

# A SURVEY OF MATRIX THEORY AND MATRIX INEQUALITIES

MARVIN MARCUS

*Professor of Computer Science  
University of California, Santa Barbara*

HENRYK MINC

*Professor of Mathematics  
University of California, Santa Barbara*

DOVER PUBLICATIONS, Inc., *New York*

# Contents

## I. SURVEY OF MATRIX THEORY

### 1. INTRODUCTORY CONCEPTS

1

*Matrices and vectors. Matrix operations. Inverse. Matrix and vector operations. Examples. Transpose. Direct sum and block multiplication. Examples. Kronecker product. Example.*

### 2. NUMBERS ASSOCIATED WITH MATRICES

9

*Notation. Submatrices. Permutations. Determinants. The quadratic relations among subdeterminants. Examples. Compound matrices. Symmetric functions; trace. Permanents. Example. Properties of permanents. Induced matrices. Characteristic polynomial. Examples. Characteristic roots. Examples. Rank. Linear combinations. Example. Linear dependence; dimension. Example.*

### 3. LINEAR EQUATIONS AND CANONICAL FORMS

30

*Introduction and notation. Elementary operations. Example. Elementary matrices. Example. Hermite normal form. Example. Use of the Hermite normal form in solving  $Ax = b$ . Example. Elementary column operations and matrices. Examples. Characteristic vectors. Examples. Conventions for polynomial and integral matrices. Determinantal divisors. Examples. Equivalence. Example. Invariant factors. Elementary divisors. Examples. Smith normal form. Example. Similarity. Examples. Elementary divisors and similarity. Example. Minimal polynomial. Companion matrix. Examples. Irreducibility. Similarity to a diagonal matrix. Examples.*

#### 4. SPECIAL CLASSES OF MATRICES, COMMUTATIVITY 59

*Bilinear functional. Examples. Inner product. Example. Orthogonality. Example. Normal matrices. Examples. Circulant. Unitary similarity. Example. Positive definite matrices. Example. Functions of normal matrices. Examples. Exponential of a matrix. Functions of an arbitrary matrix. Example. Representation of a matrix as a function of other matrices. Examples. Simultaneous reduction of commuting matrices. Commutativity. Example. Quasi-commutativity. Example. Property L. Examples. Miscellaneous results on commutativity.*

#### 5. CONGRUENCE 81

*Definitions. Triple diagonal form. Congruence and elementary operations. Example. Relationship to quadratic forms. Example. Congruence properties. Hermitian congruence. Example. Triangular product representation. Example. Conjunctive reduction of skew-hermitian matrices. Conjunctive reduction of two hermitian matrices.*

## II. CONVEXITY AND MATRICES

#### 1. CONVEX SETS 93

*Definitions. Examples. Intersection property. Examples. Convex polyhedrons. Example. Birkhoff theorem. Simplex. Examples. Dimension. Example. Linear functionals. Example.*

#### 2. CONVEX FUNCTIONS 101

*Definitions. Examples. Properties of convex functions. Examples.*

#### 3. CLASSICAL INEQUALITIES 105

*Power means. Symmetric functions. Hölder inequality. Minkowski inequality. Other inequalities. Example.*

#### 4. CONVEX FUNCTIONS AND MATRIX INEQUALITIES 112

*Convex functions of matrices. Inequalities of H. Weyl. Kantorovich inequality. More inequalities. Hadamard product.*

**5. NONNEGATIVE MATRICES****121**

*Introduction. Indecomposable matrices. Examples. Fully indecomposable matrices. Perron-Frobenius theorem. Example. Nonnegative matrices. Examples. Primitive matrices. Example. Doubly stochastic matrices. Examples. Stochastic matrices.*

**III. LOCALIZATION OF CHARACTERISTIC ROOTS****1. BOUNDS FOR CHARACTERISTIC ROOTS****139**

*Introduction. Bendixson's theorems. Hirsch's theorems. Schur's inequality (1909). Browne's theorem. Perron's theorem. Schneider's theorem.*

**2. REGIONS CONTAINING CHARACTERISTIC ROOTS OF A GENERAL MATRIX****145**

*Lévy-Desplanques theorem. Geršgorin discs. Example. Ovals of Cassini. Ostrowski's theorem. Fan's theorem.*

**3. CHARACTERISTIC ROOTS OF SPECIAL TYPES OF MATRICES****152**

*Nonnegative matrices. Example. Stability matrices. Row stochastic matrices. Normal matrices. Hermitian matrices. Jacobi, or triple diagonal matrices.*

**4. THE SPREAD OF A MATRIX****167**

*Definition. Spread of a general matrix. Spread of a normal matrix.*

**5. THE FIELD OF VALUES OF A MATRIX****168**

*Definitions. Properties of the field of values.*

**INDEX****175**