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A SUBCLASS OF PSEUDO-TYPE MEROMORPHIC BI-UNIVALENT FUNCTIONS

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Abstract. In this paper, a new subclass of pseudo-type meromorphic bi-univalent functions is defined on $\triangle = \{z \mid : z \in C \text{ and } 1 < |z| < \infty\}$, we derive estimates on the initial coefficient $|b_0|$, $|b_1|$ and $|b_2|$. Relevant connections of the new results with various well-known results are indicated.

1. Introduction

Let A denote the class of functions $f(z)$ of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the open unit open disk $U = \{z : z \in C, |z| < 1\}$. Also, let the class of univalent and normalized analytic function in the unit disc U be denoted by S with the normalization conditions

$$f(0) = 0 = f^{'}(0) - 1.$$

Furthermore, bi-univalency concept is extended to the class of meromorphic functions defined on $\triangle = \{z : z \in C, 1 < |z| < \infty\}$. For this aim, let Σ denote the class of meromorphic univalent functions g of the form

$$g(z) = z + \sum_{n=0}^{\infty} \frac{b_n}{z^n} \tag{2}$$

defined on the domain $\triangle$. It is well known that every function $g \in \Sigma$ has an inverse $g^{-1} = h$, defined by

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$$g^{-1}(g(z)) = z \quad (z \in \triangle),$$

and

$$g^{-1}(g(w)) = w \quad (M < |w| < \infty,\ M > 0),$$

where

$$g^{-1}(w) = h(w) = w + \sum_{n=0}^{\infty} \frac{B_n}{w^n} = w - b_0 - \frac{b_1}{w} - \frac{b_1 b_0 + b_2}{w^2} - \frac{b_1^2 + b_1 b_0^2 + 2b_0 b_2 + b_3}{w^3} + \dots . \tag{3}$$

A simple computation shows that

$$w = g(h(w)) = (b_0 + B_0) + w + \frac{b_1 + B_1}{w} + \frac{B_2 - b_1 B_0 + b_2}{w^2} + \frac{B_3 - b_1 B_1 + b_1 B_0^2 - 2b_2 B_0 + b_3}{w^3} + \dots . \tag{4}$$

Comparing the initial coefficients in (4), we find that

$$\begin{aligned} b_0 + B_0 = 0 \quad &\Rightarrow \quad B_0 = -b_0 \\ b_1 + B_1 = 0 \quad &\Rightarrow \quad B_1 = -b_1 \\ B_2 - b_1 B_0 + b_2 = 0 \quad &\Rightarrow B_2 = -(b_2 + b_1 b_0) \\ B_3 - b_1 B_1 + b_1 B_0^2 - 2b_2 B_0 + b_3 = 0 \quad &\Rightarrow B_3 = -(b_3 + 2b_0 b_1 + b_1 b_0^2 + b_1^2). \end{aligned}$$

A function $f \in \Sigma$ is said to be meromorphic bi-univalent if $f^{-1} \in \Sigma$. The family of all meromorphic bi-univalent functions is denoted by Σ'. Estimates on the coefficient of meromorphic univalent functions were investigated by some researchers recently; for example, Schiffer [11] obtained the estimate $|b_2| < \frac{3}{2}$ for meromorphic univalent functions $f \in S$ with $b_0 = 0$. Also, Duren [12] obtained the inequality $|b_2| < \frac{2}{n+1}$ for $f \in S$ with $b_k = 0, 1 \leq k \leq \frac{n}{2}$. Springer [8] used variational methods to prove that proved that

$$|B_3| < 1 \text{ and } |B_3 + \frac{1}{2}B_1^2| < \frac{1}{2},$$

and conjectured that

$$|B_{2n-1}| \leq \frac{(2n-2)!}{n!(n-1)!} \ (n = 1, 2, ...).$$

Later on, Kubota [16] has proved that the Springer conjecture is true for n = 3;4;5. Furthermore Schober [7] obtained sharp bounds for $|B_{2n-1}|$ if $1 \leq n \leq 7$. Recently. Kapoor and Mishra [5] found the coefficient estimates for a class consisting of inverses of meromorphic starlike univalent functions of order α in U^*.

Recently, some several researchers such as (see [1], [2], [3], [4], [6], [9], [13] [14]) introduced new subclasses of meromorphically bi-univalent functions and obtained estimates on the initial coefficients for functions belonging to these subclasses.

In 2013, Babalola [10] defined a new subclass λ-pseudo starlike function of order $0 \leq \beta < 1$ satisfying the analytic condition

$$\Re\left\{\frac{z(f(z)'^{\lambda}}{f(z)}\right\} > \beta \ (\lambda \geq 1, \ z \in U). \tag{5}$$

In particular, Babalola [10] proved that all λ-pseudo-starlike functions are Bazilevic of type $1 - \frac{1}{\lambda}$ and order $\beta^{\frac{1}{\lambda}}$ and are univalent in open unit disk U.

Motivated by the earlier work of ([9], [15]), in the present paper, we introduce a new subclasses of the class $\Sigma^{'}$ and the estimates for the coefficients $|b_0|, |b_1|$ and $|b_2|$ are investigated. Some new consequences of the new results are also pointed out.

2. Coefficient Bounds for the Function Class $\Sigma'_{h,p}(\lambda, \mu)$

We begin by introducing the function class $\Sigma'_{h,p}(\lambda, \mu)$ by means of the following definition.

Definition 2.1. Let the functions $h; p : \triangle \to C$ be analytic functions and

$$h(z) = 1 + \frac{h_1}{z} + \frac{h_2}{z^2} + \frac{h_3}{z^3} + \cdots, \quad p(z) = 1 + \frac{p_1}{z} + \frac{p_2}{z^2} + \frac{p_3}{z^3} + \cdots,$$

such that

$$\min\{\Re(h(z)), \Re(p(z))\} > 0, z \in \triangle.$$

A function $g(z) \in \Sigma'$ given by (2) is said to be in the class $\Sigma'_{h,p}(\lambda, \mu)$ if the following conditions are satisfied:

$$g \in \ \Sigma' \text{and} \ \ 1 + \frac{1}{\gamma}\left[(1-\lambda)\left(\frac{g(z)}{z}\right)^{\mu} + \lambda\left(\frac{z(g(z)'^{\mu}}{g(z)}\right) - 1\right] \in h(\triangle),$$
$$(0 < \lambda \leq 1, \ \mu \geq 1, \ z \in \triangle), \tag{6}$$

and

$$1 + \frac{1}{\gamma}\left[(1-\lambda)\left(\frac{h(w)}{w}\right)^{\mu} + \lambda\left(\frac{w(h(w)'^{\mu}}{h(w)}\right) - 1 \in p(\triangle)\right]$$
$$(0 < \lambda \leq 1, \ \mu \geq 1, \ w \in \triangle), \tag{7}$$

where $g \in \ \Sigma'$ and $\gamma \in C \setminus \{0\}$ and the function h is given by (3).

Remark 2.1. There are many choices of h and p which would provide interesting subclasses of class $\Sigma'_{h,p}(\lambda, \mu)$.

(1) If we take

$$h(z) = p(z) = \left(\frac{1 + \frac{1}{z}}{1 - \frac{1}{z}}\right)^{\alpha} = 1 + \frac{2\alpha}{z} + \frac{2\alpha^2}{z^2} + \cdots, \ (0 < \alpha \leq 1, \ z \in \triangle).$$

So it is easy to verify that the functions $h(z)$ and $p(z)$ satisfy the hypotheses of Definition 2.1. If $f \in \Sigma'_\alpha(\lambda,\mu)$. Then

$$\left|\arg\left(1+\frac{1}{\gamma}\left[(1-\lambda)\left(\frac{g(z)}{z}\right)^{\mu}+\lambda\left(\frac{z(g(z)'^{\mu}}{g(z)}\right)-1\right]\right)\right|<\frac{\alpha\pi}{2}$$
$$(0<\lambda\leq 1,\ 0<\alpha\leq 1,\ \mu\geq 1,\ z\in\triangle),$$

and

$$\left|\arg\left(1+\frac{1}{\gamma}\left[(1-\lambda)\left(\frac{h(w)}{w}\right)^{\mu}+\beta\left(\frac{w(h(w)'^{\mu}}{h(w)}\right)-1\right]\right)\right|<\frac{\alpha\pi}{2}$$
$$(0<\lambda\leq 1,\ 0<\alpha\leq 1,\ \mu\geq 1,\ w\in\triangle),$$

where $g(z)\in\Sigma'$ and $\gamma\in C\setminus\{0\}$ and the function h is given by (3).

(2) If we take

$$h(z)=p(z)=\frac{1+\frac{1-2\beta}{z}}{1-\frac{1}{z}}=1+\frac{2(1-\beta)}{z}+\frac{2(1-\beta)}{z^2},\ (0\leq\beta<1,\ z\in\triangle).$$

So it is easy to verify that the functions $h(z)$ and $p(z)$ satisfy the hypotheses of Definition 2.1. If $f \in \Sigma'_\beta(\lambda,\mu)$. Then

$$\Re\left(1+\frac{1}{\gamma}\left[(1-\lambda)\left(\frac{g(z)}{z}\right)^{\mu}+\lambda\left(\frac{z(g(z)'^{\mu}}{g(z)}\right)-1\right]\right)>\beta$$
$$(0<\lambda\leq 1,\ 0\leq\beta<1,\ \mu\geq 1,\ z\in\triangle),$$

and

$$\Re\left(1+\frac{1}{\gamma}\left[(1-\lambda)\left(\frac{h(w)}{w}\right)^{\mu}+\beta\left(\frac{w(h(w)'^{\mu}}{h(w)}\right)-1\right]\right)>\beta$$
$$(0<\lambda\leq 1,\ 0\leq\beta<1,\ \mu\geq 1,\ w\in\triangle),$$

where $g\in\ \Sigma'$ and $\gamma\in C\setminus\{0\}$ and the function h is given by (3).

Theorem 2.1. *Let $g(z)$ be given by (2) be in the class $\Sigma'_\alpha(\lambda,\mu)$. Then*

$$|b_0|\leq\min\left\{\sqrt{\frac{|\gamma|^2(|h_1|^2+|p_1|^2)}{2(\mu-\lambda\mu-\lambda)^2}},\sqrt{\frac{|\gamma|(|h_2|+|p_2|)}{|\mu(\mu-1)(1-\lambda)+2\lambda|}}\right\} \tag{8}$$

and

$$|b_1|\leq\min\{\frac{|\gamma|(|h_2|+|p_2|)}{|2(\mu(\mu-1)(1-\lambda)+2\lambda)|},$$
$$\frac{|\gamma|}{|(\mu-\lambda-2\lambda\mu)|}\left(\sqrt{\frac{|h_2|^2+|p_2|^2}{2}+\frac{[\mu(\mu-1)(1-\lambda)+2\lambda]^2[h_1^2+p_1^2]^2}{16(\mu-\lambda\mu-\lambda)^2}}\right)\}, \tag{9}$$

and

$$|b_2|\quad\leq\quad\frac{|\gamma|}{2|(\mu-\lambda-3\lambda\mu)|}\left[\frac{(\mu(\mu-1)(\mu-2)(1-\lambda)-6\lambda)\gamma^2|p_1|^3}{3|(\mu-\lambda\mu-\lambda)^3|}\right.$$

$$+\frac{2\mu(\mu-1)(1-\lambda)+8\mu\lambda-2\mu+6\lambda}{2\mu(\mu-1)(1-\lambda)-(1-\lambda)\mu+5\lambda+4\lambda\mu}|h_3| + \frac{2\mu(\mu-1)(1-\lambda)+2\mu\lambda+4\lambda}{2\mu(\mu-1)(1-\lambda)-(1-\lambda)\mu+5\lambda+4\lambda\mu}|p_3|\Bigg] \tag{10}$$

Proof. Let $g \in \Sigma'_\alpha(\lambda,\mu)$. Then, by Definition 2.1 of meromorphically bi-univalent function class $\Sigma'_\alpha(\lambda,\mu)$, the conditions (6) and (7) can be rewritten as follows:

$$1+\frac{1}{\gamma}\left[(1-\lambda)\left(\frac{g(z)}{z}\right)^{\mu}+\lambda\left(\frac{z(g(z)'^{\mu}}{g(z)}\right)-1\right]=h(z) \quad (z\in\triangle) \tag{11}$$

and

$$1+\frac{1}{\gamma}\left[(1-\lambda)\left(\frac{h(w)}{w}\right)^{\mu}+\beta\left(\frac{w(h(w)'^{\mu}}{h(w)}\right)-1\right]=p(w), \quad (w\in\triangle) \tag{12}$$

respectively. Here, and in what follows,the functions $h(z) \in P$ and $p(w) \in P$ have the following forms:

$$h(z)=1+\frac{p_1}{z}+\frac{p_2}{z^2}+\frac{p_3}{z^3}+\cdots \quad (z\in\triangle) \tag{13}$$

and

$$p(w)=1+\frac{q_1}{w}+\frac{q_2}{w^2}+\frac{q_3}{w^3}+\cdots \quad (w\in\triangle) \tag{14}$$

upon substituting from (13) and (14) into (11) and (12), respectively, and equating the coefficients, we get

$$\frac{(\mu-\lambda\mu-\lambda)}{\gamma}b_0=h_1 \tag{15}$$

$$\frac{1}{2\gamma}\left[(\mu(\mu-1)(1-\lambda)+2\lambda)b_0^2+2(\mu-\lambda-2\lambda\mu)b_1\right]=h_2 \tag{16}$$

$$\frac{1}{6\gamma}[\mu(\mu-1)(\mu-2)(1-\lambda)-\lambda]b_0^3+\frac{1}{\gamma}[\mu(\mu-1)(1-\lambda)+2\lambda+\lambda\mu]b_0b_1 + \frac{1}{\gamma}[\mu-\lambda-3\mu\lambda]b_2=h_3 \tag{17}$$

$$-\frac{(\mu-\lambda\mu-\lambda)}{\gamma}b_0=p_1 \tag{18}$$

$$\frac{1}{2\gamma}\left[(\mu(\mu-1)(1-\lambda)+2\lambda)b_0^2+2(\lambda-\mu+2\lambda\mu)b_1\right]=p_2 \tag{19}$$

and

$$\frac{1}{6\gamma}[6\lambda-(\mu(\mu-1)(\mu-2)(1-\lambda))b_0^3+6(\mu(\mu-1)(1-\lambda) - \mu(1-\lambda)+3\lambda+3\lambda\mu)b_0b_1+6(\lambda-\mu+3\mu\lambda)b_2]=p_3. \tag{20}$$

From (15) and (18), we find that

$$h_1=-q_1 \tag{21}$$

and

$$2(\mu-\lambda\mu-\lambda)^2b_0^2=\gamma^2(h_1^2+p_1^2) \tag{22}$$

that is,

$$|b_0|^2 \leq \frac{|\gamma|^2(|h_1|^2+|p_1|^2)}{2(\mu-\lambda\mu-\lambda)^2}. \tag{23}$$

Adding (16) and (19), we get

$$[(\mu(\mu-1)(1-\lambda)+2\lambda)]\, b_0^2 = \gamma(h_2+p_2) \tag{24}$$

that is,

$$|b_0|^2 \leq \frac{|\gamma|(|h_2|+|p_2|)}{|\mu(\mu-1)(1-\lambda)+2\lambda|}. \tag{25}$$

From (23) and (25) we get the desired estimate on the coefficient $|b_0|$ as asserted in (8).

Next, in order to find the bound on $|b_0|$, by subtracting the equation (16) from the equation (19), we get

$$2(\mu(\mu-1)(1-\lambda)+2\lambda)b_1 = \gamma(h_2-p_2), \tag{26}$$

that is,

$$|b_1| \leq \frac{|\gamma|(|h_2|+|p_2|)}{|2(\mu(\mu-1)(1-\lambda)+2\lambda)|}. \tag{27}$$

By squaring and adding (16) and (19), using (22) in the computation leads to

$$b_1^2 = \frac{\gamma^2}{(\mu-\lambda-2\lambda\mu)^2}\left(\frac{h_2^2+p_2^2}{2} - \frac{[\mu(\mu-1)(1-\lambda)+2\lambda]^2[h_1^2+p_1^2]^2}{16(\mu-\lambda\mu-\lambda)^2}\right). \tag{28}$$

that is,

$$|b_1| \leq \frac{|\gamma|}{|(\mu-\lambda-2\lambda\mu)|}\left(\sqrt{\frac{|h_2|^2+|p_2|^2}{2} + \frac{[\mu(\mu-1)(1-\lambda)+2\lambda]^2[h_1^2+p_1^2]^2}{16(\mu-\lambda\mu-\lambda)^2}}\right). \tag{29}$$

From (26) and (28) we get the desired estimate on the coefficient $|b_1|$ as asserted in (9).

In order to find the estimate $|b_2|$, consider the sum of (17) and (20), we have

$$b_0 b_1 = \frac{\gamma(h_3+p_3)}{2\mu(\mu-1)(1-\lambda)-(1-\lambda)\mu+5\lambda+4\lambda\mu}. \tag{30}$$

Subtracting (20) from (17) with $h_1 = -p_1$, we obtain

$$\frac{2(\mu-\lambda-3\lambda\mu)b_2}{\gamma} = h_3 - p_3 - \frac{(\mu-\lambda-3\lambda\mu)b_0b_1}{\gamma} - \frac{[\mu(\mu-1)(\mu-2)(1-\lambda)-6\lambda]b_0^3}{3\gamma}. \tag{31}$$

Using (21) and (30) in (31) give to

$$\begin{aligned} b_2 \quad = \quad & \frac{\gamma}{2(\mu-\lambda-3\lambda\mu)}\left[\frac{(\mu(\mu-1)(\mu-2)(1-\lambda)-6\lambda)\gamma^2 p_1^3}{3(\mu-\lambda\mu-\lambda)^3}\right. \\ & + \frac{2\mu(\mu-1)(1-\lambda)+8\mu\lambda-2\mu+6\lambda}{2\mu(\mu-1)(1-\lambda)-(1-\lambda)\mu+5\lambda+4\lambda\mu} h_3 \end{aligned}$$

$$-\frac{2\mu(\mu-1)(1-\lambda)+2\mu\lambda+4\lambda}{2\mu(\mu-1)(1-\lambda)-(1-\lambda)\mu+5\lambda+4\lambda\mu}p_3\Bigg]$$

This evidently completes the proof of Theorem 2.1. □

If we take $\lambda = 1$ in Theorem 2.1, we get the following Corollary.

Corollary 2.2. *Let $g(z)$ be given by (1.2) be in the class $\Sigma'_{\lambda,\beta}(\alpha)$. Then*

$$|b_0| \leq \min\left\{\sqrt{\frac{|\gamma|^2(|h_1|^2+|p_1|^2)}{2}}, \sqrt{\frac{|\gamma|(|h_2|+|p_2|)}{2}}\right\}, \tag{32}$$

$$|b_1| \leq \min\left\{\frac{|\gamma|(|h_2|+|p_2|)}{4}, \frac{|\gamma|}{|\mu+1|}\left(\sqrt{\frac{|h_2|^2+|p_2|^2}{2}+\frac{(|h_1|^2+|p_1|^2)^2}{4}}\right)\right\}. \tag{33}$$

and

$$|b_2| \leq \frac{|\gamma|}{2|(2\mu+1)|} \times \left[2\gamma^2|p_1|^3 + \frac{6(\mu+1)}{5+4\mu}|h_3| + \frac{2(\mu+2)}{5+4\mu}|p_3|\right]. \tag{34}$$

If we take

$$h(z) = p(z) = \left(\frac{1+\frac{1}{z}}{1-\frac{1}{z}}\right)^{\alpha} = 1 + \frac{2\alpha}{z} + \frac{2\alpha^2}{z^2} + \cdots, \ (0 < \alpha \leq 1, \ z \in \triangle),$$

and

$$h(z) = p(z) = \frac{1+\frac{1-2\beta}{z}}{1-\frac{1}{z}} = 1 + \frac{2(1-\mu)}{z} + \frac{2(1-\mu)}{z^2}, \ (0 < \mu \leq 1, \ z \in \triangle),$$

respectively, in the Theorem 2.1, we obtain the following results which is an improvement of estimates obtained by Srivastava et. at [9].

Corollary 2.3. *Let $g(z)$ be given by (2) be in the class $\Sigma'_{\lambda,\beta}(\alpha)$. Then*

$$|b_0| \leq 2\alpha \tag{35}$$

and

$$|b_1| \leq \frac{2\sqrt{5}\alpha^2}{\lambda+1}. \tag{36}$$

Corollary 2.4. *Let $g(z)$ be given by (2) be in the class $\Sigma'_{\lambda,\beta}(\mu)$. Then*

$$|b_0| \leq 2(1-\mu) \tag{37}$$

and

$$|b_1| \leq \frac{2(1-\mu)\sqrt{4\mu^2-8\mu+5}}{\lambda+1}. \tag{38}$$

Remark 2.2. For function $g \in \Sigma'_{h,p}(\lambda,\mu)$ given by (2) by taking $p(z) = h(z) = \frac{1+Az}{1+Bz}$ $-1 \leq B < A \leq 1)$, we obtain the initial coefficient estimates $|b_0|$, $|b_1|$, and $|b_2|$ which leads to the results discussed in Theorem 2.2 of [15].

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