Published for SISSA by Springer

Received: December 31, 2009
Accepted: January 20, 2010
Published: February 12, 2010

A supermatrix model for $\mathcal{N} = 6$ super Chern-Simons-matter theory

Nadav Drukker[a] **and Diego Trancanelli**[b,c]

[a] *Institut für Physik, Humboldt-Universität zu Berlin, Newtonstraße 15, D-12489 Berlin, Germany*
[b] *Department of Physics, University of California, Santa Barbara, CA 93106, U.S.A.*
[c] *Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, U.S.A.*

E-mail: drukker@physik.hu-berlin.de, dtrancan@physics.ucsb.edu

Abstract: We construct the Wilson loop operator of $\mathcal{N} = 6$ super Chern-Simons-matter which is invariant under half of the supercharges of the theory and is dual to the simplest macroscopic open string in $AdS_4 \times \mathbb{CP}^3$. The Wilson loop couples, in addition to the gauge and scalar fields of the theory, also to the fermions in the bi-fundamental representation of the $\mathrm{U}(N) \times \mathrm{U}(M)$ gauge group. These ingredients are naturally combined into a superconnection whose holonomy gives the Wilson loop, which can be defined for any representation of the supergroup $\mathrm{U}(N|M)$. Explicit expressions for loops supported along an infinite straight line and along a circle are presented. Using the localization calculation of Kapustin et al. we show that the circular loop is computed by a supermatrix model and discuss the connection to pure Chern-Simons theory with supergroup $\mathrm{U}(N|M)$.

Keywords: Matrix Models, Supersymmetric gauge theory, AdS-CFT Correspondence, Chern-Simons Theories

ArXiv ePrint: 0912.3006

doi:10.1007/JHEP02(2010)058

JHEP02(2010)058

Contents

JHEP02(2010)058

1 Introduction

The duality between string theory on asymptotically AdS spaces and conformal field theories has been an exciting area of research for over ten years now, with string theory providing answers to strong coupling questions in the gauge theory and vice-versa.

A year and a half ago, a new example of an AdS/CFT duality was proposed by Aharony, Bergman, Jafferis, and Maldacena for the maximally supersymmetric gauge theory in three dimensions: $\mathcal{N} = 6$ supersymmetric Chern-Simons-matter with gauge group $\mathrm{U}(N) \times \mathrm{U}(N)$ [1].[1] The proposal was inspired by a construction of the gauge theory with even more supersymmetry, $\mathcal{N} = 8$, but which applied only to the gauge group $\mathrm{SU}(2) \times \mathrm{SU}(2)$ [3, 4]. The gravity dual of this theory is M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$, where k is the level of the Chern-Simons term, or, for large enough k, type IIA string theory on $AdS_4 \times \mathbb{CP}^3$.

This gauge theory and the dual string theory have been studied extensively, but so far one of the most interesting observables in the gauge theory has not been constructed. Like in all gauge theories, one can define Wilson loop operators, which in the dual string theory are given by semi-classical string surfaces [5, 6]. The most symmetric string of this type preserves half of the supercharges of the vacuum (as well as an $\mathrm{U}(1) \times \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(3)$ bosonic symmetry) but its dual operator in the field theory has not been identified yet.

So far the most symmetric Wilson loop operators in this theory, constructed in [7–9], preserve only 1/6 of the supercharges and are therefore not viable candidates to be the dual of this classical string. In fact, these operators exist also in Chern-Simons theories

[1] In this paper we will actually deal with the generalization of this theory to the case of different ranks, $\mathrm{U}(N) \times \mathrm{U}(M)$, that was discussed in [2].

with less supersymmetry [10] and do not get any supersymmetry enhancement due to the clever quiver construction of the $\mathcal{N} = 6$ theory.

One reason to look for these operators is that Wilson loops are interesting observables in all gauge theories but in particular in Chern-Simons theories. In Chern-Simons without matter they are in fact the main observables. Beyond that, the lack of the gauge theory dual of the simplest string solution in $AdS_4 \times \mathbb{CP}^3$ is a glaring gap in our understanding of this duality.

As another motivation, recall that the analog observable in $\mathcal{N} = 4$ super Yang-Mills theory in four dimensions has the remarkable property that its expectation value is a non-trivial function of the coupling and of N which can be calculated exactly by a Gaussian matrix model and interpolates from weak to strong coupling [11–13].

Another exact interpolating function which exists in the 4-dimensional theory is the cusp anomalous dimension, also known as the universal scaling function [14–21] which captures the scaling dimension of twist two operators. Trying to compute similar quantities in the 3-dimensional theory does not go through as nicely. In the calculation of the spectrum of local operators there is a matching with the square root structure of the dispersion relation of giant magnons, but this involves one extra function of the coupling [22–24], whose value is only known at weak and at strong coupling but not in the intermediate regime.

It is therefore interesting to revisit the question of Wilson loop operators in the hope that there are exact interpolating functions for them. For the 1/6 BPS Wilson loop a matrix model has been recently derived in [25] and, despite its complexity, has been solved in the planar approximation in [26].[2] Their results indeed match the string theory calculation and provide a first non-trivial interpolation function for this theory.

Prompted by these considerations we construct here the 1/2 BPS Wilson loop for $\mathcal{N} = 6$ super Chern-Simons-matter. Furthermore we prove that the results of [25, 26] carry over to our case. The calculation of [25] uses localization with respect to a specific supercharge which is also shared by the 1/2 BPS loop. We show that the 1/2 BPS Wilson loop is cohomologically equivalent to a very specific choice of the 1/6 BPS loop and is therefore also given by a matrix model. This matrix model has a supergroup structure and the 1/2 BPS loop is the most natural observable within this model. Indeed it can be calculated for all values of the coupling also beyond the planar approximation [26].

In the coming section we present the loop and verify its symmetry. Our derivation uses in an essential way the quiver structure of the theory. In addition to the gauge fields, the Wilson loop couples to bilinears of the scalar fields and, crucially, also to the fermionic fields transforming in the bi-fundamental representation of the two gauge groups. Our loop is classified by representations of the supergroup $\mathrm{U}(N|M)$ and is defined in terms of the holonomy of a superconnection of this supergroup.[3] In our analysis we consider both a loop supported along an infinite straight line and one supported along a circle.

In section 3 we relate this Wilson loop to the 1/6 BPS one and show that it is indeed the most natural observable for the matrix model of [25]. We interpret this matrix model as

[2] This matrix model was studied also in [27].

[3] A somewhat similar construction for a topologically twisted version of $\mathcal{N} = 4$ Chern-Simons-matter was shown in [28] to be equivalent to pure Chern-Simons theory with a supergroup.

JHEP02(2010)058

that of a supermatrix which represents the semiclassical expansion of pure Chern-Simons with supergroup $\mathrm{U}(N|M)$ on the lens space $S^3/\mathbb{Z}_2$.

We conclude in section 4 with a discussion of our results and some possible extensions. An appendix contains details about our notation.

2 The loop

We introduce now the construction of the Wilson loop in the $\mathrm{U}(N)_k \times \mathrm{U}(M)_{-k}$ Chern-Simons-matter theory. We denote the gauge field of the $\mathrm{U}(N)$ factor as A_μ and the gauge field of the $\mathrm{U}(M)$ factor as $\widehat{A}_\mu$. These gauge fields are coupled to four scalar fields C_I and their complex conjugates $\bar{C}^I$, and to four fermions ψ_I^α and $\bar{\psi}_\alpha^I$, with $I = 1, 2, 3, 4$ being an $\mathrm{SU}(4)_R$ index and $\alpha = +, -$ a spinor index. The scalars and the fermions are in the bi-fundamental representation of the gauge group. Our notation is such that $C\bar{C}$ and $\bar{\psi}\psi$ are in the adjoint of $\mathrm{U}(N)$, whereas $\bar{C}C$ and $\psi\bar{\psi}$ are in the adjoint of $\mathrm{U}(M)$. In the appendix we give more details about our conventions.

The central idea of this paper is to augment the connection of $\mathrm{U}(N) \times \mathrm{U}(M)$ to a superconnection of the form

$$L \equiv \begin{pmatrix} A_\mu \dot{x}^\mu + \frac{2\pi}{k}|\dot{x}| M_J^I C_I \bar{C}^J & \sqrt{\frac{2\pi}{k}}\,|\dot{x}|\,\eta_I^\alpha \bar{\psi}_\alpha^I \\ \sqrt{\frac{2\pi}{k}}\,|\dot{x}|\,\psi_I^\alpha \bar{\eta}_\alpha^I & \widehat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k}|\dot{x}| \widehat{M}_J^I \bar{C}^J C_I \end{pmatrix}, \tag{2.1}$$

where x^μ parametrizes the curve along which the loop operator is supported and M_J^I, $\hat{M}_J^I$, η_I^α and $\bar{\eta}_\alpha^I$ are free parameters. A lot of the form of L is dictated by dimensional analysis and by the index structure of the fields. In three dimensions the scalars have dimension $1/2$, so they should appear as bi-linears, which are in the adjoint and therefore enter in the diagonal blocks together with the gauge fields. The fermions have dimension 1 and should appear linearly. Since they transform in the bi-fundamental, they are naturally placed in the off-diagonal entries of the matrix. Note that η_I and $\bar{\eta}^I$ are Grassmann even, so that the off-diagonal blocks of L are Grassmann odd and L is a supermatrix.

Although L has the structure of a $\mathrm{U}(N|M)$ superconnection, the theory has only $\mathrm{U}(N) \times \mathrm{U}(M)$ gauge symmetry. It is nevertheless possible, given a path and the extra parameters, to calculate the holonomy of this superconnection and end up with a supermatrix. For a closed curve one can then take the trace[4] in any representation $\mathcal{R}$ of the supergroup $\mathrm{U}(N|M)$. This gives the Wilson loop

$$W_{\mathcal{R}} \equiv \mathrm{Tr}_{\mathcal{R}}\, \mathcal{P} \exp\left(i \int L\, d\tau \right). \tag{2.2}$$

2.1 Infinite straight line

In order to find the maximally supersymmetric Wilson loop, we consider an operator defined along an infinite straight line in the temporal direction, parameterized by $x^\mu = (\tau, 0, 0)$.

[4] One could also take the supertrace instead of the trace. We will show later that supersymmetry imposes the latter.

JHEP02(2010)058

The supercharges of this theory are parameterized by the two-component spinors $\bar{\theta}^{IJ}_{\alpha}$ (see the appendix). Motivated by the 1/2 BPS string solution in $AdS_4 \times \mathbb{CP}^3$, we want to find a loop operator invariant under the same six supercharges. They are in fact the same supercharges also annihilated by other brane solutions dual to the vortex loop operators of [29] and are parameterized by

$$\bar{\theta}^{1I}_{+}\,, \qquad \bar{\theta}^{IJ+}\,, \qquad I,J=2,3,4\,. \tag{2.3}$$

As mentioned before, this loop should also preserve an SU(3) subgroup of the R-symmetry group. Given that and the chirality of the supercharges, this suggests the ansatz

$$M^I_J = \widehat{M}^I_J = m_1\,\delta^I_J - 2m_2\,\delta^1_1\delta^1_J\,, \qquad \eta^{\alpha}_I = \eta\,\delta^1_I\delta^{\alpha}_{+}\,, \qquad \bar{\eta}^I_{\alpha} = \bar{\eta}\,\delta^I_1\delta^{+}_{\alpha}\,. \tag{2.4}$$

We define the modified connections which appear in the diagonal blocks of L

$$\mathcal{A}_0 \equiv A_0 + \frac{2\pi}{k} M^I_J C_I \bar{C}^J\,, \qquad \widehat{\mathcal{A}}_0 \equiv \widehat{A}_0 + \frac{2\pi}{k}\widehat{M}^I_J \bar{C}^J C_I\,. \tag{2.5}$$

One can easily verify [7–9] that the supersymmetry variation of these terms does not vanish. Instead we demand that their variation contains only ψ^+_1 and $\bar{\psi}^1_+$, which appear anyhow in the Wilson loop through the couplings to η^{α}_I and $\bar{\eta}^I_{\alpha}$. Using the expressions in the appendix we find that for the particular choice[5] of $m_1 = m_2 = 1$ the variation is (noticing that $\psi^+ = \psi_-$ and $\psi_+ = -\psi^-$)

$$\begin{aligned}
\delta\mathcal{A}_0 &= \frac{8\pi}{k}\left[\bar{\theta}^{1I}_{+}\,C_I\,\psi^+_1 - \frac{1}{2}\varepsilon_{1IJK}\,\bar{\theta}^{IJ+}\,\bar{\psi}^1_+\,\bar{C}^K\right]\,,\\
\delta\widehat{\mathcal{A}}_0 &= \frac{8\pi}{k}\left[\bar{\theta}^{1I}_{+}\,\psi^+_1\,C_I - \frac{1}{2}\varepsilon_{1IJK}\,\bar{\theta}^{IJ+}\,\bar{C}^K\,\bar{\psi}^1_+\right]\,.
\end{aligned} \tag{2.6}$$

Turning to the off-diagonal entries in L, the variation of the fermions $\bar{\psi}^1$ and ψ_1 includes the covariant derivative $\gamma^\mu D_\mu$. Since the fermions appearing in the loop have specific chiralities, as do the supercharges (2.3), the covariant derivative gets projected to be along the direction of the loop by

$$(i\gamma^\mu)_+{}^+ = \delta^\mu_0\,, \qquad (i\gamma^\mu)_-{}^- = -\delta^\mu_0\,. \tag{2.7}$$

Furthermore, all the non-linear terms appearing in the variation of the fermions can be repackaged into a covariant derivative with the modified connection (2.5)

$$\begin{aligned}
\mathcal{D}_0 C_I &= \partial_0 C_I + i\big(\mathcal{A}_0\,C_I - C_I\,\widehat{\mathcal{A}}_0\big)\,,\\
\mathcal{D}_0 \bar{C}^I &= \partial_0 \bar{C}^I - i\big(\bar{C}^I\,\mathcal{A}_0 - \widehat{\mathcal{A}}_0\,\bar{C}^I\big)\,,
\end{aligned} \tag{2.8}$$

with exactly the choice (2.4) of M^I_J and $\widehat{M}^I_J$.

We finally find that

$$\begin{aligned}
\delta\bar{\psi}^1_+ &= 2\bar{\theta}^{1I}_{+}\mathcal{D}_0 C_I\,,\\
\delta\psi^+_1 &= -\varepsilon_{1IJK}\bar{\theta}^{IJ+}\mathcal{D}_0\bar{C}^K\,.
\end{aligned} \tag{2.9}$$

[5] This value of M^I_J is the same as the effective mass matrix of W-bosons arising upon higgsing the gauge symmetry [30] (see also [9]).

JHEP02(2010)058

Combining (2.6) and (2.9) the variation of L for the time-like line is given by

$$\delta L = \frac{8\pi}{k}\bar{\theta}^{1I}_{+}\begin{pmatrix} C_I\,\psi_1^+ & \sqrt{\frac{k}{8\pi}}\eta\mathcal{D}_0 C_I \\ 0 & \psi_1^+\,C_I \end{pmatrix} - \frac{4\pi}{k}\varepsilon_{1IJK}\,\bar{\theta}^{IJ+}\begin{pmatrix} \bar{\psi}^1_+\,\bar{C}^K & 0 \\ \sqrt{\frac{k}{8\pi}}\bar{\eta}\mathcal{D}_0\bar{C}^K & \bar{C}^K\,\bar{\psi}^1_+ \end{pmatrix}. \tag{2.10}$$

The proof of supersymmetry-invariance of the Wilson loop requires one additional step, namely integration by parts. Expanding to second order, the Wilson loop is

$$W_{\mathcal{R}} = \mathrm{Tr}_{\mathcal{R}}\left[1 + i\int_{-\infty}^{\infty} d\tau\, L(\tau) - \int_{-\infty}^{\infty} d\tau_1 \int_{\tau_1}^{\infty} d\tau_2\, L(\tau_1)L(\tau_2) + \ldots\right]. \tag{2.11}$$

The off-diagonal pieces of the linear term are total derivatives, as can be seen in (2.10) and integrate away. The diagonal part of the linear term does not vanish on its own, but it is canceled by the variation of the fermions in the quadratic term. To see that, we write the relevant terms for the variations with parameters $\bar{\theta}^{1I}_{+}$

$$\begin{aligned}\delta W_{\mathcal{R}} = \frac{8\pi}{k}\bar{\theta}^{1I}_{+}\,\mathrm{Tr}_{\mathcal{R}}\Bigg[& i\int_{-\infty}^{\infty} d\tau \begin{pmatrix} C_I\,\psi_1^+ & \\ & \psi_1^+\,C_I \end{pmatrix} \\ & -\frac{1}{2}\eta\bar{\eta}\int_{-\infty}^{\infty} d\tau_1 \int_{\tau_1}^{\infty} d\tau_2 \begin{pmatrix} \partial_{\tau_1} C_I(\tau_1)\psi_1^+(\tau_2) & \\ & -\psi_1^+(\tau_1)\partial_{\tau_2} C_I(\tau_2) \end{pmatrix} + \ldots\Bigg].\end{aligned} \tag{2.12}$$

The last entry on the bottom right comes from the variation of $L(\tau_2)$ and it has an extra minus sign since the supersymmetry parameter $\bar{\theta}$ was permuted through the first fermion $\psi_1^+(\tau_1)$. We have also assumed that η and $\bar{\eta}$ are constant, so we have pulled them out of the integrals.

Integrating by parts and ignoring any possible contributions from infinity, one obtains

$$\frac{8\pi}{k}\bar{\theta}^{1I}_{+}\left[i\int_{-\infty}^{\infty} d\tau \begin{pmatrix} C_I\,\psi_1^+ & \\ & \psi_1^+\,C_I \end{pmatrix} - \frac{1}{2}\eta\bar{\eta}\int_{-\infty}^{\infty} d\tau \begin{pmatrix} C_I\psi_1^+ & \\ & \psi_1^+ C_I \end{pmatrix}\right]. \tag{2.13}$$

The two integrals clearly cancel each other for $\eta\bar{\eta} = 2i$. A similar cancellation takes place for the $\bar{\theta}^{IJ+}$ supercharges.

To summarize, we have shown at leading order in the expansion (2.11) that the Wilson loop (2.1), (2.2) with

$$M^I_J = \widehat{M}^I_J = \delta^I_J - 2\delta^I_1\delta^1_J\,, \qquad \eta^\alpha_I = \eta\,\delta^1_I\delta^\alpha_+\,, \qquad \bar{\eta}^I_\alpha = \bar{\eta}\,\delta^I_1\delta^+_\alpha\,, \qquad \eta\bar{\eta} = 2i\,, \tag{2.14}$$

preserves the six Poincaré supercharges (2.3) and is therefore 1/2 BPS. We performed the same calculation to the next loop order by multiplying the diagonal part of δL with another L and the off-diagonal pieces with two more and integrating one of the three integral by parts. After including all the terms, the final result vanishes again.

This analysis can be carried over to all orders. To do that we separate L into the diagonal part L_B and the off-diagonal entries L_F. We leave the bosonic piece in the exponent and expand only L_F

$$W_{\mathcal{R}} = \mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\left[e^{i\int L_B\,d\tau}\left(1 + i\int_{-\infty}^{\infty} d\tau_1\, L_F(\tau_1) - \int_{-\infty}^{\infty} d\tau_1 \int_{\tau_1}^{\infty} d\tau_2\, L_F(\tau_1)L_F(\tau_2) + \ldots\right)\right]. \tag{2.15}$$

JHEP02(2010)058

The supersymmetry variation can act on the exponent, bringing down an extra integral of δL_B or can act on one of the L_F, giving a matrix with an off-diagonal entry of the form $\mathcal{D}_0 C_I$ (or $\mathcal{D}_0 \bar{C}^K$). As mentioned before, this $\mathcal{D}_0$ is the covariant derivative with the modified connection appearing in L_B. This allows us to integrate by parts these terms in the presence of the path ordered $\exp(i\int L_B)$. As in the case considered explicitly above, these will give non-zero contributions at the limits of integration, where in general we have

$$\begin{aligned} & i^p \int_{\tau_1<\cdots<\tau_p} d\tau_1 \cdots d\tau_n \cdots d\tau_p \, L_F(\tau_1)\cdots \delta L_F(\tau_n)\cdots L_F(\tau_p) \\ & \quad \propto (-1)^{n-1} i^p \, \bar{\theta}_+^{1I} \int_{\tau_1<\cdots<\cancel{\tau_n}<\cdots<\tau_p} d\tau_1 \cdots \cancel{d\tau_n} \cdots d\tau_p \\ & \qquad L_F(\tau_1)\cdots \left[L_F(\tau_{n-1}) \begin{pmatrix} (C_I\psi_1^+)(\tau_{n+1}) & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & (C_I\psi_1^+)(\tau_{n-1}) \end{pmatrix} L_F(\tau_{n+1}) \right] \cdots L_F(\tau_p), \end{aligned} \tag{2.16}$$

with the factor $(-1)^{n-1}$ coming from pulling out $\bar{\theta}_+^{1I}$ through the L_F insertions. Reordering the terms we see that this exactly cancels the insertion of the variation $\int \delta L_B$ into the term in (2.15) with $p-2$ integrals of L_F.

This calculation proves that this Wilson loop preserves six of the twelve Poincaré supercharges. Similarly, one can show that six conformal supercharges are also preserved.

2.2 Circle

Under a conformal transformation a line is transformed into a circle. While conformal transformations are a symmetry of the theory, they change the topology of the curve and, as it turns out, also the expectation value of the loop. In the case of the 1/2 BPS Wilson loops of $\mathcal{N}=4$ super Yang-Mills one finds that, whereas the straight line has trivial expectation value, the circular loop depends in an interesting way on the coupling constant of the theory. It is therefore of great interest to consider circular Wilson loops also in this 3-dimensional theory.

First we consider the Wick rotation of the time-like line to a space-like line. The latter can be defined either for the theory in Euclidean $\mathbb{R}^3$ or in the Lorentzian theory in $\mathbb{R}^{1,2}$, as we do here. Indeed it is simple to check that the replacement $|\dot{x}| \to -i|\dot{x}|$ gives the 1/2 BPS Wilson loop for a space-like line. This replacement affects the scalar bi-linear term and the fermionic terms.

To get the circle one should perform a conformal transformation. The path is now given by[6]

$$x^1 = \cos\tau\,, \qquad x^2 = \sin\tau\,. \tag{2.17}$$

The scalar couplings should not be affected by the conformal transformation, so for the diagonal part of the superconnection L (2.1) we again use the shorthands

$$\mathcal{A} \equiv A_\mu \dot{x}^\mu - i\frac{2\pi}{k} M_J^I C_I \bar{C}^J\,, \qquad \widehat{\mathcal{A}} \equiv \widehat{A}_\mu \dot{x}^\mu - i\frac{2\pi}{k} \widehat{M}_J^I \bar{C}^J C_I\,. \tag{2.18}$$

[6]We consider for simplicity a circle of unit radius but the result is invariant under conformal transformations and applies to an arbitrary radius circle.

JHEP02(2010)058

We still should couple only to the fermion fields ψ_1^α and $\bar\psi_\alpha^1$. The spinor index is chosen by taking $\eta_I^\alpha(\tau)$ and $\bar\eta_\alpha^I(\tau)$ to be eigenstates of the projector

$$1+\dot x^\mu(\gamma_\mu)_\alpha{}^\beta=\begin{pmatrix}1 & -ie^{-i\tau}\\ ie^{i\tau} & 1\end{pmatrix}, \tag{2.19}$$

thus

$$\eta_I^\alpha(\tau)=\begin{pmatrix}1 & -ie^{-i\tau}\end{pmatrix}\eta(\tau)\,\delta_I^1\,,\qquad \bar\eta_\alpha^I(\tau)=i\begin{pmatrix}1\\ ie^{i\tau}\end{pmatrix}\bar\eta(\tau)\,\delta_1^I\,, \tag{2.20}$$

with an arbitrary function $\eta(\tau)$ which is determined by checking the supersymmetry variation of the loop.

The loop along the line preserved six super-Poincaré symmetries and six superconformal ones, for the circle we expect to find twelve which are linear combinations of the two. The parameters of the superconformal transformations, which we label $\bar\vartheta^{IJ}$, should be related to the super-Poincaré transformation parametrized by $\bar\theta^{IJ\alpha}$. We take the ansatz

$$\bar\vartheta^{1I\alpha}=i\,\bar\theta^{1I\beta}(\sigma^3)_\beta{}^\alpha\,,\qquad \bar\vartheta^{IJ\alpha}=-i\,\bar\theta^{IJ\beta}(\sigma^3)_\beta{}^\alpha\,,\qquad I,J\neq 1\,, \tag{2.21}$$

and using the explicit superconformal transformations [31] determine the supersymmetric circular loop. Note that the choice in (2.21) is consistent with the reality condition (A.8) on $\bar\theta^{IJ}$ and the analog one for $\bar\vartheta^{IJ}$.

To do the calculation we note that, apart for one extra term in the variation of the spinors, the superconformal transformations of the fields are the same as the super-Poincaré transformations, modulo the replacement $\bar\theta^{IJ}\to\bar\vartheta^{IJ}x^\mu\gamma_\mu$. Using that $x^\mu\gamma_\mu\dot x^\nu\gamma_\nu=i\sigma^3$, with our choice of ϑ^{IJ} we find

$$\begin{aligned}&\bar\theta^{1I}+\bar\vartheta^{1I}x^\mu\gamma_\mu=\bar\theta^{1I}(1-\dot x^\mu\gamma_\mu)\,,\\ &\bar\theta^{IJ}+\bar\vartheta^{IJ}x^\mu\gamma_\mu=\bar\theta^{IJ}(1+\dot x^\mu\gamma_\mu)\,,\qquad I,J\neq 1\,.\end{aligned} \tag{2.22}$$

Another useful relation involves the change of spinor indices on the projector

$$(1\pm\dot x^\mu\gamma_\mu)_\alpha{}^\beta=(-1\pm\dot x^\mu\gamma_\mu)^\beta{}_\alpha\,. \tag{2.23}$$

As mentioned in the appendix, unless we write it explicitly, we always use the indices as in the left-hand side of this equation. Lastly, we note that

$$(1\pm\dot x^\nu\gamma_\nu)\gamma^\mu(1\pm\dot x^\rho\gamma_\rho)=\pm2(1\pm\dot x^\nu\gamma_\nu)\dot x^\mu\,. \tag{2.24}$$

Using these relations we get the variations under Poincaré and superconformal transformations of the fields in L

$$\begin{aligned}\delta\mathcal{A}&=\frac{8\pi i}{k}\bar\theta^{1I}(1-\dot x^\mu\gamma_\mu)C_I\psi_1+\frac{4\pi i}{k}\varepsilon_{1IJK}\bar\theta^{IJ}(1+\dot x^\mu\gamma_\mu)\bar\psi^1\bar C^K\,,\\ \delta\widehat{\mathcal{A}}&=\frac{8\pi i}{k}\bar\theta^{1I}(1-\dot x^\mu\gamma_\mu)\psi_1C_I+\frac{4\pi i}{k}\varepsilon_{1IJK}\bar\theta^{IJ}(1+\dot x^\mu\gamma_\mu)\bar C^K\bar\psi^1\,,\\ \delta\big(\eta_1^\alpha(\tau)\bar\psi_\alpha^1\big)&=4i\eta_1\bar\theta^{1I}\dot x^\mu\mathcal{D}_\mu C_I-2\eta_1\sigma^3\bar\theta^{1I}C_I\,,\\ \delta\big(\psi_1^\alpha\bar\eta^1(\tau)_\alpha\big)&=-\varepsilon_{1IJK}\bar\theta^{IJ}\Big[2i\bar\eta^1\dot x^\mu\mathcal{D}_\mu\bar C^K+\sigma^3\bar\eta^1\bar C^K\Big]\,.\end{aligned} \tag{2.25}$$

JHEP02(2010)058

The extra terms in the variations of ψ and $\bar{\psi}$ are written explicitly in [29]. We would like to write the last two expressions as total derivatives, which gives the equations

$$\partial_\tau \eta_1 = \frac{i}{2}\eta_1 \sigma^3 \,, \qquad \partial_\tau \bar{\eta}^1 = -\frac{i}{2}\sigma^3 \bar{\eta}^1 \,. \tag{2.26}$$

From this we deduce that the extra function $\eta(\tau)$ in (2.20) is $\eta(\tau) = e^{i\tau/2}$. The product of the two couplings is then $\eta_I^\alpha \bar{\eta}_\alpha^I = 2i$, as in the case of the line.

The superconnection for the circular Wilson loop is therefore

$$L \equiv \begin{pmatrix} \mathcal{A} & -i\sqrt{\frac{2\pi}{k}}\,\eta_I^\alpha \bar{\psi}_\alpha^I \\ -i\sqrt{\frac{2\pi}{k}}\,\psi_I^\alpha \bar{\eta}_\alpha^I & \widehat{\mathcal{A}} \end{pmatrix}, \tag{2.27}$$

with $\mathcal{A}$ and $\widehat{\mathcal{A}}$ defined in (2.18) and

$$\eta_I^\alpha(\tau) = \begin{pmatrix} e^{i\tau/2} & -ie^{-i\tau/2} \end{pmatrix} \delta_I^1 \,, \qquad \bar{\eta}_\alpha^I(\tau) = \begin{pmatrix} ie^{-i\tau/2} \\ -e^{i\tau/2} \end{pmatrix} \delta_1^I \,. \tag{2.28}$$

Collecting all the pieces, we find that the variation is

$$\begin{aligned} \delta L = {} & \frac{8\pi i}{k} \begin{pmatrix} C_I\, \psi_1 (1 + \dot{x}^\mu \gamma_\mu) & -i\sqrt{\frac{k}{2\pi}}\mathcal{D}_\tau(\eta_1 C_I) \\ 0 & \psi_1\, C_I (1 + \dot{x}^\mu \gamma_\mu) \end{pmatrix} \bar{\theta}^{1I} \\ & + \frac{4\pi i}{k} \varepsilon_{1IJK}\, \bar{\theta}^{IJ} \begin{pmatrix} (1 + \dot{x}^\mu \gamma_\mu)\bar{\psi}^1\, \bar{C}^K & 0 \\ i\sqrt{\frac{k}{2\pi}}\mathcal{D}_\tau\big(\bar{\eta}^1 \bar{C}^K\big) & (1 + \dot{x}^\mu \gamma_\mu)\bar{C}^K\, \bar{\psi}^1 \end{pmatrix} . \end{aligned} \tag{2.29}$$

It is instructive to repeat the supersymmetry analysis at leading order also for the circle. Expanding the loop as in (2.11) and varying it as in (2.29), one finds (we consider just one kind of supercharges and write explicitly only the terms in the diagonal blocks)

$$\begin{aligned} \delta W_{\mathcal{R}} \propto {} & i\,\mathrm{Tr}_{\mathcal{R}} \int_0^{2\pi} d\tau \begin{pmatrix} C_I\, \psi_1 (1 + \dot{x}^\mu \gamma_\mu)\bar{\theta}^{1I} & \\ & \psi_1\, C_I (1 + \dot{x}^\mu \gamma_\mu)\bar{\theta}^{1I} \end{pmatrix} \\ & - \mathrm{Tr}_{\mathcal{R}} \int_0^{2\pi} d\tau_1 \int_{\tau_1}^{2\pi} d\tau_2 \begin{pmatrix} -(\partial_{\tau_1}\eta_1 C_I \bar{\theta}^{1I})_{(1)} (\psi_1 \bar{\eta}^1)_{(2)} & \\ & -(\psi_1 \bar{\eta}^1)_{(1)} (\partial_{\tau_2}\eta_1 C_I \bar{\theta}^{1I})_{(2)} \end{pmatrix} . \end{aligned} \tag{2.30}$$

As done for the line, it is easy to integrate by parts and verify the cancellation of the bulk terms between the first and the second lines of this expression. From the integration by parts one has now also the following boundary terms

$$-\,\mathrm{Tr}_{\mathcal{R}} \int_0^{2\pi} d\tau \begin{pmatrix} (\eta_1 C_I \bar{\theta}^{1I})(0)\ (\psi_1 \bar{\eta}^1)(\tau) & \\ & -(\psi_1 \bar{\eta}^1)(\tau)\ \big(\eta_1 C_I \bar{\theta}^{1I}\big)(2\pi) \end{pmatrix} , \tag{2.31}$$

which cancel once taking the trace, since η_1 is antiperiodic on the circle, $\eta_1(2\pi) = -\eta_1(0)$. This calculation in fact determines that the Wilson loop is supersymmetric only when taking the trace of the holonomy, and not the supertrace.[7]

[7] The trace of a supermatrix in an arbitrary representation is defined on diagonal matrices by the supertrace as $\mathrm{Tr}_{\mathcal{R}} \left(\begin{smallmatrix} a & 0 \\ 0 & b \end{smallmatrix}\right) = \mathrm{sTr}_{\mathcal{R}} \left(\begin{smallmatrix} a & 0 \\ 0 & -b \end{smallmatrix}\right)$. With this definition, equation (2.31) vanishes for any representation.

JHEP02(2010)058

We can repeat the all-order proof outlined in (2.16). Expanding the exponential in L_F one can see again cancellations between bosons and fermions similarly to what happened for the line. The only difference from that case are the new boundary terms arising at $\tau = 0$ from integrating over the first variation $\delta L_F(\tau_1)$ and at $\tau = 2\pi$ from the last variation $\delta L_F(\tau_p)$. As in the leading order case studied above, upon taking the trace these two contributions cancel.

The same analysis carried out above applies also to the six other supercharges. We have shown then that the circle operator is invariant under the twelve supercharges in (2.22) and is therefore 1/2 BPS.

3 Localization to a matrix model

Recently, in a very nice paper [25], the evaluation of supersymmetric Wilson loop operators in Chern-Simons-matter theories with $\mathcal{N} = 2$ supersymmetry was reduced to a 0-dimensional matrix model. In this section we show how to apply the same result to the circular Wilson loop constructed in the preceding section.[8] We can then use the solution of this matrix model [26] to evaluate the Wilson loop at arbitrary values of the coupling constants.

The Wilson loop studied in [25] is the one constructed in [10]. The $\mathcal{N} = 2$ Chern-Simons-matter theories have supersymmetric Wilson loops with the gauge connection and an extra coupling to the scalar in the vector multiplet. This scalar has an algebraic equation of motion and after integrating it out we find the Wilson loop with a coupling to some of the other scalar fields of the theory.

Specializing to the case of the theory with $\mathcal{N} = 6$ supersymmetry, one ends up with the Wilson loops of the type constructed in [7–9], where the connection is given by (2.1) with $\eta_I^\alpha = \bar{\eta}_\alpha^I = 0$ and $M_J^I = \widehat{M}_J^I = \mathrm{diag}(-1, -1, 1, 1)$.[9] In the following we will denote the resulting connection matrix by $L_{1/6}$ and that for the loop constructed in section 2 by $L_{1/2}$. The reason for this notation is that while these Wilson loops preserve half of the supercharges of the $\mathcal{N} = 2$ theories, they do not see the supersymmetry enhancement of the gauge theory from $\mathcal{N} = 2$ to $\mathcal{N} = 6$, so they are 1/6 BPS. The loops constructed in section 2 preserve instead half of the supercharges of the $\mathcal{N} = 6$ theory.

3.1 Relation between the different Wilson loops

The calculation of [25] uses localization with respect to a single supercharge, which is also shared by the 1/2 BPS Wilson loop. We will show now that the 1/2 BPS Wilson loop is related to the 1/6 BPS loop — they are in the same cohomology class under this supercharge. Hence the localization calculation immediately applies also to the 1/2 BPS Wilson loop.

[8] The calculation of [25] was done for a Wilson loop on the equator of S^3, while here we discuss a circle in $\mathbb{R}^3$ or $\mathbb{R}^{1,2}$. These operators should have the same expectation value when normalizing by the partition function.

[9] The choice of 1/6 BPS Wilson loop is not unique. The constructions in [7–9] all preserve the same supercharges but have slight differences. We use this definition, since it will turn out to be related to the loop constructed in section 2, as we show below.

JHEP02(2010)058

We start by analyzing the case of the infinite straight line. We notice that the 1/2 BPS Wilson loop shares all four supercharges preserved by the 1/6 BPS one. These are the ones parameterized by $\bar{\theta}^{12}_{+}$ and $\bar{\theta}^{34+}$ and their superconformal counterparts. For the 1/6 BPS Wilson loop the couplings of the scalars is given by the matrices $M^I_J = \widehat{M}^I_J = \text{diag}(-1,-1,1,1)$ and there is no coupling to the fermions. One can therefore write the difference between the superconnection for the 1/2 BPS loop and the connection of the 1/6 BPS one as

$$\tilde{L} = L_{1/2} - L_{1/6} = \begin{pmatrix} \frac{4\pi}{k} C_2 \bar{C}^2 & \sqrt{\frac{2\pi}{k}}\,\eta\,\bar{\psi}^1_+ \\ \sqrt{\frac{2\pi}{k}}\,\bar{\eta}\,\psi^+_1 & \frac{4\pi}{k}\bar{C}^2 C_2 \end{pmatrix} . \tag{3.1}$$

The off-diagonal term in $\tilde{L}$ is the same as L_F defined above in (2.15). The diagonal piece comes from the difference in the scalar couplings M^I_J and $\widehat{M}^I_J$ between the two loops.

We want now to show that the 1/2 BPS loop, $W_{1/2}$, and the 1/6 BPS one, $W_{1/6}$, are cohomologically equivalent with respect to the aforementioned supercharges. This means that the difference between the two loops is exact with respect to a linear combination Q of the supersymmetries with parameters $\bar{\theta}^{12}_{+}$ and $\bar{\theta}^{34+}$, namely that there exists a V such that

$$W_{1/2} - W_{1/6} = \text{Tr}_{\mathcal{R}}\,\mathcal{P}\left[e^{i\int L_{1/2}} - e^{i\int L_{1/6}}\right] = Q\,V\,, \qquad Q \equiv Q^+_{12} + Q_{34+}\,. \tag{3.2}$$

To find V it is useful to rewrite the difference between the loops as

$$W_{1/2} - W_{1/6} = \text{Tr}_{\mathcal{R}}\,\mathcal{P}\left[e^{i\int_{-\infty}^{\infty} L_{1/6}(\tau)d\tau} \sum_{p=1}^{\infty} i^p \int_{-\infty<\tau_1<\cdots<\tau_p<\infty} d\tau_1\cdots d\tau_p\,\tilde{L}(\tau_1)\cdots\tilde{L}(\tau_p)\right] . \tag{3.3}$$

We take

$$V = i\,\text{Tr}_{\mathcal{R}}\,\mathcal{P}\left[\int_{-\infty}^{\infty} d\tau\, e^{i\int_{-\infty}^{\tau} L_{1/6}(\tau_1)d\tau_1}\,\Lambda(\tau)\,e^{i\int_{\tau}^{\infty} L_{1/2}(\tau_2)d\tau_2}\right] , \tag{3.4}$$

where

$$\Lambda = \sqrt{\frac{\pi}{2k}} \begin{pmatrix} 0 & -\eta\,C_2 \\ \bar{\eta}\,\bar{C}^2 & 0 \end{pmatrix} \tag{3.5}$$

is such that $Q\,\Lambda = L_F$. Acting with Q on V and recalling that $Q\,L_{1/6} = 0$, one finds the following two terms

$$QV = i\,\text{Tr}_{\mathcal{R}}\,\mathcal{P}\left[\int_{-\infty}^{\infty} d\tau e^{i\int_{-\infty}^{\tau} L_{1/6}(\tau_1)d\tau_1}\left(L_F(\tau)e^{i\int_{\tau}^{\infty} L_{1/2}(\tau_2)d\tau_2} + \Lambda(\tau)\,Q\,e^{i\int_{\tau}^{\infty} L_{1/2}(\tau_2)d\tau_2}\right)\right] . \tag{3.6}$$

To evaluate the second contribution we can use a similar logic to the all-order proof (2.16) and Taylor expand the L_F in the exponent, the difference being that the integral in the exponent is now between τ and infinity rather than between minus infinity and infinity. The cancellation between bosons and fermions is therefore incomplete and when Q acts on the first L_F the integration by parts introduces an extra boundary term

$$i\Lambda(\tau)\int_{\tau}^{\infty} d\tau_1\,Q\,L_F(\tau_1) = \begin{pmatrix} \frac{4\pi}{k}(C_2\bar{C}^2)(\tau) & 0 \\ 0 & \frac{4\pi}{k}(\bar{C}^2 C_2)(\tau) \end{pmatrix} . \tag{3.7}$$

JHEP02(2010)058

This is nothing else than the diagonal part of $\tilde{L}$. Combining it with the term in L_F in (3.6), one finds

$$\begin{aligned} Q\,V &= i\,\mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\left[\int_{-\infty}^{\infty} d\tau\, e^{i\int_{-\infty}^{\tau} L_{1/6}(\tau_1)d\tau_1}\,\tilde{L}(\tau)\,e^{i\int_{\tau}^{\infty} L_{1/2}(\tau_2)d\tau_2}\right] \\ &= i\,\mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\left[e^{i\int_{-\infty}^{\infty} L_{1/6}(\tau_1)d\tau_1}\int_{-\infty}^{\infty} d\tau\,\tilde{L}(\tau)\,e^{i\int_{\tau}^{\infty}\tilde{L}(\tau_2)d\tau_2}\right], \end{aligned} \tag{3.8}$$

which, upon Taylor expansion, can be seen to be exactly equal to (3.3).

We analyze now the circular loop. The difference between the connections is now

$$\tilde{L} = L_{1/2} - L_{1/6} = \begin{pmatrix} -i\frac{4\pi}{k}C_2\bar{C}^2 & -i\sqrt{\frac{2\pi}{k}}\,\eta_1^{\alpha}(\tau)\,\bar{\psi}_{\alpha}^{1} \\ -i\sqrt{\frac{2\pi}{k}}\,\psi_1^{\alpha}\,\bar{\eta}_{\alpha}^{1}(\tau) & -i\frac{4\pi}{k}\bar{C}^2C_2 \end{pmatrix} \equiv \tilde{L}_B + L_F\,. \tag{3.9}$$

So we can write $W_{1/2} - W_{1/6}$ in a power series of terms with the $W_{1/6}$ connection

$$W_{1/2} - W_{1/6} = \mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\left[e^{i\int_0^{2\pi} L_{1/6}d\tau}\left(i\int_0^{2\pi} d\tau_1\,\tilde{L}(\tau_1) - \int_{\tau_1<\tau_2} d\tau_1 d\tau_2\,\tilde{L}(\tau_1)\tilde{L}(\tau_2) + \cdots\right)\right]. \tag{3.10}$$

As we saw in the supersymmetry analysis, terms with different numbers of integrals mix. It will be therefore useful to separate this sum into terms with different numbers of field insertions. First L_F, then $\tilde{L}_B$ and L_F^2, next $L_F\tilde{L}_B$ and L_F^3, etc.

Before finding V we should choose one of the supercharges annihilating the 1/6 BPS Wilson loop. We take[10]

$$Q = (Q_{12+} + iS_{12+}) + (Q_{34+} - iS_{34+}) \tag{3.11}$$

and define

$$\Lambda = i\sqrt{\frac{\pi}{2k}}\,e^{i\tau/2}\begin{pmatrix} 0 & C_2 \\ \bar{C}^2 & 0 \end{pmatrix}. \tag{3.12}$$

It is easy to check that

$$Q\,\Lambda = L_F\,, \qquad Q\,L_F = -8\mathcal{D}_{\tau}(e^{-i\tau}\Lambda)\,, \qquad 8ie^{-i\tau}\Lambda\Lambda = \tilde{L}_B\,. \tag{3.13}$$

The covariant derivative acting on Λ in $Q\,L_F$ has the generalized connection with $\mathcal{A}$ and $\hat{\mathcal{A}}$ in $L_{1/2}$ (2.18), but its action on Λ is the same as a covariant derivative in the $L_{1/6}$ connection, since the difference between the two, involving $C_2\bar{C}^2$ and $\bar{C}^2C_2$, cancels when acting on Λ. We can therefore integrate the total derivative inside a Wilson loop with either the $L_{1/2}$ or $L_{1/6}$ connection.

[10] For consistency with the analysis of [25] we consider here only one specific chirality.

JHEP02(2010)058

We now solve for V in a power series. We take $V = \sum_{p=1}^{\infty} V_p$, where the term V_p has p field insertions into the Wilson loop with connection $L_{1/6}$. The first few are

$$\begin{aligned}
V_1 &= i\,\mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\left[e^{i\int_0^{2\pi} L_{1/6}d\tau}\int_0^{2\pi} d\tau_1\,\Lambda(\tau_1)\right],\\
V_2 &= -\frac{1}{2}\,\mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\left[e^{i\int_0^{2\pi} L_{1/6}d\tau}\int_{\tau_1<\tau_2} d\tau_1\,d\tau_2\left(\Lambda(\tau_1)L_F(\tau_2)-L_F(\tau_1)\Lambda(\tau_2)\right)\right],\\
V_3 &= \mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\Bigg[e^{i\int_0^{2\pi} L_{1/6}d\tau}\Bigg(-\int_{\tau_1<\tau_2} d\tau_1\,d\tau_2\left(\tilde{L}_B(\tau_1)\Lambda(\tau_2)+\Lambda(\tau_1)\tilde{L}_B(\tau_2)\right)\\
&\quad -i\int_{\tau_1<\tau_2<\tau_3} d\tau_1\,d\tau_2\,d\tau_3\left(\Lambda(\tau_1)L_F(\tau_2)L_F(\tau_3)+L_F(\tau_1)\Lambda(\tau_2)L_F(\tau_3)+L_F(\tau_1)L_F(\tau_2)\Lambda(\tau_3)\right)\Bigg)\Bigg].
\end{aligned} \tag{3.14}$$

Using (3.13) it is easy to check that

$$\begin{aligned}
Q\,V_1 &= i\,\mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\left[e^{i\int_0^{2\pi} L_{1/6}d\tau}\int_0^{2\pi} d\tau_1\,L_F(\tau_1)\right],\\
Q\,V_2 &= \mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\left[e^{i\int_0^{2\pi} L_{1/6}d\tau}\left(i\int_0^{2\pi} d\tau_1\,\tilde{L}_B(\tau_1)-\int_{\tau_1<\tau_2} d\tau_1\,d\tau_2\,L_F(\tau_1)L_F(\tau_2)\right)\right],\\
Q\,V_3 &= \mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\Bigg[e^{i\int_0^{2\pi} L_{1/6}d\tau}\Bigg(-\int_{\tau_1<\tau_2} d\tau_1\,d\tau_2\left(\tilde{L}_B(\tau_1)L_F(\tau_2)+L_F(\tau_1)\tilde{L}_B(\tau_2)\right)\\
&\qquad -i\int_{\tau_1<\tau_2<\tau_3} d\tau_1\,d\tau_2\,d\tau_3\,L_F(\tau_1)L_F(\tau_2)L_F(\tau_3)\Bigg)\Bigg].
\end{aligned} \tag{3.15}$$

These indeed are the terms in the expansion of $W_{1/2}-W_{1/6}$ around the $L_{1/6}$ connection with one, two and three fields insertions. We have also checked the next term in the expansion and expect this pattern to extend to all orders.

This comparison with the 1/6 BPS loop allows for an alternative, immediate verification that the loop constructed in section 2 is indeed 1/2 BPS. This derivation shows that the Wilson loop is invariant under the single supercharge used above. Then we note that from inspecting $L_{1/2}$, the Wilson loop preserves an SU(3) subgroup of the SU(4) R-symmetry group of the theory. Note though that the supercharge is not invariant under this SU(3), so acting with this symmetry we automatically generate more supercharges preserved by this loop. In a similar fashion one can generate the full supergroup with twelve supercharges preserved by the loop as the minimal one containing the SU(3) generators and the four preserved by the 1/6 BPS loop.

3.2 Supermatrix model and supergroup Chern-Simons

Since the 1/2 BPS loop and the 1/6 BPS one are in the same cohomology class with respect to Q, we can immediately conclude that the localization argument used in [25] for the 1/6 BPS circular loop will also apply unaltered to our operator.

Generalizing the matrix model derived in [25] to the case of $M \neq N$ gives the following expression for the partition function

$$Z = \int \prod_{a=1}^{N} d\lambda_a\, e^{ik\pi\lambda_a^2} \prod_{\hat{a}=1}^{M} d\hat{\lambda}_{\hat{a}}\, e^{-ik\pi\hat{\lambda}_{\hat{a}}^2}\, \frac{\prod_{a<b}\sinh^2(\pi(\lambda_a-\lambda_b))\prod_{\hat{a}<\hat{b}}\sinh^2\left(\pi\left(\hat{\lambda}_{\hat{a}}-\hat{\lambda}_{\hat{b}}\right)\right)}{\prod_{a,\hat{a}}\cosh^2\left(\pi\left(\lambda_a-\hat{\lambda}_{\hat{a}}\right)\right)}. \tag{3.16}$$

JHEP02(2010)058

Here λ_a $(a = 1, \ldots, N)$ and $\hat{\lambda}_{\hat{a}}$ $(\hat{a} = 1, \ldots, M)$ are two sets of eigenvalues corresponding to the two gauge groups of the theory. Our 1/2 BPS Wilson loop in the fundamental representation is evaluated by inserting into the integral above

$$W = \sum_{a=1}^{N} e^{2\pi\lambda_a} + \sum_{\hat{a}=1}^{M} e^{2\pi\hat{\lambda}_{\hat{a}}} \,. \tag{3.17}$$

For a general representation the insertion is (see footnote 7)

$$W_{\mathcal{R}} = \mathrm{Tr}_{\mathcal{R}} \begin{pmatrix} \mathrm{diag}(e^{2\pi\lambda_a}) & 0 \\ 0 & \mathrm{diag}\left(e^{2\pi\hat{\lambda}_{\hat{a}}}\right) \end{pmatrix} = \mathrm{sTr}_{\mathcal{R}} \begin{pmatrix} \mathrm{diag}(e^{2\pi\lambda_a}) & 0 \\ 0 & -\mathrm{diag}\left(e^{2\pi\hat{\lambda}_{\hat{a}}}\right) \end{pmatrix} . \tag{3.18}$$

Examining these expressions one sees that if the cosh functions in (3.16) were in the numerator rather than in the denominator, this would be the matrix model for pure Chern-Simons theory with gauge group $\mathrm{SU}(N+M)$ on a lens space $S^3/\mathbb{Z}_2$, where an $\mathrm{SU}(N)$ subgroup is expanded around the trivial vacuum and an $\mathrm{SU}(M)$ subgroup around the other flat connection [32–34].[11] Compared to the trivial saddle point with unbroken $\mathrm{SU}(N+M)$, the non-trivial connection is represented by the shift of the eigenvalue $\lambda \to \lambda + i/2$. This replaces some of the sinh functions with cosh functions and also gives the $(-)$ factor in the lower-right block on the right-hand side of (3.18).

In (3.16) the cosh functions are in the denominator rather than in the numerator. This arises naturally when considering instead the matrix model and Chern-Simons theory (and the same saddle point) for the gauge supergroup $\mathrm{SU}(N|M)$. For a fuller discussion see [26].

4 Discussion

In this paper we have found the so far elusive 1/2 BPS Wilson loop operator of $\mathcal{N} = 6$ super Chern-Simons-matter. We have considered both a loop supported along an infinite straight line and one supported along a circle. The former preserves separately six Poincaré supercharges and six conformal supercharges, whereas the latter preserves twelve linear combinations of the two. The proof of the invariance of our operator under these supercharges is quite novel, for it requires to Taylor expand the exponential and to integrate by parts some of the variations in order to have cancellations between terms of different order. While the theory has only $\mathrm{U}(N) \times \mathrm{U}(M)$ gauge symmetry, our loop can be defined for arbitrary representations of the supergroup $\mathrm{U}(N|M)$.

We have shown that this loop is related to another one, which is only 1/6 BPS, by the addition of a term exact under a supercharge Q. This in turn implies that the expectation values of the two loops are equal and allows us to use the matrix model (3.16), derived using localization in [25]. Beyond this formal derivation it would be interesting to check this expression by an explicit perturbative computation. It is straightforward to do the matrix model calculation perturbatively, plugging (3.17) into (3.16). The first few orders are

$$\langle W \rangle_{MM} = 1 + i\frac{\pi}{k}(N-M) - \frac{2\pi^2}{3k^2}\left(N^2 - \frac{5}{2}NM + M^2 - \frac{1}{4}\right) + \mathcal{O}\left(\frac{1}{k^3}\right) . \tag{4.1}$$

[11] We are grateful to Marcos Mariño for discussions about this point.

JHEP02(2010)058

This is a prediction for a corresponding computation to be performed directly in the gauge theory (with framing one) by summing Feynman diagrams.

The matrix model gives, in principle, an exact expression for the expectation value of the Wilson loop valid for all values of N, M and k. Unfortunately, unlike the 4-dimensional analog [11–13], this matrix model is quite complicated (for similar models see e.g. [32–36]). Still, it can be solved [26] and gives the correct expression for the expectation value of the Wilson loop at strong coupling as evaluated by a macroscopic fundamental string extending in $AdS_4 \times \mathbb{CP}^3$ and ending along the circular loop on the boundary of AdS_4 (or by an M2 brane wrapping the orbifolded direction of $S^7/\mathbb{Z}_k$)

$$\langle W \rangle \simeq \exp\left(\pi\sqrt{\frac{2N}{k}}\right) \simeq \exp\left(\pi\sqrt{\frac{N+M}{k}}\right) . \tag{4.2}$$

Note that while the matrix model is related to the supergroup Chern-Simons theory, it is not exactly the same. The matrix model calculates the contribution of a single saddle point in a perturbative expansion of the Chern-Simons theory. This is reminiscent of the situation for the Wilson loops on S^2 in $\mathcal{N} = 4$ supersymmetric Yang-Mills in four dimensions and their relation to two-dimensional Yang-Mills [37, 38]. They are not given by the full answer in the 2-dimensional theory, but rather by a semiclassical expansion around the zero-instanton sector [39–41]. Recently an interpretation was given for the other saddle points, as the correlation function of Wilson and 't Hooft loops [42]. It would be interesting to understand if there are any observables in $\mathcal{N} = 6$ Chern-Simons-matter theory which give the other saddle points in the perturbative expansion of the U($N|M$) pure Chern-Simons theory.

Another direction worth investigating is related to the construction of Wilson loops via the higgsing of membranes, as done in four dimensions in [43]. In [9, 30] the coupling of the Wilson loop to the scalar fields was found by separating membranes and computing the mass of the resulting off-diagonal modes stretching between them. This indeed gives the scalar couplings in (2.4), but did not include the fermions, that, as we have seen, are crucial to enhance the supersymmetry of the loop operator. It would be therefore interesting to repeat the calculation considering also fermionic off-diagonal modes and to reproduce the couplings η_I^α and $\bar{\eta}_\alpha^I$ in this way.

There are other objects in this theory which are very closely related to the Wilson loops constructed here. These are the vortex loop operators of [29], which have a semiclassical description in the gauge theory. Along the loop the gauge symmetry is broken to some subgroup and different U(1) factors have vortices. In addition, the scalar fields can have square-root branch cuts. So, parameterizing the transverse plane to the line by complex coordinates z and $\bar{z}$, the field configuration (in one U(1) factor) is

$$A = \widehat{A} = -i\frac{\alpha}{2k}\left(\frac{dz}{z} - \frac{d\bar{z}}{\bar{z}}\right), \qquad C_1 = \frac{\beta}{\sqrt{z}}, \tag{4.3}$$

with α and β being two real parameters. The vortex loops carry k unit of electric flux and should be an alternative description for k coincident Wilson loops. In fact, while one can construct the Wilson loop in any anti-symmetric representation, in Chern-Simons theory

JHEP02(2010)058

the dimension of the symmetric representation should be smaller than k. We expect the vortex loops to take over as the description of the object carrying k units of electric flux. In the M-theory picture the 1/2 BPS Wilson loop is an M2-brane wrapping the orbifolded direction of $S^7/\mathbb{Z}_k$. The k-th symmetric loop is the brane wrapping the circle in the covering space. This brane then develops extra allowed deformations, including opening up in AdS_4, which corresponds to the β parameter above, and rotating on S^7, thus leading to the 1/3 BPS vortex of [29].

Still, a fuller classification of all 1-dimensional defects in this theory is in order. In $\mathcal{N} = 4$ super Yang-Mills in four dimensions the classification of Wilson and 't Hooft loops gives rise to a rich structure of objects in the dual string theory, including probe branes wrapping various cycles (see e.g. [43–46]) and fully backreacted geometries, the so-called "bubbling" solutions (see e.g. [47–50]). So far this classification has only been partially undertaken in the M-theory dual of $\mathcal{N} = 6$ super Chern-Simons-matter. We plan to complete this in a future publication.

Apart for the theory with $\mathcal{N} = 6$ supersymmetry, there are closely related 3-dimensional Chern-Simons-matter theories with $\mathcal{N} = 4$ and $\mathcal{N} = 5$ supersymmetry [51–53]. These theories are based on more complicated quivers and have a richer structure of allowed gauge groups and matter representations. We expect that constructions similar to ours will give the 1/2 BPS loops of these theories. These should probably be related to Wilson loop observables in the topological theories discussed in [28].

It would be also of some importance to find other loop operators of $\mathcal{N} = 6$ super Chern-Simons-matter preserving reduced amounts of supersymmetry, following the spirit of [54, 55]. In particular, note that the couplings satisfy the relations $\bar{\eta}^I_\alpha \eta^\beta_I = i(1 + \dot{x}^\mu \gamma_\mu)_\alpha{}^\beta$ and $M^I_J = \delta^I_J + i\eta^\alpha_J \bar{\eta}^I_\alpha$. Such relations may play a role in a more comprehensive analysis as do the pure spinors in the treatment of [56].

Acknowledgments

This project grew out of discussions with Georgios Michalogiorgakis and Vasilis Niarchos to whom we are extremely grateful for the initial collaboration. We would also like to thank David Berenstein, Giulio Bonelli, Jaume Gomis, Johannes Henn, Juan Maldacena, Jan Plefka, and especially Marcos Mariño for interesting discussions. We also thank Marcos Mariño and Pavel Putrov for sharing their results [26] prior to publication. We thank the KITP for hospitality during the "Fundamental Aspects of Superstring Theory" workshop. N.D. would also like to thank the hospitality of the École Polytechnique in Paris, the Benasque Science Center, and the hospitality and financial support of Perimeter Insititute. D.T. is grateful to the Theory Group at CERN, the LPTHE in Paris, the Niels Bohr Institute, and the 7th Simons Workshop in Stony Brook for the very nice hospitality during the completion of this work. N.D. and D.T. are partly supported by the NSF grant PHY-05-51164 and D.T. also by the Department of Energy under Contract DE-FG02-91ER40618.

JHEP02(2010)058

A Notation and conventions

For the supersymmetry analysis of the loop, we consider the theory in $\mathbb{R}^{1,2}$ with metric $g_{\mu\nu} = \mathrm{diag}(-1,1,1)$ and space-time indices $\mu,\nu,\ldots = 0,1,2$. The spinor indices are denoted with lower case letters from the beginning of the Greek alphabet, $\alpha,\beta,\ldots = +,-$. Spinor indices are raised and lowered according to the following rules

$$\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta\,, \qquad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta\,, \qquad \varepsilon^{+-} = -\varepsilon_{+-} = 1\,, \tag{A.1}$$

and we choose for our basis of gamma matrices

$$(\gamma^\mu)_\alpha{}^\beta = \{-i\sigma^3, \sigma^1, \sigma^2\}\,, \tag{A.2}$$

obeying the relation $\gamma^\mu\gamma^\nu = g^{\mu\nu} + \varepsilon^{\mu\nu\rho}\gamma_\rho$ (with $\varepsilon^{012} = 1$). Lowering the upper index, these matrices become symmetric

$$(\gamma^\mu)_{\alpha\beta} = \{-i\sigma^1, -\sigma^3, i1\}\,. \tag{A.3}$$

When not written explicitly, the spinor indices are contracted as

$$\begin{aligned}\theta\psi &\equiv \theta^\alpha\psi_\alpha = -\theta_\alpha\psi^\alpha = \psi^\alpha\theta_\alpha = \psi\theta\,,\\ \theta\gamma^\mu\psi &\equiv \theta^\alpha(\gamma^\mu)_\alpha{}^\beta\psi_\beta = -\psi\gamma^\mu\theta\,,\end{aligned} \tag{A.4}$$

where θ and ψ are arbitrary spinors.

Regarding the gauge index structure, if we denote by a, $\hat{a}$ the gauge indices in the fundamental of the first and the second gauge group, respectively, we have

$$(C_I)_a{}^{\hat{a}}\,, \qquad (\bar{C}^I)_{\hat{a}}{}^a\,, \qquad (\psi_I)_{\hat{a}}{}^a\,, \qquad (\bar{\psi}^I)_a{}^{\hat{a}}\,. \tag{A.5}$$

Under supersymmetry the fields transform as (for clarity we indicate here all the spinor indices explicitly)

$$\begin{aligned}
\delta A_\mu &= \frac{4\pi i}{k}\bar{\theta}^{IJ\alpha}(\gamma_\mu)_\alpha{}^\beta\left(C_I\psi_{J\beta} + \frac{1}{2}\varepsilon_{IJKL}\bar{\psi}^K_\beta\bar{C}^L\right),\\
\delta\widehat{A}_\mu &= \frac{4\pi i}{k}\bar{\theta}^{IJ\alpha}(\gamma_\mu)_\alpha{}^\beta\left(\psi_{J\beta}C_I + \frac{1}{2}\varepsilon_{IJKL}\bar{C}^L\bar{\psi}^K_\beta\right),\\
\delta C_K &= \bar{\theta}^{IJ\alpha}\varepsilon_{IJKL}\bar{\psi}^L_\alpha\,,\\
\delta\bar{C}^K &= 2\bar{\theta}^{KL\alpha}\psi_{L\alpha}\,,\\
\delta\psi^\beta_K &= -i\bar{\theta}^{IJ\alpha}\varepsilon_{IJKL}(\gamma^\mu)_\alpha{}^\beta D_\mu\bar{C}^L\\
&\quad + \frac{2\pi i}{k}\bar{\theta}^{IJ\beta}\varepsilon_{IJKL}\big(\bar{C}^L C_P\bar{C}^P - \bar{C}^P C_P\bar{C}^L\big) + \frac{4\pi i}{k}\bar{\theta}^{IJ\beta}\varepsilon_{IJML}\bar{C}^M C_K\bar{C}^L,\\
\delta\bar{\psi}^K_\beta &= -2i\bar{\theta}^{KL\alpha}(\gamma^\mu)_{\alpha\beta}D_\mu C_L - \frac{4\pi i}{k}\bar{\theta}^{KL}_\beta(C_L\bar{C}^M C_M - C_M\bar{C}^M C_L) - \frac{8\pi i}{k}\bar{\theta}^{IJ}_\beta C_I\bar{C}^K C_J\,,
\end{aligned} \tag{A.6}$$

where we have written the transformations only in terms of the parameters $\bar{\theta}$ and not θ, by using the following relation

$$\theta_{IJ} = \frac{1}{2}\varepsilon_{IJKL}\bar{\theta}^{KL}\,. \tag{A.7}$$

The supersymmetry parameters are antisymmetric, $\bar{\theta}^{IJ} = -\bar{\theta}^{JI}$, and obey the reality condition

$$\bar{\theta}^{IJ} = (\theta_{IJ})^*\,. \tag{A.8}$$

JHEP02(2010)058

References

[1] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, $\mathcal{N} = 6$ *superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [SPIRES].

[2] O. Aharony, O. Bergman and D.L. Jafferis, *Fractional M2-branes*, *JHEP* **11** (2008) 043 [arXiv:0807.4924] [SPIRES].

[3] A. Gustavsson, *Algebraic structures on parallel M2-branes*, *Nucl. Phys.* **B 811** (2009) 66 [arXiv:0709.1260] [SPIRES].

[4] J. Bagger and N. Lambert, *Gauge Symmetry and Supersymmetry of Multiple M2-Branes*, *Phys. Rev.* **D 77** (2008) 065008 [arXiv:0711.0955] [SPIRES].

[5] J.M. Maldacena, *Wilson loops in large-N field theories*, *Phys. Rev. Lett.* **80** (1998) 4859 [hep-th/9803002] [SPIRES].

[6] S.-J. Rey and J.-T. Yee, *Macroscopic strings as heavy quarks in large-N gauge theory and anti-de Sitter supergravity*, *Eur. Phys. J.* **C 22** (2001) 379 [hep-th/9803001] [SPIRES].

[7] N. Drukker, J. Plefka and D. Young, *Wilson loops in 3-dimensional $\mathcal{N} = 6$ supersymmetric Chern-Simons Theory and their string theory duals*, *JHEP* **11** (2008) 019 [arXiv:0809.2787] [SPIRES].

[8] B. Chen and J.-B. Wu, *Supersymmetric Wilson Loops in $\mathcal{N} = 6$ Super Chern-Simons- matter theory*, *Nucl. Phys.* **B 825** (2010) 38 [arXiv:0809.2863] [SPIRES].

[9] S.-J. Rey, T. Suyama and S. Yamaguchi, *Wilson Loops in Superconformal Chern-Simons Theory and Fundamental Strings in Anti-de Sitter Supergravity Dual*, *JHEP* **03** (2009) 127 [arXiv:0809.3786] [SPIRES].

[10] D. Gaiotto and X. Yin, *Notes on superconformal Chern-Simons-matter theories*, *JHEP* **08** (2007) 056 [arXiv:0704.3740] [SPIRES].

[11] J.K. Erickson, G.W. Semenoff and K. Zarembo, *Wilson loops in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 582** (2000) 155 [hep-th/0003055] [SPIRES].

[12] N. Drukker and D.J. Gross, *An exact prediction of $\mathcal{N} = 4$ SUSYM theory for string theory*, *J. Math. Phys.* **42** (2001) 2896 [hep-th/0010274] [SPIRES].

[13] V. Pestun, *Localization of gauge theory on a four-sphere and supersymmetric Wilson loops*, arXiv:0712.2824 [SPIRES].

[14] G.P. Korchemsky, *Asymptotics of the Altarelli-Parisi-Lipatov Evolution Kernels of Parton Distributions*, *Mod. Phys. Lett.* **A 4** (1989) 1257 [SPIRES].

[15] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *A semi-classical limit of the gauge/string correspondence*, *Nucl. Phys.* **B 636** (2002) 99 [hep-th/0204051] [SPIRES].

[16] S. Frolov and A.A. Tseytlin, *Semiclassical quantization of rotating superstring in $AdS_5 \times S^5$*, *JHEP* **06** (2002) 007 [hep-th/0204226] [SPIRES].

JHEP02(2010)058

[17] M. Kruczenski, *A note on twist two operators in $\mathcal{N} = 4$ SYM and Wilson loops in Minkowski signature*, *JHEP* **12** (2002) 024 [hep-th/0210115] [SPIRES].

[18] N. Beisert, B. Eden and M. Staudacher, *Transcendentality and crossing*, *J. Stat. Mech.* (2007) P01021 [hep-th/0610251] [SPIRES].

[19] Z. Bern, M. Czakon, L.J. Dixon, D.A. Kosower and V.A. Smirnov, *The Four-Loop Planar Amplitude and Cusp Anomalous Dimension in Maximally Supersymmetric Yang-Mills Theory*, *Phys. Rev.* **D 75** (2007) 085010 [hep-th/0610248] [SPIRES].

[20] M.K. Benna, S. Benvenuti, I.R. Klebanov and A. Scardicchio, *A test of the AdS/CFT correspondence using high-spin operators*, *Phys. Rev. Lett.* **98** (2007) 131603 [hep-th/0611135] [SPIRES].

[21] F. Cachazo, M. Spradlin and A. Volovich, *Four-Loop Cusp Anomalous Dimension From Obstructions*, *Phys. Rev.* **D 75** (2007) 105011 [hep-th/0612309] [SPIRES].

[22] T. Nishioka and T. Takayanagi, *On Type IIA Penrose Limit and $\mathcal{N} = 6$ Chern-Simons Theories*, *JHEP* **08** (2008) 001 [arXiv:0806.3391] [SPIRES].

[23] D. Gaiotto, S. Giombi and X. Yin, *Spin Chains in $\mathcal{N} = 6$ Superconformal Chern-Simons-Matter Theory*, *JHEP* **04** (2009) 066 [arXiv:0806.4589] [SPIRES].

[24] G. Grignani, T. Harmark and M. Orselli, *The* SU(2) $\times$ SU(2) *sector in the string dual of $\mathcal{N} = 6$ superconformal Chern-Simons theory*, *Nucl. Phys.* **B 810** (2009) 115 [arXiv:0806.4959] [SPIRES].

[25] A. Kapustin, B. Willett and I. Yaakov, *Exact Results for Wilson Loops in Superconformal Chern-Simons Theories with Matter*, arXiv:0909.4559 [SPIRES].

[26] M. Mariño and P. Putrov, *Exact Results in ABJM Theory from Topological Strings*, arXiv:0912.3074 [SPIRES].

[27] T. Suyama, *On Large-N Solution of ABJM Theory*, arXiv:0912.1084 [SPIRES].

[28] A. Kapustin and N. Saulina, *Chern-Simons-Rozansky-Witten topological field theory*, *Nucl. Phys.* **B 823** (2009) 403 [arXiv:0904.1447] [SPIRES].

[29] N. Drukker, J. Gomis and D. Young, *Vortex Loop Operators, M2-branes and Holography*, *JHEP* **03** (2009) 004 [arXiv:0810.4344] [SPIRES].

[30] D. Berenstein and D. Trancanelli, *Three-dimensional $\mathcal{N} = 6$ SCFT's and their membrane dynamics*, *Phys. Rev.* **D 78** (2008) 106009 [arXiv:0808.2503] [SPIRES].

[31] M.A. Bandres, A.E. Lipstein and J.H. Schwarz, *Studies of the ABJM Theory in a Formulation with Manifest* SU(4) *R-Symmetry*, *JHEP* **09** (2008) 027 [arXiv:0807.0880] [SPIRES].

[32] M. Mariño, *Chern-Simons theory, matrix integrals and perturbative three-manifold invariants*, *Commun. Math. Phys.* **253** (2004) 25 [hep-th/0207096] [SPIRES].

[33] M. Aganagic, A. Klemm, M. Mariño and C. Vafa, *Matrix model as a mirror of Chern-Simons theory*, *JHEP* **02** (2004) 010 [hep-th/0211098] [SPIRES].

[34] N. Halmagyi and V. Yasnov, *The spectral curve of the lens space matrix model*, *JHEP* **11** (2009) 104 [hep-th/0311117] [SPIRES].

[35] N. Halmagyi, T. Okuda and V. Yasnov, *Large-N duality, lens spaces and the Chern-Simons matrix model*, *JHEP* **04** (2004) 014 [hep-th/0312145] [SPIRES].

JHEP02(2010)058

[36] M. Mariño, S. Pasquetti and P. Putrov, *Large-N duality beyond the genus expansion*, arXiv:0911.4692 [SPIRES].

[37] N. Drukker, S. Giombi, R. Ricci and D. Trancanelli, *Wilson loops: From four-dimensional SYM to two-dimensional YM*, *Phys. Rev.* **D 77** (2008) 047901 [arXiv:0707.2699] [SPIRES].

[38] V. Pestun, *Localization of the four-dimensional $\mathcal{N} = 4$ SYM to a two- sphere and 1/8 BPS Wilson loops*, arXiv:0906.0638 [SPIRES].

[39] E. Witten, *Two-dimensional gauge theories revisited*, *J. Geom. Phys.* **9** (1992) 303 [hep-th/9204083] [SPIRES].

[40] M. Staudacher and W. Krauth, *Two-dimensional QCD in the Wu-Mandelstam-Leibbrandt prescription*, *Phys. Rev.* **D 57** (1998) 2456 [hep-th/9709101] [SPIRES].

[41] A. Bassetto and L. Griguolo, *Two-dimensional QCD, instanton contributions and the perturbative Wu-Mandelstam-Leibbrandt prescription*, *Phys. Lett.* **B 443** (1998) 325 [hep-th/9806037] [SPIRES].

[42] S. Giombi and V. Pestun, *The 1/2 BPS 't Hooft loops in $\mathcal{N} = 4$ SYM as instantons in 2D Yang-Mills*, arXiv:0909.4272 [SPIRES].

[43] J. Gomis and F. Passerini, *Holographic Wilson loops*, *JHEP* **08** (2006) 074 [hep-th/0604007] [SPIRES].

[44] N. Drukker and B. Fiol, *All-genus calculation of Wilson loops using D-branes*, *JHEP* **02** (2005) 010 [hep-th/0501109] [SPIRES].

[45] S. Yamaguchi, *Wilson loops of anti-symmetric representation and D5-branes*, *JHEP* **05** (2006) 037 [hep-th/0603208] [SPIRES].

[46] J. Gomis and F. Passerini, *Wilson loops as D3-branes*, *JHEP* **01** (2007) 097 [hep-th/0612022] [SPIRES].

[47] S. Yamaguchi, *Bubbling geometries for half BPS Wilson lines*, *Int. J. Mod. Phys.* **A 22** (2007) 1353 [hep-th/0601089] [SPIRES].

[48] O. Lunin, *On gravitational description of Wilson lines*, *JHEP* **06** (2006) 026 [hep-th/0604133] [SPIRES].

[49] E. D'Hoker, J. Estes and M. Gutperle, *Gravity duals of half-BPS Wilson loops*, *JHEP* **06** (2007) 063 [arXiv:0705.1004] [SPIRES].

[50] T. Okuda and D. Trancanelli, *Spectral curves, emergent geometry and bubbling solutions for Wilson loops*, *JHEP* **09** (2008) 050 [arXiv:0806.4191] [SPIRES].

[51] D. Gaiotto and E. Witten, *Janus Configurations, Chern-Simons Couplings, And The Theta-Angle in $\mathcal{N} = 4$ Super Yang-Mills Theory*, arXiv:0804.2907 [SPIRES].

[52] K. Hosomichi, K.-M. Lee, S. Lee, S. Lee and J. Park, *$\mathcal{N} = 4$ Superconformal Chern-Simons Theories with Hyper and Twisted Hyper Multiplets*, *JHEP* **07** (2008) 091 [arXiv:0805.3662] [SPIRES].

[53] K. Hosomichi, K.-M. Lee, S. Lee, S. Lee and J. Park, *$\mathcal{N} = 5, 6$ Superconformal Chern-Simons Theories and M2-branes on Orbifolds*, *JHEP* **09** (2008) 002 [arXiv:0806.4977] [SPIRES].

[54] K. Zarembo, *Supersymmetric Wilson loops*, *Nucl. Phys.* **B 643** (2002) 157 [hep-th/0205160] [SPIRES].

JHEP02(2010)058

[55] N. Drukker, S. Giombi, R. Ricci and D. Trancanelli, *Supersymmetric Wilson loops on S^3*, *JHEP* **05** (2008) 017 [arXiv:0711.3226] [SPIRES].

[56] A. Dymarsky and V. Pestun, *Supersymmetric Wilson loops in $\mathcal{N} = 4$ SYM and pure spinors*, arXiv:0911.1841 [SPIRES].

JHEP02(2010)058