A Supply Chain Generalized Network Oligopoly Model for Pharmaceuticals Under Brand Differentiation and Perishability

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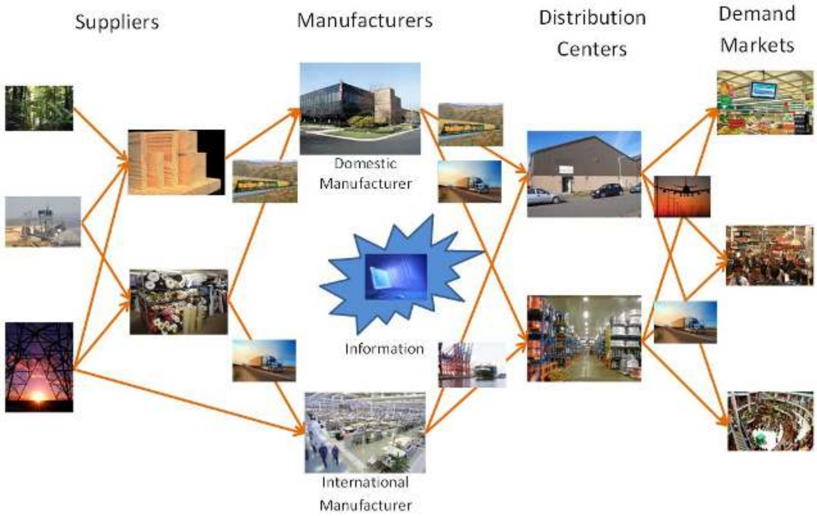
Outline

- Background and Motivation
- Literature Review
- A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains
- A Case Study
- Summary

Background and Motivation



The Pharmaceutical Industry

Pharmaceutical, that is, medicinal drug, manufacturing is an immense global industry.

In 2003, worldwide pharmaceutical industry sales were at $491.8 billion, an increase in sales volume of 9% over the preceding year with the US being the largest national market, accounting for 44% of global industry sales.

In 2011, the global pharmaceutical industry was expected to record growth of $5-7\%$ on sales of approximately $880 billion (Zacks Equity Research (2011)).

The Pharmaceutical Industry

Although pharmaceutical supply chains have begun to be coupled with sophisticated technologies in order to improve both the quantity and the quality of their associated products, despite all the advances in manufacturing, storage, and distribution methods, pharmaceutical drug companies are far from effectively satisfying market demands on a consistent basis.

In fact, it has been argued that pharmaceutical drug supply chains are in urgent need of efficient optimization techniques in order to reduce costs and to increase productivity and responsiveness (Shah (2004) and Papageorgiou (2009)).

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- In 2007, in a warehouse belonging to the Health Department of Chicago, over one million dollars in drugs, vaccines, and other medical supplies were found spoiled, stolen, or unaccounted for.
- In 2009, CVS pharmacies in California, as a result of a settlement of a lawsuit filed against the company, had to offer promotional coupons to customers who had identified expired drugs, including expired baby formula and children's medicines, in more than 42 percent of the stores surveyed the year before.

Pharmaceutical Product Perishability

Other instances of medications sold more than a year past their expiration dates have occurred in other pharmacies across the US.

According to the Harvard Medical School (2003), since a law was passed in the US in 1979, drug manufacturers are required to stamp an expiration date on their products. This is the date at which the manufacturer can still guarantee the full, that is, 100%, potency and safety of the drug, assuming, of course, that proper storage procedures have been followed.

For example, certain medications, including insulin, must be stored under appropriate environmental conditions, and exposure to water, heat, humidity or other factors can adversely affect how certain drugs perform in the human body.

Waste and Environmental Impacts

The environmental impact of the medical waste includes the perished excess medicine, and inappropriate disposal on the retailer / consumer end.

Abundant amounts of unused or expired drugs have been found in 41 million American people's drinking water due to improper disposal in domestic trash or in the waste water.

Pharmaceutical Product Shortages

Ironically, whereas some drugs may be unsold and unused and / or past their expiration dates, the number of drugs that were reported in short supply in the US in the first half of 2011 has risen to 211 – close to an all-time record – with only 58 in short supply in 2004.

According to the Food and Drug Administration (FDA), hospitals have reported shortages of drugs used in a wide range of applications, ranging from cancer treatment to surgery, anesthesia, and intravenous feedings.

Some Consequences of Product Shortages

The consequences of such shortages include the postponement of surgeries and treatments, and may also result in the use of less effective or costlier substitutes.

According to the American Hospital Association, all US hospitals have experienced drug shortages, and 82% have reported delayed care for their patients as a consequence (Szabo (2011)).

Shortages of some lifesaving drugs have resulted in huge spikes in prices, ranging from a 100% to a 4,500% increase with an average of 650% (Schneider (2011)).

Some Possible Causes of Shortages

While the real causes of such shortages are complex, most cases appear to be related to manufacturers' decisions to cease production in the presence of financial challenges.

It is interesting to note that, among curative cancer drugs, only the older generic, yet, less expensive, ones, have experienced shortages.

As noted by Shah (2004), pharmaceutical companies secure notable returns solely in the early lifetime of a successful drug, before competition takes place. This competition-free time-span, however, has been observed to be shortening, from 5 years to only $1-2$ years.

Some Possible Causes of Shortages

Pharmaceutical companies are expected to suffer a significant decrease in their revenues as a result of losing patent protection for ten of the best-selling drugs by the end of 2012 (De la Garza (2011)).

Several pharmaceutical products, including Lipitor and Plavix, that, presently, generate more than $142 billion in sales, are expected, over the next five years, to be faced with generic competition.

In 2011, pharmaceutical products valued at more than $30 billion are losing patent protection, with such products generating more than $15 billion in sales in 2010.

Some Possible Causes of Shortages

Hence, the low profit margins associated with such drugs may be forcing pharmaceutical companies to make a difficult decision: whether to lose money by continuing to produce a lifesaving product or to switch to a more profitable drug.

Unfortunately, the FDA cannot force companies to continue to produce low-profit medicines even if millions of lives rely on them.

Safety Issues

- More than 80% of the ingredients of drugs sold in the US are made overseas, mostly in remote facilities located in China and India that are rarely – if not ever – visited by government inspectors.
- Supply chains of generic drugs, which account for 75% of the prescription medicines sold in the US, are, typically, more susceptible to falsification with the supply chains of some of the over-the-counter products, such as vitamins or aspirins, also vulnerable to adulteration.
- The amount of counterfeit drugs in the European pharmaceutical supply chains has considerably increased.

The emergence of counterfeit products has resulted in major reforms in the relationships among various tiers in pharmaceutical supply chain.

Relevant Literature

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A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

The supply chain generalized network oligopoly model has the following novel features:

1. it handles the perishability of the pharmaceutical product through the introduction of arc multipliers;

2. it allows each firm to minimize the discarding cost of waste / perished medicine;

3. it captures product differentiation under oligopolistic competition through the branding of drugs, which can also include generics as distinct brands.

Our proposed framework can also applied to similar cases of oligopolistic competition in which a finite number of firms provide perishable products.

However, proper minor modifications may have to be made in order to address differences in the supply chain network topologies in related industries.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

I pharmaceutical firms are considered, with a typical firm denoted by i.

The firms compete noncooperatively, in an oligopolistic manner, and the consumers can differentiate among the products of the pharmaceutical firms through their individual product brands.

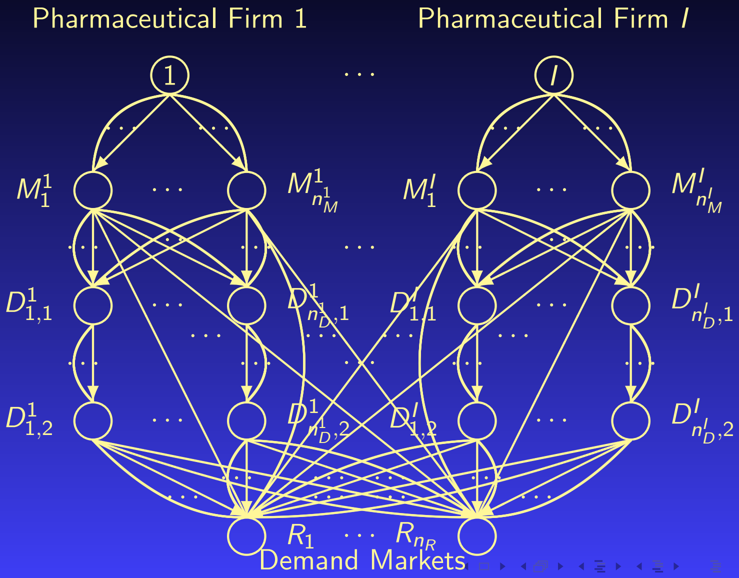
The supply chain network activities include manufacturing, shipment, storage, and, ultimately, the distribution of the brand name drugs to the demand markets.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

Each pharmaceutical firm i; $i = 1, \ldots, I$, utilizes n^i_M manufacturing plants and n^i_D distribution / storage facilities, and the goal is to serve n_R demand markets consisting of pharmacies, retail stores, hospitals, and other medical centers.

L^i denotes the set of directed links corresponding to the sequence of activities associated with firm i. Also, $G = [N, L]$ denotes the graph composed of the set of nodes N, and the set of links L, where L contains all sets of L_is: $L \equiv \cup_{i=1,\ldots,I} L^i$.

The Pharmaceutical Supply Chain Network Topology



A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

There are alternative shipment links to denote different possible modes of transportation.

Since drugs may require different storage conditions / technologies before being ultimately shipped to the demand markets, we represent these alternatives through multiple links at this tier.

There are direct links connecting manufacturing units with various demand markets in order to capture the possibility of direct mail shipments from manufacturers.

How We Handle Perishability

Although pharmaceutical products may have different life-times, we can assign a multiplier to each activity / link of the supply chain to represent the fraction of the product that may perish / be wasted / be lost over the course of that activity.

Also, such multipliers can capture pilferage / theft, a significant issue in drug supply chains.

How We Handle Perishability

The fraction of lost product depends on the type of the activity since various processes of manufacturing, shipment, storage, and distribution may result in dissimilar amounts of losses.

In addition, this fraction need not be the same among various links of the same tier in the supply chain network since different firms and even different units of the same firm may experience non-identical amounts of waste, depending on the brand of drug, the efficiency of the utilized technology, and the experience of the staff, etc.

How We Handle Perishability

Note that the arc multipliers may be obtained from historical and statistical data.

They may also, in the case of certain perishable products, be related to an exponential time decay function where the time, in our framework, is associated with each specific link activity (see, for instance, Blackburn and Scudder (2009) and Bai and Kendall (2009)).

How We Handle Perishability

We associate with every link a in the supply chain network, a multiplier α_a, which lies in the range of (0,1]. The parameter α_a may be interpreted as a throughput factor corresponding to link a meaning that $\alpha_a \times 100\%$ of the initial flow of product on link a reaches the successor node of that link.

Let f_a denote the (initial) flow of product on link a with f'_a denoting the final flow on link a; i.e., the flow that reaches the successor node of the link after wastage has taken place. Therefore, we have:

$$f'_a = \alpha_a f_a, \qquad \forall a \in L. \tag{1}$$

Consequently, the waste / loss on link a is the difference between the initial and the final flow, $f_a - f'_a$, where

$$f_a - f'_a = (1 - \alpha_a) f_a, \qquad \forall a \in L. \tag{2}$$

How We Handle Perishability

Associated with this waste is a discarding total cost function, $\hat{z}_a$, which, in view of (2), is a function of flow on the link, f_a, that is

$$\hat{z}_a = \hat{z}_a(f_a), \qquad \forall a \in L, \tag{3}$$

and which is assumed to be convex and continuously differentiable.

How We Handle Perishability

We define the multiplier, α_{ap}, which is the product of the multipliers of the links on path p that precede link a in that path, as follows:

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{a' < a} \alpha_{a'}, & \text{if } \{a' < a\} \neq \emptyset, \\ \\ \delta_{ap}, & \text{if } \{a' < a\} = \emptyset, \end{cases} \tag{4}$$

where $\{a' < a\}$ denotes the set of the links preceding link a in path p, and $\emptyset$ denotes the null set. As a result, α_{ap} is equal to the product of all link multipliers preceding link a in path p. Hence, the relationship between the link flow, f_a, and the path flows can be expressed as:

$$f_a = \sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} x_p \, \alpha_{ap}, \qquad \forall a \in L. \tag{5}$$

How We Handle Perishability

Let μ_p denote the multiplier corresponding to the throughput on path p, defined as the product of all link multipliers on links comprising that path, that is,

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \qquad \forall p \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R. \tag{6}$$

Let d_{ik} denote the demand for pharmaceutical firm i's brand drug; $i = 1, \ldots, I$, at demand market R_k; $k = 1, \ldots, n_R$. The following equation reveals the relationship between the path flows and the demands in the supply chain network:

$$\sum_{p \in P_k^i} x_p \mu_p = d_{ik}, \quad i = 1, \ldots, I;\ k = 1, \ldots, n_R, \tag{7}$$

that is, the demand for a brand drug at the demand market R_k is equal to the sum of all the final flows – subject to perishability – on paths joining (i, R_k).

The Demand Price Functions

A demand price function is associated with each firm's pharmaceutical at each demand market. We denote the demand price of firm i's product at demand market R_k by ρ_{ik} and assume that

$$\rho_{ik} = \rho_{ik}(d), \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{8}$$

These demand price functions are assumed to be continuous, continuously differentiable, and monotone decreasing.

The Total Cost Functions

The total operational cost on link a may, in general, depend upon the product flows on all the links, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L, \tag{9}$$

where f is the vector of all the link flows. The total cost on each link is assumed to be convex and continuously differentiable.

The Profit Function of Firm i

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{z}_a(f_a). \qquad (10)$$

X_i: the vector of path flows associated with firm i; $i = 1, \ldots, I$, where $X_i \equiv \{\{x_p\}|p \in P^i\}\} \in R_+^{n_{P^i}}$.

X: the vector of all the firm' strategies, that is, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}$.

$\hat{U}_i(X) = U_i$ for all firms i; $i = 1, \ldots, I$.

$$\hat{U} = \hat{U}(X). \qquad (11)$$

Supply Chain Generalized Network Cournot-Nash Equilibrium

In the Cournot-Nash oligopolistic market framework, each firm selects its product path flows in a noncooperative manner, seeking to maximize its own profit, until an equilibrium is achieved.

Definition 1: Supply Chain Generalized Network Cournot-Nash Equilibrium

A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ constitutes a supply chain generalized network Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{12}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ and $K_i \equiv \{X_i | X_i \in R_+^{n_{P^i}}\}$.

The Variational Inequality Formulation

Theorem 1

Assume that, for each pharmaceutical firm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i, and is continuously differentiable. Then $X^* \in K$ is a supply chain generalized network Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*)^T, X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \tag{13}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i.

The Variational Inequality Formulation

Variational Inequality (Path Flows)

Determine $x^* \in K^1$ such that:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}+\frac{\partial \hat{Z}_p(x^*)}{\partial x_p}\right]\times[x_p-x_p^*]$$

$$+\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[-\rho_{ik}(d^*)-\sum_{l=1}^{n_R}\frac{\partial \rho_{il}(d^*)}{\partial d_{ik}}d_{il}^*\right]\times[d_{ik}-d_{ik}^*]\geq 0,$$

$$\forall (x,d)\in K^1, \tag{14}$$

where $K^1 \equiv \{(x,d) | x \in R_+^{n_P}$ *and* (7) *holds*$\}$.

The Variational Inequality Formulation

Variational Inequality (Link Flows)

Determine the vector of equilibrium link flows and the vector of equilibrium demands $(f^*, d^*) \in K^2$, *such that:*

$$\sum_{i=1}^{I} \sum_{a \in L^i} \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

$$+ \sum_{i=1}^{I} \sum_{k=1}^{n_R} \left[-\rho_{ik}(d^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d^*)}{\partial d_{ik}} d_{il}^* \right] \times [d_{ik} - d_{ik}^*] \geq 0,$$

$$\forall (f, d) \in K^2, \qquad (15)$$

where $K^2 \equiv \{(f, d) | x \geq 0$, *and* (5), *and* (7) *hold*$\}$.

Corollaries

Corollary 1: Homogeneous Drug

Let d_k and ρ_k denote the demand for the homogeneous drug and its demand price at demand market R_k, respectively. One can derive:

$$\sum_{i=1}^{I} \sum_{p \in P_k^i} x_p \mu_p = d_k, \quad k = 1, \ldots, n_R. \tag{16}$$

Then, the profit function can be rewritten as:

$$U_i = \sum_{k=1}^{n_R} \rho_k(d) \sum_{p \in P_k^i} \mu_p x_p - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{z}_a(f_a). \tag{17}$$

Corollaries

Corollary 1 (cont'd): Homogeneous Drug

The corresponding variational inequality in terms of path flows can be rewritten as: determine $x^* \in K^1$ such that:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}+\frac{\partial \hat{Z}_p(x^*)}{\partial x_p}-\sum_{l=1}^{n_R}\frac{\partial \rho_l(d^*)}{\partial d_k}\mu_p\sum_{p\in P_l^i}\mu_p x_p^*\right]$$

$$\times[x_p - x_p^*]$$

$$+\sum_{k=1}^{n_R}[-\rho_k(d^*)]\times[d_k-d_k^*]\geq 0, \quad \forall (x,d)\in K^3, \qquad (18)$$

where $K^3 \equiv \{(x,d)|x\in R_+^{n_P}$ *and* (16) *holds*$\}$.

Corollaries

Corollary 2: Fixed Demand

Assume that the demand d_{ik} for firm i's pharmaceutical is fixed. Then, the demand price of this product at demand market R_k will then also be fixed. One can derive:

$$U_i = \sum_{k=1}^{n_R} \bar{\rho}_{ik} d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{z}_a(f_a), \tag{19}$$

Corollaries

Corollary 2 (cont'd): Fixed Demand

Therefore, the corresponding variational inequality in terms of path flows simplifies, in this case, to: determine $x^* \in K^3$ such that:

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial(\sum_{q \in \mathcal{P}} \hat{C}_q(x^*))}{\partial x_p} + \frac{\partial(\sum_{q \in \mathcal{P}} \hat{Z}_q(x^*))}{\partial x_p} \right] \times [x_p - x_p^*] \geq 0,$$

$$\forall x \in K^4, \tag{20}$$

where

$$K^4 \equiv \{x | x \geq 0, \textit{ and (7) is satisfied with the } d_{ik}\textit{s known and fixed}, \forall i, k.\}$$

Corollaries

Corollary 3: Homogeneous Drug and Fixed Demand

Assume that the firms produce a homogeneous drug for which the demand d_k at market R_k is fixed, as well as the demand price $\bar{\rho}_k$. One has:

$$U_i = \sum_{k=1}^{n_R} \bar{\rho}_k \sum_{p \in P_k^i} \mu_p x_p - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{z}_a(f_a). \tag{21}$$

The corresponding variational inequality is: determine $x^* \in K^5$ such that:

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial(\sum_{q \in \mathcal{P}} \hat{C}_q(x^*))}{\partial x_p} + \frac{\partial(\sum_{q \in \mathcal{P}} \hat{Z}_q(x^*))}{\partial x_p} \right] \times [x_p - x_p^*] \geq 0, \ \forall x \in K^5, \tag{22}$$

where $K^5 \equiv \{x | x \geq 0$, and (7) is satisfied with the d_ks known and fixed, $\forall k.\}$.

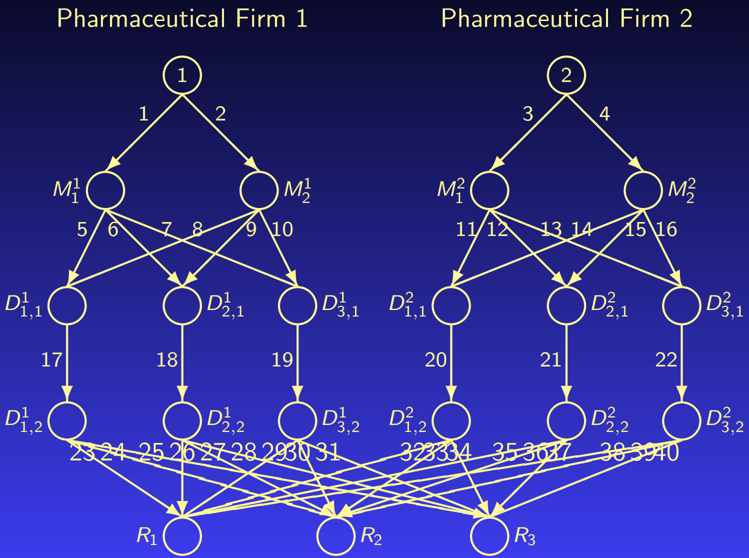
Case Study – Case I

We consider the case of these two competing brands in three demand markets located across the US. Each of these two firms is assumed to have two manufacturing units and three storage / distribution centers.

Firm 1 represents a multinational pharmaceutical giant, hypothetically, Pfizer, Inc., which currently possesses the patent for Lipitor, the most popular brand of cholesterol-lowering drug.

Firm 2, on the other hand, which might represent, for example, Merck & Co., Inc., also is one of the largest global pharmaceutical companies, and has been producing Zocor, another cholesterol regulating brand, whose patent expired in 2006.

The Pharmaceutical Supply Chain Network Topology for Case I



Case I

The demand price functions corresponding to the three demand markets for each of the two brands 1 and 2 were as follows:

$$\rho_{11}(d) = -1.1d_{11} - 0.9d_{21} + 275; \quad \rho_{21}(d) = -1.2d_{21} - 0.7d_{11} + 210;$$

$$\rho_{12}(d) = -0.9d_{12} - 0.8d_{22} + 255; \quad \rho_{22}(d) = -1.0d_{22} - 0.5d_{12} + 200;$$

$$\rho_{13}(d) = -1.4d_{13} - 1.0d_{23} + 265; \quad \rho_{23}(d) = -1.5d_{23} - 0.4d_{13} + 186.$$

These cost functions have been selected based on the average values of the data corresponding to the prices, the shipping costs, etc., available on the web. The values of arc multipliers, in turn, although hypothetical, are constructed in order to reflect the percentage of perishability / waste / loss associated with the various supply chain network activities in medical drug supply chains.

Link Multipliers, Total Cost Functions and Link Flow Solution for Case I

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.95	$5f_1^2 + 8f_1$	$.5f_1^2$	13.73
2	.97	$7f_2^2 + 3f_2$	$.4f_2^2$	10.77
3	.96	$6.5f_3^2 + 4f_3$	$.3f_3^2$	8.42
4	.98	$5f_4^2 + 7f_4$	$.35f_4^2$	10.55
5	1.00	$.7f_5^2 + f_5$	$.5f_5^2$	5.21
6	.99	$.9f_6^2 + 2f_6$	$.5f_6^2$	3.36
7	1.00	$.5f_7^2 + f_7$	$.5f_7^2$	4.47
8	.99	$f_8^2 + 2f_8$	$.6f_8^2$	3.02
9	1.00	$.7f_9^2 + 3f_9$	$.6f_9^2$	3.92
10	1.00	$.6f_{10}^2 + 1.5f_{10}$	$.6f_{10}^2$	3.50
11	.99	$.8f_{11}^2 + 2f_{11}$	$.4f_{11}^2$	3.10
12	.99	$.8f_{12}^2 + 5f_{12}$	$.4f_{12}^2$	2.36
13	.98	$.9f_{13}^2 + 4f_{13}$	$.4f_{13}^2$	2.63
14	1.00	$.8f_{14}^2 + 2f_{14}$	$.5f_{14}^2$	3.79
15	.99	$.9f_{15}^2 + 3f_{15}$	$.5f_{15}^2$	3.12
16	1.00	$1.1f_{16}^2 + 3f_{16}$	$.6f_{16}^2$	3.43
17	.98	$2f_{17}^2 + 3f_{17}$	$.45f_{17}^2$	8.20
18	.99	$2.5f_{18}^2 + f_{18}$	$.55f_{18}^2$	7.25
19	.98	$2.4f_{19}^2 + 1.5f_{19}$	$.5f_{19}^2$	7.97
20	.98	$1.8f_{20}^2 + 3f_{20}$	$.3f_{20}^2$	6.85

Link Multipliers, Total Cost Functions and Solution for Case I (cont'd)

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
21	.98	$2.1f_{21}^2 + 3f_{21}$	$.35f_{21}^2$	5.42
22	.99	$1.9f_{22}^2 + 2.5f_{22}$	$.5f_{22}^2$	6.00
23	1.00	$.5f_{23}^2 + 2f_{23}$	$.6f_{23}^2$	3.56
24	1.00	$.7f_{24}^2 + f_{24}$	$.6f_{24}^2$	1.66
25	.99	$.5f_{25}^2 + .8f_{25}$	$.6f_{25}^2$	2.82
26	.99	$.6f_{26}^2 + f_{26}$	$.45f_{26}^2$	3.34
27	.99	$.7f_{27}^2 + .8f_{27}$	$.4f_{27}^2$	1.24
28	.98	$.4f_{28}^2 + .8f_{28}$	$.45f_{28}^2$	2.59
29	1.00	$.3f_{29}^2 + 3f_{29}$	$.55f_{29}^2$	3.45
30	1.00	$.75f_{30}^2 + f_{30}$	$.55f_{30}^2$	1.28
31	1.00	$.65f_{31}^2 + f_{31}$	$.55f_{31}^2$	3.09
32	.99	$.5f_{32}^2 + 2f_{32}$	$.3f_{32}^2$	2.54
33	.99	$.4f_{33}^2 + 3f_{33}$	$.3f_{33}^2$	3.43
34	1.00	$.5f_{34}^2 + 3.5f_{34}$	$.4f_{34}^2$	0.75
35	.98	$.4f_{35}^2 + 2f_{35}$	$.55f_{35}^2$	1.72
36	.98	$.3f_{36}^2 + 2.5f_{36}$	$.55f_{36}^2$	2.64
37	.99	$.55f_{37}^2 + 2f_{37}$	$.55f_{37}^2$	0.95
38	1.00	$.35f_{38}^2 + 2f_{38}$	$.4f_{38}^2$	3.47
39	1.00	$.4f_{39}^2 + 5f_{39}$	$.4f_{39}^2$	2.47
40	.98	$.55f_{40}^2 + 2f_{40}$	$.6f_{40}^2$	0.00

Case I: Result Analysis

The values of the equilibrium link flows in Table 1 demonstrate the impact of perishability of the product throughout the supply chain network links of each pharmaceutical firm. Under the above demand price functions, the computed equilibrium demands for each of the two brands were:

$$d^*_{11} = 10.32, \quad d^*_{12} = 4.17, \quad d^*_{13} = 8.41;$$

$$d^*_{21} = 7.66, \quad d^*_{22} = 8.46, \quad d^*_{23} = 1.69.$$

The incurred equilibrium prices associated with the branded drugs at each demand market were as follows:

$$\rho_{11} = 256.75, \quad \rho_{12} = 244.48, \quad \rho_{13} = 251.52;$$

$$\rho_{21} = 193.58, \quad \rho_{22} = 189.46, \quad \rho_{23} = 180.09.$$

Case I: Result Analysis

Firm 1, which produces the top-selling product, captures the majority of the market share at demand markets 1 and 3, despite the higher price. While this firm has a slight advantage over its competitor in demand market 1, it has almost entirely seized demand market 3. Consequently, several links connecting Firm 2 to demand market 3 have insignificant flows including link 40 with a flow equal to zero.

Firm 2 dominates demand market 2, due to the consumers' willingness to lean towards this product there, perhaps as a consequence of the lower price, or the perception of quality, etc., as compared to the product of Firm 1.

The profits of the two firms are:

$$U_1 = 2,936.52 \text{ and } \quad U_2 = 1,675.89.$$

Case Study – Case II

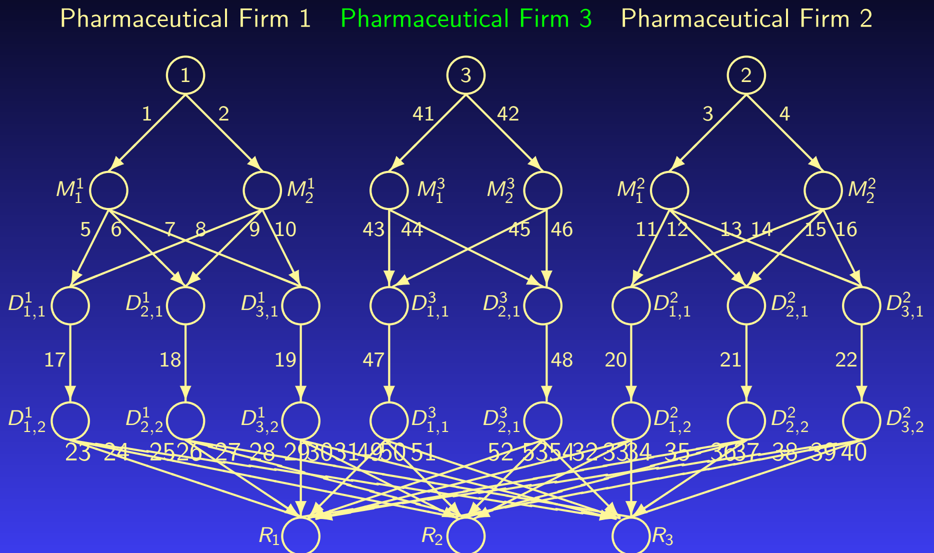
We consider the scenario in which Firm 1 has just lost the exclusive patent right of its highly popular cholesterol regulator. A manufacturer of generic drugs, say, Sanofi, here denoted by Firm 3, has recently introduced a generic substitute for Lipitor by reproducing its active ingredient Atorvastatin.

Since, in Case II, the new generic drug has just been released, we assume that the demand price functions for the products of Firm 1 and 2 will stay the same as in Case I. On the other hand, the demand price functions corresponding to the product of Firm 3 for demand markets 1, 2, and 3 are as follows:

$$\rho_{31}(d) = -0.9d_{31} - 0.6d_{11} - 0.8d_{21} + 150;$$

$$\rho_{32}(d) = -0.8d_{32} - 0.5d_{12} - 0.6d_{22} + 130;$$

$$\rho_{33}(d) = -0.9d_{33} - 0.7d_{13} - 0.5d_{23} + 133.$$


Pharmaceutical Firm 1
Pharmaceutical Firm 3
Pharmaceutical Firm 2
1
3
2
1 2
41 42
3 4
M_1^1 M_2^1 M_1^3 M_2^3 M_1^2 M_2^2
5 6 7 8 9 10
43 44 45 46
11 12 13 14 15 16
$D_{1,1}^1$ $D_{2,1}^1$ $D_{3,1}^1$ $D_{1,1}^3$ $D_{2,1}^3$ $D_{1,1}^2$ $D_{2,1}^2$ $D_{3,1}^2$
17 18 19 47 48 20 21 22
$D_{1,2}^1$ $D_{2,2}^1$ $D_{3,2}^1$ $D_{1,1}^3$ $D_{2,1}^3$ $D_{1,2}^2$ $D_{2,2}^2$ $D_{3,2}^2$
23 24 25 26 27 28 29 30 31 49 50 51 52 53 54 32 33 34 35 36 37 38 39 40
R_1 R_2 R_3

Link Multipliers, Total Cost Functions and Link Flow Solution for Case II

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.95	$5f_1^2 + 8f_1$	$.5f_1^2$	13.73
2	.97	$7f_2^2 + 3f_2$	$.4f_2^2$	10.77
3	.96	$6.5f_3^2 + 4f_3$	$.3f_3^2$	8.42
4	.98	$5f_4^2 + 7f_4$	$.35f_4^2$	10.55
5	1.00	$.7f_5^2 + f_5$	$.5f_5^2$	5.21
6	.99	$.9f_6^2 + 2f_6$	$.5f_6^2$	3.36
7	1.00	$.5f_7^2 + f_7$	$.5f_7^2$	4.47
8	.99	$f_8^2 + 2f_8$	$.6f_8^2$	3.02
9	1.00	$.7f_9^2 + 3f_9$	$.6f_9^2$	3.92
10	1.00	$.6f_{10}^2 + 1.5f_{10}$	$.6f_{10}^2$	3.50
11	.99	$.8f_{11}^2 + 2f_{11}$	$.4f_{11}^2$	3.10
12	.99	$.8f_{12}^2 + 5f_{12}$	$.4f_{12}^2$	2.36
13	.98	$.9f_{13}^2 + 4f_{13}$	$.4f_{13}^2$	2.63
14	1.00	$.8f_{14}^2 + 2f_{14}$	$.5f_{14}^2$	3.79
15	.99	$.9f_{15}^2 + 3f_{15}$	$.5f_{15}^2$	3.12
16	1.00	$1.1f_{16}^2 + 3f_{16}$	$.6f_{16}^2$	3.43
17	.98	$2f_{17}^2 + 3f_{17}$	$.45f_{17}^2$	8.20
18	.99	$2.5f_{18}^2 + f_{18}$	$.55f_{18}^2$	7.25
19	.98	$2.4f_{19}^2 + 1.5f_{19}$	$.5f_{19}^2$	7.97
20	.98	$1.8f_{20}^2 + 3f_{20}$	$.3f_{20}^2$	6.85
21	.98	$2.1f_{21}^2 + 3f_{21}$	$.35f_{21}^2$	5.42
22	.99	$1.9f_{22}^2 + 2.5f_{22}$	$.5f_{22}^2$	6.00
23	1.00	$.5f_{23}^2 + 2f_{23}$	$.6f_{23}^2$	3.56
24	1.00	$.7f_{24}^2 + f_{24}$	$.6f_{24}^2$	1.66
25	.99	$.5f_{25}^2 + .8f_{25}$	$.6f_{25}^2$	2.82
26	.99	$.6f_{26}^2 + f_{26}$	$.45f_{26}^2$	3.34
27	.99	$.7f_{27}^2 + .8f_{27}$	$.4f_{27}^2$	1.24

Link Multipliers, Total Cost Functions and Solution for Case II (cont'd)

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
28	.98	$.4f_{28}^2 + .8f_{28}$	$.45f_{28}^2$	2.59
29	1.00	$.3f_{29}^2 + 3f_{29}$	$.55f_{29}^2$	3.45
30	1.00	$.75f_{30}^2 + f_{30}$	$.55f_{30}^2$	1.28
31	1.00	$.65f_{31}^2 + f_{31}$	$.55f_{31}^2$	3.09
32	.99	$.5f_{32}^2 + 2f_{32}$	$.3f_{32}^2$	2.54
33	.99	$.4f_{33}^2 + 3f_{33}$	$.3f_{33}^2$	3.43
34	1.00	$.5f_{34}^2 + 3.5f_{34}$	$.4f_{34}^2$	0.75
35	.98	$.4f_{35}^2 + 2f_{35}$	$.55f_{35}^2$	1.72
36	.98	$.3f_{36}^2 + 2.5f_{36}$	$.55f_{36}^2$	2.64
37	.99	$.55f_{37}^2 + 2f_{37}$	$.55f_{37}^2$	0.95
38	1.00	$.35f_{38}^2 + 2f_{38}$	$.4f_{38}^2$	3.47
39	1.00	$.4f_{39}^2 + 5f_{39}$	$.4f_{39}^2$	2.47
40	.98	$.55f_{40}^2 + 2f_{40}$	$.6f_{40}^2$	0.00
41	.97	$3f_{41}^2 + 12f_{41}$	$.3f_{41}^2$	6.17
42	.96	$2.7f_{42}^2 + 10f_{42}$	$.4f_{42}^2$	6.23
43	.98	$1.1f_{43}^2 + 6f_{43}$	$.45f_{43}^2$	3.23
44	.98	$.9f_{44}^2 + 5f_{44}$	$.45f_{44}^2$	2.75
45	.97	$1.3f_{45}^2 + 6f_{45}$	$.5f_{45}^2$	3.60
46	.99	$1.5f_{46}^2 + 7f_{46}$	$.55f_{46}^2$	2.38
47	.98	$1.5f_{47}^2 + 4f_{47}$	$.4f_{47}^2$	6.66
48	.98	$2.1f_{48}^2 + 6f_{48}$	$.45f_{48}^2$	5.05
49	.99	$.6f_{49}^2 + 3f_{49}$	$.55f_{49}^2$	3.79
50	1.00	$.7f_{50}^2 + 2f_{50}$	$.7f_{50}^2$	1.94
51	.98	$.6f_{51}^2 + 7f_{51}$	$.45f_{51}^2$	0.79
52	.99	$.9f_{52}^2 + 9f_{52}$	$.5f_{52}^2$	1.43
53	1.00	$.55f_{53}^2 + 6f_{53}$	$.55f_{53}^2$	1.23
54	.98	$.8f_{54}^2 + 4f_{54}$	$.5f_{54}^2$	2.28

Case II: Result Analysis

The equilibrium product flows of Firms 1 and 2 on links 1 through 40 are identical to the corresponding values in Case I.

When the new product produced by Firm 3 is just introduced, the manufacturers of the two existing products will not experience an immediate impact on their respective demands of branded drugs.

The equilibrium computed demands for the products of Firms 1 and 2 at the demand markets will remain as in Case I. However, the equilibrium amounts of demand for the new product of Firm 3 at each demand market is equal to:

$$d_{31}^* = 5.17, \quad d_{32}^* = 3.18, \quad \text{and } d_{33}^* = 3.01.$$

Case II: Result Analysis

The equilibrium prices associated with the branded drugs 1 and 2 at the demand markets will not change, whereas the incurred equilibrium prices of generic drug 3 are as follows:

$$\rho_{31} = 133.02, \quad \rho_{32} = 120.30, \quad \text{and } \rho_{33} = 123.55,$$

which is significantly lower than the respective prices of its competitors in all the demand markets.

The profit that Firm 3 derived from manufacturing and delivering the new generic substitute to these 3 markets is:

$$U_3 = 637.38,$$

while the profits of Firms 1 and 2 remain unchanged.

Case Study – Case III

The generic product of Firm 3 has now been well established, and, thus, has affected the behavior of the consumers through the demand price functions of the relatively more recognized products of Firms 1 and 2. The demand price functions are now given by:

$$\text{Firm 1:}\ \rho_{11}(d) = -1.1d_{11} - 0.9d_{21} - 1.0d_{31} + 192;$$

$$\rho_{12}(d) = -0.9d_{12} - 0.8d_{22} - 0.7d_{32} + 166;$$

$$\rho_{13}(d) = -1.4d_{13} - 1.0d_{23} - 0.5d_{33} + 173;$$

$$\text{Firm 2:}\ \rho_{21}(d) = -1.2d_{21} - 0.7d_{11} - 0.8d_{31} + 176;$$

$$\rho_{22}(d) = -1.0d_{22} - 0.5d_{12} - 0.8d_{32} + 146;$$

$$\rho_{23}(d) = -1.5d_{23} - 0.4d_{13} - 0.7d_{33} + 164;$$

$$\text{Firm 3:}\ \rho_{31}(d) = -0.9d_{31} - 0.6d_{11} - 0.8d_{21} + 170;$$

$$\rho_{32}(d) = -0.8d_{32} - 0.5d_{12} - 0.6d_{22} + 153;$$

$$\rho_{33}(d) = -0.9d_{33} - 0.7d_{13} - 0.5d_{23} + 157.$$

Link Multipliers, Total Cost Functions and Link Flow Solution for Case III

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.95	$5f_1^2 + 8f_1$	$.5f_1^2$	8.42
2	.97	$7f_2^2 + 3f_2$	$.4f_2^2$	6.72
3	.96	$6.5f_3^2 + 4f_3$	$.3f_3^2$	6.42
4	.98	$5f_4^2 + 7f_4$	$.35f_4^2$	8.01
5	1.00	$.7f_5^2 + f_5$	$.5f_5^2$	3.20
6	.99	$.9f_6^2 + 2f_6$	$.5f_6^2$	2.07
7	1.00	$.5f_7^2 + f_7$	$.5f_7^2$	2.73
8	.99	$f_8^2 + 2f_8$	$.6f_8^2$	1.85
9	1.00	$.7f_9^2 + 3f_9$	$.6f_9^2$	2.44
10	1.00	$.6f_{10}^2 + 1.5f_{10}$	$.6f_{10}^2$	2.23
11	.99	$.8f_{11}^2 + 2f_{11}$	$.4f_{11}^2$	2.42
12	.99	$.8f_{12}^2 + 5f_{12}$	$.4f_{12}^2$	1.75
13	.98	$.9f_{13}^2 + 4f_{13}$	$.4f_{13}^2$	2.00
14	1.00	$.8f_{14}^2 + 2f_{14}$	$.5f_{14}^2$	2.84
15	.99	$.9f_{15}^2 + 3f_{15}$	$.5f_{15}^2$	2.40
16	1.00	$1.1f_{16}^2 + 3f_{16}$	$.6f_{16}^2$	2.60
17	.98	$2f_{17}^2 + 3f_{17}$	$.45f_{17}^2$	5.02
18	.99	$2.5f_{18}^2 + f_{18}$	$.55f_{18}^2$	4.49
19	.98	$2.4f_{19}^2 + 1.5f_{19}$	$.5f_{19}^2$	4.96
20	.98	$1.8f_{20}^2 + 3f_{20}$	$.3f_{20}^2$	5.23
21	.98	$2.1f_{21}^2 + 3f_{21}$	$.35f_{21}^2$	4.11
22	.99	$1.9f_{22}^2 + 2.5f_{22}$	$.5f_{22}^2$	4.56
23	1.00	$.5f_{23}^2 + 2f_{23}$	$.6f_{23}^2$	2.44
24	1.00	$.7f_{24}^2 + f_{24}$	$.6f_{24}^2$	1.47
25	.99	$.5f_{25}^2 + .8f_{25}$	$.6f_{25}^2$	1.02
26	.99	$.6f_{26}^2 + f_{26}$	$.45f_{26}^2$	2.48
27	.99	$.7f_{27}^2 + .8f_{27}$	$.4f_{27}^2$	1.31

Link Multipliers, Total Cost Functions and Solution for Case III (cont'd)

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
28	.98	$.4f_{28}^2 + .8f_{28}$	$.45f_{28}^2$	0.66
29	1.00	$.3f_{29}^2 + 3f_{29}$	$.55f_{29}^2$	2.29
30	1.00	$.75f_{30}^2 + f_{30}$	$.55f_{30}^2$	1.29
31	1.00	$.65f_{31}^2 + f_{31}$	$.55f_{31}^2$	1.28
32	.99	$.5f_{32}^2 + 2f_{32}$	$.3f_{32}^2$	2.74
33	.99	$.4f_{33}^2 + 3f_{33}$	$.3f_{33}^2$	0.00
34	1.00	$.5f_{34}^2 + 3.5f_{34}$	$.4f_{34}^2$	2.39
35	.98	$.4f_{35}^2 + 2f_{35}$	$.55f_{35}^2$	1.82
36	.98	$.3f_{36}^2 + 2.5f_{36}$	$.55f_{36}^2$	0.00
37	.99	$.55f_{37}^2 + 2f_{37}$	$.55f_{37}^2$	2.21
38	1.00	$.35f_{38}^2 + 2f_{38}$	$.4f_{38}^2$	3.46
39	1.00	$.4f_{39}^2 + 5f_{39}$	$.4f_{39}^2$	0.00
40	.98	$.55f_{40}^2 + 2f_{40}$	$.6f_{40}^2$	1.05
41	.97	$3f_{41}^2 + 12f_{41}$	$.3f_{41}^2$	8.08
42	.96	$2.7f_{42}^2 + 10f_{42}$	$.4f_{42}^2$	8.13
43	.98	$1.1f_{43}^2 + 6f_{43}$	$.45f_{43}^2$	4.21
44	.98	$.9f_{44}^2 + 5f_{44}$	$.45f_{44}^2$	3.63
45	.97	$1.3f_{45}^2 + 6f_{45}$	$.5f_{45}^2$	4.62
46	.99	$1.5f_{46}^2 + 7f_{46}$	$.55f_{46}^2$	3.19
47	.98	$1.5f_{47}^2 + 4f_{47}$	$.4f_{47}^2$	8.60
48	.98	$2.1f_{48}^2 + 6f_{48}$	$.45f_{48}^2$	6.72
49	.99	$.6f_{49}^2 + 3f_{49}$	$.55f_{49}^2$	3.63
50	1.00	$.7f_{50}^2 + 2f_{50}$	$.7f_{50}^2$	3.39
51	.98	$.6f_{51}^2 + 7f_{51}$	$.45f_{51}^2$	1.41
52	.99	$.9f_{52}^2 + 9f_{52}$	$.5f_{52}^2$	1.12
53	1.00	$.55f_{53}^2 + 6f_{53}$	$.55f_{53}^2$	2.86
54	.98	$.8f_{54}^2 + 4f_{54}$	$.5f_{54}^2$	2.60

Case III: Results

The computed equilibrium demands and sales prices for the products of Firms 1, 2, and 3 are as follows:

$$d^*_{11} = 7.18, \quad d^*_{12} = 4.06, \quad d^*_{13} = 2.93,$$

$$d^*_{21} = 7.96, \quad d^*_{22} = 0.00, \quad d^*_{23} = 5.60,$$

$$d^*_{31} = 4.70, \quad d^*_{32} = 6.25, \quad \text{and } d^*_{33} = 3.93.$$

$$\rho_{11} = 172.24, \quad \rho_{12} = 157.97, \quad \rho_{13} = 161.33,$$

$$\rho_{21} = 157.66, \quad \rho_{22} = 138.97, \quad \rho_{23} = 151.67,$$

$$\rho_{31} = 155.09, \quad \rho_{32} = 145.97, \quad \text{and } \rho_{33} = 148.61.$$

The computed amounts of profit for each of the three competitors are as follows:

$$U_1 = 1,199.87, \quad U_2 = 1,062.73, \quad \text{and } U_3 = 980.83.$$

Case III: Result Analysis

As a result of the consumers' growing inclination towards the generic substitute of the previously popular Lipitor, Firm 2 has lost its entire share of market 2 to its competitors, resulting in zero flows on several links. Similarly, Firm 1 now has declining sales of its brand in demand markets 1 and 3.

As expected, the introduction of the generic substitute of cholesterol regulators has also caused remarkable drops in the prices of the existing brands. Interestingly, the decrease in the price of Firm 1's product - Lipitor - in demand markets 2 and 3 exceeds 35%.

Note that simultaneous declines in the amounts of demand and sales price has caused a severe reduction in the profits of Firms 1 and 2. This decline for Firm 1 is observed to be as high as 60%.

Summary

A new supply chain network model for the study of oligopolistic competition among the producers of a perishable product – that of pharmaceuticals. The contributions of this paper are:

- a new oligopolistic supply chain network model, based on variational inequality theory, that captures the perishability of pharmaceuticals through the use of arc multipliers, that assesses the discarding cost associated with the disposal of waste / perished products in the supply chain network activities, and that includes product differentiation by the consumers, capturing, for example, as to whether or not the products are branded or generic; and
- a case study focused on a real-world scenario of cholesterol-lowering drugs, with the investigation of the impacts of patent rights expiration and generic drug competition.

Thank You!

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