

A SUPPLY FUNCTION OF FIRST-TERM REENLISTEES TO THE AIR FORCE

John McCall  
Neil Wallace

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John McCall and Neil Wallace<sup>†</sup>

The RAND Corporation and the University of Minnesota

The management of military personnel is in many ways similar to the management of human resources within any large organization. Attracting competent new people, providing the training necessary to efficiently operate the organization, and guarding against the loss of skilled personnel are problems that arise in both military and civilian organizations. The methods used by the military to resolve these human resource problems, however, differ from those used by non-military organizations. As will be evident, some of these differences inhibit the efficient operation of the Air Force Personnel system. In this study our attention is restricted to one positive aspect of the retention problem, the reenlistment response of first-term airmen, Air Force enlisted men, to changes in the relative pecuniary attractiveness of civilian versus Air Force employment.

The retention of trained personnel is, perhaps, the personnel problem that has received the most attention within the military. In general, the skills that are most valuable when transferred to the civilian sector are those that are most costly for the Air Force to acquire in terms of recruit training; and while there is some correlation between skill level and rate of promotion in the Air Force, the difference in salaries between high and low skill airmen seems to be much smaller than that existing in the civilian sector.

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In this study we examine the reenlistment behavior of airmen who are trained to be electronic specialists. Among technically trained airmen, electronic specialists stand out as the group for whom there seems to be the greatest disparity between the supply of reenlistees and the number the Air Force would like to retain. Electronic specialists, after receiving about a year of fairly general electronics training and then on-the-job training, perform a variety of maintenance tasks. They comprise about 20 percent of all airmen. Roughly two-thirds of all electronic specialists were leaving the Air Force in 1965 rather than reenlisting, and today (mid-1967) less than 15 percent are reenlisting after their first term. At present training costs and pay scales, the departure of these skilled airmen has been and is very costly to the Air Force. Consequently, many proposals have been made to increase the retention of these men. The Cordiner Committee in 1957 recommended a system of proficiency pay that would reward the highly skilled. However, the implementation of this proposal did not seem to be very effective, possibly because the dollar amounts were small and because the timing was such that it had little impact on the reenlistment decision. The recently inaugurated reenlistment bonus program may overcome these deficiencies. The bonuses are relatively large and they are awarded when the airmen makes his reenlistment decision.

The effectiveness of such proposals for increasing the reenlistment rate depends on the responsiveness of airmen to increases in salary. If the supply curve is very inelastic, so that the reenlistment rate is increased only slightly by relatively large pay increases, then it might be more economical to keep the reenlistment low and to continue to incur high training costs. On the other hand, if the supply curve is fairly elastic, relatively small increases in pay will result in large increases in the reenlistment rate and will reduce the total cost of acquiring the services of skilled airmen.

As this suggests, a full analysis of retention must recognize the interrelation among recruitment, training, and retention. Recruiting policies have a direct impact on both training and retention. An influx of highly skilled personnel would reduce training requirements and would, undoubtedly, necessitate revisions in the retention policy. The provision

of general training facilitates recruitment while making retention more difficult, whereas the provision of specific training (not transferable to the civilian sector) has the opposite effect. Finally, a successful retention policy lowers both recruitment and training requirements. Appendix A, within the context of given skill requirements, investigates some of these interrelations and shows how a supply function of reenlistees fits into a framework for determining an optimal Air Force retention policy. Of course, in a more general analysis, the mix of skill requirements should depend upon the costs of acquiring the different skills. Finally, account should also be taken of the fact that from society's viewpoint, the minimization of dollar cost to the Air Force, the criterion used in Appendix A, ignores some by-products of the Air Force's personnel policies. For example, if the Air Force undertakes a policy that leads to higher retention rates, this and the accompanying decline in recruitment means that over time fewer civilians in any cohort group will have had a tour of duty in the Air Force. Whether this would be a source of social gain or loss is a very complicated issue that to some extent depends on one's values. One would, however, go a long way toward arriving at an agreed upon measure of the social gain or loss involved if one could determine whether ex-airmen are more productive as civilians than comparable people who have not experienced a tour of duty in the Air Force.<sup>†</sup>

The rest of this paper is devoted to an investigation of the relationship between the probability of reenlisting and remuneration. The analysis is based on cross-sectional data obtained from the Personnel Research Laboratory at Lackland Air Force Base. The data consist of information on two groups of randomly selected first-term airmen--a group who left the Air Force after completing their first term and a group who reenlisted. We find that the difference between civilian earnings opportunities, as inferred from demographic attributes and measures of educational attainment, and opportunities within the Air Force is an important determinant of reenlistment behavior.

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<sup>†</sup> For a more complete discussion of this point see McCall and Wallace [3].

THE HYPOTHESIZED SUPPLY FUNCTION

Our basic hypothesis is that for all first-term airmen the probability of reenlisting is a decreasing function of the difference between civilian earnings and Air Force remuneration. The following logit model was used to represent this relationship.<sup>†</sup>

$$(1) \quad \ln[P_i/(1 - P_i)] = \alpha + \beta(E_i^* - A_i) ,$$

where  $P_i$  is the probability that the  $i$ th airmen reenlists,  $E_i^*$  is his potential earnings as a civilian, and  $A_i$  is his prospective Air Force earnings if he reenlists.

We further assume that  $A_i$  depends only on the airman's rank at the end of his first term and possibly on his skill category. Since we are investigating the reenlistment behavior of a group of electronic specialists, we initially assume that for this group  $A_i$  depends only on the airman's rank.

This allows us to rewrite the function as

$$\ln[P_i/(1 - P_i)] = \alpha - \beta A_r + \beta E_i^*$$

or as

$$\ln[P_i/(1 - P_i)] = \alpha_r + \beta E_i^*,$$

where  $\alpha_r$  is a constant that depends only on the  $i$ th airman's rank. Thus, in estimating this function, we will not require information on earnings in the Air Force, but will simply allow the constant term to depend on rank by using dummy variables to represent rank.

This function asserts that the change in the probability of reenlisting  $dP_i$ , induced by a change in potential civilian earnings  $dE_i^*$ , is

$$dP_i = \beta P_i(1 - P_i)dE_i^*,$$

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<sup>†</sup>For a discussion of the logit model, see Zellner and Lee [7].

and that the effect of a change in Air Force remuneration  $dA$  is

$$dP_i = -\beta P_i (1 - P_i) dA,$$

where these are valid only for small changes in pay, and where  $P_i$  is the initial probability of reenlisting. We, of course, hypothesize that  $\beta$  is negative.

These equations assert that as  $P_i$  approaches either 1 or 0, the response to a change in the pay differential approaches zero. For first-term airmen, the shape of the hypothesized relationship between the pay differential  $E^*-A$ , and  $P$  is shown in Fig. 1, where the graph of the equation--the solution of equation (1)--

$$P = \frac{1}{1 + e^{[-\alpha - \beta(E^*-A)]}}$$

is plotted for  $\alpha = 0$ ,  $\beta = -1$ ; and for  $\alpha = \beta = -1$ . The first approximates the estimated supply function for airmen of rank E4 at the end of the first term, while the second approximates that estimated for airmen of rank E3 at the end of the first term.<sup>†</sup>

The first step in estimating this supply function is to obtain an estimate of potential civilian earnings for each airman in the sample. This is accomplished in the following section. Then airmen are classified into  $n$  groups by their estimated potential civilian earnings and the proportion who reenlist is calculated for each group. Finally, the logit of the proportion in each group that reenlists,  $\ln[P_j/(1 - P_j)]$ , is regressed on average potential earnings  $E_j^*$ , and averages of the dummy variables that represent the constant term  $\alpha_j$ , where these averages are calculated for each of the  $n$  groups, i.e.,  $j = 1, 2, \dots, n$ .

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<sup>†</sup>The results given below are not presented in this form, because an estimate of prospective Air Force pay is required in order to do so, and a good deal of uncertainty is attached to any such estimate. See the discussion of Air Force pay, p. 8-9.

Ranks for enlisted men range from E1, which is rank upon entering the services, to E9 and then to warrant officer. Most airmen attain E3 or E4 by the end of their first four-year term.

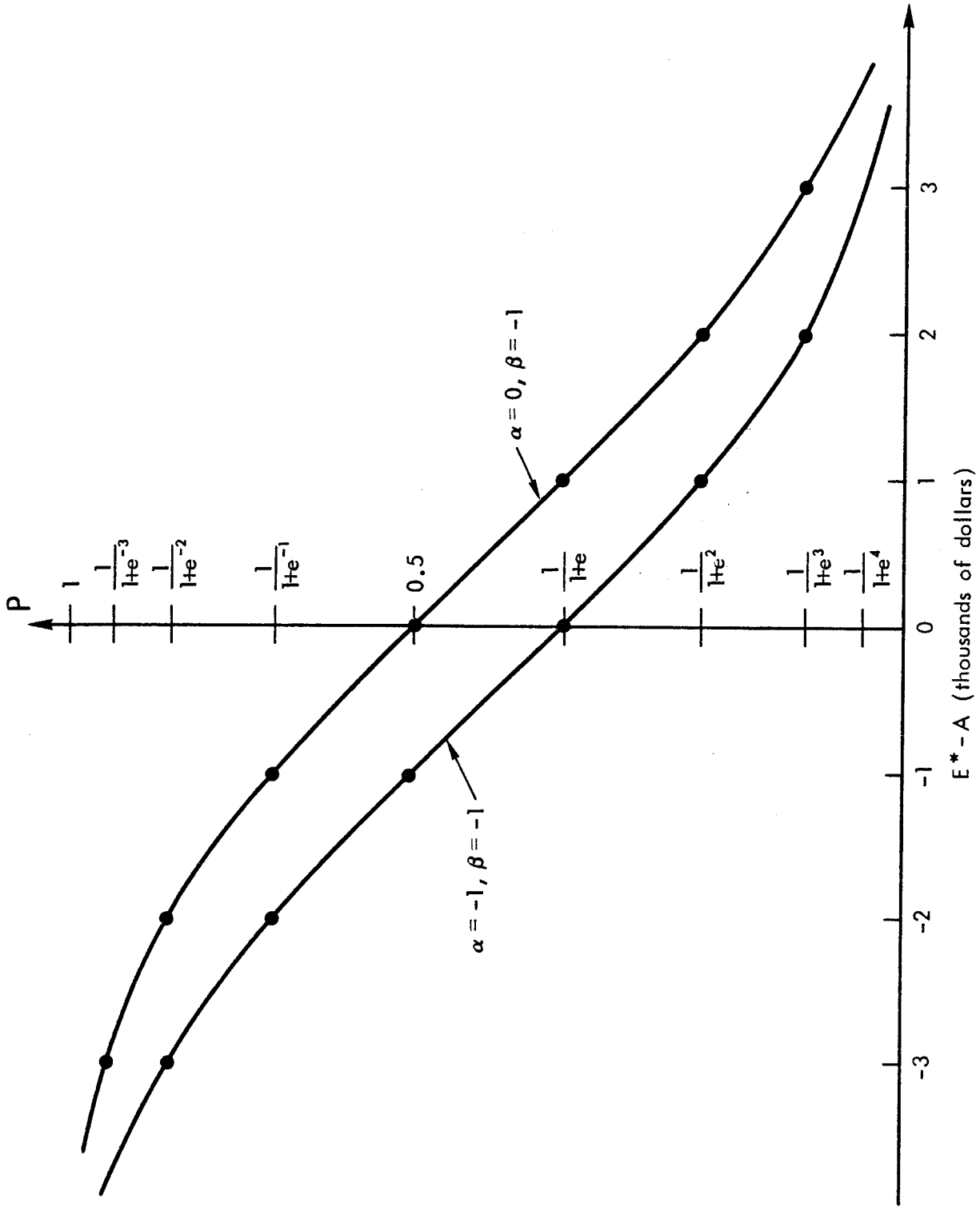


Fig. 1—Relation between  $P$ , the probability of reenlisting, and  $E^* - A$ , the pay differential



THE DETERMINATION OF POTENTIAL EARNINGS

Estimated potential civilian earnings for each airman,  $\hat{E}_i$ , are inferred from the airman's years of schooling prior to enlistment; his high school average as he reported it; his scores on a set of Air Force administered tests; and the region in the country from which he entered the Air Force. This is done via a relationship of the form

$$\log_{10} \hat{E} = \gamma_0 + \sum_1^6 \gamma_j x_j,$$

where  $x_1$  is 1 if the airman is from North-urban and 0 otherwise;  $x_2$  is 1 if the airman is from North-rural and 0 otherwise;  $x_3$  is 1 if the airman is from South-urban and 0 otherwise;  $x_4$  denotes years of schooling upon entering the Air Force;  $x_5$  denotes high school average; and  $x_6$  is the average score on Air Force tests. <sup>††</sup>

We estimate the  $\gamma$ 's in this relationship by using observations on the  $x$ 's and on reported post-Air Force civilian earnings of a group of 397 airmen who did not reenlist after their first tour of duty. Denoting these reported earnings by  $E_m$ , our hypothesis is that

$$\log E_m = \log \hat{E} + z,$$

where  $z$  is a random component of reported earnings that has mean zero and is distributed independently of  $\log \hat{E}$ . It follows that a regression, hereafter called the earnings regression, of  $\log E_m$  on the  $x$ 's yields estimates of the  $\gamma$ 's.

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<sup>†</sup>The South by our definition consists of the following 12 states; Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, Texas, and Virginia. The rest of the country is defined as the North.

A rural community is one with a population less than 10,000. All others are urban communities.

<sup>††</sup> $x_6$  is the sum of the score on the Armed Forces Qualifying Test and of the average score on other Air Force tests. The Armed Forces Qualifying Test has been used since 1951 for classifying Selective Service registrants and enlistees into the four services. The purpose of the test is to measure an individual's potential with regard to

The earnings data were obtained from questionnaires sent by the Personnel Research Laboratory at Lackland Air Force Base in 1964 to 800 ex-airmen who had all been electronic specialists during their first term and who had been in the Air Force from about 1958 to 1962. Aside from the restriction to electronic specialists and to the 1958-1962 period, the 800 men are a random sample of airmen who did not reenlist after their first tour of duty. Among the information requested was current salary. For the 568 from whom responses were obtained, additional biographical information, information on the x's, was sought from Air Force records. We were able to obtain the additional information for 505, of whom 397 had reported full-time civilian earnings. The difference is accounted for by 80 who reported spending a significant portion of their time in school and by 28 who did not report earnings.

The fact that the sample size for the earnings regression is only 397 out of a potential 800 is of no special concern, since there is no reason to expect the estimated  $\gamma$ 's of the earnings regression to be systematically affected by this diminution in sample size. The estimated  $\gamma$ 's would be so affected only if the relationship between civilian earnings and the x's were different for the 397 airmen for whom we have observations from that for the remaining airmen. But there is no reason why this should be.

Some aspects of the reported civilian earnings are worth noting. Reported full-time annual earnings ranged from \$528 to \$25,000. Average and median annual earnings were, respectively, \$5,920 and \$5,760. It is interesting that the Air Force earnings of these men had they reenlisted would have been approximately the same as their reported median earnings as civilians. It is fair to assume that in 1964, the time at which the ex-airmen reported earnings and two years after the reenlistment decision, most of the ex-airmen would have attained the E5 rank.

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military training. The test consists of 100 questions equally divided among the following four areas: vocabulary, arithmetic, spatial relations, and mechanical ability. The other tests include the Airman Qualifying Exams and other Air Force aptitude tests. These tests are used to measure aptitudes in the areas of mechanics, administration, general and electronics.

On average, an E5 would earn about \$5,600, where this includes pay, allowances, and the value of services and items provided without charge. It includes an estimate of the value of the retirement benefits and of medical services provided to servicemen, but not of the lower effective tax rate on Air Force earnings.

The estimated regression of the  $\log_{10}$  of reported earnings on the six biographical variables is

$$\begin{aligned} \log_{10} E = & 3.62387 + 0.05645(x_1) + 0.01008(x_2) - 0.01145(x_3) \\ & \quad (0.0246)^{\dagger} \quad (0.0264) \quad (0.0295) \\ & + 0.00289(x_4) - 0.01122(x_5) + 0.00059(x_6). \\ & \quad (0.0072) \quad (0.0133) \quad (0.0003) \end{aligned}$$

The coefficient of determination is 0.061 and the F-ratio is  $F(6,390) = 4.24$ , which is significant at the one percent level.

The estimated coefficients suggest that potential earnings are greater if the airman is from the North than from the South, and are greater if he is from a Northern urban area rather than from a Northern rural area. If he is from the South, however, the regression suggests that potential earnings are greater if he is from a rural area, a puzzling result. In addition, the regression suggests that potential earnings are greater the greater the airman's years of schooling prior to entering the Air Force,  $x_4$ ; the higher his reported high school average,  $x_5$  (ranges from a possible high of 1 to a possible low of 4); and the higher his scores on Air Force administered tests,  $x_6$  (ranges from a possible low of 0 to a possible high of 200).

One noteworthy finding is that the coefficient of years of schooling, while having the right sign, is statistically insignificant and is very small in absolute value. Its magnitude implies that an additional year of schooling raises earnings by less than one-third of one percent per year. Two things may account for this finding. First, two measures of ability are held constant in the regression. Other investigators of

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<sup>†</sup>The numbers in parenthesis are standard errors of the regression coefficients.

the effect on earnings of years of schooling are rarely able to do this. Second, it would not be too surprising if the common Air Force experience of these men washed out the effects of small differences in pre-Air Force years of schooling.

The low degree of explanatory power of this regression may lead the reader to question whether  $\hat{E}_i$  as estimated from it is a good estimate of  $E_i^*$ , the relevant earnings concept for the supply function. There is certainly a possibility that this earnings regression does not include all the variables that determine  $E_i^*$ , either because these could not be measured for all the airmen, or because we could not identify all the variables that influence  $E_i^*$ . It turns out that such omission does not prevent us from obtaining a consistent estimate of the supply function parameter,  $\beta$ , although the omission may produce an earnings regression in which the coefficients of some of the included variables contain the influence of some of the omitted variables. The reader may also wonder why  $\hat{E}_i$  is used below instead of reported earnings,  $E_m$ , for the 397 who reported civilian earnings. The answer is that the use of  $E_m$  would lead to an inconsistent estimator of  $\beta$ . A technical discussion of both of these points is contained in Appendix B.

A separate word may be in order for why race is not included as a determinant of estimated potential civilian earnings. Of the 505 men who did not reenlist only two were Negroes.<sup>†</sup> Thus, had we wanted to estimate the effect of race on  $\hat{E}_i$ , it would have to have been inferred from the reported earnings of the two Negroes who did not reenlist, and this seemed too small a sample from which to draw inferences.

For use in the supply function we calculate  $\hat{E}_i$ , estimated potential civilian earnings, from the earnings regression for each of the 505 airmen who did not reenlist and for each of a random sample of 393 airmen (also first-term electronic specialists during 1958-1962) who did reenlist. Since any differences in potential earnings between airmen in

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<sup>†</sup> However, in the random sample of 393 men who reenlisted there were 22 Negroes. The proportion of Negroes in the group that did not reenlist was 0.004 whereas the proportion in the group that reenlisted was 0.04. These two proportions are significantly different at the 0.1-percent level.

the two groups must result from differences in the values of the x's for the two groups, we first examine in Table 1 the average values of the biographical variables of the two groups.

The unavailability of measures of the x's for all 800 of the original sample of those who did not reenlist may be a source of bias in Table 1 and in the supply function estimated below. The bias may arise because of the initial diminution of the sample from 800 to 568. This diminution resulted because the addresses of 232 of the original sample could not be traced. Bias may arise because the x values for these men may not be a random sample of the x values for the original 800 men.<sup>†</sup> The further diminution from 568 to 505 can be assumed to occur in a random way with respect to values of the x's, since it occurs because of missing information on Air Force records for some of these men.

Subject to this possible source of bias, Table 1 shows that those who reenlist have fewer years of schooling, have a slightly lower reported high school average, and perform less well on Air Force tests. In addition, those who come from the South reenlist more frequently than those who come from the North. When these average values are inserted into the earnings regression they imply average potential earnings for the 505 men who did not reenlist equal to \$6,053 and average potential earnings for the 393 who did reenlist equal to \$5,768.

Table 1

EX-AIRMEN GROUP COMPARED TO REENLISTED GROUP

Group	Proportion From				x <sub>4</sub> , Years Schooling Before AF	x <sub>5</sub> , High School Average	x <sub>6</sub> , Test Scores
	North- urban	North- rural	South- urban	South- rural			
505 ex-airmen	.416	.372	.101	.111	12.3	2.38	152.0
393 reenlistees	.382	.241	.155	.122	11.9	2.45	146.0

<sup>†</sup>Unfortunately, we have not been able to obtain Air Force records for the 232 non-responders.

THE ESTIMATED SUPPLY FUNCTION

As noted, the earnings regression supplies estimates of potential civilian earnings for the 898 airmen in the sample, 393 who reenlisted and 505 who did not. In addition, we form the following dummy variables for each airman:

$$Y_1 = \begin{cases} 1 & \text{if E1 or E2 at end of first term} \\ 0 & \text{otherwise} \end{cases}$$
$$Y_2 = \begin{cases} 1 & \text{if E4 at end of first term} \\ 0 & \text{otherwise} \end{cases}$$
$$Y_3 = \begin{cases} 1 & \text{if E5 at end of first term} \\ 0 & \text{otherwise} \end{cases}$$

Most of the airmen had rank E3 or E4 at the end of the first term; only four were E1 or E2, and only 12 were E5. Finally, each of the 393 men who reenlisted is assigned a probability of reenlisting equal to 1, while each of the 505 men who did not is assigned a probability equal to 0.

The logit function given above cannot be estimated from these individual observations, because one cannot form the logit of  $p$ ,  $\ln[p/(1 - p)]$ , if  $p$  takes the value zero or one. This suggests that observations be grouped. We have done this by ranking the observations by  $\hat{E}$ , and by forming 44 groups of size 20 and one group of size 18, thus exhausting the 898 observations. Then for each group the average values of the variables are computed. These average values are the data used to compute the logit supply function.

Our hypothesis is that

$$p_j = P_j + u_j,$$

where  $p_j$  is the observed proportion who reenlist in the  $j$ th group of size  $n_j$ ,  $P_j$  is the true proportion, and  $u_j$  is a random variable, binomially distributed with mean zero and variance  $P_j(1 - P_j)/n_j$ . This hypothesis together with the hypothesized logit relation between  $P_j$  and  $\hat{E}_j$  and the  $Y_j$ 's implies a model of the form

$$\ln[p_j/(1 - p_j)] = \alpha_0 + \beta \hat{E}_j + \alpha_1 Y_{1j} + \alpha_2 Y_{2j} + \alpha_e Y_{3j} + z_j,$$

where  $z_j = u_j/[P_j(1 - P_j)]$ , which has mean 0 and variance  $1/[n_j P_j(1 - P_j)]$ .

Since the variance of the disturbance  $z_j$  varies across observations and depends on unknown true parameters, we adopt a two-stage procedure. First we regress  $\ln[p_j/(1 - p_j)]$  on  $\hat{E}_j$  and the  $Y_j$ 's. Then using the estimated values of  $\beta$  and the  $\alpha$ 's, we compute estimated  $P_j$ 's and construct the weights

$$w_j = [n_j \hat{P}_j (1 - \hat{P}_j)]^{1/2}.$$

We then regress  $w_j \ln[p_j/(1 - p_j)]$  on  $w_j \hat{E}_j$  and the  $w_j Y_j$ 's and  $w_j$ , the last replacing what had been a constant in the unweighted regression.

Since the weighted regression is virtually identical to the unweighted one, we relegate the weighted regression to a footnote. In the regressions,  $\hat{E}_j$  is measured in thousands of dollars.

The unweighted logit is<sup>†</sup>

$$\ln[p/(1 - p)] = 4.430 - 1.154 \hat{E} - 5.12 Y_1 + 2.12 Y_2 + 8.37 Y_3.$$

(0.199)      (6.02)      (1.07)      (3.86)

The F-ratio is 9.3, and the coefficient of determination is 0.48.

Loosely speaking, the coefficient of  $\hat{E}$  implies that if the reenlistment rate is initially 0.5, an annual across-the-board 100-dollar raise in Air Force pay would increase the reenlistment rate from 0.5 to about 0.53. Before proceeding to a more detailed discussion of this estimated function and its uses, we present a linear regression of  $p_i$  on  $\hat{E}_i$  and the  $Y_i$ 's. Here  $p_i$  is either 0 or 1, and the regression is computed across the 898 individual observations. We do this to check on the

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<sup>†</sup>The corresponding weighted logit is

$$w \ln(p/1 - p) = 4.406 w - 1.170 w \hat{E} - 5.84 w Y_1 + 2.264 w Y_2 + 8.625 w Y_3$$

(1.242)      (0.203)      (6.04)      (1.076)      (3.829)

estimated logit function which was estimated from data that were grouped in a somewhat arbitrary way. The estimated regression is

$$p = 1.60294 - 0.246 \hat{E} + 0.015 Y_1 + 0.265 Y_2 + 0.367 Y_3.$$

(0.037)      (0.240)      (0.040)      (0.142)

The F-ratio and coefficient of determination are, respectively, 21.33 and 0.295. It is satisfying to note that the implied response to a change in the pay differential is consistent with that implied by the logit function.

#### THE EFFECT OF CHANGES IN AIR FORCE REMUNERATION

Although the supply function has been estimated from a particular set of data--data generated by the reenlistment behavior of a group of first-term electronic specialists who completed their first term during 1962--the estimated slope of the supply function may accurately measure the response of reenlistments to changes in pay differentials independent of the time the change occurs and independent of the skill class of the airmen for whom the change in pay is to occur. Thus, let us show how it could be used to predict the effect of a change in annual Air Force pay.

Given a group of airmen for whom a change in Air Force pay is being considered, and given the initial reenlistment rate for them,  $p^0$ , an estimate of the reenlistment rate,  $p^1$ , that will result if the change in pay is put into effect is found as follows:

$$\ln[p^0/(1 - p^0)] = \alpha + \beta(E_0 - A_0)$$

and

$$\ln[p^1/(1 - p^1)] = \alpha + \beta(E_0 - A_1),$$

where

$$A_1 - A_0 = \Delta A$$



is the contemplated change in pay. Then

$$\ln[p^1/(1 - p^1)] - \ln[p^0/(1 - p^0)] = -\beta[\Delta A],$$

and

$$p^1/(1 - p^1) = e^{-\beta\Delta A} [p^0/(1 - p^0)],$$

so

$$p^1 = \left[ 1 + e^{\beta\Delta A} \frac{(1 - p^0)}{p^0} \right]^{-1}.$$

Then if  $\beta = -1.154$ , its estimated value, and  $p_0 = 1/2$  and  $\Delta A = 1$  (thousand dollars),

$$p^1 = \frac{1}{1 + e^{-1.154}} \cong 0.76$$

However this calculation is strictly correct only if the group for whom it is made is homogeneous in the sense of being composed of individuals all of whom initially reenlist with probability 1/2. To illustrate this, suppose it is instead composed of two groups of equal size, the first of men all of whom initially reenlist with probability 0.1; the second of men all of whom initially reenlist with probability 0.9. If for each of these subgroups we separately compute the effect of the pay change, we find that for the first group the proportion rises from 0.1 to about 0.26, while for the second group it rises from 0.9 to about 0.97. The combined result is a rise in the proportion who reenlist from 0.5 to 0.62, a change less than half as large as that found under the assumption that the whole group was homogeneous. †

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† Given the assumed shape of the supply function, the response to a pay raise half as large would be more than half as large, while the response to a pay raise twice as large would be less than twice as large.

This, of course, is an extreme that is not likely to be encountered. Nevertheless, if the data are available, classifying a group for whom a predicted reenlistment rate is desired into subgroups will help. The group should be classified by the variables that enter significantly into the earnings regression and by rank attained in the Air Force.

Assuming that all first-term airmen are homogeneous, which of course they are not, we may compute an elasticity of supply from the estimated logit supply function. The elasticity of supply may be written

$$\mathcal{E}_s = \frac{dR}{dA} \frac{A}{R}$$

where R is the number who reenlist, and A is the level of Air Force pay. But

$$R = Np$$

and

$$dR = Ndp$$

where N is the total number of first-term airmen, and p is the proportion who reenlist. Then,

$$\mathcal{E}_s = \frac{dp}{dA} \frac{A}{p}$$

From the logit supply function

$$dp/dA = -\beta p(1 - p) = 1.154p(1 - p)$$

so that

$$\mathcal{E}_s = 1.154(1 - p)A$$

Taking A to be somewhere between 4 and 5 (thousand dollars) and p to be somewhere between .1 and .3, the estimated elasticity ranges from about 3.2 to 5.2.

RANK AND REENLISTMENT RATES

Our result suggests that rank has a substantial effect on reenlistment rates. Table 2 indicates the simple relation between the two.

Table 2

PROPORTION OF REENLISTEES, BY RANK

Rank	Number	Proportion Who Reenlist
E1 or E2	4	0.250
E3	175	0.353
E4	707	0.486
E5	12	0.583

Behind differences in rank lie differences in prospective Air Force pay, and differences in the amenities of Air Force life. Thus, we would not expect the pay differences between ranks to completely explain the size of the rank dummy coefficients. This certainly seems the case. For example, if an E3 and an E4 both have civilian potential earnings equal to \$6,000, the E3 according to the unweighted logit supply function reenlists with probability about 0.07, while the E4 with probability about 0.40. It would take an Air Force pay raise of approximately \$2,400 per year to the E3 to get his probability of reenlisting up to 0.40. The actual or prospective pay differences between the two ranks is not this large.

REENLISTMENT RATES OVER TIME

While our supply function may provide an accurate measure of the response of reenlistment rates to changes in pay differentials at any point in time, it cannot, by itself, be used to predict the course of

reenlistment rates over time. This is so because we have provided no way to estimate how potential civilian earnings vary over time, nor have we provided a way to take account of the abrupt changes in the nonpecuniary attractiveness of reenlisting in the Air Force that occur when the country passes from peacetime to wartime or vice versa.

#### CONCLUSION

This study has investigated the first-term reenlistment behavior of Air Force airmen, in particular, electronic specialists. Primary attention has been given to drawing inferences about the responsiveness of the reenlistment rate to changes in Air Force remuneration. We have found the reenlistment rate to be quite sensitive to differences in estimated potential civilian earnings among the airmen. This implies an elasticity of supply of first-term airmen with respect to annual Air Force remuneration equal to about four.

This study can be extended in several directions. Analysis of the temporal behavior of the supply of reenlistees is the most obvious. This would require the collection of time series data on the earnings of ex-airmen. We expect these earnings to depend upon the general state of the economy, the economic health of those industries in which Air Force training is likely to be used, and on the characteristics of the airmen themselves--on the biographical variables we've examined and on skill level attained in the Air Force. The relationship between earnings and these variables together with the supply function estimated in this paper could be used to predict the supply of reenlistees over time, and, in addition, would provide further tests of the supply function.

Finally, the supply curve analysis used here is germane to broader issues involved in comparing the draft with an all volunteer military force. Since the reenlistment decision, as contrasted with the initial enlistment decision, is not influenced by the presence of the draft, it is determined by the same considerations that would determine the supply of recruits in the absence of the draft.

Appendix A

OPTIMAL RETENTION POLICIES FOR DIFFERENT SKILL CATEGORIES

The relationship between Air Force remuneration and the reenlistment rate undoubtedly varies among skill categories. The main reason for this variability is that civilian job opportunities differ across skill categories. Training costs also differ from one skill category to another. These factors suggest that both remuneration and retention should vary across skill categories. This appendix outlines a procedure that can be used to determine the best mix of reenlistment rates.

Let  $c_i$  and  $k_i$  denote, respectively, average training cost and average training time for the  $i$ th skill category; also let  $P_i(x)$  denote the probability of a first term reenlistment in the  $i$ th skill category when annual remuneration is  $x$  dollars. This is the function that is estimated on page 5 for electronic specialists and is assumed to have the following form:

$$P_i(x) = \frac{e^{-\alpha-\beta x}}{1 + e^{\alpha-\beta x}},$$

where  $\alpha$  and  $\beta$  are the parameters to be estimated. Neglecting discounting considerations, assume that the Air Force wishes to choose  $x$  to maximize the output to cost ratio for specialty  $i$  over the next four years.<sup>†</sup> If the airman reenlists, the output from job  $i$  over the next four years is  $t_4$ , which represents the total time spent on job  $i$  over the four-year period. On the other hand, if the airman does not reenlist, four-year output from job  $i$  is  $t_4 - k_i$ . If the airman reenlists, the cost of the four-year output is  $w_i$ , where  $w_i$  is the airman's

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<sup>†</sup>Discounting factors are probably of considerable importance, which implies that the time profile of remuneration is a key variable. In particular, reenlistment bonuses may have a greater influence on retention than comparable increases in remuneration. The composition of remuneration (cash versus fringe benefits) will also affect the retention decision. Finally, other criteria may be more appropriate like maximizing total discounted net benefits accruing to the Air Force.

four-year salary for performing job  $i$ . If the airman does not reenlist, the cost of the four-year output is  $c_i + w_i'$ , where  $w_i'$  is the four-year salary of a new recruit. It follows then that the expected four-year output from job  $i$  and the expected cost of job  $i$  are, respectively,

$$E(Q) = t_4 P_i(x) + (t_4 - k_i)(1 - P_i(x))$$

and

$$E(C) = (w_i')P_i(x) + (c_i + w_i')(1 - P_i(x)).$$

The Air Force chooses  $x$  to maximize the ratio.

$$\frac{E(Q)}{E(C)} .$$

A more complete model could also take account of second and third reenlistments. Using methods of Markov chains, the age distribution of the force could be predicted and, depending upon the assumptions made with regard to productivity and age, this distribution could be altered (in the appropriate way) by changing remuneration. For an analysis of the problem of differential pay see Smith [4].

Appendix B

ON THE RELATION BETWEEN ESTIMATED POTENTIAL  
CIVILIAN EARNINGS AND THE ESTIMATED SUPPLY FUNCTION

The earnings regression, whose predictions are used as estimates of potential civilian earnings, explains only about 6 percent of the variance of the logarithm of post-Air Force reported civilian earnings. For this and other reasons, it is quite likely that predictions from this regression constitute only part of the potential civilian earnings variable that is relevant to the reenlistment decision. The purpose of this appendix is to explore the implications of this possibility for the estimated supply function.

We write

$$E_m = \hat{E} + u + z$$

and

$$E^* = \hat{E} + u + v,$$

where  $E_m$  is reported post-Air Force civilian earnings;  $E^*$  is the earnings variable relevant to the reenlistment decision, a variable we do not observe;  $\hat{E}$  is the prediction from the earnings regression;  $u$  is the unidentified part of  $E^*$  that is also a component of  $E_m$  and that arises because of left-out variables in the earnings regression;  $z$  is a random component of  $E_m$  that is not related to  $E^*$  or its components; and  $v$  is a random component of  $E^*$  that is not related to  $E_m$  or its components.

In this discussion, we use a simplified version of the supply function, one for airmen all of whom face the same Air Force opportunities. Such a "true" supply function may be written

$$Q = \alpha + \beta E^* + w,$$

where  $Q$  is an increasing function of the probability of reenlisting and  $w$  is a disturbance that is assumed independent of  $\hat{E}$ ,  $u$ ,  $z$  and  $v$ .

In what follows,  $q$ ,  $e^*$ , and  $\hat{e}$  represent deviations from the means of  $Q$ ,  $E^*$ , and  $\hat{E}$ , respectively, while,  $z$ ,  $v$ ,  $w$ , and  $y$  represent both levels and deviations because all have zero means-- $z$ ,  $v$ , and  $w$  by assumption,  $u$  because  $\hat{E}$  is a prediction from a least squares regression.

The questions taken up are:

1. What is the relation between the parameter  $\beta$  and the regression coefficient of  $\hat{E}$  in a regression of  $Q$  on  $\hat{E}$ ?

2. Why is not  $E_m$  used as an estimate of  $E^*$  for those airmen for whom these data are available?

To begin, suppose  $Q$  is regressed on  $\hat{E}$ , instead of  $Q$  on  $E^*$ . The computed coefficient is

$$b_{q\hat{e}} = \frac{\Sigma q\hat{e}}{\Sigma \hat{e}^2} = \frac{\Sigma[\beta(\hat{e} + u + v) + w]\hat{e}}{\Sigma \hat{e}^2}$$

Taking probability limits, we obtain

$$\text{plim } b_{q\hat{e}} = \beta \left[ 1 + \frac{\text{plim } \Sigma \hat{e}u + \text{plim } \Sigma \hat{e}v}{\text{plim } \Sigma \hat{e}^2} \right].$$

Thus,  $b_{q\hat{e}}$  converges in probability to  $\beta$ , the true supply function parameter, if  $\Sigma \hat{e}u$  and  $\Sigma \hat{e}v$  both converge to zero.

Now, it is quite reasonable to assume, as we do, that  $v$ , the component of  $E^*$  that is not part of  $E_m$ , is independent of all components of  $E_m$ . Thus consistency of  $b_{q\hat{e}}$  depends on  $\Sigma \hat{e}u$ .

Since  $u$  arises because of left out variables in the earnings regression, it seems clear that it will tend to be uncorrelated with  $\hat{e}$ , the prediction from that regression. This is so because the included variables in that regression pick up the effect of the excluded or left out variables to the extent that the latter are correlated with the former. This leaves for  $u$  only the effect on  $E_m$  of the part of the left out variables that is uncorrelated with the included variables, and, hence, implies that  $u$  is uncorrelated with  $\hat{e}$ .

Thus, regressing  $Q$  on  $\hat{E}$  instead of  $Q$  on  $E^*$  need not give rise to an inconsistent estimate of  $\beta$ .



Now, for the second question. An alternative to using  $\hat{E}$  as an estimate of  $E^*$  is to use  $E_m$  for those airmen for whom this information is available, and  $\hat{E}$  for the others. This may seem a reasonable procedure, because given the relationship among  $E_m$ ,  $E^*$ , and  $\hat{E}$ , we seem to be throwing information away by not using  $E_m$  which, after all, does contain information about  $u$ .

To investigate this, define  $E'$  as  $E_m$  for those for whom we have such data and as  $\hat{E}$  for everyone else. Further, let  $e'$  be the deviation of  $E'$  from its mean. Then, the regression coefficient obtained by regressing  $Q$  on  $E'$  is

$$b_{qe'} = \frac{\sum qe'}{\sum e'^2} = \frac{\sum_0 qe' + \sum_1 qe'}{\sum_0 e'^2 + \sum_1 e'^2},$$

where the subscript "1" means summation over those observations for which  $E'$  equals  $E_m$  and the subscript "0" means summation over all other observations.

For observations "0,"

$$e' = \hat{E} - (1/n)\sum E' = \hat{E} - (1/n)\left[\sum_0 \hat{E} + \sum_1 \hat{E} + \sum_1 u + \sum_1 z\right] = \hat{e} - (1/n)\left[\sum_1 u + \sum_1 z\right]$$

while, for observations "1,"

$$e' = E_m - (1/n)\sum E' = \hat{E} + u + z - (1/n)\sum E' = \hat{e} + u + z - (1/n)\left[\sum_1 u + \sum_1 z\right],$$

where  $n$  is the total sample size and summation without a subscript means summation over all observations.

When the probability limit of  $b_{qe'}$  is taken, any term that involved  $(1/n)\left[\sum_1 u + \sum_1 z\right]$  would be zero since both  $u$  and  $z$  converge to zero. Therefore, to simplify the following display, we simply drop  $(1/n)\left[\sum_1 u + \sum_1 z\right]$  from the expressions for  $e'$ .

Then,  $b_{qe'}$  may be written in terms of  $\hat{e}$ ,  $u$ ,  $z$ ,  $v$ , and  $w$  as

$$b_{qe'} = \frac{\sum_o [\beta(\hat{e} + u + v) + w] \hat{e} + \sum_1 [\beta(\hat{e} + u + v) + w] [\hat{e} + u + z]}{\sum_o \hat{e}^2 + \sum_1 (\hat{e} + u + z)^2}$$

Assuming that  $z$ ,  $v$ , and  $w$  are each independent of  $\hat{e}$  and  $u$ , and of each other

$$\text{plim } b_{qe'} = \beta \left[ \frac{\text{plim}(\sum_o \hat{e}^2 + \sum_1 \hat{e}u + 2 \sum_1 \hat{e}u + \sum_1 u^2)}{\text{plim}(\sum_o e^2 + 2 \sum_1 eu + \sum_1 u^2 + \sum_1 z^2)} \right]$$

The presence of  $\sum_1 z^2$  in the denominator implies that  $b_{qe'}$  is an inconsistent estimate of  $\beta$ . For this reason we used  $b_{q\hat{e}}$  as our estimate of  $\beta$ .

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