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A STUDY ON VOLTAGE COLLAPSE MECHANISM IN ELECTRIC POWER SYSTEMS

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Abstract – The heart of the voltage stability problem is the voltage drop that occurs when the power system experiences a heavy load, and one serious type of voltage instability is voltage collapse. In this paper, a mechanism of the dynamic phenomenon of voltage collapse is presented from the physical point of view. It is shown that an iterative reaction of voltage drop between a dynamic load and the system network can cause voltage collapse. The dynamic phenomenon of voltage collapse is analyzed on a simple power system model which includes a synchronous motor.

1. INTRODUCTION

The voltage stability problem is now a serious concern to the electric utility industry. Many large interconnected power systems are increasingly experiencing abnormally high or low voltages and voltage collapse. These voltage problems are associated with the increased loading of transmission lines, insufficient local reactive supply, and the shipping of power across long distances.

The heart of the voltage stability problem is the voltage drop that occurs when the power system experiences a heavy load, and one serious type of voltage instability is voltage collapse. Voltage collapse is characterized by an initial slow progressive decline in the voltage magnitude of the power system buses and a final rapid decline in the voltage magnitude.

There is a body of literature on voltage collapse with both static and dynamic considerations. Venikov et al. [1] suggested a criterion for voltage stability based on a steady state sensitivity analysis using a simple two bus system. Kwatny et al. [2] studied the problem by applying the bifurcation analysis to the load flow equations. They showed that a static bifurcation associated with voltage collapse exists and at that point the load voltages are infinitely sensitive to parameter variations. Tamura et al. [3,4] explained the voltage collapse by multiple load flow solutions and showed that load flow solutions undergo saddle node bifurcations as reactive power supply parameters are varied. Medanic et al. [5] studied the voltage stability of discrete models of multiple tap changers. Liu [6], and Liu and Vu [7] presented a nonlinear on-line tap-changer model for a dynamical description of voltage collapse using simple two or three bus models, and analyzed the voltage stability by characterizing the voltage stability region in terms of the tap-changer model.

Schlueter et al. [8-10] introduced PQ and PV stabilities and controllabilities based on how the bus voltage at every load bus and the reactive generation at every generator bus should react to voltage changes at generator buses and reactive load changes at load buses. They presented a unified theoretical foundation for determining tests for voltage security conditions. These conditions are for static voltage stability and Lee et al. [11] demonstrated their usefulness for security economic operation. Rajagopalan et al. [12] included the excitation system for voltage stability analysis and investigated the eigenvalues of the linearized system matrix for the dynamic voltage stability. Sekine et al. [13] suggested an eigenvalue method to judge voltage instability. It was shown that voltage instability is influenced by multiple solutions and by dynamic characteristics of loads and control equipments. They showed that a static var compensator improves voltage stability. Sekine et al.[14] also analyzed the dynamic phenomena of voltage collapse using induction motor models. Dobson et al. [15-16] and Chiang et al. [17] explained the dynamics of voltage collapse as a dynamic consequence of the bifurcation, using a simple three bus model including a dynamic load. They analyzed voltage collapse based on a center manifold voltage collapse model.

Although there is extensive literature on voltage collapse, very few deal with the physical mechanism of the voltage collapse phenomenon. This paper presents a mechanism of voltage collapse using a simple power system model. It is evident that the system is unstable when the linearized system matrix has a positive eigenvalue, even if it may be infinitesimally small. But the mechanism by which its eigenvalue changes from a small negative value to a small positive value by some disturbance, and consequently leads to voltage instability, has not yet been well understood.

In this paper, a synchronous motor operating at fixed excitation is introduced as a load for voltage collapse analysis. This model for voltage collapse analysis is also applicable to more general power system because the synchronous motor model can easily be changed to a generator model. A detailed mechanism of the voltage collapse phenomenon is analyzed from the physical point of view rather than the mathematical point of view.

When the system is operating near the bifurcation point and its linearized system matrix has a very small negative eigenvalue, dynamic equations describing the mechanism of voltage collapse caused by a very small disturbance are derived and the physical explanation of the voltage collapse mechanism is presented in detail. It is shown that an iterative reaction of voltage drop between the machine and the system network can cause voltage collapse.

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2. DYNAMIC STABILITY

The mathematical model for the dynamic stability study of a power system comprises a) differential equations, b) stator algebraic equations, and c) network equations. The model can be shown to be in the following differential-algebraic form:

$$\dot{x} = f(x, y), \tag{2.1}$$

$$0 = g(x, y), \tag{2.2}$$

where x represents the state variables of the machines and the excitation systems, and y represents the stator currents of each machine, the injected real and reactive powers, and the voltage magnitudes and angles at each of the network buses. The steady state values or equilibriums of the dynamic system states are evaluated by setting the derivative in equation (2.1) to zero.

In the neighborhood of an equilibrium point, equations (2.1) and (2.2) can be linearized to give

$$\frac{d\triangle x}{dt} = A\triangle x + B\triangle y \ , \tag{2.3}$$

$$0 = C\triangle x + D\triangle y \ . \tag{2.4}$$

Furthermore, the incremental algebraic variables can be eliminated and the resulting dynamic system is

$$\frac{d\triangle x}{dt} = (A - BD^{-1}C)\triangle x = \tilde{A}\triangle x \ . \tag{2.5}$$

The static bifurcation can occur when $det[D] = 0$. However, since only the dynamic bifurcation phenomenon is considered here, it is assumed that $det[D] \neq 0$ and D^{-1} exists.

By monitoring the eigenvalues of $\tilde{A}$ matrix, dynamic stability is analyzed. When the determinant of $\tilde{A}$ is zero, that is, $\tilde{A}$ has a zero eigenvalue, the equilibrium point becomes a bifurcation point and voltage collapse will occur. The matrix A reflects electrical and mechanical variables whose time constants are relatively small, and its eigenvalues are large. However, matrix $\tilde{A}$ reflects the interactive reaction between machines and network and its eigenvalues may become very small resulting in a very slow process of voltage collapse. Dynamic stability analysis for this type of model was also presented in [12,13].

3. MODELING FOR DYNAMIC VOLTAGE STABILITY ANALYSIS

A simple 3-bus power system is shown in Figure 1. Two load buses are fed by one generating unit and one of the load buses is connected to a cylindrical-rotor synchronous motor as a dynamic equivalent at the bus. This system resembles the case when power is transfered over a long transmission line to a receiving-end which is in synchronism with the sending-end but often experiences voltage collapse. The receiving-end may have its own generating units but unable to meet all power demand in its own service area. Although the service area may have many induction motors as industrial loads, its *dynamic equivalent* will be better represented by a synchronous machine rather than an induction machine since the area has other generating units which are in synchronism with the rest of the network. The characteristics of induction machines, however, can be included in a composite load model as in usual stability studies.

The mathematical model for dynamic analysis of a power system comprises 1) differential equations, 2) stator algebraic equations, and 3) network equations. The generator bus is a slack bus at which the voltage magnitude is maintained at a constant value. The load model at bus 3 is a composite of constant power, constant current, and constant impedance as follows:

$$P = P_A + P_B \cdot V + P_C \cdot V^2 \ , \tag{3.1}$$

$$Q = Q_A + Q_B \cdot V + Q_C \cdot V^2 \ . \tag{3.2}$$

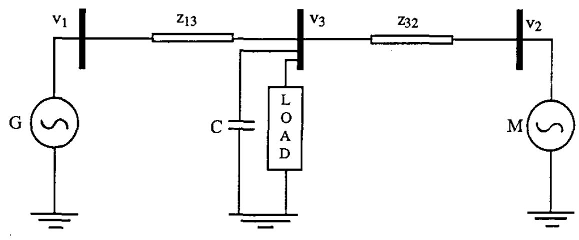

Fig. 1. A simple power system model

It is assumed that the synchronous motor of load bus 2 is operating under fixed excitation with unity power-factor and that the losses of the motor can be neglected. For simplicity of analysis on voltage collapse, only the dynamic model of the synchronous motor is considered while the terminal voltage V_1 of the generator bus 1 is being held constant.

Differential equations of a synchronous motor in a large network are expressed normally as follows:

$$\frac{d\delta_m}{dt} = \omega_m - \omega_s, \tag{3.3}$$

$$M_m \frac{d\omega_m}{dt} = T_{M_m} - [E'_{qm} - X'_{dm} I_{dm}] I_{qm} - [E'_{dm} - X'_{qm} I_{qm}] I_{dm} - D_m(\omega_m - \omega_s), \tag{3.4}$$

$$T'_{dm} \frac{dE'_{qm}}{dt} = -E'_{qm} - (X_{dm} - X'_{dm}) I_{dm} + E_{fdm}, \tag{3.5}$$

$$T'_{qm} \frac{dE'_{dm}}{dt} = -E'_{dm} + (X_{qm} - X'_{qm}) I_{qm}. \tag{3.6}$$

where M, D and T_M are the moment of inertia, damping coefficient and mechanical torque, respectively; T', X and X' are the machine constants; subscripts d and q denote the direct- and quadrature-axis components; and the subscript m represents the synchronous motor bus and is 2 in Figure 1. Also, stator algebraic equations of the motor are as follows:

$$V_i cos\theta_i + R_{si}(I_{di} sin\delta_i + I_{qi} cos\delta_i) - X'_{di}(I_{qi} sin\delta_i - I_{di} cos\delta_i) - [E'_{di} sin\delta_i + (X'_{qi} - X'_{di}) I_{qi} sin\delta_i + E'_{qi} cos\delta_i] = 0, \quad i = m \tag{3.7}$$

$$V_i sin\theta_i + R_{si}(I_{qi} sin\delta_i - I_{di} cos\delta_i) + X'_{di}(I_{di} sin\delta_i + I_{qi} cos\delta_i) - [E'_{qi} sin\delta_i - (X'_{qi} - X'_{di}) I_{qi} cos\delta_i - E'_{di} cos\delta_i] = 0, \quad i = m \tag{3.8}$$

Finally the network equations of the power system are expressed as follows:

$$P_i = V_i[I_{di}sin(\delta_i-\theta_i)+I_{qi}cos(\delta_i-\theta_i)]+P_{Li}(V_i), \quad i=m, \quad (3.9)$$

$$Q_i = V_i[I_{di}cos(\delta_i-\theta_i)-I_{qi}sin(\delta_i-\theta_i)]+Q_{Li}(V_i), \quad i=m, \quad (3.10)$$

$$P_i = P_{Li}(V_i), \qquad i = m+1,...,n, \qquad (3.11)$$

$$Q_i = Q_{Li}(V_i), \qquad i = m+1,...,n, \qquad (3.12)$$

where n is the number of total buses, P_i and Q_i are net injected real and reactive powers at each of the buses, and $P_{Li}(V_i)$ and $Q_{Li}(V_i)$ are real and reactive power loads at bus i, respectively, which are given in equations (3.1) and (3.2). For the system in Figure 1, there is only one load bus that is not expressed by a synchronous machine model and thus $i = 3$ in equations (3.11) and (3.12). The net injected real and reactive powers are given as

$$P_i = \sum_{j=1}^{n} |Y_{ij}|V_jV_icos(-\theta_{ij} - \theta_j + \theta_i), \qquad (3.13)$$

$$Q_i = \sum_{j=1}^{n} |Y_{ij}|V_jV_isin(-\theta_{ij} - \theta_j + \theta_i), \qquad (3.14)$$

where Y_{ij} is a complex element of the bus admittance matrix.

In general, E'_d is very small compared to E'_q, and R_s is very small compared to X_d and X_q. Therefore, their effects are neglected for simplicity of analysis. Then the stator algebraic equations (3.7) and (3.8) are simplified as follows:

$$I_{dm} = \frac{E'_{qm} - V_mcos(\delta_m - \theta_m)}{X'_{dm}}, \qquad (3.15)$$

$$I_{qm} = \frac{V_msin(\delta_m - \theta_m)}{X'_{qm}}. \qquad (3.16)$$

These current equations are substituted into the differential equations and the network equations in order to reduce the number of variables. Then differential equations of the motor are obtained as follows:

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s, \qquad i = m, \qquad (3.17)$$

$$M_i\frac{d\omega_i}{dt} = T_{M_i} - \Big[\frac{E'_{qi}V_isin(\delta_i - \theta_i)}{X'_{di}} - \frac{1}{2}V_i{}^2\frac{X'_{qi} - X'_{di}}{X'_{di}X'_{qi}} \cdot sin2(\delta_i - \theta_i)\Big] - D_i(\omega_i - \omega_s), \quad i = m, \quad (3.18)$$

$$T'_{di}\frac{dE'_{qi}}{dt} = -E'_{qi} - (X_{di} - X'_{di})\frac{E'_{qi} - V_icos(\delta_i - \theta_i)}{X'_{di}} + E_{fdi}, \qquad i = m. \qquad (3.19)$$

And the network equations of the power system can be rewritten as follows:

$$P_i = V_i\Big[\frac{E'_{qi} - V_icos(\delta_i - \theta_i)}{X'_{di}}sin(\delta_i - \theta_i) + \frac{V_isin(\delta_i - \theta_i)}{X'_{qi}}cos(\delta_i - \theta_i)\Big] + P_{Li}(V_i), \quad i = m, (3.20)$$

$$Q_i = V_i\Big[\frac{E'_{qi} - V_icos(\delta_i - \theta_i)}{X'_{di}}cos(\delta_i - \theta_i) - \frac{V_isin(\delta_i - \theta_i)}{X'_{qi}}sin(\delta_i - \theta_i)\Big] + Q_{Li}(V_i), \quad i = m. (3.21)$$

where P_i and Q_i are from equations (3.13) and (3.14).

Equations (3.17)-(3.21) are now in the general form of equations (2.1)-(2.2). Following the procedure of section 2, eigenvalues of $\tilde{A}$ in equation (2.5) and thus a possible bifurcation point can be found.

4. DYNAMIC ANALYSIS OF VOLTAGE COLLAPSE AROUND THE BIFURCATION POINT

Assumptions

It is assumed that the power system is at a steady state with a very small negative eigenvalue at the initial time, the generator of bus 1 holds its bus voltage constant, and the real power load of the motor is constant. The motor is in synchronism and the change in speed is very small. It is also assumed that $\tilde{A}$ matrix of the power system in equation (2.5) has the infinitesimally small negative eigenvalue right after a small disturbance of $\triangle E'_q$ takes place. That is, the equilibrium point of the power system is near the bifurcation point. Since the synchronous machine is of the cylindrical rotor, $X_d = X_q$ and $X'_d = X'_q$, and since the machine operates at fixed excitation, E_{fd} is constant.

Dynamic Analysis of Voltage Collapse Phenomenon

For the convenience of notation, the subscript m that represents the synchronous motor bus is deleted in the following developments. To understand the voltage collapse mechanism, consider equation (3.19):

$$T'_d\dot{E}'_q = [X'_dE_{fd} - X_dE'_q + (X_d - X'_d)V\,cos(\delta - \theta)] \cdot \frac{1}{X'_d}, \qquad (4.1)$$

where $\dot{E}'_q$ means dE'_q/dt.

Solving this for V, the terminal voltage of the motor bus is represented by

$$V = \frac{X_dE'_q - X'_dE_{fd} + X'_dT'_d\dot{E}'_q}{(X_d - X'_d)\;cos(\eta)}, \qquad (4.2)$$

where $\eta = \delta - \theta$. This equation reveals an important fact that voltage collapse is closely related to the collapse of E'_q and η. Thus, it is necessary to develop a dynamic model describing the changes of these two variables. Therefore, the static formulation alone cannot give an adequate answer to the voltage collapse problem.

When the loss of the machine is neglected and the machine is of the cylindrical rotor type, its real power can be expressed as follows:

$$P = \frac{E'_qV}{X'_d}\;sin(\eta), \qquad (4.3)$$

where the positive power and the negative power represent the generation and the load, respectively. When the change in speed is very small $\dot{\omega}$ in equation (3.18) is negligible, and from the condition that the real power load of the machine is constant, equation (3.18) yields

$$E'_q V\, sin(\eta) = c, \tag{4.4}$$

where c is a constant.

Equation (4.4) also indicates that the change in voltage V is due to the changes in E'_q and η. In order to develop expressions for these changes, consider the magnitude of the armature current of the machine which can be expressed as follows:

$$|I| = \sqrt{I_d^2 + I_q^2}. \tag{4.5}$$

From equations (3.15), (3.16), (4.2) and (4.4), this can be rewritten as follows:

$$|I(E'_q, \dot{E}'_q)| = \frac{1}{X'_d(X_d - X'_d)E'_q} \cdot F(E'_q, \dot{E}'_q), \tag{4.6}$$

where

$$F(E'_q, \dot{E}'_q) = \sqrt{{X'_d}^2 {E'_q}^2 (E'_q - E_{fd} + T'_d \dot{E}'_q)^2 + (X_d - X'_d)^2 c^2}.$$

The equation (4.6) is linearized about an equilibrium point to yield the following sensitivity relationship:

$$\begin{aligned}\triangle|I_A| = &\frac{1}{X'_d(X_d - X'_d){E'_q}^2} \cdot \\ &\left[\frac{{X'_d}^2 {E'_q}^3 (E'_q - E_{fd} + T'_d \dot{E}'_q) - (X_d - X'_d)^2 c^2}{F(E'_q, \dot{E}'_q)} \cdot \triangle E'_q \right. \\ &\left. + \frac{{X'_d}^2 T'_d {E'_q}^3 (E'_q - E_{fd} + T'_d \dot{E}'_q)}{F(E'_q, \dot{E}'_q)} \cdot \triangle \dot{E}'_q \right],\end{aligned} \tag{4.7}$$

where $\triangle|I_A|$ means the variation of the armature current I due to $\triangle E'_q$ and $\triangle \dot{E}'_q$ when the effect of voltage drop is not considered. However, due to the variation of the armature current, voltage drop will occur through the transmission line:

$$\triangle \underline{V} = -Z \cdot \triangle I_A, \tag{4.8}$$

where Z is the impedance of the transmission line from the generator bus to the motor bus and $\underline{V}$ is a phasor of the voltage. Since the motor operates at unity power-factor and $\triangle|V|$ is very small compared to $|V|$, the magnitude of $\triangle V$ can be approximated as follows:

$$\triangle|V| \approx -R \cdot \triangle|I_A|, \tag{4.9}$$

where R is the resistance of the transmission line from the generator bus to the motor bus.

To see the effect of this voltage drop on the armature current, consider the real power absorbed by the motor bus as follows:

$$|P| = |V||I|\, cos(\phi), \tag{4.10}$$

where ϕ is the angle between the phase voltage V and the phase current I and its value is 0 from the assumption that the motor operates at unity power-factor.

The reduction in terminal voltage, $\triangle V$, causes further increase in the armature current in order to maintain the constant power in equation (4.10). Let the variation of the armature current I due to $\triangle V$ be $\triangle I_B$. Then from equation (4.10),

$$\triangle|I_B| = -\frac{|P|}{|V|^2 cos\phi} \cdot \triangle|V|. \tag{4.11}$$

The change in E'_q also causes η to change. To see this effect, equation (4.2) is substituted into equation (4.3):

$$P = \frac{tan(\eta)}{X'_d(X_d - X'_d)}(X_d E'_q - X'_d E_{fd} + X'_d T'_d \dot{E}'_q)E'_q. \tag{4.12}$$

Expanding this in a Taylor series and neglecting higher order terms, the following sensitivity relationship is obtained:

$$\begin{aligned}\triangle\eta = &\frac{X'_d E_{fd} - 2X_d E'_q - X'_d T'_d \dot{E}'_q}{(X_d E'_q - X'_d E_{fd} + X'_d T'_d \dot{E}'_q)E'_q} sin(\eta)cos(\eta) \cdot \triangle E'_q \\ &- \frac{X'_d T'_d}{X_d E'_q - X'_d E_{fd} + X'_d T'_d \dot{E}'_q} sin(\eta)cos(\eta) \cdot \triangle \dot{E}'_q.\end{aligned} \tag{4.13}$$

Thus far, we have shown how the change in E'_q affects armature current and angle η. We now show that E'_q is affected dynamically by the armature current and angle η.

The decrease in flux across the air gap of the machine caused by armature current cannot take place immediately. This means we can assume that the time constant of the armature circuit is very small compared to the time constant of the field winding and thus it can be neglected. Then equation (3.5) can be approximated by the following difference equation:

$$\begin{aligned}T'_d\left(\frac{E'_q((k+1)T) - E'_q(kT)}{T}\right) = &E_{fd} - E'_q(kT) \\ &- (X_d - X'_d)I_d(kT),\end{aligned} \tag{4.14}$$

where k is an integer, and the time interval T is related to the time constant of the field winding and can be set to T'_d. Then this equation can be expressed as follows:

$$\begin{aligned}E'_q((k+1)T) &= E_{fd} - (X_d - X'_d) \cdot [I_d((k-1)T) + \triangle I_d(kT)] \\ &= E'_q(kT) - (X_d - X'_d) \cdot \triangle I_d(kT).\end{aligned} \tag{4.15}$$

The direct axis component of the armature current, I_d, is expressed as follows:

$$I_d = -|I|\, sin(\eta + \phi). \tag{4.16}$$

Since $\phi = 0$, the variation of $\triangle I_d$ is: $\triangle I_d = -sin(\eta) \cdot \triangle|I| - |I|cos(\eta) \cdot \triangle\eta$. Thus, equation (4.15) can be rewritten as follows:

$$\begin{aligned}\triangle E'_q((k+1)T) &= -(X_d - X'_d) \cdot \triangle I_d(kT) \\ &= (X_d - X'_d)\Big[sin(\eta(kT)) \cdot (\triangle|I_A(kT)| + \triangle|I_B(kT)|) \\ &\quad + |I(kT)|cos(\eta(kT)) \cdot \triangle\eta(kT)\Big],\end{aligned} \tag{4.17}$$

where the variation of armature current $\triangle|I|$ is the superposition of both variations in equations (4.7) and (4.11). $\dot{E}'_q$ and $\triangle \dot{E}'_q$ in equations (4.2), (4.7) and (4.13) can be approximated as follows:

$$\dot{E}'_q = \frac{E'_q(kT) - E'_q((k-1)T)}{T}, \tag{4.18a}$$

$$\triangle \dot{E}'_q = \frac{\triangle E'_q(kT) - \triangle E'_q((k-1)T)}{T}. \tag{4.18b}$$

The dynamic change in E'_q is now modeled by equation (4.17) as a function of the changes in I and η, which are given in equations (4.7), (4.9), (4.11) and (4.13). Then, by combining these equations together with (4.2), equation (4.17) can be shown to be in the following form:

$$\triangle E'_q((k+1)T) = G_1(E'_q(kT), E'_q((k-1)T), \eta(kT)) \cdot \triangle E'_q(kT) + G_2(E'_q(kT), E'_q((k-1)T), \eta(kT)) \cdot \triangle \dot{E}'_q(kT), \quad (4.19)$$

where E'_q and η are updated as follows:

$$E'_q((k+1)T) = E'_q(kT) + \triangle E'_q((k+1)T), \quad (4.20)$$

$$\eta((k+1)T) = \eta(kT) + \triangle\eta((k+1)T). \quad (4.21)$$

with $\triangle\eta$ being given in equation (4.13).

Finally, in order to demonstrate the voltage collapse phenomenon, the value of V can be obtained by substituting the values of E'_q and η into equation (4.2) together with equation (4.18a).

The key equation governing the dynamics of voltage collapse is equation (4.19), where the gains G_1 and G_2 determine the characteristics of the voltage collapse mechanism. In order to see the mechanism more clearly, we can make the following simplifications. The effect of $\dot{E}'_q$ is relatively small in the initial stage of voltage collapse since the voltage collapse proceeds very slowly. Neglecting the effect of $\dot{E}'_q$, equation (4.19) can be approximated in the following form:

$$\triangle E'_q((k+1)T) = (X_d - X'_d)\left[\frac{(|P|R + V^2cos(\phi))sin(\eta)}{V^2cos(\phi)X'_d(X_d - X'_d)E'^2_q} \cdot \frac{X'^2_d E'^3_q(E'_q - E_{fd}) - (X_d - X'_d)^2c^2}{\sqrt{X'^2_d E'^2_q(E'_q - E_{fd})^2 + (X_d - X'_d)^2c^2}} + \frac{|P|cos(\eta)^2 sin(\eta)}{Vcos(\phi)} \cdot \frac{X'_d E_{fd} - 2X_d E'_q}{X_d E'^2_q - X'_d E_{fd} E'_q}\right] \cdot \triangle E'_q(kT) = G(E'_q, \eta) \cdot \triangle E'_q(kT), \quad (4.22)$$

where the argument kT is deleted in variables in the coefficient $G(E'_q, \eta)$. Thus, the gain $G(E'_q, \eta)$ plays the vital role in the analysis of the voltage collapse mechanism.

5. DISCUSSION AND RESULTS OF SIMULATION

Initial Data

The data of the power system model shown in Figure 1 are as follows:

$X_d = 1.18$, $X'_d = 0.22$, $X_q = X_d$, $X'_q = X'_d$, $T'_d = 1.28$,
$M = 0.0498$, $D = 0.0053$, $\omega_s = 377.$, $V_1 = 1.0$, $\theta_1 = 0$,
$Z_{13} = 0.012 + j0.08$, $Z_{32} = 0.014 + j0.092$, $R = 0.026$.

where the above data are in p.u., except that ω_s is in rad/sec and θ_1 is in radian.

Bifurcation Point

At a bifurcation point the determinant of $\tilde{A}$ in equation (2.5) is zero and the $\tilde{A}$ matrix has a zero eigenvalue. A dynamic bifurcation of the power system can be found by adjusting real and reactive powers of each bus and the value of the capacitor in the load bus 3 so that the $\tilde{A}$ matrix may have a zero eigenvalue. Figure 2 shows the variation of a critical eigenvalue as the real power load of bus 3 is increased in steps from a value of -1.0 p.u. to -3.0 p.u. It is shown that the critical eigenvalue drifts towards the positive value as the loading increases progressively.

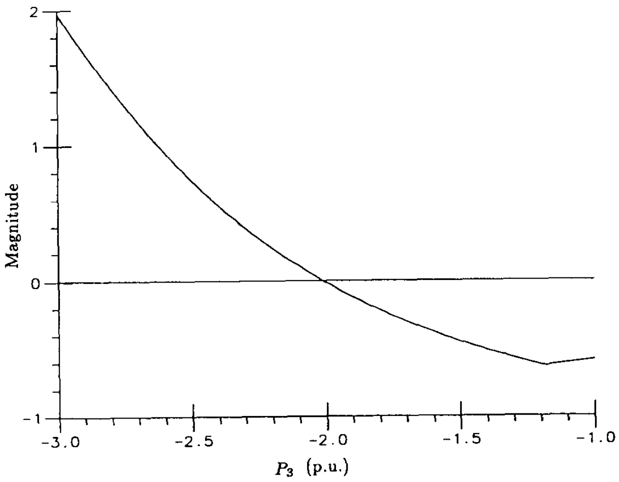

Fig. 2. Variation of a critical eigenvalue due to loading effect

The data for a case when the $\tilde{A}$ matrix has a zero eigenvalue are shown using per unit values as follows:

$P_2 = -1.12$, $P_3 = -2.0032$, $Q_2 = 0$, $Q_3 = -0.4976$,
$P_A = -1.8115$, $P_B = -0.1006$, $P_C = -0.1006$.
$Q_A = -0.45$, $Q_B = -0.025$, $Q_C = -0.025$, $C = 1.2221$.

From the output of the load flow, $V_2 = 0.9450$, $V_3 = 0.9678$, $\theta_2 = -0.3824$, and $\theta_3 = -0.2695$, where angles are in radians. The power of the swing bus 1 is 3.2728+j0.3486.

For the case considered the bifurcating equilibrium point $\underline{x}^o = (\delta^o, \omega^o, E'^o_q)$ is (-0.6516, 377., 0.9803), where ω is in rad/sec. The eigenvalues at the bifurcation are 0.0, -0.9624+j5.3321, and -0.9624 -j5.3321. The eigenvector corresponding to the zero eigenvalue is $\underline{e}$= (-0.7801, +0.12D-6, -0.6257). Here the component values of δ and E'_q are relatively large. It is well known that the bus voltages are very closely related to δ and E'_q. From equation (4.3), the real power is also very closely related to η and E'_q, where $\eta = \delta - \theta$. Therefore, due to the strong coupling between voltage and angle, the voltage collapse will inevitably accompany the angle collapse.

Explanation of Voltage Collapse Mechanism

Suppose that the power system, which is initially at a steady state with a very small negative eigenvalue, is perturbed by (0.1D-3)$\underline{e}$ in the direction of the eigenvector $\underline{e}$ so that the equilibrium may be near $\underline{x}^o$. Suppose also that the parameters of the power system do not vary during the entire period of the voltage collapse. Right after the very small perturbation, the $\tilde{A}$ matrix of this power system has an infinitesimally small negative eigenvalue. Then the scenario of voltage collapse is as follows:

1) By the above assumption, a very small perturbation took place and right after this small perturbation, the equilibrium point of the power system is near the bifurcating equilibrium point $\underline{x}^o$. That is, the $\tilde{A}$ matrix in equation (2.5) has an almost zero eigenvalue. The eigenvector with an almost zero eigenvalue dominates the dynamics of the system, while the components as-

sociated with other relatively larger negative eigenvalues diminish very fast. Thus, the value of each variable will still remain displaced by the initial perturbation due to the effect of the almost zero eigenvalue. Since E_q' was perturbed in the direction of decreasing magnitude however, the magnitude of armature current should increase in order to supply the constant power to the motor from equation(4.7). This is verified from the fact that the values of the coefficients of $\triangle E_q'$ and $\triangle \dot{E}_q'$ in equation (4.7) are negative when the motor operates at unity power-factor.

2) The increase in the magnitude of armature current causes the voltage drop through the transmission line except for leading power factors. This voltage drop becomes an initial motive force that makes the dynamic trajectory move even further. The equation (4.9) represents the voltage drop through the transmission line.

3) This voltage drop in turn causes the increase of armature current in order to supply the constant power to the motor. The equation (4.11) represents this increase of armature current in order to compensate for the terminal voltage drop at the motor bus.

4) The effect of the disturbance reacts again to the machine. This increase of armature current causes a decrease in the airgap flux, which in turn causes the magnitude of E_q' to decrease. However, since the airgap flux cannot change instantaneously, a time delay occurs by the amount of the time constant of the field winding. The equation (4.17) represents this relationship.

The internal voltage of the machine, E_q', thus decreases after the time delay and the above procedures from 1) to 4) are repeated. Thus, an iterative reaction of voltage drop between the machine and the system network can lead to voltage collapse. When the system is operating near the bifurcation point and its linearized system matrix has a very small negative eigenvalue, dynamic equations for describing the mechanism of voltage collapse, caused by a very small disturbance, were derived in Section 4 following the above sequence of events, i.e., the difference equations (4.19), (4.20) and (4.21) represent this iterative process.

Criteria for Dynamic Voltage Stability

We can also use equation (4.22) to approximate equation (4.19). The coefficient $G(E_q', \eta)$ in equation (4.22) is obtained by neglecting the effect of $\dot{E}_q'$. This approximation is very good, especially during the initial stage of voltage collapse, since voltage collapse proceeds very slowly and the magnitude of $\dot{E}_q'$ is very small in the initial stage of voltage collapse. In the case of a very small perturbation, the system will remain at equilibrium if the magnitude of $G(E_q', \eta)$ is less than 1. However, if the magnitude of $G(E_q', \eta)$ is larger than 1, the power system will experience voltage collapse. The function $G(E_q', \eta)$ depends on E_q' and initially its value increases very slowly as E_q' decreases very slowly, and then increases very sharply near the voltage collapse point when E_q' decreases very sharply (See Figure 6). From equation (4.22) we also note that the magnitude of $G(E_q', \eta)$ increases if the real power load of the motor, $|P|$, increases.

Here we conclude the following criteria for preventing a voltage collapse:

a) The linearized system matrix ($\tilde{A}$) has all negative eigenvalues.

b) The magnitude of $G(E_q', \eta)$ at an equilibrium point is less than 1.

Extension of the Model to General Power System

The voltage collapse model is developed for a radial power system. It has been reported that a voltage collapse starts at the weakest node and then spreads out to other weak nodes[14]. Therefore, the weakest node is the most important in the voltage collapse analysis. For the general power system, the weakest node can be handled as one bus and the rest of the system can be handled as another bus. Then the criteria suggested above can also be applied for voltage stability.

This voltage collapse analysis model can be extended to include synchronous generators because the synchronous motor model can be easily changed to the generator model. The fixed excitation implies that the generator operates with fixed excitation in the case of an emergency or that the generator operates as a PQ bus when its reactive power hits its limit. Then the generator does not have reactive power reserve and loses the capability of voltage control. This can happen during the peak load of the power system. When the generator supports constant power as a PQ bus during the peak load, this voltage collapse analysis model can be applied similarly. This also agrees with the general theory that the voltage instability occurs if the reactive demand is not met.

Results of Simulation

From equations (4.19), (4.20), (4.21), and (4.2), the dynamical trajectories of several variables are drawn, with the initial value of $G(E_q', \eta)$ as 1.036. Figures 3, 4 and 5 show the dynamical behaviour of E_q', η and V, respectively. From Figures 3 and 5 we see that E_q' is very closely related to V. Figure 4 shows that the angle collapse is accompanied by the voltage collapse. Figure 5 shows the dynamical characteristics of the voltage collapse of bus 2. The voltage decreases very slowly until it approaches a certain value, and then it collapses abruptly near a voltage collapse point.

The voltage collapse model described by difference equations not only gives a systematic explanation of the voltage collapse mechanism, but also produces anticipated results, which have been sought by many researchers. These results can also be obtained by applying similar assumptions to more detailed differential equations (3.17)-(3.19), but with much extensive simulation effort.

For an approximate analysis, equations (4.22), (4.20), (4.21), and (4.2) can be used. Figure 6 shows the trajectory of the magnitude of $G(E_q', \eta)$. From Figure 6 we observe that the magnitude of $G(E_q', \eta)$ increases very sharply around the voltage collapse point and accelerates the dynamics of the voltage collapse. Figures 7, 8 and 9 show the approximated trajectories of E_q', η and V, respectively. Since the effect of $\dot{E}_q'$ is neglected in the approximation, we note from the figures that the trajectories of these variables proceed rather slowly. Finally, it is also observed that if the initial magnitude of $G(E_q', \eta)$ or the magnitude of the initial perturbation increases, the time required for voltage collapse decreases.

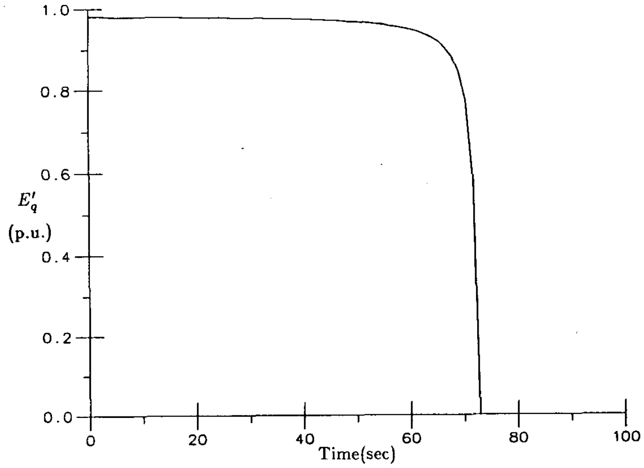

Fig. 3. Dynamical trajectory of internal voltage E_q'

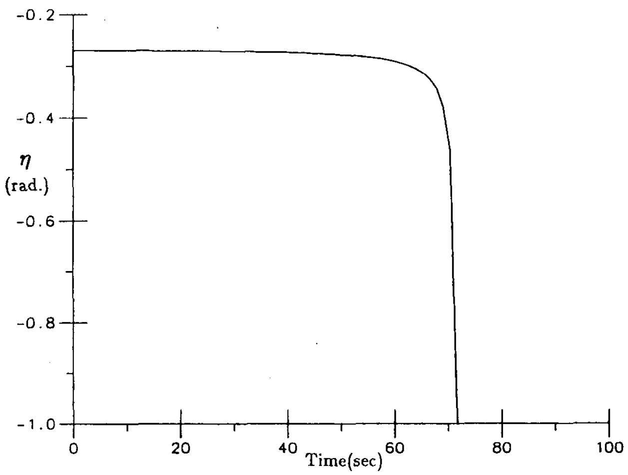

Fig. 4. Dynamical trajectory of η: Angle collapse phenomenon

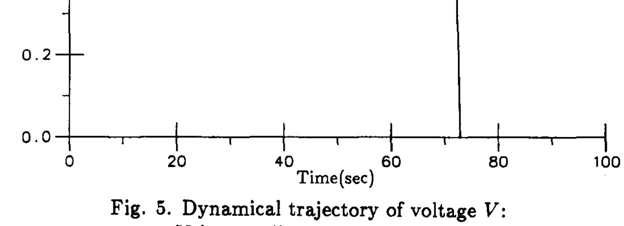

Fig. 5. Dynamical trajectory of voltage V: Voltage collapse phenomenon

6. CONCLUSIONS

This paper has demonstrated a detailed dynamic mechanism of the voltage collapse phenomenon using a simple power system model which includes a synchronous motor. Especially in this paper, the mechanism of voltage collapse phenomenon was analyzed from the physical point of view rather than from the mathematical point of view, and some meaningful physical interpretations are given.

When the system is operating near the bifurcation point and its linearized system matrix has a very small negative eigenvalue, dynamic equations describing the mechanism of voltage collapse caused by a very small disturbance are derived and the physical explanation of the voltage collapse mechanism is presented. It has been verified that the voltage collapse phenomenon accelerates very fast near the voltage collapse point because the magnitude of $G(E_q', \eta)$ increases very sharply around a voltage collapse point. It is also shown that from the simulation results of the derived dynamic equations an iterative reaction of voltage drop between the dynamic load and the system network can cause voltage collapse.

This paper presents an alternative model to describe a voltage collapse mechanism. It is believed that this paper may provide a deeper insight into the dynamical mechanism of voltage collapse phenomenon which has not been well understood.

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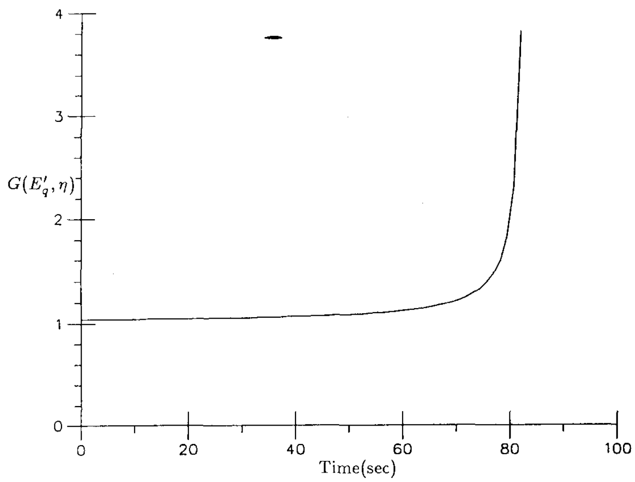

Fig. 6. Trajectory of the coefficient $G(E_q', \eta)$

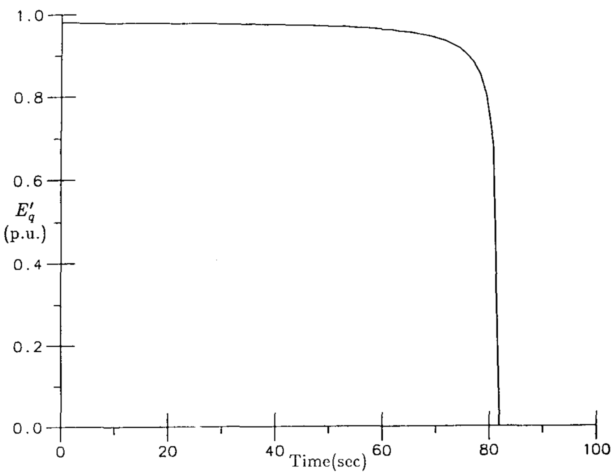

Fig. 7. Approximated trajectory of internal voltage E_q'

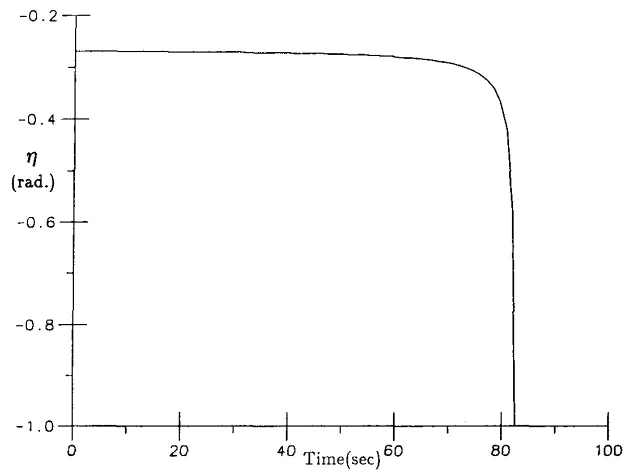

Fig. 8. Approximated trajectory of η: Angle collapse phenomenon

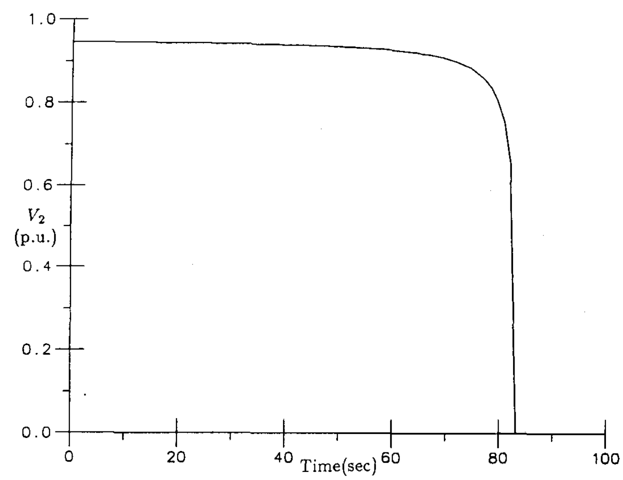

Fig. 9. Approximated trajectory of voltage V: Voltage collapse phenomenon

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