Search is defined as a set of solutions which can be reached with one move from the current solution. In this paper, move is defined to change the status of one DC candidate site (i.e., if the DC is open, then close it and *vice versa*). Therefore, there are m neighborhood solutions at each step. The same definition of neighborhood has been used by Michel and Hentenryck [21] and Sun [28] to solve FLM.

Tabu Search moves the current solution to the neighborhood solution which provides the best objective function value. If the neighborhood solution is to open a new DC, then the assignment cost of each retailer to the new DC is compared to the existing assignment cost and the DC providing the lower cost is selected for each retailer. If the neighborhood solution is to close a DC, then the assigned retailers to the closed DC needs to be re-assigned to another DC. Those retailers are assigned to the DC providing the lowest cost except the closed DC.

Tabu list

Tabu Search records some characteristics on the searched solution in a short term memory called Tabu list and excludes the neighborhood solution having the characteristics in the Tabu list from the search process. In this model, the Tabu list records the DC whose status has been changed by the previous move. That is, if the status of DC j has been changed by the previous move, then the status of DC j would remain the same for the time being. This allows the algorithm to search various solution areas and may improve the performance of the algorithm. Size of the Tabu list is set to $m/2$ which showed good results at the pretest. Moreover, as an aspiration condition, if the objective function value of the neighborhood solution in the Tabu list is better than the current best objective function value, then the neighborhood solution is set free from the tabu status and can be a candidate for the next solution.

To improve the performance of Tabu Search, various methodologies such as diversification strategy and intensification strategy have been developed. Diversification strategy is to search diverse solution areas if the algorithm seems to be stuck at some specific solution area. And intensification strategy is to focus on the promising solution area. By adapting those strategies, the performance of Tabu Search might be improved. However, those strategies increase the complexity of the algorithm and result in a longer time to solve the problem.

From the result of the pretest, the procedure described in this paper seems to be effective without those additional methodologies. Therefore, this paper does not consider those methodologies.

3.3. Whole solution procedure

This subsection summarizes the whole solution procedure described in the above two subsections as follows:

Step 1. Initialization

1.1. Initializing the upper and lower bound.

- Set the upper bound of the objective function value *Best_Upper_Bound* to *Big_M* and the lower bound of the objective function value *Best_Lower_Bound* to 0.

1.2. Initializing the Lagrangian multipliers.

- Set the Lagrangian multiplier λ_i to $\min_{j \in M}(C_{ij}\chi\mu_i)$.

Step 2. Lower bound update.

2.1. Solving problem (LR j).

- Solve problem (LR j) for all j in M, and obtain its objective function value V_j.

2.2. Solving problem (LR Y).

- Solve problem (LR Y). If the objective function value of problem (LR Y) Z^L is bigger than *Best_Lower_Bound*, then replace *Best_Lower_Bound* with Z^L.

Step 3. Upper bound update

3.1. Constructing the feasible solution.

- Construct a feasible solution of the original problem by using the construction phase described in Section 3.2.1.

3.2. Improving the solution.

– Improve the feasible solution by using the improvement phase described in Section 3.2.2. If the objective function value of the improved solution Z^U is less than *Best_Upper_Bound*, then update *Best_Upper_Bound* with Z^U and update *Current_Best_Solution* with the current solution.

Step 4. Dual gap update

4.1. Updating the Lagrangian multipliers.

– Update the Lagrangian multiplier as follows.

$$\lambda_i^{k+1} = \lambda_i^k + t_k\left(1 - \sum_{j\in M} X_{ij}\right), \text{ where } t_k = \alpha\frac{Z^U - Z^L}{\sum\limits_{i\in N}\left(1 - \sum\limits_{j\in M} X_{ij}\right)^2}$$

for $\forall i \in N$, and k is the number of iteration.

4.2. Updating the dual gap.

– Update the dual gap with $(Z^U - Z^L)/Z^L$.

Step 5. Termination condition.

– If the number of iteration is greater than the predetermined value or the dual gap is less than the predetermined value ε, then stop the algorithm.

– If the termination condition is not satisfied, go to Step 2.

In the above, *big_M* in Step 1.1 means a big value which can be used as an initial upper bound. In the experimental test, *big_M* is set to be $\sum_{i\in N}\sum_{j\in M} C_{ij}\chi\mu_i + \sum_{j\in M}\left\{\sqrt{2O_j h_j \sum_{i\in N}\chi\mu_i} + h_j Z_\alpha\sqrt{L_j\sum_{i\in N}\mu_i}\right\}$, which is the upper bound of any feasible solution.

Step 4.1. shows the process which updates Lagrangian multipliers λ_i by the subgradient method. The Lagrangian multiplier of demand node i after kth iteration, λ_i^{k+1}, is updated by using the previous value λ_i^k and the infeasibility of the newly obtained solution of problem (LR). The positive stepsize t_k is updated by summing the infeasibility of the whole demand nodes.

4. Experimental results

To evaluate the performance of the solution algorithm described in Section 3, the experimental tests under various environments were performed. The algorithm was coded in C++ language on a Pentium IV CPU 2.0 GHz 2 GB RAM desktop computer.

To test the algorithm effectiveness under various environments, the following design factors are considered to generate the data set; the size of network, the level of demand variation between retailers, the level of cost variation between DC candidates and the level of budget.

The size of network is defined by the number of retailers and the number of DC candidates. In this paper, four different network sizes are considered to have (30, 20), (50, 30), (80, 40), (100, 50), where (n, m) means that the data set has n retailers and m DC candidates. Given the network size, the coordinate of each

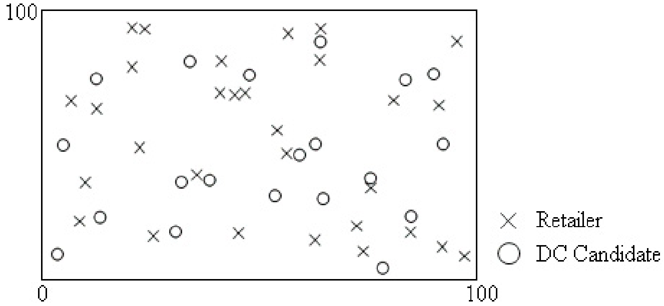

FIGURE 2. Sample data set having 30 retailers and 20 DC candidates.

retailer and DC candidate is randomly generated within a predetermined rectangular area. The delivery cost from a DC to a retailer is assumed to be proportional to its Euclidean distance. Figure 2 shows a sample data set having 30 retailers and 20 DC candidates.

The level of demand variation between retailers is defined as $\max_{i\in N}(\mu_i)/\min_{i\in N}(\mu_i)$. The level is set to small if the ratio is between 1.0 and 2.0, medium if the ratio is between 2.0 and 5.0 and large if the ratio is between 5.0 and 10.0. Demand of each retailer is randomly generated as follows.

$$\mu_i = \begin{cases} \frac{\text{Uniform}(0.5,1)}{\sum_{i\in N}\mu_i}(30\times n), & \text{when the level is set to small,} \\ \frac{\text{Uniform}(0.2,1)}{\sum_{i\in N}\mu_i}(30\times n), & \text{when the level is set to medium,} \\ \frac{\text{Uniform}(0.1,1)}{\sum_{i\in N}\mu_i}(30\times n), & \text{when the level is set to large.} \end{cases}$$

The level of cost variation between DC candidates is defined as the ratio of the maximum and minimum value of each cost (ordering cost and inventory holding cost) between DC candidates. The level is set to small if the ratio is between 1.0 and 2.0, medium if the ratio is between 2.0 and 5.0 and large if the ratio is between 5.0 and 10.0. Costs at each DC are randomly generated as follows.

$$O_j = \begin{cases} \frac{\text{Uniform}(0.5,1)}{\sum_{j\in M}O_j}(150\times m), & \text{when the level is set to small,} \\ \frac{\text{Uniform}(0.2,1)}{\sum_{j\in M}O_j}(150\times m), & \text{when the level is set to medium,} \\ \frac{\text{Uniform}(0.1,1)}{\sum_{j\in M}O_j}(150\times m), & \text{when the level is set to large.} \end{cases}$$

$$h_j = \begin{cases} \frac{\text{Uniform}(0.5,1)}{\sum_{j\in M}h_j}(50\times m), & \text{when the level is set to small,} \\ \frac{\text{Uniform}(0.2,1)}{\sum_{j\in M}h_j}(50\times m), & \text{when the level is set to medium,} \\ \frac{\text{Uniform}(0.1,1)}{\sum_{j\in M}h_j}(50\times m), & \text{when the level is set to large.} \end{cases}$$

TABLE 1. Test results with the randomly generated data sets.

Design factor	Level	Dual gap (%)		Elapsed time (s)	
		Avg.	Max.	Avg.	Max.
Network size	(30, 20)	1.13	4.95	0.16	0.59
	(50, 30)	1.13	4.59	0.51	2.11
	(80, 40)	1.30	5.81	1.57	5.02
	(100, 50)	1.13	5.70	2.87	12.86
Level of demand variation	S	1.16	5.54	1.26	12.86
	M	1.15	5.81	1.21	6.69
	L	1.21	4.95	1.37	9.13
Level of cost variation	S	1.25	5.00	1.65	12.86
	M	1.27	5.81	1.16	7.49
	L	1.00	5.54	1.02	8.94
Level of budget	S	1.72	5.81	1.04	5.06
	M	0.89	4.95	1.01	5.67
	L	0.91	3.99	1.78	12.86

The level of budget is defined as the ratio of the budget to the sum of the setup cost. The level is set to large if budget is randomly generated between $0.3\sum_{j\in M} F_j$ and $0.5\sum_{j\in M} F_j$, medium if budget is randomly generated between $0.15\sum_{j\in M} F_j$ and $0.3\sum_{j\in M} F_j$, small if budget is randomly generated between $2 \times \max_{j\in M}(F_j)$ and $0.15\sum_{j\in M} F_j$.

The performance of the proposed algorithm is evaluated in terms of the dual gap and the elapsed time. The dual gap is used to measure the effectiveness of the algorithm since it calculates the difference between the upper bound and the lower bound of the optimal solution. The elapsed time is used to measure the efficiency of the algorithm.

For reliable tests, 5 data sets are randomly generated at each combination of the design factors and the average and the worst case of the performances at each combination are recorded. Since there are 4 levels of network sizes, 3 levels of the demand variation, 3 levels of the cost variation and 3 levels of the budget, 108 combinations of the design factors are made. Moreover, since 5 data sets are generated at each combination, 540 data sets are used for the test. Table 1 shows the summary of the experimental test.

The average dual gap of 540 data sets is 1.17% and the average elapsed time is 1.28 seconds. The maximum dual gap is 5.81%, which is found in the data set having the medium level of demand variation, the medium level of cost variation, the small level of budget with 80 retailers and 40 DC candidates. The maximum elapsed time is 12.86 seconds which is found in the data set having the small level of demand variation, the small level of cost variation, the large level of budget with 100 retailers and 50 DC candidates.

Considering errors in the data estimates, 1% of the dual gap might be almost optimal in the real life problem [20]. Since the average dual gap of the proposed algorithm is 1.17%, it might be claimed that the proposed algorithm is acceptable in terms of the effectiveness.

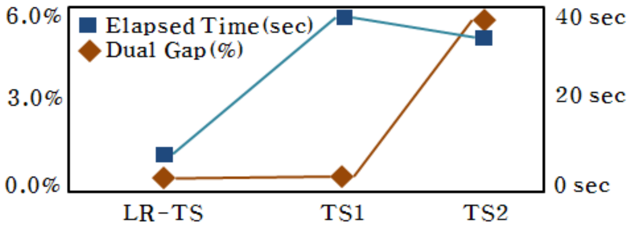

FIGURE 3. Comparison with simple Tabu Search heuristic algorithms.

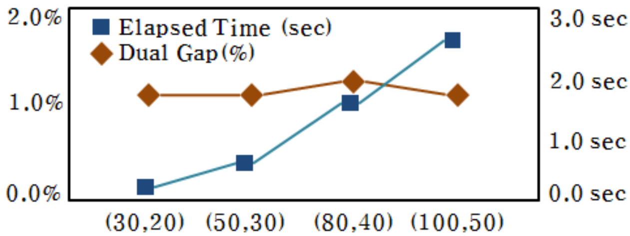

FIGURE 4. Test results in terms of the network size.

Since this paper deals with the new problem, the author could not find any algorithms to be compared with the proposed algorithm in the literature. Therefore, 2 simple algorithms, TS1 and TS2, are used for the comparison. TS1 is a simple Tabu Search heuristic algorithm described in Section 3.2.2. TS2 is another simple Tabu Search heuristic algorithm similar to TS1, but allowing swap move [3]. Initial solution of TS1 and TS2 is obtained by the construction phase described in Section 3.2.1. Test results are summarized in Figure 3 as follows.

Test results show that the proposed algorithm (LR-TS) outperforms other simple algorithms in terms of the average dual gap and the average elapsed time. Therefore, it might be claimed that incorporating Lagrangian relaxation procedure described in Section 3.1 with Tabu Search heuristic generates better results than Tabu Search algorithm itself. Comparing TS1 and TS2, the average dual gap of TS2 (5.38%) is slightly less than TS1 (5.97%). However, TS2 takes much longer time (35.17 s) than TS1 (1.29 s), since TS2 searches wider area of neighborhood.

The performance of the proposed algorithm in terms of the design factors can be shown from Figures 4 to 7. Figure 4 shows the performance of the proposed algorithm in terms of the network size.

Figure 4 shows the elapsed time of the proposed algorithm increases as the network size increases. It means that the complexity of the problem increases because the number of variables increases. However, the dual gap did not increase even though the network size increased. Therefore, it can be inferred that the increase of the network size reduces the efficiency of the proposed algorithm by

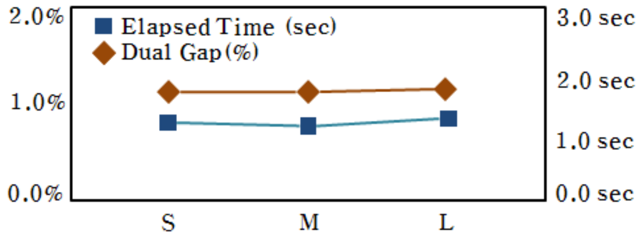

FIGURE 5. Test results in terms of the demand variation.

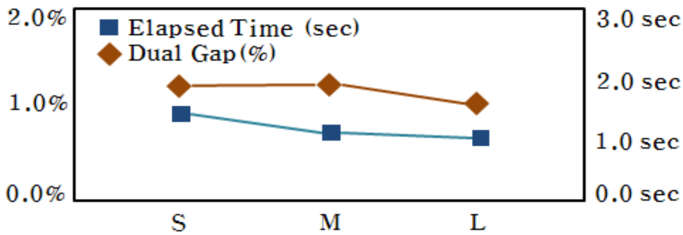

FIGURE 6. Test results in terms of the cost variation.

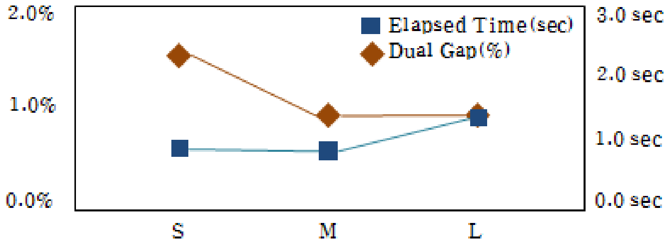

FIGURE 7. Test results in terms of the level of the given budget.

increasing the complexity of the problem but does not affect the effectiveness of the proposed algorithm.

Figure 5 shows that demand variation across retailers does not affect the effectiveness (dual gap) nor the efficiency (elapsed time) of the proposed algorithm. Therefore, it might be claimed that the proposed algorithm is robust against the demand variation.

Figure 6 shows the effect of cost variation on the effectiveness and the efficiency of the proposed algorithm. Based on the test results, the effect of cost variation on the dual gap seems to be small but the elapsed time tends to decrease as the level of cost variation increases.

Figure 7 shows the effect of the level of the given budget on the dual gap and the elapsed time. Based on the test results, the dual gap is large when the level of budget is small and the elapsed time is large when the level of budget is large.

5. Conclusions

This paper deals with the distribution network design problem considering the inventory management cost. Given the locations and demands of each retailer as well as the locations of DC candidates, the proposed model makes both the location related decisions and the inventory related decisions. The objective of the model is to minimize the total expected network operating cost including the expected delivery cost and the expected inventory management cost. The cost for installing a DC is considered to be restricted by a given budget.

The problem is formulated as a nonlinear integer program and a solution approach based on the Lagrangian relaxation method is proposed. The proposed algorithm relaxes the retailer assignment constraints and solves each sub problems. Then, a solution of the original problem is induced from the solution of the sub problems.

The performance of the proposed algorithm is evaluated in terms of the dual gap and the elapsed time. By considering the design factors, 540 data sets are randomly generated to test the proposed algorithm under various environments. Test results show that the average dual gap is less than 2% and the average elapsed time in the largest network is less than 3 seconds.

The contribution of this paper would be to propose a new inventory-location model considering the cost segmentation as well as to develop a solution procedure for the new model. An extension of the proposed problem considering the capacity of each DC may be an interesting subject for further study as well as multiple item distribution network.

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